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Fixed Point Results for Hardy Roger Type Contraction in Ordered Complete Dislocated G_d Metric Space

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Abstract In this paper we discuss the fixed points of mappings satisfying a contractive condition on a closed ball in an ordered complete dislocated quasi G_d-metric space. The notion of dominated mappings is applied to approximate the unique solution of non linear functional equations. An example is given to show the validity of our work. Our results improve/generalize several well known recent and classical results.

Keywords: *fixed point, contractive dominated mappings, closed ball, crdered complete dislocated quasi G_d-metric spaces*

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1. Introduction

Let $T: X \to X$ be a mapping. A point $x \in X$ is called a fixed point of T if $x = Tx$. Let x_0 be an arbitrary chosen point in X. Define a sequence $\{x_n\}$ in X by a simple iterative method given by $x_{n+1} = Tx_n$, where $n \in \{0,1,2,3,...\}$. Such a sequence is called a picard iterative sequence and its convergence plays a very important role in proving existence of fixed point of a mapping T. A self mapping T on a metric space X is said to be a Banach contraction mapping if, $d(Tx,Ty) \le kd(x,y)$ holds for all $x, y \in X$ where $0 \le k < 1$. Recently, many results appeared in literature related to fixed point results in complete metric spaces endowed with a partial ordering. Ran and Reurings [21] proved an analogue of Banach's fixed point theorem in metric space endowed with partial order and gave applications to matrix equations. Subsequently, Nieto et. al. [16] extended the results of [21] for non decreasing mappings and applied this result to obtain a unique solution for a 1st order ordinary differential equation with periodic boundary conditions. Mustafa and Sims in [18] introduce the notion of a generalized metric space as a generalization of the usual metric space. Mustafa and others studied fixed point theorems for mappings satisfying different contractive conditions. Further useful results can be seen in [3,12,13,14,19,20,27,28]. Recently, Arshad et. al. [4] proved a result concerning the existence of fixed points of a mapping satisfying a contractive condition on closed ball in a complete dislocated metric space. For further results on closed ball we refer the reader to see [5,6,7,24,25,26]. The dominated mapping [2] which satis.es the condition $fx \preceq x$ occurs very naturally in several practical problems. For example x denotes the total quantity of food produced over a certain period of time and $f(x)$ gives the quantity of food consumed over the same period in a certain town, then we must have $fx \preceq x$.

In this paper we have obtained fixed point results for dominated self-mappings in an ordered complete dislocated symmetric G_d-metric space on a closed ball satisfying Hardy Roger type contractive condition. In the process we extend and improve several recent and classical fixed point results. We have used weaker contractive condition and weaker restrictions to obtain unique fixed point. Our results do not exists even yet in metric spaces. An example is given to show the validity of our result.

Definition 1.1. Let X be a nonempty set and let $G_d : X \times X \times X \to R^+$ be a function satisfying the following axioms

(i) If $G_d(x,y,z) = G_d(y,z,x) = G_d(z,x,y) = 0$, then $x = y = z$,

(ii) $G_d(x,y,z) \le G_d(x,a,a) + G_d(a,y,z)$ for all $x,y,z,a \in X$ (rectangle inequality).

Then the pair (X, G_d) is called the dislocated quasi G_d-metric space. It is clear that if

$G_d(x,y,z) = G_d(y,z,x) = G_d(z,x,y) = 0$ then from (i) $x = y = z$. But if $x = y = z$ then $G_d(x,y,z)$ may not be 0: It is observed that if $G_d(x,y,z) = G_d(y,z,x) = G_d(z,x,y)$ for all $x,y,z \in X$, then (X, G_d) becomes a dislocated G_d-metric space.

Definition 1.2. If X be a set of non-negative real numbers, then $G_d(x,y,z) = y + \max\{x,y,z\}$ defines a dislocated quasi metric G on X.

Definition 1.3. A G_d -metric space (X,G_d) is called symmetric if $G_d(x,y,y) = G_d(y,x,x)$ for all $x,y \in X$.

Definition 1.4. Let (X,G_d) be a G_d -metric space, and let $\{x_n\}$ be a sequence of points in X, a point x in X is said to be the limit of the sequence $\{x_n\}$ if $\lim_{m,n\to\infty} G_d(x,x_n,x_m) = 0$, and one says that sequence $\{x_n\}$ is G_d -convergent to x. Thus, if $x_n \to x$ in a G_d-metric space (X,G_d), then for any $\in > 0$, there exist $N \in \mathbb{N}$ such that $G_d(x,x_n,x_m) < \in$, for all $n,m \ge N$.

Definition 1.5. Let (X,G_d) be a G_d − metric space. A sequence $\{x_n\}$ is called G_d-Cauchy sequence if, for each $\in > 0$ there exists a positive integer $n^* \in \mathbb{N}$ such that $G_d(x_n,x_m,x_l) < \in$ for all $n,l,m \ge n^*$; i.e. if $G_d(x_n,x_m,x_l) \to 0$ as $n,m,l \to \infty$.

Definition 1.6. A G_d-metric space (X,G_d) is said to be G_d-complete if every G_d-Cauchy sequence in (X,G_d) is G_d-convergent in X.

Lemma 1.7. Let (X,G_d) be a G_d-metric space, then the following are equivalent:

(i) $\{x_n\}$ is Gd convergent to x.

(ii) $G_d(x_n,x_n,x) \to 0$ as $n \to \infty$.

