Turkish Journal of Analysis and Number Theory, 2018, Vol. 6, No. 6, 155-158
Available online at http://pubs.sciepub.com/tjant/6/6/2

DOI:10.12691/tjant-6-6-2

On a Class of P-Kenmotsu Manifolds Admitting Weyl-projective Curvature Tensor of Type (1, 3)

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Received September 11, 2018; Revised October 23, 2018; Accepted November 24, 2018

Abstract We study a class of para-Kenmotsu manifolds admitting Weyl-projective curvature tensor of type (1, 3). At the end, it is shown that an n-dimensional (n > 2) P-Kenmotsu manifold is Ricci semisymmetric if and only if it is an Einstein manifold.

Keywords: *para kenmotsu manifold, recurrent manifold, W_2 - Curvatute tensor, ricci tensor, einstein manifold*

Cite This Article: K. L. Sai Prasad, S. Sunitha Devi, and G. V. S. R. Deekshitulu, "On a Class of P-Kenmotsu Manifolds Admitting Weyl-projective Curvature Tensor of Type (1, 3)." *Turkish Journal of Analysis and Number Theory*, vol. 6, no. 6 (2018): 155-158. doi: 10.12691/tjant-6-6-2.

1. Introduction

In [1,2], Sato introduced the notions of an almost para contact Riemannian manifold. In 1977, Adati and Matsumoto defined para-Sasakian and special para-Sasakian manifolds, which are regarded as a special kind of an almost contact Riemannian manifolds [3]. Para-Sasakian manifolds have been studied by Adati and Miyazawa [4], De and Avijit [5], Matsumoto, Ianus and Mihai [6] and many others. Before Sato, Kenmotsu defined a class of almost contact Riemannian manifolds [7]. In 1995, Sinha and Sai Prasad defined a class of almost para contact metric manifolds namely para-Kenmotsu (briefly P-Kenmotsu) and special para-Kenmotsu (briefly SP-Kenmotsu) manifolds [8].

In 1970, Pokhariyal and Mishra introduced new tensor fields, called W_2 and E tensor fields, on a Riemannian manifold [9]. Later, in [10], Pokhariyal studied some of the properties of these tensor fields on a Sasakian manifold. In 1986, Matsumoto, Ianus and Mihai have extended these concepts to almost para-contact structures and studied para-Sasakian manifolds admitting these tensor fields [6]. These results were further generalised by De and Sarkar, in [5]. Motivated by these studies, in 2015, Sai Prasad and Satyanarayana studied W_2-tensor field in an SP-Kenmotsu manifold [11]. In the present work, we investigate a class of para-Kenmotsu manifolds admitting Weyl-projective curvature tensor W_2 of type (1, 3). The present work is organised as follows: Section 2 is equipped with some prerequisites about P-Kenmotsu manifolds. In Section 3, we define W_2-recurrent and semisymmetric para-Kenmotsu manifolds and shown that W_2-recurrent para-Kenmotsu manifold is a semisymmetric manifold. Further, it is shown that the curvature of W2-semisymmetric para-Kenmotsu manifold is constant and hence we establish that a W_2-recurrent para-Kenmotsu manifold is an SP-Kenmotsu manifold. Section 4 is devoted to study Ricci semisymmetric P-Kenmotsu manifold.

2. Preliminaries

Let M_n be an n-dimensional differentiable manifold equipped with structure tensors (Φ, ξ, η) where Φ is a tensor of type (1, 1), ξ is a vector field, η is a 1-form such that

$$\eta(\xi)=1$$
$$\Phi^2(X)=X-\eta(X)\xi;\ \bar{X}=\Phi X. \tag{2.1}$$

Then the manifold M_n is called an almost para-contact manifold.

Let g be a Riemannian metric such that, for all vector fields X and Y on M_n

$$g(X,\xi)=\eta(X)$$
$$\Phi\xi=0, \eta(\Phi X)=0, rank\Phi=n-1 \tag{2.2}$$
$$g(\Phi X,\Phi Y)=g(X,Y)-\eta(X)\eta(Y).$$

Then the manifold M_n [1] is said to admit an almost para-contact Riemannian structure (Φ,ξ,η,g).

