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# Weak Compatibility and Related Fixed Point Theorem for Six Maps in Multiplicative Metric Space

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**Abstract** We consider six self-maps satisfying the condition of commuting and weak compatibility of mappings and the purpose of this paper is to give some common fixed points theorems for complete multiplicative metric space.

***Keywords:*** *commuting mapping, complete multiplicative metric spaces, weakly compatible maps and common fixed points*

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## 1. Introduction

The concept of multiplicative metric spaces is introduced by M. Özavsar [8].They also gave some topological properties of the relevant multiplicative metric space and now it's more general than well-known metric space. Fixed point theorems are admirable tool for Existence and uniqueness of the solutions to various mathematical models like differential, integral and partial differential equations and vibrational inequalities etc. The study of metric space plays very important role to many fields both in pure and applied science [4]. Abounding researchers extended the notion of a metric space such as vector valued metric space of Perov [3], a cone metric spaces of Huang and Zhang [7], a modular metric spaces of Chistyakov [17], for details about multiplicative metric space and related concepts, we refer the reader to[8] etc.

It is well know that the set of positive real numbers $\mathbb{R}_+$ is not complete according to the usual metric. To overcome this problem, In 2008, Bashirov [2] Introduced the concept of multiplicative metric spaces as follows:

**Definition 1.1. [8]** Let X be a nonempty set. Multiplicative metric is a mapping d: $X \times X \to \mathbb{R}_+$ satisfying the following conditions

(1.1) $d(x, y) \geq 1$ for all $x, y \in X$ and $d(x, y) = 1$ if and only if $x = y$,

(1.2) $d(x, y) = d(y, x)$ for all $x, y \in X$,

(1.3) $d(x, z) \leq d(x, y) \cdot d(y, z)$ for all $x, y, z \in X$ (multiplicative triangle inequality)

To articulate the importance of this study, we should first note that $\mathbb{R}_+$is a complete multiplicative metric space with respect to the multiplicative metric. Furthermore, we introduce concept of multiplicative contraction mapping and prove some fixed point theorems of multiplicative contraction mappings on multiplicative metric spaces.

**Definition 1.2.** [8] (Multiplicative ball) Let (X, d) be a multiplicative metric space, $x \in X$ and $\varepsilon > 1$. We now define a set $B_\varepsilon(x) = \{y \in X \mid d(x, y) < \varepsilon\}$, which is called multiplicative open ball of radius ε with centre x. Similarly, one can describe multiplicative closed ball as

$$\overline{B\varepsilon(x)} = \{y \in X \mid d(x, y) \leq \varepsilon\}.$$

**Definition 1.3.** [8] (Multiplicative interior point): Let (X, d) be a multiplicative metric space and $A \subset X$. Then we call $x \in A$ a multiplicative interior point of A if there exists an $\varepsilon > 1$ such that $B_\varepsilon(x) \subset A$. The collection of all interior points of A is called multiplicative interior of A and denoted by int(A).

**Definition 1.4.** [8] (Multiplicative open set): Let (X, d) be a multiplicative metric space and $A \subset X$. If every point of A is a multiplicative interior point of A, i.e., A = int(A), then A is called a multiplicative open set.

**Definition 1.5.** [8] Let (X, d) be a multiplicative metric space. A point $x \in X$ is said to be multiplicative limit point of $S \subset X$ if and only if $(B_\varepsilon(x) \setminus \{x\}) \cap S \neq \emptyset$ for every $\varepsilon > 1$. The set of all multiplicative limit points of the set S is denoted by $S'$.

**Definition 1.6.** [8] Let (X, d) be a multiplicative metric space. We call a set $S \subset X$ multiplicative closed in (X, d) if S contains all of its multiplicative limit points.

**Definition 1.7.** [8] (Multiplicative convergence): Let (X, d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X and $x \in X$. If for every multiplicative open ball $B_\varepsilon(x)$, there exists a natural number N such that $n \geq N \Rightarrow x_n \in B_\varepsilon(x)$, then the sequence $\{x_n\}$ is said to be multiplicative convergent to x, denoted by $x_n \to x$ $(n \to \infty)$.