(iii) $G_d(x_n,x,x) \to 0$ as $n \to \infty$.

(iv) $G_d(x_n,x_m,x) \to 0$ as $m\, n \to \infty$.

Definition 1.8. Let (X,G_d) be a G_d-metric space then for $x_0 \in X$, $r > 0$, the G_d-ball with centre x_0 and radius r is,

$$B(x_0,r) = \{y \in X : G_d(x_0,y,y) < r\}.$$

Definition 1.9. Let $(X,\preceq)$ be a partial ordered set. Then $x,y \in X$ are called comparable if $x \preceq y$ or $y \preceq x$ holds.

Definition 1.10. [2] Let $(X,\preceq)$ be a partially ordered set. A self mapping f on X is called dominated if $fx \preceq x$ for each x in X.

Example 1.11. [2] Let $X = [0,1]$ be endowed with usual ordering and $f: X \to X$ be defined by $fx = x^n$ for some $n \in \mathbb{N}$. Since $fx = x^n \le x$ for all $x \in X$, therefore f is a dominated map.

2. Main Result

Theorem 2.1: Let $(X,\preceq,G_d)$ be an ordered complete dislocated symmetric G_d metric space, $x_0 \in X$, $r > 0$, and $S: X \to X$ be a dominated mapping. Suppose there exists α,β and γ such that $0 < \alpha + 3\beta + 10\gamma < 1$ and for all comparable elements x,y and z in $\overline{B(x_0,r)}$.

$$G_d(Sx,Sy,Sz) \le \alpha G_d(x,y,z) + \beta\left[G_d(x,Sx,Sx) + G_d(y,Sy,Sy) + G_d(z,Sz,Sz)\right] + \gamma\begin{bmatrix} G_d(x,Sy,Sy) + G_d(x,Sz,Sz) + G_d(y,Sx,Sx) \\ +G_d(y,Sz,Sz) + G_d(z,Sx,Sx) + G_d(z,Sy,Sy) \end{bmatrix} \quad (2.1)$$

and

$$G_d(x_0,Sx_0,Sx_0) \le (1-\lambda)\gamma. \quad (2.2)$$

where $\lambda = \dfrac{\alpha+\beta+6\gamma}{1-2\beta-4\gamma}$.

If for a nonincreasing sequence $\{x_n\}$ in $\overline{B(x_0,r)}$, $\{x_n\} \to u$ implies that $u \preceq x_n$, then there exists a point x^* in $\overline{B(x_0,r)}$ such that $G_d(x^*,x^*,x^*) = 0$ and $x^* = Sx^*$.

Proof. Consider a picard sequence $x_{n+1} = Sx_n$ with initial guess x_0 . As $x_{n+1} = Sx_n \preceq x_n$ for all $n \in \{0\} \cup \mathbb{N}$. Now by using inequality (2.2) we have,

$$G_d(x_0,x_1,x_1) \le r,$$

it implies that $x_1 \in \overline{B(x_0,r)}$. Now by using inequality (2.1) we have,

$$G_d(x_1,x_2,x_2) \le \alpha G_d(x_0,x_1,x_1) + \beta\begin{bmatrix} G_d(x_0,Sx_0,Sx_0) + G_d(x_1,Sx_1,Sx_1) \\ +G_d(x_1,Sx_1,Sx_1) \end{bmatrix} + \gamma\begin{bmatrix} G_d(x_0,Sx_1,Sx_1) + G_d(x_0,Sx_1,Sx_1) \\ +G_d(x_1,Sx_0,Sx_0) + G_d(x_1,Sx_1,Sx_1) \\ +G_d(x_1,Sx_0,Sx_0) + G_d(x_1,Sx_1,Sx_1) \end{bmatrix}$$

$$G_d(x_1,x_2,x_2) \le (\alpha+\beta)G_d(x_0,x_1,x_1) + 2\beta G_d(x_1,x_2,x_2) + \gamma\begin{bmatrix} 2G_d(x_0,x_2,x_2) + 2G_d(x_1,x_1,x_1) \\ +2G_d(x_1,x_2,x_2) \end{bmatrix}.$$

$$G_d(x_1,x_2,x_2) \le (\alpha+\beta)G_d(x_0,x_1,x_1) + 2\beta G_d(x_1,x_2,x_2) + \gamma\begin{bmatrix} 2G_d(x_0,x_1,x_1) + 2G_d(x_1,x_2,x_2) \\ +2G_d(x_1,x_0,x_0) + 2G_d(x_0,x_1,x_1) \\ +2G_d(x_1,x_2,x_2) \end{bmatrix}$$

$$G_d(x_1,x_2,x_2) \le (\alpha+\beta+4\gamma)G_d(x_0,x_1,x_1) + 2\beta G_d(x_1,x_2,x_2) + 4\gamma G_d(x_1,x_2,x_2) + 2\gamma G_d(x_0,x_1,x_1)$$

$$(1-2\beta-4\gamma)G_d(x_1,x_2,x_2) \le (\alpha+\beta+6\gamma)G_d(x_0,x_1,x_1)$$

$$G_d(x_1,x_2,x_2) \le \left(\frac{\alpha+\beta+6\gamma}{1-2\beta-4\gamma}\right) G_d(x_0,x_1,x_1)$$

