Journal of Mathematical Sciences and Applications, 2013, Vol. 1, No. 1, 1-5
Available online at http://pubs.sciepub.com/jmsa/1/1/1

DOI:10.12691/jmsa-1-1-1

Existence and Uniqueness Theorem for Fuzzy Integral Equation

Andrej V. Plotnikov[1,2,*], Natalia V. Skripnik[2]

[1]Department of Applied Mathematics, Odessa State Academy Civil Engineering and Architecture, Odessa, Ukraine
[2]Department of Optimal Control and Economic Cybernetics, Odessa National University named after I.I. Mechnikov, Odessa, Ukraine
*Corresponding a-plotnikov@ukr.net

Received December 20, 2012; Revised January 29, 2013; Accepted March 02, 2013

Abstract In this article we consider fuzzy integral equations and prove the existence and uniqueness theorem.

Keywords: *fuzzy integral equation, existence, uniqueness, fuzzy differential equation*

1. Introduction

In recent years, the fuzzy set theory introduced by Zadeh [1] has emerged as an interesting and fascinating branch of pure and applied sciences. The applications of fuzzy set theory can be found in many branches of science as physical, mathematical, differential equations and engineering sciences. Recently there have been new advances in the theory of fuzzy differential equations [2-7], fuzzy integral equations [8-16], fuzzy integrodifferential equations [17,18,19,20], differential inclusions with fuzzy right-hand side [21-24] and fuzzy differential inclusions [25,26,27] as well as in the theory of control fuzzy differential equations [28,29,30], control fuzzy integrodifferential equations [31-36], control fuzzy differential inclusions [37,38,39,40], and control fuzzy integrodifferential inclusions [41].

Almost in all papers mentioned above the authors also considered equivalent fuzzy integral equations. However, integral equations are encountered in various fields of science and in numerous applications, including elasticity, plasticity, heat and mass transfer, oscillation theory, fluid dynamics, filtration theory, electrostatics, electrodynamics, biomechanics, game theory, control, queuing theory, electrical engineering, economics, and medicine. Therefore, in this article we consider fuzzy integral equations and prove the existence and uniqueness theorem.

2. Preliminaries

Let $comp(R^n)$ $\left(conv(R^n)\right)$ be a set of all nonempty (convex) compact subsets from the space R^n,

$$h(A,B)=\min_{r\ge 0}\{S_r(A)\supset B, S_r(B)\supset A\}$$

be Hausdorff distance between sets A and B, $S_r(A)$ is r-neighborhood of set A.

Let E^n be the set of all $u:R^n\to[0,1]$ such that u satisfies the following conditions:

i) u is normal, that is, there exists an $x_0\in R^n$ such that $u(x_0)=1$;

ii) u is fuzzy convex, that is,

$$u(\lambda x+(1-\lambda)y)\ge\min\{u(x),u(y)\}$$

for any $x,y\in R^n$ and $0\le\lambda\le 1$;

iii) u is upper semicontinuous,

iv) $[u]^0=cl\{x\in R^n: u(x)>0\}$ is compact.

If $u\in E^n$, then u is called a fuzzy number, and E^n is said to be a fuzzy number space. For $0<\alpha\le 1$, denote

$$[u]^\alpha=\{x\in R^n: u(x)\ge\alpha\}.$$

Then from i)-iv), it follows that the α-level set $[u]^\alpha\in conv(R^n)$ for all $0\le\alpha\le 1$.

Let $\hat{\theta}$ be the fuzzy mapping defined by $\hat{\theta}(x)=0$ if $x\ne 0$ and $\hat{\theta}(0)=1$.

Define $D:E^n\times E^n\to[0,\infty)$ by the relation

$$D(u,v)=\sup_{0\le\alpha\le 1} h\left([u]^\alpha,[v]^\alpha\right).$$

Then D is a metric in E^n. Further we know that [42]:

1) $\left(E^n,D\right)$ is a complete metric space,

2) $D(u+w,v+w)=D(u,v)$ for all $u,v,w\in E^n$,

3) $D(\lambda u,\lambda v)=|\lambda|D(u,v)$ for all $u,v\in E^n$ and $\lambda\in R$.

Definition 1. [5] A mapping $F:[0,T]\to E^n$ is measurable (continuous) if for all $\alpha\in[0,1]$ the set-valued map $F_\alpha:[0,T]\to conv\left(R^n\right)$ defined by $F_\alpha(t)=[F(t)]^\alpha$ is Lebesgue measurable (continuous).