**Lemma 1.8.** [8] Let (X, d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X and $x \in X$. Then $x_n \to x$ $(n \to \infty)$ if and only if $d(x_n, x) \to 1$ $(n \to \infty)$.

$$\frac{1}{\varepsilon} > d(x_n, x) < 1, \varepsilon \text{ for all } n \geq N.$$

**Lemma 1.9.** [8] Let (X, d) be a multiplicative metric space, $\{x_n\}$ be a sequence in X. If the sequence $\{x_n\}$ is multiplicative convergent, then the multiplicative limit point is unique.

**Definition 1.10.** [8] Let (X, d) be a multiplicative metric space and $\{x_n\}$ be a sequence in X. The sequence is called a multiplicative Cauchy sequence if it holds that, for all $\varepsilon > 1$, there exists $N \in \mathbb{N}$ such that $d(x_m, x_n) < \varepsilon$ for $m, n \geq N$.

**Definition 1.11.** [8] Let (X, d) be a multiplicative metric space and $A \subset X$. The set A is called multiplicative bounded if there exist $x \in X$ and $M > 1$ such that $A \subseteq B_M(x)$.

**Lemma 1.12.** [8] Let (X, d) be a multiplicative metric space and $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a multiplicative Cauchy sequence if and only if $d(x_n, x_m) \to 1$ $(m, n \to \infty)$.

**Definition 1.13.** [5] Let S and T be self-maps of multiplicative metric space a non-empty set X. then

i. Any point $x \in X$ is said to be fixed point of T if Tx=x.
ii. Any point $x \in X$ is said to be coincidence point of T and S if Sx=Tx and we shall called w=Sx=Tx that a point of coincidence of S and T.
iii. Any point $x \in X$ is said to be fixed point of T and S if Sx=Tx=x

**Definition 1.14.** [14] Let S and T be self-maps of multiplicative metric space (X, d), then S,T are said to be weakly commuting if $d(STx, TSx) \leq d(Sx, Tx)$, for all $x \in X$

**Definition 1.15.** [5] Two self-maps of multiplicative metric space S, T of a non-empty set X are said to be commuting is TSx=STx for all $x \in X$.

**Definition 1.16.** [5] Let S, T be self-maps of multiplicative metric space (X,d), then S,T are said to be compatible if $\lim_{n\to\infty} d(STx_n, TSx_n)=0$, Whenever $\{x_n\}$ is a sequence in X such that$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$, for some $z \in X$

**Definition 1.17.** [5] Two self-maps of multiplicative metric space S, T of a non-empty set X are said to be weakly compatible is STx = TSx whenever Sx = Tx.

## 2. Main Results

**Theorem 2.1** let (X,d) be a complete multiplicative metric space and P,Q,R,S,T,U be self-maps of X satisfying the following condition

$$TU(X) \subseteq P(X) \ \textit{and} \ RS(X) \subseteq Q(X) \ \textit{and} \tag{2.1}$$

$$d(RSx, TUy) \leq \begin{pmatrix} d(Px, TUy).d(Px, Qy) \\ .d(Qy, RSx).d(TUy, Qy) \\ .d(Px, RSx).d(TUy, RSx) \\ .\dfrac{[d(RSx, Px) + d(Qy, TUy)]}{d(RSx, TUy) + d(Px, Qy)} \end{pmatrix}^{\frac{\lambda}{3}} \tag{2.2}$$

for all $x, y \in X, \lambda \in (0, \frac{1}{2})$ is a constant. Assume that the pairs (TU, Q), (RS, P) are weakly compatible. Pairs (T, U), (T, Q), (U, Q), (R, S), (R, P), (S, P) are commuting pairs of maps. Then P, Q, R, S, T, U have a unique common fixed point in X.