$$G_d(x_1,x_2,x_2) \le \lambda G_d(x_0,x_1,x_1). \tag{2.3}$$

Now by using (2.2) and (2.3) we get,

$$\begin{aligned}&G_d(x_0,x_2,x_2)\\ &\le G_d(x_0,x_1,x_1)+G_d(x_1,x_2,x_2)\\ &\le G_d(x_0,x_1,x_1)+\lambda G_d(x_0,x_1,x_1)\\ &\le (1+\lambda)G_d(x_0,x_1,x_1)\\ &\le \left(1-\lambda^2\right)r \le r,\end{aligned}$$

it implies that $x_2 \in \overline{B(x_0,r)}$. Let $x_3,\ldots,x_j \in \overline{B(x_0,r)}$ for some $j \in \mathbb{N}$. Now by using inequality (2.1) we have,

$$\begin{aligned}&G_d(x_2,x_3,x_3) \le \alpha G_d(x_1,x_2,x_2)\\ &+\beta\begin{bmatrix}G_d(x_1,Sx_1,Sx_1)+G_d(x_2,Sx_2,Sx_2)\\ +G_d(x_2,Sx_2,Sx_2)\end{bmatrix}\\ &+\gamma\begin{bmatrix}G_d(x_1,Sx_2,Sx_2)+G_d(x_1,Sx_2,Sx_2)\\ +G_d(x_2,Sx_1,Sx_1)+G_d(x_2,Sx_2,Sx_2)\\ +G_d(x_2,Sx_1,Sx_1)+G_d(x_2,Sx_2,Sx_2)\end{bmatrix}.\end{aligned}$$

$$\begin{aligned}&G_d(x_2,x_3,x_3) \le \alpha G_d(x_1,x_2,x_2)\\ &+\beta\left[G_d(x_1,x_2,x_2)+2G_d(x_2,x_3,x_3)\right]\\ &+\gamma\begin{bmatrix}2G_d(x_1,x_3,x_3)+G_d(x_2,x_2,x_2)\\ +G_d(x_2,x_3,x_3)+G_d(x_2,x_2,x_2)\\ +G_d(x_2,x_3,x_3)\end{bmatrix}.\end{aligned}$$

$$\begin{aligned}&G_d(x_2,x_3,x_3) \le (\alpha+\beta)G_d(x_1,x_2,x_2)\\ &+2\beta G_d(x_2,x_3,x_3)+2\gamma G_d(x_1,x_3,x_3)\\ &+2\gamma G_d(x_2,x_2,x_2)+2\gamma G_d(x_2,x_3,x_3).\end{aligned}$$

$$\begin{aligned}&G_d(x_2,x_3,x_3) \le (\alpha+\beta)G_d(x_1,x_2,x_2)\\ &+2\beta G_d(x_2,x_3,x_3)+2\gamma_d G_d(x_1,x_2,x_2)\\ &+2\gamma G_d(x_2,x_3,x_3)+2\gamma G_d(x_2,x_1,x_1)\\ &+2\gamma G_d(x_1,x_2,x_2)+2\gamma G_d(x_2,x_3,x_3)\end{aligned}$$

$$\begin{aligned}&(1-2\beta-4\gamma)G_d(x_2,x_3,x_3)\\ &\le (\alpha+\beta+6\gamma)G_d(x_1,x_2,x_2)\end{aligned}$$

$$\begin{aligned}G_d(x_2,x_3,x_3) &\le \left(\frac{\alpha+\beta+6\gamma}{1-2\beta-4\gamma}\right)G_d(x_1,x_2,x_2)\\ &\le \lambda G_d(x_1,x_2,x_2)\end{aligned}$$

$$G_d(x_2,x_3,x_3) \le \lambda^2 G_d(x_0,x_1,x_1). \tag{2.4}$$

Similarly we get,

$$G_d\left(x_j,x_{j+1},x_{j+1}\right) \le \lambda^j G_d(x_0,x_1,x_1). \tag{2.5}$$

By using (2.3), (2.4) and (2.5) we get,

$$\begin{aligned}&G_d\left(x_0,x_{j+1},x_{j+1}\right)\\ &\le G_d(x_0,x_1,x_1)+G_d(x_1,x_2,x_2)\\ &\quad+\ldots+G_d\left(x_j,x_{j+1},x_{j+1}\right)\\ &\le \left(1+\lambda+\lambda^2+\ldots+\lambda^j\right)G_d(x_0,x_1,x_1)\\ &\le \left(\frac{1-\lambda^{j+1}}{1-\lambda}\right)G_d(x_0,x_1,x_1)\\ &\le \left(\frac{1-\lambda^{j+1}}{1-\lambda}\right)(1-\lambda)r = \left(1-\lambda^{j+1}\right)r\end{aligned}$$

$$G_d\left(x_0,x_{j+1},x_{j+1}\right) \le r,$$

which further implies $x_{j+1} \in \overline{B(x_0,r)}$. Hence by induction $x_n \in \overline{B(x_0,r)}$ for all $n \in \mathbb{N}$. Using inequality (2.5) we get

$$\begin{aligned}&G_d(x_n,x_{n+i},x_{n+i})\\ &\le G_d(x_n,x_{n+1},x_{n+1})+\ldots+G_d(x_{n+i-1},x_{n+i},x_{n+i})\end{aligned}$$

$$\begin{aligned}&G_d(x_n,x_{n+i},x_{n+i})\\ &\le \lambda^n G_d(x_0,x_1,x_1)+..+\lambda^{n+i-1}G_d(x_0,x_1,x_1)\end{aligned}$$

$$\begin{aligned}&G_d(x_n,x_{n+i},x_{n+i})\\ &\le \left(\lambda^n+\lambda^{n+1}+..+\lambda^{n+i-1}\right)G_d(x_0,x_1,x_1)\end{aligned}$$

$$\begin{aligned}&G_d(x_n,x_{n+i},x_{n+i})\\ &\le \lambda^n\left(1+\lambda+\ldots+\lambda^{i-1}\right)G_d(x_0,x_1,x_1)\\ &\le \lambda^n\left(\frac{1-\lambda^i}{1-\lambda}\right)G_d(x_0,x_1,x_1)\\ &\le \lambda^n\left(\frac{1-\lambda^i}{1-\lambda}\right)(1-\lambda)r\\ &= \lambda^n\left(1-\lambda^i\right)r \to 0 \ as\ n \to \infty.\end{aligned}$$