In addition, if (Φ,ξ,η,g) satisfies the conditions

$$(\nabla_X\eta)Y-(\nabla_Y\eta)X=0,$$
$$(\nabla_X\nabla_Y\eta)Z=\left[-g(X,Z)+\eta(X)\eta(Z)\right]\eta(Y)$$
$$+\left[-g(X,Y)+\eta(X)\eta(Y)\right]\eta(Z), \tag{2.3}$$
$$\nabla_X\xi=X-\eta(X)\xi,$$
$$(\nabla_X\Phi)Y=-g(X,\Phi Y)\xi-\eta(Y)\Phi X;$$

then M_n is called para-Kenmotsu manifold or briefly a P-Kenmotsu manifold [8].

A P-Kenmotsu manifold admitting a 1-form η satisfying

$$\begin{aligned}(\nabla_X \eta)Y &= g(X,Y) - \eta(X)\eta(Y) \\ (\nabla_X \eta)Y &= \varphi(\bar{X},Y);\end{aligned} \tag{2.4}$$

where φ is an associate of Φ, is called special para-Kenmotsu manifold or briefly SP- Kenmotsu manifold [8].

Let (M_n, g) be an n-dimensional, $n \geq 3$, differentiable manifold of class C^∞ and let ∇ be its Levi-Civita connection. Then the Riemannian Christoffel curvature tensor R of type (1, 3) is given by:

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z. \tag{2.5}$$

The Ricci operator S and the (0, 2)- tensor S^2 are defined by

$$g(SX,Y) = S(X,Y); \tag{2.6}$$

and

$$S^2(X,Y) = S(SX,Y). \tag{2.7}$$

It is known [8] that in a P-Kenmotsu manifold the following relations hold:

$$\begin{aligned}&S(X,\xi) = -(n-1)\eta(X), \\ &g[R(X,Y)Z,\xi] = \eta[R(X,Y,Z)] \\ &= g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \\ &R(\xi,X)Y = \eta(Y)X - g(X,Y)\xi, \\ &R(X,Y,\xi) = \eta(X)Y - \eta(Y)X;\end{aligned} \tag{2.8}$$

when X is orthogonal to $\boldsymbol{\xi}$.

An n-dimensional (n > 2) Riemannian manifold M_n is said to be Einstein manifold if the Ricci curvature tensor S(X, Y) of the Levi-Civita connection satisfies the condition

$$S(X,Y) = \lambda g(X,Y) \tag{2.9}$$

where λ is a constant.

3. W_2- Recurrent P-Kenmotsu Manifolds

The Weyl-projective curvature tensor W_2 of type (1, 3) of a Riemannian manifold M_n with respect to Riemannian connection is given by [9]:

$$\begin{aligned}&W_2(X,Y,Z,U) \\ &= R(X,Y,Z,U) + \frac{1}{n-1}\begin{bmatrix} g(X,Z)S(Y,U) \\ -g(Y,Z)S(X,U)\end{bmatrix}.\end{aligned} \tag{3.1}$$

Now, we define a W_2-semisymmetric para-Kenmotsu manifold as:

Definition 3.1: An n-dimensional para-Kenmotsu manifold is called W_2-semisymmetric if its W_2-curvature tensor satisfies the condition

$$R(X,Y).W_2 = 0, \tag{3.2}$$

where $R(X,Y)$ is considered to be a derivation of the tensor algebra at each point of the manifold for tangent vectors X and Y.

It can be easily shown that on a P-Kenmotsu manifold the W_2-curvature tensor satisfies the condition

$$W_2(X,Y,Z,\xi) = 0. \tag{3.3}$$

Further, we define a W_2 -recurrent para-Kenmotsu manifold as:

Definition 3.2: An n-dimensional para-Kenmotsu manifold with respect to the Levi-Civita connection is called W_2-recurrent manifold if its W_2-curvature tensor satisfies the condition

$$(\nabla_U W_2)(X,Y)Z = A(U)W_2(X,Y)Z, \tag{3.4}$$

where A is some non-zero 1-form.