**Proof**. Let $x_0 \in X$ by (2.1) we can define inductively a sequence $\{y_n\}$ in X such that $y_{2n} = RSx_{2n} = Qx_{2n+1}$ and $TUx_{2n+1} = Px_{2n+2} = y_{2n+1}$ for all n=0,1, 2, 3 …

By (2.1), we have

$$d(y_{2n}, y_{2n+1}) = d(RSx_{2n}, TUx_{2n+1})$$

$$\leq \begin{pmatrix} d(Px_{2n}, TUx_{2n+1}).d(Px_{2n}, Qx_{2n+1}) \\ .d(Qx_{2n+1}, RSx_{2n}).d(TUx_{2n+1}, Qx_{2n+1}) \\ .d(Px_{2n}, RSx_{2n}).d(TUx_{2n+1}, RSx_{2n}) \\ .\dfrac{[d(RSx_{2n}, Px_{2n}) + d(Qx_{2n+1}, TUx_{2n+1})]}{d(RSx_{2n}, TUx_{2n+1}) + d(Px_{2n}, Qx_{2n+1})} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$\leq \begin{pmatrix} d(y_{2n-1}, y_{2n+1}).d(y_{2n-1}, y_{2n}) \\ .d(y_{2n}, y_{2n}).d(y_{2n+1}, y_{2n}) \\ .d(y_{2n-1}, y_{2n}).d(y_{2n+1}, y_{2n}) \\ .\dfrac{[d(y_{2n}, y_{2n-1}) + d(y_{2n}, y_{2n+1})]}{d(y_{2n}, y_{2n+1}) + d(y_{2n-1}, y_{2n})} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$d(y_{2n}, y_{2n+1}) \leq \begin{pmatrix} d(y_{2n-1}, y_{2n+1}).d(y_{2n-1}, y_{2n}).1 \\ .d(y_{2n+1}, y_{2n}).d(y_{2n-1}, y_{2n}) \\ .d(y_{2n+1}, y_{2n}).1 \end{pmatrix}^{\frac{\lambda}{3}}$$

$$d(y_{2n}, y_{2n+1}) \leq \left(d^3(y_{2n-1}, y_{2n}).d^3(y_{2n}, y_{2n+1})\right)^{\frac{\lambda}{3}}$$

(using (1.2) and (1.3))

$$d(y_{2n}, y_{2n+1}) \leq d^{\lambda}(y_{2n-1}, y_{2n}).d^{\lambda}(y_{2n}, y_{2n+1})$$

$$d(y_{2n}, y_{2n+1}) \leq d^{\frac{\lambda}{1-\lambda}}(y_{2n-1}, y_{2n})$$

$$d(y_{2n}, y_{2n+1}) \leq d^{h}(y_{2n-1}, y_{2n}), \tag{2.3}$$

where $h = \dfrac{\lambda}{1-\lambda}$.

Similarly, we have

$$d(y_{2n+1}, y_{2n+2}) = d(TUx_{2n+1}, RSx_{2n+2})$$
$$= d(RSx_{2n+2}, TUx_{2n+1})$$

$$\leq \begin{pmatrix} d(Px_{2n+2}, TUx_{2n+1}).d(Px_{2n+2}, Qx_{2n+1}) \\ .d(Qx_{2n+1}, RSx_{2n+2}).d(TUx_{2n+1}, Qx_{2n+1}) \\ .d(Px_{2n+2}, RSx_{2n+2}).d(TUx_{2n+1}, RSx_{2n+2}) \\ .\dfrac{[d(RSx_{2n+2}, Px_{2n+2}) + d(Qx_{2n+1}, TUx_{2n+1})]}{d(RSx_{2n+2}, TUx_{2n+1}) + d(Px_{2n+2}, Qx_{2n+1})} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$\leq \begin{pmatrix} d(y_{2n+1}, y_{2n+1}).d(y_{2n+1}, y_{2n}) \\ .d(y_{2n}, y_{2n+2}).d(y_{2n+1}, y_{2n}) \\ .d(y_{2n+1}, y_{2n+2}).d(y_{2n+1}, y_{2n+2}) \\ .\frac{[d(y_{2n+2}, y_{2n+1}) + d(y_{2n}, y_{2n+1})]}{d(y_{2n+2}, y_{2n+1}) + d(y_{2n+1}, y_{2n})} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$\leq \begin{pmatrix} 1.d(y_{2n+1}, y_{2n}).d(y_{2n}, y_{2n+2}) \\ .d(y_{2n+1}, y_{2n}).d(y_{2n+1}, y_{2n+2}) \\ .d(y_{2n+1}, y_{2n+2}).1 \end{pmatrix}^{\frac{\lambda}{3}}$$