Hence the sequence $\{x_n\}$ is Cauchy sequence in $\left(\overline{B(x_0,r)},G_d\right)$. Therefore, there exists a point $x^* \in \overline{B(x_0,r)}$ such that

$$\begin{aligned}&\lim_{n\to\infty} G_d\left(x_n,x^*,x^*\right)\\ &= \lim_{n\to\infty} G_d\left(x^*,x^*,x_n\right)=0.\end{aligned} \tag{2.6}$$

Now,

$$\begin{aligned}&G_d\left(x^*,Sx^*,Sx^*\right)\\ &\le G_d\left(x^*,x_n,x_n\right)+G_d\left(Sx_{n-1},Sx^*,Sx^*\right).\end{aligned}$$

By assumption $x^* \preceq x_n \preceq x_{n-1}$, therefore

$$G_d\left(x^*, Sx^*, Sx^*\right)$$
$$\leq G_d\left(x^*, x_n, x_n\right) + \alpha G_d\left(x_{n-1}, x^*, x^*\right)$$
$$+\beta\begin{bmatrix} G_d\left(x_{n-1}, Sx_{n-1}, Sx_{n-1}\right) + G_d\left(x^*, Sx^*, Sx^*\right) \\ +G_d\left(x^*, Sx^*, Sx^*\right) \end{bmatrix}$$
$$+\gamma\begin{bmatrix} G_d\left(x_{n-1}, Sx^*, Sx^*\right) + G_d\left(x_{n-1}, Sx^*, Sx^*\right) \\ +G_d\left(x^*, Sx_{n-1}, Sx_{n-1}\right) + G_d\left(x^*, Sx^*, Sx^*\right) \\ +G_d\left(x^*, Sx_{n-1}, Sx_{n-1}\right) + G_d\left(x^*, Sx^*, Sx^*\right) \end{bmatrix}$$

$$G_d\left(x^*, Sx^*, Sx^*\right)$$
$$\leq G_d\left(x^*, x_n, x_n\right) + \alpha G_d\left(x_{n-1}, x^*, x^*\right)$$
$$+\beta\left[G_d\left(x_{n-1}, x_n, x_n\right) + 2G_d\left(x^*, Sx^*, Sx^*\right)\right]$$
$$+\gamma\begin{bmatrix} 2G_d\left(x_{n-1}, Sx^*, Sx^*\right) + 2G_d\left(x^*, x_n, x_n\right) \\ +2G_d\left(x^*, Sx^*, Sx^*\right) \end{bmatrix}$$

$$G_d\left(x^*, Sx^*, Sx^*\right)$$
$$\leq G_d\left(x^*, x_n, x_n\right) + \alpha G_d\left(x_{n-1}, x^*, x^*\right)$$
$$+\beta\begin{bmatrix} G_d\left(x_{n-1}, x^*, x^*\right) + G_d\left(x^*, x_n, x_n\right) \\ 2G_d\left(x^*, Sx^*, Sx^*\right) \end{bmatrix}$$
$$+\gamma\begin{bmatrix} 2G_d\left(x_{n-1}, x^*, x^*\right) + 2G_d\left(x^*, Sx^*, Sx^*\right) \\ +2G_d\left(x^*, x_n, x_n\right) + 2G_d\left(x^*, Sx^*, Sx^*\right) \end{bmatrix}$$

$$G_d\left(x^*, Sx^*, Sx^*\right)$$
$$\leq G_d\left(x^*, x_n, x_n\right) + (\alpha+\beta+2\gamma) G_d\left(x_{n-1}, x^*, x^*\right)$$
$$+2\beta G_d\left(x^*, Sx^*, Sx^*\right) + \beta G_d\left(x^*, x_n, x_n\right)$$
$$+4\gamma G_d\left(x^*, Sx^*, Sx^*\right) + 2\gamma G_d\left(x^*, x_n, x_n\right)$$

$$G_d\left(x^*, Sx^*, Sx^*\right)$$
$$\leq G_d\left(x^*, x_n, x_n\right) + (\alpha+\beta+2\gamma) G_d\left(x_{n-1}, x^*, x^*\right)$$
$$+(2\beta+4\gamma) G_d\left(x^*, Sx^*, Sx^*\right) + \beta G_d\left(x^*, x_n, x_n\right)$$
$$+2\gamma G_d\left(x^*, x_n, x_n\right)$$

$$(1-2\beta-4\gamma) G_d\left(x^*, Sx^*, Sx^*\right)$$
$$\leq (1+\beta+2\gamma) G_d\left(x^*, x_n, x_n\right)$$
$$+(\alpha+\beta+2\gamma) G_d\left(x_{n-1}, x^*, x^*\right).$$

Taking $\lim_{n\to\infty}$ on both sides and by using (2.6) we have,

$$(1-2\beta-4\gamma) G_d\left(x^*, Sx^*, Sx^*\right)$$
$$\leq (0) + (\alpha+\beta+2\gamma)(0) + (\beta+2\gamma)(0)$$
$$G_d\left(x^*, Sx^*, Sx^*\right) \leq 0.$$

Also,

$$G_d\left(Sx^*, x^*, x^*\right) = G_d\left(x^*, Sx^*, Sx^*\right) \leq 0.$$

Hence, $Sx^* = x^*$.

