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Conceptual Understanding vs. Procedural Fluency: A Literature Review on the Mathematics Teachers Emphasis in Teaching of Rational Algebraic Expressions

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Abstract This paper presents a review of published research articles related to teachers' conceptual understanding and procedural fluency in teaching rational algebraic expressions. The study reviewed practices in mathematics teachers emphasis in teaching rational algebraic expressions through content analysis. Findings revealed that mathematics teachers place a greater emphasis on conceptual understanding in teaching rational algebraic expressions than on developing procedural skills. Giving more emphasis on the conceptual understanding of rational algebraic expressions was essential than focusing on procedural fluency. It is suggested to consider some common strategies that were utilized by mathematics teachers to clear up misconceptions by students in simplifying rational algebraic expressions such as; probing, collaborative learning, differentiated learning. and small group instruction. Moreover, mathematics teachers are also encouraged to innovate strategies that they can utilize to help improve students' learning of rational algebraic expressions.

Keywords: *rational algebraic expressions, conceptual understanding, procedural fluency*

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1. Introduction

The study of teaching and learning has long been a popular topic in the realm of education. New perspectives on teachers' conceptual understanding and procedural fluency emerged in the 1980s [1-3]. However, studies have shown opposing views as to teachers' emphasis on conceptual understanding and procedural fluency in teaching rational algebraic expressions. Particularly, it is argued that procedural fluency is built on the basis of conceptual understanding, in which greater emphasis on teaching concepts is more essential [35]. Throughout the last few decades, studies in mathematics have now concentrated on the significance of conceptual mathematics instruction [5]. Emphasizing that conceptual understanding is not just knowing information but it is the initial capital of students in solving mathematical problems [6]. Conceptual understanding (knowing why) supports understanding mathematical principles that are considered as the product of a process that connects prior knowledge with new knowledge [35]. As stated, mathematics teachers must focus on the students-their needs, interests and particularly their existing mathematical conceptual understanding.

However, there were also studies which gave more emphasis on procedural fluency in teaching some mathematics lessons. Teachers perform better on tasks requiring procedural fluency than on those demanding conceptual understanding [7,8]. Doyle *et al.* (2003) [9] pointed out that when students were asked to perform a procedure such as solving an equation, students can often follow an example and get a correct answer. Majority of assessment in math learning has been based on the students' abilities to manipulate procedural fluency. With these viewpoints, it merits a closer look at the question of whether conceptual understanding or procedural fluency should be emphasized first in teaching rational algebraic expressions.

Moreover, studies have shown that many students have challenges in solving mathematical problems involving rational algebraic expressions [10]. It is observed that students pay little attention when it comes to math lessons, especially when fraction is involved. Several studies claimed [11,12] that students in high school find simplifying rational algebraic expressions a challenging one and perceive fractions as a difficult task to accomplish. For instance, Keiran *et al.* (2007) [13] pointed out how students are weak in algebra involving factoring algebraic expressions which seemed as a serious problem since students were not able to generate explanations of the solutions to their answers. Similarly, in the study of

Crooks, et al. (2014) [14] about students errors when trying to simplify rational expressions, it was found that students make frequent, common, and persistent errors when simplifying rational expressions, if these issues and problems are ignored, it will be difficult for students to comprehend the idea and even harder for them to solve complex problems. Moreover, misconceptions and errors will continuously be made by students unintentionally if these problems will not be addressed [11].

With the aforementioned views, this paper reviews: (1) mathematics teachers' conceptual understanding or procedural fluency emphasis in teaching rational algebraic expressions; (2) the challenges encountered by students when learning rational algebraic expressions; and (3) strategies to help students overcome their challenges encountered when simplifying rational algebraic expressions.

2. Methodology

The study utilized a systematic review method [27]. The review employed content analysis [16] techniques to get a comprehensive review [17]. The Google Scholar database was used for sampling of the publications. The following initial requirements: must be indexed in Scopus, ACI and other peer-reviewed journals; has a relevant search term in the title, abstract, or keywords; and must be written in English. There were 40 articles examined and scrutinized, and these articles were not limited to their year of publication or county of origin.

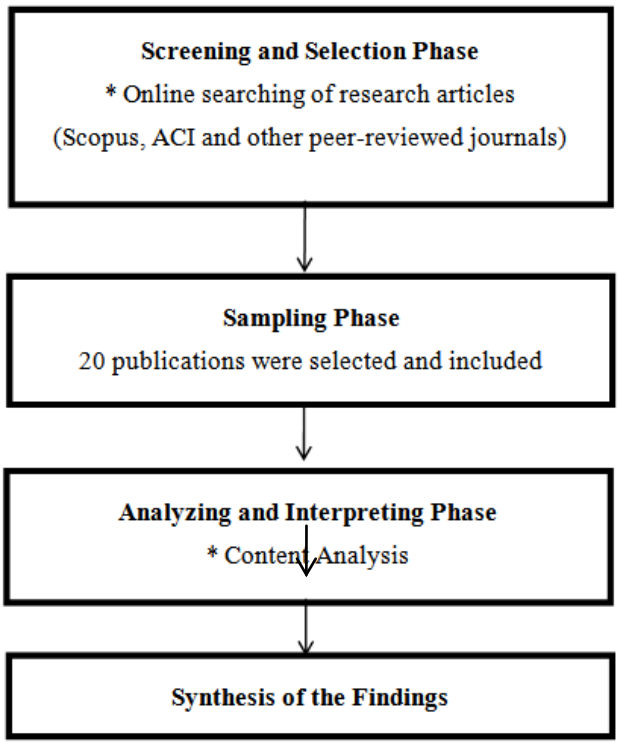

Figure 1. The flow diagram of the study (Adopted from Koseogtu et al., 2018)

The review process involved four stages, which were systematically executed (see Figure 1). The first stage focused on the screening and selection of research articles through title, abstract, and keywords, which were related to teachers' conceptual knowledge and procedural expertise and rational algebraic expressions. The second stage was the sampling phase: the Google Scholar database was used as a tool for sampling. A total of 25 papers passed through the initial screening. Then, reading of all the paper abstracts and full texts were requisite procedures needed to identify the papers relevance to this review. Any duplicate resources were removed. After the cautious screening, five articles were excluded. The third stage involved analyzing and interpreting the full articles selected. After a thorough assessment, 20 articles, which concentrated on teachers' conceptual understanding and procedural fluency in rational algebraic expressions, were analyzed and interpreted through content analysis. The last phase focused on synthesizing the findings.