$$\leq \begin{pmatrix} 1.d(y_{2n+1}, y_{2n}).d(y_{2n}, y_{2n+1}) \\ .d(y_{2n+1}, y_{2n+2}).d(y_{2n+1}, y_{2n}) \\ .d(y_{2n+1}, y_{2n+2}).d(y_{2n+1}, y_{2n+2}).1 \end{pmatrix}^{\frac{\lambda}{3}}$$

$$\begin{aligned} & d(y_{2n+1}, y_{2n+2}) \\ & \leq \left(d^3(y_{2n}, y_{2n+1}).d^3(y_{2n+1}, y_{2n+2})\right)^{\frac{\lambda}{3}} \\ & \leq \left(d(y_{2n}, y_{2n+1}).d(y_{2n+1}, y_{2n+2})\right)^{\lambda} \qquad (2.4) \\ & \leq d^{\frac{\lambda}{1-\lambda}}(y_{2n}, y_{2n+1}) \\ & \leq d^{h}(y_{2n}, y_{2n+1}) \leq d^{h^2}(y_{2n-1}, y_{2n}) \end{aligned}$$

where $h = \frac{\lambda}{1-\lambda} < 1$ as $\lambda \in (0, \frac{1}{2})$

Therefore, using (2.3) and (2.4), we have

$$\begin{aligned} & d(y_{n+1}, y_{n+2}) \leq d^{h}(y_n, y_{n-1}) \\ & \leq d^{h^2}(y_{n-1}, y_{n-2}) \leq \ldots \leq d^{h^{n+1}}(y_0, y_1) \end{aligned} \qquad (2.5)$$

for n=0,1, 2, 3…

Now, for all m > n

$$\begin{aligned} d(y_n, y_m) & \leq d(y_n, y_{n+1}).d(y_{n+1}, y_{n+2}) \\ & .d(y_{n+2}, y_{n+3})...d(y_{m-1}, y_m) \\ & \leq d^{(h^n + h^{n+1} + h^{n+2} + \ldots + h^{m-1})}(y_0, y_1) \\ & [\text{using } (2.5)] \\ & \leq d^{(\frac{h^n}{h-1})}(y_0, y_1) \end{aligned}$$

which implies that, $d(y_n, y_m) \to 1$ as $(n, m \to \infty)$. Hence $\{y_n\}$ is a Cauchy sequence, by the completeness of X, there exist $z \in X$ such that,

$$\begin{aligned} & \lim_{n\to\infty} RSx_{2n} = \lim_{n\to\infty} Qx_{2n+1} \\ & = \lim_{n\to\infty} TUx_{2n+1} = \lim_{n\to\infty} Px_{2n+2} = z. \end{aligned} \qquad (2.6)$$

Since, $TU(X) \subseteq P(X)$ there exist $u \in X$ such $z = Pu$, we claim that $RSu = z$, if possible $RSu \neq z$, if possible then by using (2.2), we have

$$d(RSu, z) = d(RSu, TUx_{2n-1}).d(TUx_{2n-1}, z)$$

$$\leq \begin{pmatrix} d(Pu, TUx_{2n-1}).d(Pu, Qx_{2n-1}) \\ .d(Qx_{2n-1}, RSu).d(TUx_{2n-1}, Qx_{2n-1}) \\ .d(Pu, RSu).d(TUx_{2n-1}, RSu) \\ .\frac{[d(RSu, Pu) + d(Qx_{2n-1}, TUx_{2n-1})]}{d(RSu, TUx_{2n-1}) + d(Pu, Qx_{2n-1})} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$.d(TUx_{2n-1}, z)$$

taking limit as $n \to \infty$, we have

$$d(RSu, z) \leq \begin{pmatrix} d(z,z).d(z,z).d(z, RSu) \\ .d(z,z).d(z, RSu).d(z, RSu) \\ .\frac{[d(RSu, z) + d(z,z)]}{d(RSu, z) + d(z,z)} \end{pmatrix}^{\frac{\lambda}{3}} .d(z,z)$$