If we take $\beta = \gamma = 0$ in inequality then we obtain the following corollary.

Corollary 2.2. Let $(X, \preceq, G_d)$ be an ordered complete dislocated symmetric G_d metric space, and $S: X \to X$ be a dominated mapping and x_0 be any arbitrary point in X. Suppose there exists $\alpha \in [0,1)$ with,

$$G_d(Sx, Sy, Sz) \leq \alpha G_d(x, y, z),$$
$$\text{for all } x, y \text{ and } z \in \overline{B(x_0, r)}$$

and

$$G_d(Sx, Sy, Sz) \leq (1-\alpha) r.$$

If for a nonincreasing sequence $\{x_n\} \to u$ implies that $u \preceq x_n$. Then there exists a point x^* in $\overline{B(x_0, r)}$ such that $x^* = Sx^*$ and $G_d\left(x^*, x^*, x^*\right) = 0$. Moreover if for any three points x, y and z in $\overline{B(x_0, r)}$ such that there exists a point $v \in \overline{B(x_0, r)}$ such that $v \preceq x, v \preceq y$ and $v \preceq z$, that is, every three of elements in $\overline{B(x_0, r)}$ has a lower bound, then the point x^* is unique.

Similarly if we take $\alpha = \gamma = 0$ in inequality then we obtain the following corollary.

Corollary 2.3. Let $(X, \preceq, G_d)$ be an ordered complete dislocated symmetric G_d-metric space $S: X \to X$ be a mapping and x_0 be an arbitrary point in X. Suppose there exists $\beta \in \left[0, \frac{1}{3}\right)$ with

$$G_d(Sx, Sy, Sz)$$
$$\leq b\left[G_d(x, Sx, Sx) + G_d(y, Sy, Sy) + G_d(z, Sz, Sz)\right]$$

for all elements $x, y, z \in \overline{B(x_0, r)}$ and

$$G_d(x_0, Sx_0, Sx_0) \leq (1-\Phi) r,$$

where $\Phi = \dfrac{\beta}{1-2\beta}$. If for nonincreasing sequence $\{x_n\} \to u$ implies that $u \preceq x_n$. Then there exists a point x^* in $B(x_0, r)$ such that $x^* = Sx^*$ and $G_d\left(x^*, x^*, x^*\right) = 0$.

If we take $\gamma = 0$ in inequality then we obtain the following Corollary.

Corollary 2.4. Let $(X, \preceq, G_d)$ be an ordered complete dislocated symmetric G_d metric space, $x_0 \in X$, $r > 0$, and $S : X \to X$ be a dominated mapping. Suppose there exists α, β and for all elements x, y and z in $\overline{B(x_0, r)}$.

$$G_d(Sx, Sy, Sz) \le \alpha G_d(x, y, z) + \beta\left[G_d(x, Sx, Sx) + G_d(y, Sy, Sy) + G_d(z, Sz, Sz)\right]$$

where $\lambda = \dfrac{\alpha+\beta}{1-2\beta}$ and $\alpha + 3\beta < 1$ and

$$G_d(x_0, Sx_0, Sx_0) \le (1-\lambda) r.$$

If for a nonincreasing sequence $\{x_n\}$ in $\overline{B(x_0, r)}$, $\{x_n\} \to u$ implies that $u \preceq x_n$ there exists a point x^* in $\overline{B(x_0, r)}$ such that $G_d(x^*, x^*, x^*) = 0$ and $x^* = Sx^*$.

Example 2.5. Let $X = \mathbb{R}^+ \cup \{0\}$ be endowed with usual order and $G_d : X \times X \times X \to X$ be a complete dislocated symmetric G_d metric space defined by,

$$G_d(x, y, z) = \max\left\{\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\right\}$$

Then (X, G_d) is a complete dislocated symmetric G_d metric space. Let $S : X \to X$ be defined by,

$$Sx = \begin{cases} \dfrac{x}{4} & \text{if } x \in [0,3] \\ x - \dfrac{1}{7} & \text{if } x \in (3, \infty) \end{cases}.$$

Clearly, S is a dominated mapping. Take $x_0 = \dfrac{1}{2}$, $r = \dfrac{3}{2}$, $\overline{B(x_0, r)} = [0,3]$ and $\lambda = \dfrac{2}{3}$, $\lambda = \dfrac{2}{3}, \alpha + 3\beta < 1$, where $\alpha = \dfrac{1}{5}$, and $\beta = \dfrac{1}{5}$.

$$G_d(x_0, Sx_0, Sx_0) = G_d\left(\frac{1}{2}, S\frac{1}{2}, S\frac{1}{2}\right) = G_d\left(\frac{1}{2}, \frac{1}{8}, \frac{1}{8}\right)$$
$$= \max\left\{\frac{1}{4}, \frac{1}{16}, \frac{1}{16}\right\} = \frac{1}{4} < (1-\lambda) r = \left(1 - \frac{2}{3}\right)\frac{3}{2} = \frac{1}{2}.$$