Definition 2. [5] A mapping $F:[0,T]\to E^n$ is said to be integrably bounded if there is an integrable function $h(t)$ such that $\|x(t)\|\le h(t)$ for every $x(t)\in F_0(t)$.

Definition 3. [5] The integral of a fuzzy mapping $F:[0,T]\to E^n$ is defined levelwise by $\left[\int_0^T F(t)dt\right]^\alpha = \int_0^T F_\alpha(t)dt = \{\int_0^T f(t)dt : f:[0,T]\to R^n$ is a measurable selection of $F_\alpha:[0,T]\to conv(R^n)\}$ for all $\alpha\in[0,1]$.

Definition 4. [5] A measurable and integrably bounded mapping $F:[0,T]\to E^n$ is said to be integrable over $[0,T]$ if $\int_0^T F(t)dt\in E^n$.

Note that if $F:[0,T]\to E^n$ is measurable and integrably bounded, then F is integrable. Further if $F:[0,T]\to E^n$ is continuous, then it is integrable.

Proposition 1. [2] Let $F,G:[0,T]\to E^n$ be integrable and $\lambda\in R$. Then

1) $\int_0^T F(t)+G(t)dt=\int_0^T F(t)dt+\int_0^T G(t)dt$;

2) $\int_0^T \lambda F(t)dt=\lambda\int_0^T F(t)dt$;

3) $D(F(t),G(t))$ is integrable;

4) $D\left(\int_0^T F(t)dt,\int_0^T G(t)dt\right)\le\int_0^T D(F(t),G(t))dt$.

3. Main Result

Consider the fuzzy integral equation

$$X(t)=A(t)\left[X_0+\int_0^t F\left(s,A^{-1}(s)X(s)\right)ds\right], \qquad (1)$$

where $t\in[0,d]\subset R_+$ is time, $X\in E^n$ is a phase variable, $A(t)$ is $n\times n$ -dimensional matrix-valued function, $F:R_+\times E^n\to E^n$ is a fuzzy mapping, $X_0\in E^n$.

Definition 5. A fuzzy mapping $X:[0,d]\to E^n$ is called a solution of integral equation (1) if it is continuous and satisfies integral equation (1) on interval $[0,d]$.

Theorem. Let in the domain $Q=\{(t,X)\in[0,d]\times E^n\}$ the following conditions hold:

i) for any fixed X the fuzzy mapping $F(\cdot,X)$ is continuous;

ii) there exists a positive constant L such that

$$D(F(t,X'),F(t,X''))\le LD(X',X'')$$

for all $(t,X'),(t,X'')\in Q$;

iii) there exists a positive constant K such that

$$D(F(t,X),\hat{\theta})\le K(1+D(X,\hat{\theta}))$$

for all $(t,X)\in Q$;

iv) the matrix-valued functions $A(t)$, $A^{-1}(t)$ are continuous;

v) there exist positive constants a_1,a_2 such that

$$\|A(t)\|\le a_1,\quad \|A^{-1}(t)\|\le a_2$$

for all $t\in[0,d]$.

Then equation (1) has a unique solution on the interval $[0,d]$.

Proof. Let us build the successive approximations of the solution:

$$X^0(t)=A(t)X_0 \text{ for } 0\le t\le d,$$

$$X^k(t)=A(t)\left[X_0+\int_0^t F(s,A^{-1}(s)X^{k-1}(s))ds\right]$$

for $0\le t\le d$.

By conditions i), ii) and iv) of the theorem $X^k(t)$ is continuous on $[0,d]$ for all $k\in N$. Besides

$$D(X^0(t),X_0)=D(A(t)X_0,X_0)\le$$

$$\le D(A(t)X_0,\hat{\theta})+D(X_0,\hat{\theta})\le(a_1+1)D(X_0,\hat{\theta});$$

$$D(X^1(t),X^0(t))=$$

$$=D(A(t)[X_0+\int_0^t F(s,A^{-1}(s)X^0(s))ds],A(t)X_0)=$$

$$=D(A(t)[X_0+\int_0^t F(s,A^{-1}(s)A(s)X_0)ds],A(t)X_0)=$$

$$=D(A(t)[X_0+\int_0^t F(s,X_0)ds],A(t)X_0)\le$$

$$\le a_1D([X_0+\int_0^t F(s,X_0)ds],X_0)\le$$

$$\le a_1D(\int_0^t F(s,X_0)ds,\hat{\theta})\le a_1\int_0^t D(F(s,X_0),\hat{\theta})ds\le$$