$[\text{using } (2.6)]$

$$d(RSu, z) \leq (1.1.d(z, RSu).1.d(z, RSu).d(z, RSu).1)^{\frac{\lambda}{3}}$$

$$d(RSu, z) \leq \left(d^3(RSu, z)\right)^{\frac{\lambda}{3}},$$

$$d(RSu, z) \leq \left(d(RSu, z)\right)^{\lambda},$$

which is a contradiction. Therefore,

$$RSu = Pu = z.$$

Since, $RS(X) \subseteq Q(X)$ there exist $v \in X$ such that $z = Qv$.

We claim that $TUv = z$, if possible $TUv \neq z$, then by using (2.2), we have

$$d(z, TUv) = d(RSu, TUv)$$

$$\leq \begin{pmatrix} d(Pu, TUv).d(Pu, Qv).d(Qv, RSu) \\ .d(TUv, Qv).d(Pu, RSu) \\ .d(TUv, RSu).\frac{[d(RSu, Pu) + d(Qv, TUv)]}{d(RSu, TUv) + d(Pu, Qv)} \end{pmatrix}^{\frac{\lambda}{3}}$$

we have,

$$d(z, TUv) \leq \begin{pmatrix} d(z, TUv).d(z,z).d(z,z) \\ .d(TUv, z).d(z,z).d(TUv, z) \\ .\frac{[d(z,z) + d(z, TUv)]}{d(z, TUv) + d(z,z)} \end{pmatrix}^{\frac{\lambda}{3}}$$

$[\text{using } (2.7)]$

$$\begin{aligned} & \leq (d(z, TUv).1.1.d(TUv, z).1.d(TUv, z).1)^{\frac{\lambda}{3}} \\ & \leq \left(d^3(z, TUv)\right)^{\frac{\lambda}{3}} \end{aligned} \qquad (2.8)$$

which is a contradiction.
Therefore, TUv=Qv=z.

Here, Q and TU are weakly compatibles, we have TUz=Qz.

Now we claim that z is a fixed point of TU. If TUz≠z, then by (2.2), we have

$$d(z,TUz) = d(RSu,TUz)$$

$$\le \begin{pmatrix} d(Pu,TUz).d(Pu,Qz).d(Qz,RSu) \\ .d(TUz,Qz).d(Pu,RSu). \\ d(TUz,RSu).\dfrac{[d(RSu,Pu)+d(Qz,TUz)]}{d(RSu,TUz)+d(Pu,Qz)} \end{pmatrix}^{\frac{\lambda}{3}}$$

therefore,

$$d(z,TUz) \le \begin{pmatrix} d(z,TUz).d(z,TUz).d(TUz,z) \\ .d(TUz,TUz).d(z,z) \\ .d(TUz,z).\dfrac{[d(z,z)+d(TUz,TUz)]}{d(z,TUz)+d(z,TUz)} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$[\text{using } (2.7)]$$

$$\le \begin{pmatrix} d(z,TUz).d(z,TUz).d(TUz,z) \\ .1.1.d(TUz,z).\dfrac{1}{d(z,TUz)} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$\le \left(d^3(z,TUv)\right)^{\frac{\lambda}{3}},$$

which is a contradiction.

Therefore, TUz=z, hence Qz=z. so we have

$$TUz = Qz = z. \qquad (2.9)$$

So, z is a common fixed point of TU and Q.

Similarly, P and RS are weakly compatible maps, we have RSz=Pz.