Also if x, y and $z \in (3, \infty)$. We assume that $x = 4$, $y = 4$, $z = 4$, then

$$G_d(Sx, Sy, Sz) = G_d\left(4 - \frac{1}{7}, 4 - \frac{1}{7}, 4 - \frac{1}{7}\right)$$
$$= \max\{3,3,3\} = 3 > \frac{1}{5}\max\{2,2,2\}$$
$$+ \frac{1}{5}\left[\max\{2,3,3\} + \max\{2,3,3\} + \max\{2,3,3\}\right]$$
$$= aG_d(x, y, z) + b\begin{bmatrix} G_d(x, Sx, Sx) + G_d(y, Sy, Sy) \\ + G_d(z, Sz, Sz) \end{bmatrix} = \frac{11}{5}.$$

So the contractive condition does not holds in X. Now if x, y and $z \in \overline{B(x_0, r)}$ then,

$$G_d(Sx, Sy, Sz) = G_d\left(\frac{x}{4}, \frac{y}{4}, \frac{z}{4}\right) = \max\left\{\frac{x}{8}, \frac{y}{8}, \frac{z}{8}\right\}$$

$$G_d(Sx, Sy, Sz) = \max\left\{\frac{x}{8}, \frac{y}{8}, \right\} \le \frac{1}{5}\max\left\{\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\right\}$$
$$+ \frac{1}{5}\left[\max\left\{\frac{x}{2}, \frac{x}{8}, \frac{x}{8}\right\} + \max\left\{\frac{y}{2}, \frac{y}{8}, \frac{y}{8}\right\} + \max\left\{\frac{z}{2}, \frac{z}{8}, \frac{z}{8}\right\}\right]$$

$$G_d(Sx, Sy, Sz) = \max\left\{\frac{x}{8}, \frac{y}{8}, \frac{z}{8}\right\} \le \frac{1}{5}\max\left\{\frac{x}{2}, \frac{y}{2}, \frac{z}{2}\right\}$$
$$+ \frac{1}{5}\left[\max\left\{\frac{x}{2}, \frac{x}{8}, \frac{x}{8}\right\} + \max\left\{\frac{y}{2}, \frac{y}{8}, \frac{y}{8}\right\} + \max\left\{\frac{z}{2}, \frac{z}{8}, \frac{z}{8}\right\}\right]$$
$$= aG_d(x, y, z) + b\begin{bmatrix} G_d(x, Sx, Sx) + G_d(y, Sy, Sy) \\ + G_d(z, Sz, Sz) \end{bmatrix}.$$

Hence it satisfies all the requirements of Corollary 2.4 and 0 is the fixed point of *S*.

Theorem 2.6: Let $(X, \preceq, G_d)$ be an ordered complete dislocated symmetric G_d metric space, and $S : X \to X$ be a dominated map and x_0 be an arbitrary point in X. Suppose there exists $\lambda \in [0,1)$ with,

$$\begin{aligned} &G_d(Sx, Sy, Sz) \\ &\le \alpha G_d(x, y, z) + \beta\begin{bmatrix} G_d(x, Sx, Sx) + G_d(y, Sy, Sy) \\ + G_d(z, Sz, Sz) \end{bmatrix} \\ &+ \gamma\begin{bmatrix} G_d(x, Sy, Sy) + G_d(z, Sz, Sz) + G_d(y, Sx, Sx) \\ + G_d(y, Sz, Sz) + G_d(z, Sx, Sx) + G_d(z, Sy, Sy) \end{bmatrix}. \end{aligned} \quad (2.7)$$

for all comparable elements x, y and z in X. If, for a nonincreasing sequence $\{x_n\}$ in X, $\{x_n\} \to u$ implies that $u \preceq \{x_n\}$, then there exists a point x^* in X such that $x^* = Sx^*$ and $S(x^*, x^*, x^*) = 0$. Moreover, x^* is unique, for every triple of elements x, y and z in X if there exist a point $v \in X$ such that $v \preceq x$, $v \preceq y$ and $v \preceq x$.

Proof. From the proof of Theorem 2.1, we can find x^* such that $x^* = Sx^*$. Now if x^* and y are not comparable then there exists a point $v \in X$ which is a lower bound of both x^* and y that is $v \preceq x^*$ and $v \preceq y$. As $S^n v \preceq S^{n-1} v \preceq S^{n-2} v \ldots \preceq v$, then by inequality 2.7, we have

$$G_d\left(S^n v, S^{n+1} v, S^{n+1} v\right) = G_d\left(S\left(S^{n-1} v, S^n v, S^n v\right)\right)$$
$$\le \alpha G_d\left(S^{n-1} v, S^n v, S^n v\right)$$
$$+ \beta\begin{bmatrix} G_d\left(S^{n-1} v, S^n v, S^n v\right) \\ + G_d\left(S^n v, S^{n+1} v, S^{n+1} v\right) \\ + G_d\left(S^n v, S^{n+1} v, S^{n+1} v\right) \end{bmatrix}$$

$$+\gamma\begin{bmatrix} G_d\left(S^{n-1}v,S^{n+1}v,S^{n+1}v\right)+G_d\left(S^{n-1}v,S^{n+1}v,S^{n+1}v\right)\\ +G_d\left(S^nv,S^nv,S^nv\right)+G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)\\ +G_d\left(S^nv,S^nv,S^nv\right)+G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)\end{bmatrix},$$