$$\le a_1\int_0^t K(1+D(X_0,\hat{\theta}))ds\le a_1K(1+D(X_0,\hat{\theta}))t;$$

$$D(X^2(t),X^1(t))=$$

$$=D(A(t)[X_0+\int_0^t F(s,A^{-1}(s)X^1(s))ds],$$

$$A(t)[X_0+\int_0^t F(s,A^{-1}(s)X^0(s))ds])\le$$

$$\le a_1 D(\int_0^t F(s,A^{-1}(s)X^1(s))ds, \int_0^t F(s,A^{-1}(s)X^0(s))ds)\le$$

$$\le a_1\int_0^t D(F(s,A^{-1}(s)X^1(s)),F(s,A^{-1}(s)X^0(s)))ds\le$$

$$\le a_1\int_0^t LD(A^{-1}(s)X^1(s),A^{-1}(s)X^0(s))ds\le$$

$$\le a_1\int_0^t La_2 D(X^1(s),X^0(s))ds\le$$

$$\le a_1a_2L\int_0^t a_1K(1+D(X_0,\hat{\theta})s\,ds=$$

$$=a_1^2a_2LK(1+D(X_0,\hat{\theta})\frac{t^2}{2!};$$

$$D(X^3(t),X^2(t))=$$

$$=D\left(A(t)[X_0+\int_0^t F(s,A^{-1}(s)X^2(s))ds],\right.$$

$$\left.A(t)[X_0+\int_0^t F(s,A^{-1}(s)X^1(s))ds]\right)\le$$

$$\le a_1\int_0^t La_2 D(X^2(s),X^1(s))ds\le$$

$$\le a_1a_2L\int_0^t a_1^2a_2K(1+D(X_0,\hat{\theta})\frac{s^2}{2!}ds=$$

$$=a_1^3a_2^2LK(1+D(X_0,\hat{\theta})\frac{t^3}{3!}$$

and so on.

Therefore,

$$D(X^{n+1}(t),X^n(t))\le$$

$$\le a_1^{n+1}a_2^nL^nK(1+D(X_0,\hat{\theta}))\frac{t^{n+1}}{(n+1)!}.$$

Then

$$\max_{t\in[0,d]} D(X^{n+1}(t),X_0)\le$$

$$\le \max_{t\in[0,d]} D(X^{n+1}(t),X^n(t))+...+\max_{t\in[0,d]} D(X^2(t),X^1(t))+$$

$$+\max_{t\in[0,d]} D(X^1(t),X^0(t))+\max_{t\in[0,d]} D(X^0(t),X_0)\le$$

$$\le\frac{K(1+D(X_0,\hat{\theta}))}{a_2L}\sum_{i=1}^{n+1}\frac{(a_1a_2Ld)^i}{i!}+(a_1+1)D(X_0,\hat{\theta})\le$$

$$\le\frac{K(1+D(X_0,\hat{\theta}))}{a_2L}e^{a_1a_2Ld}+(a_1+1)D(X_0,\hat{\theta})=b.$$

Hence, it follows that the sequence of the fuzzy mappings $\{X^k(t)\}_{k=0}^{\infty}$ in uniformly bounded:

$$D(X^k(t),X_0)\le b$$

for all $t\in[0,d]$.

Let us show that the sequence of the fuzzy mappings $\{X^k(t)\}_{k=0}^{\infty}$ is a Cauchy sequence. For any $m,p\in N$ we have

$$D(X^{m+p}(t),X^p(t))\le$$

$$\le a_1D\left(\int_0^t F(s,A^{-1}(s)X^{m+p-1}(s))ds,\right.$$

$$\left.\int_0^t F(s,A^{-1}(s)X^{p-1}(s))ds\right)\le$$

$$\le a_1La_2\int_0^t D(X^{m+p-1}(s),X^{p-1}(s))ds.$$

Hence,

$$D(X^{m+p}(t),X^p(t))\le$$

$$\le(a_1a_2L)^p\overbrace{\int_0^t...\int_0^{t_{p-1}}}^{p}D(X^m(t_p),X_0)dt_p...dt_1\le$$

$$\le\frac{b(a_1a_2L)^pt^p}{p!}\le\frac{b(a_1a_2L)^pd^p}{p!}$$

Therefore, the sequence $\{X^k(t)\}_{k=0}^{\infty}$ is a Cauchy sequence. Its limit is a continuous fuzzy mapping that we will denote by $X(t)$. Owing to the theorem conditions in (1) it is possible to pass to the limit under the sign of the integral. We receive that the fuzzy mapping $X(t)$ satisfies equation (1), i.e. $X(t)$ is the solution of (1) on the interval $[0,d]$.