Now, let us establish a relation between W_2-recurrent and W_2-semisymmetric para-Kenmotsu manifolds.

For that, let us suppose that $W_2 \neq 0$. Now, we define a function by

$$f^2 = g(W_2, W_2). \tag{3.5}$$

Using the fact that $\nabla_U g = 0$, from (3.5) we get $2f(Uf) = 2f^2(A(U))$.

Since $f \neq 0$, we have

$$Uf = f(A(U)). \tag{3.6}$$

Then, from (3.6), we get

$$X(Uf) = \frac{1}{f}(Xf)(Uf) + (XA(U))f, \tag{3.7}$$

and hence, we have

$$X(Uf) - U(Xf) = [XA(U) - UA(X)]f. \tag{3.8}$$

Therefore,

$$\begin{aligned}&(\nabla_X \nabla_U - \nabla_U \nabla_X - \nabla_{[X,U]})f \\ &= [XA(U) - UA(X) - A([X,U])]f \\ &= 2[dA(X,U)]f.\end{aligned} \tag{3.9}$$

Since the left hand side of (3.9) is zero and $f \neq 0$, we deduce that $dA(X,Y) = 0$ and it shows that the 1-form A is closed.

Then from (3.4), we get that

$$\begin{aligned}&(\nabla_V \nabla_U W_2)(X,Y)Z \\ &= [VA(U) + A(V)A(U)]W_2(X,Y)Z,\end{aligned} \tag{3.10}$$

and hence, we get that

$$\begin{aligned}&(\nabla_V \nabla_U W_2)(X,Y)Z - (\nabla_U \nabla_V W_2)(X,Y)Z \\ &-(\nabla_{[U,V]} W_2)(X,Y)Z \\ &= 2dA(V,U)W_2(X,Y)Z = 0;\end{aligned} \tag{3.11}$$

i.e., $R(V,U).W_2 = 0$, where $R(V,U)$ is considered to be a derivation of tensor algebra at each point of the manifold for the tangent vectors V and U.

This shows that a W_2-recurrent P-Kenmotsu manifold is W_2-semisymmetric and hence we state that:

Theorem 3.1: A W_2-recurrent para-Kenmotsu manifold is W_2-semisymmetric.

Further we determine the curvature value of W_2-semisymmetric P-Kenmotsu manifold.

From (3.2), we have

$$R(X,Y)W_2(Z,U)V - W_2(R(X,Y)Z,U)V \\ -W_2(Z,R(X,Y)U)V - W_2(Z,U)R(X,Y)V = 0, \quad (3.12)$$

which implies

$$g(R(X,Y)W_2(Z,U)V,\xi) \\ -g(W_2(R(X,Y)Z,U)V,\xi) \\ -g(W_2(Z,R(X,Y)U)V,\xi) \\ -g(W_2(Z,U)R(X,Y)V,\xi) = 0. \quad (3.13)$$

By putting $X = \xi$ in the above equation, we get

$$R((\xi,Y)W_2(Z,U)V,\xi) \\ -W_2(R(\xi,Y)Z,U,V,\xi) \\ -W_2(Z,R(\xi,Y)U,V,\xi) \\ -W_2(Z,U,R(\xi,Y)V,\xi = 0). \quad (3.14)$$

Now, by using (2.8) and (3.3), the above equation reduces to:

$$\eta(Y)\eta(W_2(Z,U)V) - g(Y,W_2(Z,U)V) = 0. \quad (3.15)$$

Again on using (3.3), we get that $W_2(Z,U,V,Y) = 0$.
Therefore, from (3.1) we have

$$R(X,Y,Z,V) = \frac{1}{n-1}\begin{bmatrix} g(Y,Z)S(X,V) \\ -g(X,Z)S(Y,V) \end{bmatrix}. \quad (3.16)$$

On contracting the above equation, we get

$$S(Y,Z) = \frac{r}{n}g(Y,Z). \quad (3.17)$$

Then, from equations (3.16) and (3.17), we have

$$R(X,Y,Z,V) = \frac{r}{n(n-1)}\begin{bmatrix} g(Y,Z)g(X,V) \\ -g(X,Z)g(Y,V) \end{bmatrix}. \quad (3.18)$$

This shows that the curvature of W_2-semisymmetric P-Kenmotsu manifold is constant.