$$G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)\le \alpha G_d\left(S^{n-1}v,S^nv,S^nv\right)$$
$$+\beta\left[G_d\left(S^{n-1}v,S^nv,S^nv\right)+2G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)\right]$$
$$+\gamma\begin{bmatrix}2G_d\left(S^{n-1}v,S^{n+1}v,S^{n+1}v\right)+2G_d\left(S^nv,S^nv,S^nv\right)\\ +2G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)\end{bmatrix},$$

$$G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)$$
$$\le(\alpha+\beta)G_d\left(S^{n-1}v,S^nv,S^nv\right)$$
$$+2\beta G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)+2\gamma G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)$$
$$+2\gamma G_d\left(S^nv,S^nv,S^nv\right)++2\gamma G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right).$$

$$G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)$$
$$\le(\alpha+\beta)G_d\left(S^{n-1}v,S^nv,S^nv\right)$$
$$+2\beta G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)+2\gamma G_d\left(S^nv,S^nv,S^nv\right)$$
$$+2\gamma G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)+2\gamma G_d\left(S^nv,S^{n-1}v,S^{n-1}v\right)$$
$$+2\gamma G_d\left(S^{n-1}v,S^nv,S^nv\right)+2\gamma G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)$$

$$G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)$$
$$\le(\alpha+\beta+4\gamma)G_d\left(S^{n-1}v,S^nv,S^nv\right)$$
$$+2\beta G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)+2\gamma G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)$$
$$+2G_d\left(S^{n-1}v,S^nv,S^nv\right)+2\gamma G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)$$

$$(1-2\beta-4\gamma)G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)$$
$$\le(\alpha+\beta+6\gamma)G_d\left(S^{n-1}v,S^nv,S^nv\right)$$

$$G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)$$
$$\le\left(\frac{\alpha+\beta+6\gamma}{1-2\beta-4\gamma}\right)G_d\left(S^{n-1}v,S^nv,S^nv\right)$$

$$G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)\le\lambda G_d\left(S^{n-1}v,S^nv,S^nv\right).$$

Which implies that,

$$G_d\left(S^nv,S^{n+1}v,S^{n+1}v\right)$$
$$\le\lambda G_d\left(S^{n-1}v,S^{n-2}v,S^{n-2}v\right)\tag{2.8}$$
$$\le\ldots\le\lambda^nG_d\left(v,Sv,Sv\right)\to 0 \text{ as } n\to\infty.$$

As $S^nv\preceq S^{n-1}v\preceq S^{n-2}v\preceq\ldots\preceq v\preceq x^*$, then,

$$G_d\left(Sx^*,S^nv,S^nv\right)=G_d\left(Sx^*,S\left(S^{n-1}v\right),S\left(S^{n-1}v\right)\right)$$
$$\le\alpha G_d\left(x^*,S^{n-1}v,S^{n-1}v\right)$$
$$+\beta\begin{bmatrix}G_d\left(x^*,Sx^*,Sx^*\right)+G_d\left(S^{n-1}v,S^nv,S^nv\right)\\ +G_d\left(S^{n-1}v,S^nv,S^nv\right)\end{bmatrix}$$
$$+\gamma\begin{bmatrix}G_d\left(x^*,S^nv,S^nv\right)+G_d\left(x^*,S^nv,S^nv\right)\\ +G_d\left(S^{n-1}v,Sx^*,Sx^*\right)+G_d\left(S^{n-1}v,S^nv,S^nv\right)\\ +G_d\left(S^{n-1}v,Sx^*,Sx^*\right)+G_d\left(S^{n-1}v,S^nv,S^nv\right)\end{bmatrix}.$$

$$G_d\left(Sx^*,S^nv,S^nv\right)$$
$$\le\alpha G_d\left(x^*,S^{n-1}v,S^{n-1}v\right)$$
$$+\beta\begin{bmatrix}G_d\left(Sx^*,S^nv,S^nv\right)+G_d\left(S^nv,Sx^*,Sx^*\right)\\ +2G_d\left(S^{n-1}v,S^nv,S^nv\right)\end{bmatrix}$$
$$+\gamma\begin{bmatrix}2G_d\left(Sx^*,S^nv,S^nv\right)+2G_d\left(S^n-1v,Sx^*,Sx^*\right)\\ +2G_d\left(S^{n-1}v,S^nv,S^nv\right)\end{bmatrix}$$

$$G_d\left(Sx^*,S^nv,S^nv\right)\le\alpha G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)$$
$$+\beta\begin{bmatrix}G_d\left(Sx^*,S^nv,S^nv\right)+G_d\left(Sx^*,S^nv,S^nv\right)\\ +2G_d\left(S^{n-1}v,S^nv,S^nv\right)\end{bmatrix}$$
$$+\gamma\begin{bmatrix}2G_d\left(Sx^*,S^nv,S^nv\right)+2G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)\\ +2G_d\left(S^{n-1}v,S^nv,S^nv\right)\end{bmatrix}$$

$$(1-2\beta-2\gamma)G_d\left(Sx^*,S^nv,S^nv\right)$$
$$\le(\alpha+2\gamma)G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)$$
$$+(2\beta+2\gamma)G_d\left(S^{n-1}v,S^nv,S^nv\right).$$

$$G_d\left(Sx^*,S^nv,S^nv\right)$$
$$\le\left(\frac{\alpha+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)\tag{2.9}$$
$$+\left(\frac{2\beta+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(S^{n-1}v,S^nv,S^nv\right).$$