To prove the uniqueness, suppose that there exist at least two different solutions $X(t)$ and $Y(t)$ of (1) on $[0,d]$. Then $\rho=\max_{t\in[0,d]} D(X(t),Y(t))>0$.

As

$$X(t)=A(t)\left[X_0+\int_0^t F(s,A^{-1}(s)X(s))ds\right],$$

$$Y(t)=A(t)\left[X_0+\int_0^t F(s,A^{-1}(s)Y(s))ds\right]$$

then

$$D(X(t),Y(t)) \le a_1a_2L\int_0^t D(X(s),Y(s))ds .$$

So

$$D(X(t),Y(t)) \le a_1a_2L\int_0^t D(X(s),Y(s))ds \le$$

$$\le a_1a_2L\int_0^t \rho ds \le a_1a_2L\rho t \le a_1a_2L\rho d,$$

$$D(X(t),Y(t)) \le a_1a_2L\int_0^t La_1a_2\rho s ds =$$

$$= (a_1a_2L)^2\rho\frac{t^2}{2!} \le (a_1a_2L)^2\rho\frac{d^2}{2!},$$

$$\vdots$$

$$D(X(t),Y(t)) \le (a_1a_2L)^m\rho\frac{d^m}{m!}.$$

Then $\rho = \max_{t\in[0,d]} D(X(t),Y(t)) \le (a_1a_2L)^m\rho\frac{d^m}{m!}$ for any $m \in N$ that contradicts $\lim_{m\to\infty}\frac{(a_1a_2Ld)^m}{m!} = 0$.

This concludes the proof.

Remark 1. If $A(t) \equiv I$ then fuzzy integral equation (1) is equivalent to the Cauchy problem

$$D_HX = F(t,X), \quad X(0) = X_0,$$

where D_HX is the fuzzy Hukuhara derivative of a fuzzy mapping $X: R_+ \to E^n$ [2].

Remark 2. Solutions of integral equation (1) can be not fuzzy differentiable in the sense of Hukuhara. For example, if

$$A(t) = \begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}, \; F(t, A^{-1}(t)X) = A^{-1}(t)X,$$

$X_0 = K_1 \in E^2$, where K_μ such, that for all $\alpha \in [0,1]$ $[K_\mu]^\alpha = \{x \in R^2 : |x_i| \le (1-\alpha)\mu, i = 1,2\}$, then we get the fuzzy integral equation

$$X(t) = A(t)\left[X_0 + \int_0^t A^{-1}(s)X(s)ds\right]$$

It is obvious that its solution is $X(t) = A(t)K_{e^t}$ and $X(t)$ is not fuzzy differentiable in the sense of Hukuhara for all $t \ge 0$. However fuzzy integral system (1) will be equivalent to the following fuzzy hybrid system

$$D_HY = Y, \; Y(0) = K_1, \; X(t) = A(t)Y(t)$$

4. Conclusion

In 1982, D. Dubois and H. Prade [43,44] first introduced the concept of integration of fuzzy functions. O. Kaleva [2] studied the measurability and integrability for the fuzzy set-valued mappings of a real variable whose values are normal, convex, upper semicontinuous, and compactly supported by fuzzy sets in R^n. Existence of solutions of fuzzy integral equations has been studied by several authors. They have used the embedding theorem of Kaleva, which is a generalization of the classical Rådström embedding theorem, and the Darbo fixed point theorem in the convex cone. In this article we prove the existence and uniqueness theorem without using the embedding theorem of Kaleva.

References

[1] Zadeh, L.A., "Fuzzy sets," *Inf. Control*, (8), 338-353, 1965.

[2] Kaleva, O., "Fuzzy differential equations," *Fuzzy Sets Syst.*, 24 (3), 301-317, 1987.

[3] Lakshmikantham, V., Gnana Bhaskar, T. and Vasundhara, Devi J. *Theory of set differential equations in metric spaces*, Cambridge Scientific Publishers, Cambridge, 2006.