Now we claim that z is a fixed point of RS. If RSz≠z, then by (2.2), we have

$$d(RSz,z) = d(RSz,TUz)$$

$$\le \begin{pmatrix} d(Pz,TUz).d(Pz,Qz).d(Qz,RSz) \\ .d(TUz,Qz).d(Pz,RSz).d(TUz,RSz) \\ .\dfrac{[d(RSz,Pz)+d(Qz,TUz)]}{d(RSz,TUz)+d(Pz,Qz)} \end{pmatrix}^{\frac{\lambda}{3}}$$

we have,

$$d(RSz,z) \le \begin{pmatrix} d(RSz,z).d(RSz,z).d(z,RSz) \\ .d(z,z).d(RSz,RSz).d(z,RSz) \\ .\dfrac{[d(RSz,RSz)+d(z,z)]}{d(RSz,z)+d(RSz,z)} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$[\text{using } (2.9)]$$

$$d(RSz,z) \le \begin{pmatrix} d(RSz,z).d(RSz,z).d(z,RSz) \\ .1.1.d(z,RSz).\dfrac{1}{d(RSz,z)} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$\le \left(d^3(z,RSz)\right)^{\frac{\lambda}{3}},$$

which is a contradiction.

Therefore, RSz = z, hencePz = z. so, we have

$$RSz = Pz = Qz = TUz = z. \qquad (2.10)$$

So z is a common fixed point of TU, Q, P and RS.

By commuting condition of pairs

$$Tz = T(TUz) = T(UTz) = TU(Tz)$$
$$Tz = T(Pz) = P(Tz)$$
$$\text{and } Uz = U(TUz) = (UT)(Uz) = (TU)(Uz)$$
$$Uz = U(Pz) = P(Uz)$$

which follows that, Tz and Uz are common fixed points of TU and P, then

$$\begin{aligned} &Rz = R(RSz) = R(SRz) = RS(Rz) \\ &Rz = R(Qz) = Q(Rz) \\ &\text{and } Sz = S(RSz) = (SR)(Sz) = (RS)(Sz) \\ &(Sz) = S(Qz) = Q(Sz) \end{aligned} \qquad (2.11)$$

Similarly, by commuting property,

$$Rz = R(RSz) = R(SRz) = RS(Rz)$$
$$Rz = R(Qz) = Q(Rz)$$
$$\text{and } Sz = S(RSz) = (SR)(Sz) = (RS)(Sz)$$
$$(Sz) = S(Qz) = Q(Sz)$$

which follows that, Rz and Sz are common fixed points of RS and Q, then

Then Rz = z = Sz = Qz = RSz.

Therefore z is a common fixed point of T, U, R, S, P and Q.

## 2.1. Uniqueness

Let w be other common fixed point of T, U, R, P, S and Q. if possible w≠ z, we have

$$d(z,w) = d(RSz,TUw)$$

$$\le \begin{pmatrix} d(Pz,TUw).d(Pz,Qw).d(Qw,RSz) \\ .d(TUw,Qw).d(Pz,RSz).d(TUw,RSz) \\ .\dfrac{[d(RSz,Pz)+d(Qw,TUw)]}{d(RSz,TUw)+d(Pz,Qw)} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$= \begin{pmatrix} d(z,w).d(z,w).d(w,z).d(w,w) \\ .d(z,z).d(w,z).\dfrac{[d(z,z)+d(w,w)]}{d(z,w)+d(w,z)} \end{pmatrix}^{\frac{\lambda}{3}}$$

$$\left[\text{using } (2.10)\right]$$

$$= \left( d(z,w).d(z,w).d(z,w).1.1.d(w,z).\frac{1}{d(z,w)} \right)^{\frac{\lambda}{3}}$$

$$\le \left( d^3(z,w) \right)^{\frac{\lambda}{3}},$$

a contradiction.

So z = w.

In Theorem 2.1, if we put S = U= 1, then we obtain the following corollary.