Now let,

$$G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)$$
$$=G_d\left(Sx^*,S\left(S^{n-2}v\right),S\left(S^{n-2}v\right)\right)$$
$$\le\alpha G_d\left(x^*,S^{n-2}v,S^{n-2}v\right)$$
$$+\beta\begin{bmatrix}G_d\left(x^*,Sx^*,Sx^*\right)+G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right)\\ +G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right)\end{bmatrix}$$

$$+\gamma\begin{bmatrix} G_d\left(x^*,S^{n-1}v,S^{n-1}v\right)+G_d\left(x^*,S^{n-1}v,S^{n-1}v\right) \\ +G_d\left(S^{n-2}v,Sx^*,Sx^*\right)+G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right) \\ +G_d\left(S^{n-2}v,Sx^*,Sx^*\right)+G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right) \end{bmatrix}.$$

$$G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)\le \alpha G_d\left(Sx^*,S^{n-2}v,S^{n-2}v\right)$$

$$+\beta\begin{bmatrix} G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)+G_d\left(S^{n-1}v,Sx^*,Sx^*\right) \\ +2G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right) \end{bmatrix}$$

$$+\gamma\begin{bmatrix} 2G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)+2G_d\left(S^{n-2}v,Sx^*,Sx^*\right) \\ +2G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right) \end{bmatrix}$$

$$G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)\le \alpha G_d\left(Sx^*,S^{n-2}v,S^{n-2}v\right)$$

$$+\beta\begin{bmatrix} G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)+G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right) \\ +2G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right) \end{bmatrix}$$

$$+\gamma\begin{bmatrix} 2G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)+2G_d\left(Sx^*,S^{n-2}v,S^{n-2}v\right) \\ +2G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right) \end{bmatrix}$$

$$\left(1-2\beta-2\gamma\right)G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)$$

$$\le\left(\alpha+2\gamma\right)G_d\left(Sx^*,S^{n-2}v,S^{n-2}v\right)$$

$$+\left(2\beta+2\gamma\right)G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right)$$

$$G_d\left(Sx^*,S^{n-1}v,S^{n-1}v\right)$$

$$\le\left(\frac{\alpha+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(Sx^*,S^{n-2}v,S^{n-2}v\right) \qquad (2.10)$$

$$+\left(\frac{2\beta+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right).$$

Using inequality (2.10) we get

$$G_d\left(Sx^*,S^nv,S^nv\right)$$

$$\le\left(\frac{\alpha+2\gamma}{1-2\beta-2\gamma}\right)\begin{bmatrix} \left(\frac{\alpha+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(Sx^*,S^{n-2}v,S^{n-2}v\right) \\ +\left(\frac{2\beta+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right) \end{bmatrix}$$

$$+\left(\frac{2\beta+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(S^{n-1}v,S^nv,S^nv\right)$$

$$G_d\left(Sx^*,S^nv,S^nv\right)$$

$$\le\left(\frac{\alpha+2\gamma}{1-2\beta-2\gamma}\right)^2 G_d\left(Sx^*,S^{n-2}v,S^{n-2}v\right)$$

$$+\left(\frac{\alpha+2\gamma}{1-2\beta-2\gamma}\right)\left(\frac{2\beta+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right)$$

$$+\left(\frac{2\beta+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(S^{n-1}v,S^nv,S^nv\right).$$

Continuing in this way we get

$$G_d\left(Sx^*,S^nv,S^nv\right)$$

$$\le\left(\frac{\alpha+2\gamma}{1-2\beta-2\gamma}\right)^n G_d\left(Sx^*,v,v\right)$$

$$+\left(\frac{\alpha+2\gamma}{1-2\beta-2\gamma}\right)^{n-1}\left(\frac{2\beta+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(v,Sv,Sv\right)+\ldots \qquad (2.11)$$

$$+\left(\frac{\alpha+2\gamma}{1-2\beta-2\gamma}\right)\left(\frac{2\beta+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(S^{n-2}v,S^{n-1}v,S^{n-1}v\right)$$

$$+\left(\frac{2\beta+2\gamma}{1-2\beta-2\gamma}\right)G_d\left(S^{n-1}v,S^nv,S^nv\right).$$

On taking limit $n\to\infty$ and by using (2.8) we get,

$$G_d\left(Sx^*,S^nv,S^nv\right)\to 0 \text{ as } n\to\infty.$$

Also

$$G_d\left(S^nv,Sx^*,Sx^*\right)=G_d\left(Sx^*,S^nv,S^nv\right)\to 0 \text{ as } n\to\infty.$$

Similarly

$$G_d\left(S^nv,Sy,Sy\right)=G_d\left(Sy,S^nv,S^nv\right)\to 0 \text{ as } n\to\infty.$$

Now,

$$G_d\left(x^*,y,y\right)=G_d\left(Sx^*,Sy,Sy\right)$$

$$G_d\left(x^*,y,y\right)\le G_d\left(Sx^*,S^nv,S^nv\right)+G_d\left(S^nv,Sy,Sy\right)$$

$$G_d\left(x^*,y,y\right)\to 0 \text{ as } n\to\infty.$$

Also

$$G_d\left(x^*,y,y\right)=G_d\left(x^*,y,y\right)\to 0 \text{ as } n\to\infty.$$

Hence $x^*=y$.

Competing Interests

The authors declare that they have no competing interests.

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