[4] Lakshmikantham, V. and Mohapatra, R. *Theory of fuzzy differential equations and inclusions*, Taylor - Francis, 2003.

[5] Park, J.Y., and Han, H.K., "Existence and uniqueness theorem for a solution of fuzzy differential equations," *Int. J. Math. Math. Sci.*, 22 (2), 271-279, 1999.

[6] Park, J.Y., and Han, H.K., "Fuzzy differential equations," *Fuzzy Sets Syst.*, 110 (1), 69-77, 2000.

[7] Plotnikov, A.V. and Skripnik, N.V., *Differential equations with "clear" and fuzzy multivalued right-hand sides. Asymptotics Methods (in Russian)*, AstroPrint, Odessa, 2009.

[8] Sadigh Behzadi, Sh., "Solving Fuzzy Nonlinear Volterra-Fredholm Integral Equations by Using Homotopy Analysis and Adomian Decomposition Methods," *Journal of Fuzzy Set Valued Analysis*, Volume 2011 Article ID jfsva-00067, 13 Pages, 2011.

[9] Jahantigh, M., Allahviranloo, T. and Otadi, M., "Numerical Solution of Fuzzy Integral Equations," *Applied Mathematical Sciences*, 2 (1), 33-46, 2008.

[10] Friedman, M., Ma, M. and Kandel, A., "Numerical Solutions of fuzzy differential equations and integral equations," *Fuzzy Sets and Systems*, 106, 35-48, 1999.

[11] Ghanbari, M., Toushmalni, R. and Kamrani, E., "Numerical Solution of Linear Fredholm Fuzzy Integral Equation of the Second Kind by Block-pulse Functions," *Australian Journal of Basic and Applied Sciences*, 3 (3), 2637-2642, 2009.

[12] Mordeson J. and Newman, W., "Fuzzy Integral Equations," *Information Sciences*, 87 (4), 215-229, 1995.

[13] Park, J.Y., Kwun, Y.C. and Jeong, J.U., "Existence of solutions of fuzzy integral equations in Banach spaces," *Fuzzy Sets and Systems*, 72, 373-378, 1995.

[14] Parandin, N. and Fariborzi Araghi, M. A., "The Approximate Solution of Linear Fuzzy Fredholm Integral Equations of the Second Kind by Using Iterative Interpolation," *World Academy of Science, Engineering and Technology*, 25, 978-984, 2009.

[15] Shamivand, M.M., Shahsavaran, A. and Tari, S.M., "Solution to Fredholm Fuzzy Integral Equations with Degenerate Kernel," *Int. J. Contemp. Math. Sciences*, 6 (11), 535-543, 2011.

[16] Wu, C. and Ma, M., "On the integrals,series and integral equations of fuzzy set-valued functions," *J. Harbin Inst. Technol.*, 21, 11-19, 1990.

[17] Allahviranloo, T., Amirteimoori, A., Khezerloo, M., and Khezerloo, S., "A new method for solving fuzzy volterra integro-differential equations," *Australian Journal of Basic and Applied Sciences*, 5 (4), 154-164, 2011.

[18] Balachandran, K., and Kanagarajan, K., "Existence of solutions of fuzzy delay integrodifferential equations with nonlocal condition," *Journal of Korea Society for Industrial and Applied Mathematics*, 9 (2), 65-74, 2005.

[19] Balasubramaniam, P., and Muralisankar, S., "Existence and uniqueness of fuzzy solution for the nonlinear fuzzy integrodifferential equations," *Appl. Math. Lett.*, 14 (4), 455-462, 2001.

[20] Balasubramaniam, P., and Muralisankar, S., "Existence and uniqueness of fuzzy solution for semilinear fuzzy integrodifferential equations with nonlocal conditions," *Comput. Math. Appl.*, 47, 1115-1122, 2004.

[21] Aubin, J.-P., "Fuzzy differential inclusions," *Probl. Control Inf. Theory*, 19 (1), 55-67, 1990.

[22] Baidosov, V. A., "Differential inclusions with fuzzy right-hand side," *Sov. Math.*, 40 (3), 567-569, 1990.

[23] Baidosov, V. A., "Fuzzy differential inclusions," *J. Appl. Math. Mech.*, 54 (1), 8-13, 1990.