**Corollary 2.2.** Let (X, d) be a complete multiplicative metric space and P,Q,R ,T be self-maps of X satisfying the following condition

$$T(X) \subseteq P(X) \textit{ and } R(X) \subseteq Q(X) \qquad (2.13)$$

$$d(Rx,Ty) \le \left( \begin{array}{l} d(Px,Ty).d(Px,Qy).d(Qy,Rx) \\ .d(Ty,Qy).d(Px,Rx).d(Ty,Rx) \\ .\dfrac{[d(Rx,Px)+d(Qy,Ty)]}{d(Rx,Ty)+d(Px,Qy)} \end{array} \right)^{\frac{\lambda}{3}} \qquad (2.14)$$

for all x, y ∈ X, $\lambda \in (0,\frac{1}{2})$ is a constant. Assume that the pairs (T, Q), (R, P) are weakly compatible. Pairs (T,Q), (U,Q), (R,P), (S,P) are commuting pairs of maps. Then P, Q, R, T have a unique common fixed point in X.

# References


[1] A. Azam, B. Fisher and M. Khan: Common fixed point theorems in Complex valued metric spaces. Numerical Functional Analysis and Optimization. 32(3): 243-253(2011).

[2] A. E. Bashirov, E. M. Kurplnara and A. Ozyapici, Multiplicative calculus and its applications, J. Math. Anal. Appl., 337 (2008).

[3] Al Pervo: On the Cauchy problem for a system of ordinary differential equations. Pvi-blizhen met Reshen Diff Uvavn. Vol. 2, pp. 115-134, 1964.

[4] C. Semple, M. Steel: Phylogenetics, Oxford Lecture Ser. In Math Appl, vol. 24, Oxford Univ. Press, Oxford, 2003.

[5] Dr. Yogita R. Sharma,Common Fixed Point Theorem in Complex Valued Metric Spaces ISSN: 2319-8753 International Journal of Innovative Research in Science, Engineering and Technology (An ISO 3297: 2007 Certified Organization) Vol. 2, Issue 12, December 2013

[6] G. Junck,Commuting maps and fixed points. Am Math Monthly. vol. 83, pp. 261-263,1976.

[7] L.G. Huang, X. Zhang: Cone metric spaces and fixed point theorem for contractive mappings. J Math Anal Appl. Vol. 332, pp. 1468-1476, 2007.

[8] MuttalipÖzavsar and adem C. ceviket, fixed points of multiplicative contraction mapping on multiplivate metric spaces arXiv:1205.5131v1 [math.GM]

[9] R.P.Agarwal, M. A. El-Gebeily and D. O'Regan, Generalized contractions in partially ordered metric spaces, Applicable Analysis. 87(2008), 109-116

[10] R. H. Haghi, Sh. Rezapour and N. Shahzadb; Some fixed point generalizations are not real generalization. Nonlinear Anal.Vol. 74, pp. 1799- 1803, 2011.

[11] R. Tiwari, D. P. Shukla: Six maps with a common fixed point in complex valued metric spaces. Research J of Pure Algebra. Vol. 2issue 12 pp.365-369, 2012. ISSN: 2319-8753 International Journal of Innovative Research in Science, Engineering and Technology (An ISO 3297: 2007 Certified Organization) Vol. 2, Issue 12, December 2013.

[12] S. A. Mohiuddine, M. Cancan and H. Sevli, Intuitionistic fuzzy stability of a Jensen functional equation via fixed point technique, Math. Comput.Model. 54 (2011), 2403-2409.

[13] S. Sessa, On a weak commutativity condition of mappings in fixed point consideration. PublInst Math, 32(46): 149-153(1982).

[14] S.B.Nadler, Multivalued nonlinear contraction mappings, Pacific J.Math. 30(1969) 475-488.

[15] Soon-Mo Jung, A Fixed Point Approach to the Stability of Differential Equations y ′ = F(x, y), Bull of the Malys. Math.Sci. Soc. (33) (2010), 47-56.

[16] T.Suzuki, Subrahmanyam's fixed point theorem, Nonlinear Analysis, 71(2009) 1678-1683

[17] W. Chistyakov, Modular metric spaces, I: basic concepts. Nonlinear Anal. Vol. 72, pp. 1-14, 2010.

[18] W.Takahashi, Nonlinear Functional Analysis:Fixed point theory and its applications, Yokohama Publishers, 2000.

[19] Y. Kimura and W. Takahashi, Weak convergence to common fixed points of countable nonexpansive mappings and its applications, Journal of the Korean Mathematical Society 38 (2001), 1275-1284.