As it is known [8] that a P-Kenmotsu manifold with constant curvature is an SP-Kenmotsu manifold and using the above shown result, we state that:

Theorem 3.2: A W_2 -semisymmetric P-Kenmotsu manifold is an SP-Kenmotsu manifold.

Therefore, form theorems (3.1) and (3.2), we have the following result:

Theorem 3.3: *A W_2-recurrent P-Kenmotsu manifold is an SP-Kenmotsu manifold.*

4. Ricci Semisymmetric Para-Kenmotsu Manifolds

Definition 4.1: An n-dimensional Riemannian manifold is said to be Ricci semisymmetric if its Ricci tensor $S(X,Y)$ of the Levi-Civita connection satisfies the condition

$$R(X,Y).S = 0. \quad (4.1)$$

Theorem 4.1: An n-dimensional (n > 2) P-Kenmotsu manifold M_n is Ricci semisymmetric if and only if it is an Einstein Manifold.

Proof: Let us suppose that a P-Kenmotsu manifold be Ricci semisymmetric. Then from (4.1), we have

$$S(R(X,Y)U,V) + S(U,R(X,Y)V) = 0. \quad (4.2)$$

By putting $X = \xi$ in (4.2), we get

$$S(R(\xi,Y)U,V) + S(U,R(\xi,Y)V) = 0. \quad (4.3)$$

Now by using the equations (2.8) (a) and (2.8) (c), the above equation reduces to

$$\eta(U)S(Y,V) + (n-1)\eta(V)g(Y,U) \\ +\eta(V)S(U,Y) + (n-1)\eta(U)g(Y,V) = 0. \quad (4.4)$$

Again by putting $X = \xi$ in (4.4), we get

$$S(Y,V) = -(n-1)g(Y,V). \quad (4.5)$$

This proves that the manifold M_n is an Einstein manifold.

As an every Einstein manifold is Ricci semisymmetric, the converse of the theorem is trivial.

This completes the proof.

Statement of Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgements

The authors acknowledge Prof. Kalpana of Banaras Hindu University, Varanasi for her valuable suggestions in preparation of the manuscript.

References

[1] Sato, I, On a structure similar to the almost contact structure, *Tensor (N.S.),* 30, 219-224, 1976.

[2] Sato, I, On a structure similar to the almost contact structure II, *Tensor (N.S.),* 31, 199-205, 1977.

[3] Adati, T. and Matsumoto, K, On conformally recurrent and conformally symmetric P-Sasakian Manifolds, *TRU Math.,* 13, 25-32, 1977.

[4] Adati, T. and Miyazawa, T, On P-Sasakian manifolds admitting some parallel and recurrent tensors, *Tensor (N.S.),* 33, 287-292, 1979.

[5] De, U. C. and Avijit Sarkar, On a type of P-Sasakian manifolds, *Math. Reports,* 11(2), 139-144, 2009.

[6] Matsumoto, K., Ianus, S. and Ion Mihai, On P-Saskian manifolds which admit certain tensor fields, *Publ. Math. Debrecen*, 33, 61-65, 1986.

[7] Kenmotsu, K, A class of almost contact Riemannian manifolds, *Tohoku Math. Journal*, 24, 93-103, 1972.

[8] Sinha, B. B. and Sai Prasad, K. L, A class of almost para contact metric Manifold, *Bulletin of the Calcutta Math. Soc.,* 87, 307-312, 1995.

[9] Pokhariyal, G. P. and Mishra, R. S, The curvature tensors and their relativistic significance, *Yokohoma Math. J.,* 18, 105-108, 1970.

[10] Pokharial, G. P, Study of a new curvature tensor in a Sasakian manifold, *Tensor (N.S.),* 36, 222-225, 1982.

[11] Sai Prasad, K. L. and Satyanarayana, T, Some curvature properties on a Special paracontact Kenmotsu manifold with respect to Semi-symmetric connection, *Turkish Journal of Analysis and Number Theory*, 3(4), 94-96, 2015.