[24] Hullermeier, E., "An approach to modeling and simulation of uncertain dynamical system," *Int. J. Uncertain. Fuzziness Knowl.-Based Syst.*, 7, 117-137, 1997.

[25] Plotnikov, A. V., and Skripnik, N. V., 2009, The generalized solutions of the fuzzy differential inclusions, Int. J. Pure Appl. Math., 56(2), 165-172.

[26] Skripnik, N.V., "The full averaging of fuzzy differential inclusions," *Iranian Journal of Optimization*, 1, 302-317, 2009.

[27] Skripnik, N.V., "The partial averaging of fuzzy differential inclusions," *J. Adv. Res. Differ. Equ.*, 3 (1), 52-66, 2011.

[28] Skripnik, N.V., "The partial averaging of fuzzy impulsive differential inclusions," *Differential and Integral Equations*, 24 (7-8), 743-758, 2011.

[29] Kwun, Y.C., and Park, D.G., "Optimal control problem for fuzzy differential equations," *Proceedings of the Korea-Vietnam Joint Seminar*, 103-114, 1998.

[30] Phu, N.D., and Tung, T.T., "Some results on sheaf-solutions of sheaf set control problems," *Nonlinear Anal.*, 67(5), 1309-1315, 2007.

[31] Plotnikov, A.V., Komleva, T.A., and Arsiry, A.V., "Necessary and sufficient optimality conditions for a control fuzzy linear problem," *Int. J. Industrial Mathematics*, 1 (3), 197-207, 2009.

[32] Kwun, Y.C., Kim, M.J., Lee, B.Y., and Park, J.H., "Existence of solutions for the semilinear fuzzy integrodifferential equations using by successive iteration," *Journal of Korean Institute of Intelligent Systems*, 18, 543-548, 2008.

[33] Kwun, Y.C., Kim, J.S., Park, M.J., and Park, J.H., "Nonlocal controllability for the semilinear fuzzy integrodifferential equations in n-dimensional fuzzy vector space," *Adv. Difference Equ.*, vol. 2009, Article ID 734090, 16 pages, 2009.

[34] Kwun, Y.C., Kim, J.S., Park, M.J., and Park, J.H., "Controllability for the impulsive semilinear nonlocal fuzzy integrodifferential equations in n-dimensional fuzzy vector space," *Adv. Difference Equ.*, vol. 2010, Article ID 983483, 22 pages, 2010.

[35] Park, J.H., Park, J.S., and Kwun, Y.C., "Controllability for the semilinear fuzzy integrodifferential equations with nonlocal conditions," *Fuzzy Systems and Knowledge Discovery, Lecture Notes in Computer Science*, vol. 4223/2006, 221-230, 2006.

[36] Park, J.H., Park, J.S., Ahn, Y.C., and Kwun, Y.C., "Controllability for the impulsive semilinear fuzzy integrodifferential equations," *Adv. Soft Comput.*, 40, 704-713, 2007.

[37] Molchanyuk, I.V., and Plotnikov, A.V., "Linear control systems with a fuzzy parameter," *Nonlinear Oscil.*, 9 (1), 59-64, 2006.

[38] Molchanyuk, I.V., and Plotnikov, A.V., "Necessary and sufficient conditions of optimality in the problems of control with fuzzy parameters," *Ukr. Math. J.*, 61 (3), 457-463, 2009.

[39] Plotnikov, A.V., and Komleva, T.A., "Linear problems of optimal control of fuzzy maps," *Intelligent Information Management*, 1 (3), 139-144, 2009.

[40] Plotnikov, A.V., Komleva, T.A., and Molchanyuk, I.V., "Linear control problems of the fuzzy maps," *J. Software Engineering & Applications*, 3 (3), 191-197, 2010.

[41] Vasil'kovskaya, V.S., and Plotnikov, A.V., "Integrodifferential systems with fuzzy noise," *Ukr. Math. J.*, 59(10), 1482-1492, 2007.

[42] Puri, M.L., and Ralescu, D.A., "Fuzzy random variables," *J. Math. Anal. Appl.*, 114(2), 409-422, 1986.

[43] Dubois, D. and Prade, H., "Towards fuzzy differential calculus. I. Integration of fuzzy mappings," *Fuzzy Sets and Systems*, 8 (1), 1-17, 1982.

[44] Dubois, D. and Prade, H., "Towards fuzzy differential calculus. II. Integration on fuzzy intervals," *Fuzzy Sets and Systems*, 8 (2), 105-116, 1982.