3. Discussion of the Findings

The content analysis revealed the following themes: Teachers' emphasis on conceptual understanding or procedural fluency in teaching rational algebraic expressions, strategies in the teaching of rational algebraic expressions, and challenges faced by the students in learning rational expressions.

3.1. Conceptual Understanding vs. Procedural Fluency Emphasis in Teaching Rational Algebraic Expressions

Conceptual understanding refers to the underlying structure of ideas which explain and give meaning on its processes [2]. When addressing students mathematical difficulties, a teacher with good conceptual understanding can use a variety of mathematical ideas [18].

A study have been made about mathematics teachers' conceptual understanding on the context of rational expressions, it revealed that majority of the teachers were preferred to put more emphasis in the concept of the lesson than focusing on the process in solving [19]. Also, the same study conducted by Ibrahim, (2003) [20] about teachers' conceptual understanding in selected community high school teachers from New York who were teaching mathematics subject, it revealed that 70.7% of mathematics teachers have placed more emphasis on the concepts in teaching mathematics lessons compared to 29.3% who are focused on procedural fluency. They believed that accenting the concept of the lessons more would give students a better chance of understanding it. The outcome was also affirmed in the study of Bryan, *et al*. (2002) [21] in Turkey that focused on conceptual understanding on several topics such as: rational expressions, quadratic equations, trigonometry, and geometry, it revealed that about 77% of the respondents were able to explain or justify their answers the concepts of the lessons than on giving explanation on its procedures. It was indicated that a bigger percentage of the respondents succeeded in emphasizing the concepts behind the solutions of their answers [21]. It is well acknowledged that teachers with deep conceptual understanding of the lesson will affect their effectiveness in teaching the lesson [22]

Furthermore, emphasizing the concept of the lesson in the first place is evident through the research of Rittle-

Johnson, *et al.* (2015) and Star, J. (2002) [23,24] which stated that it would be advantageous for the early introduction of procedures to take place after the introducing the concept of the lesson, in which conceptual understanding often supports procedural fluency. Similarly, Conley (2014) [24] stressed that introducing concepts will allow the students the opportunity to grasp the 'big picture', that is, developing their conceptual understanding. Other study also posits that although 'experts' do know more 'facts', crucially it is that the facts are connected and organized into meaningful patterns, a characteristic of conceptual understanding which is very important [26].

However, it has been argued that giving emphasis on the procedural fluency in teaching the lesson can lead teachers and students to better explain or justify the way an answer is obtained without knowing why a certain step, method, operation, or formula is used in the problem-solving process [7]. Further, it is said that it can boost the mastery of skills and familiarity with procedures for identifying mathematical components, algorithms, and definitions [27].

Yet, several studies about the emphasis on procedural fluency in teaching the lesson have conducted and the results were found to be contradicting. As revealed in a study about teachers' procedural fluency in solving problems involving algebraic expression, it was found that most teachers have mastered conceptual understanding more thoroughly than they have mastered procedural fluency of the topic [7]. Similarly, another study also found that teachers managed to solve operations involving rational expressions but failed to give emphasis on the reasoning behind the algorithm. Teachers are also unable to provide deep procedural explanations for and algorithms used to complete a mathematical problem [28].

Moreover, a research by Torbeyns, J. *et al.* (2015) [29] about three college teachers from the state of California who were teaching an algebra course required respondents to solve a mathematical equation involving conceptual understanding and procedural fluency. It was found that procedural fluency of mathematics teachers in solving mathematical equations can be increased when conceptual understanding and meta-cognitive reflection are both at high levels [30]. Additionally, in a study of Leung, F., *et al.* (2002) [31] where teachers from Korea, Hong Kong, and the United States were presented with algebra problems and were asked to explain to the researcher on how to solve them in the same way that they would explain them to their students. Findings showed that nearly all the teachers manifests deep conceptual understanding, and few were able to emphasized procedure behind the concept clearly. Concepts could only be the vehicle for understanding to assist teachers to deeply learn procedures as they grasp its concepts [32].

These viewpoints entail that mathematics teachers have placed greater emphasis on conceptual understanding of the subject for students to avoid misconceptions and for them to better grasp the idea of rational algebraic expressions. This further indicates that mathematics teachers' significant emphasis on the concepts of rational expressions may lead students in providing sufficient explanations of ideas needed concerning the procedures of solving [18,22]. As Rittle-Johson *et al.* (2015) [23] stressed that thoughtful and deep-understanding of concepts was required on the teachers' behalf, to support students' procedural fluency in the processes of solving.

3.2. Challenges Faced by Students When Simplifying Rational Algebraic Expressions

Knowing the algebraic fractions concept is essential for students to succeed in algebra and other mathematics subjects. However, it has been observed that students pay little attention on mathematical problems especially when fraction is involved [33]. Most of them perceived fraction as a difficult task to accomplished, with these reasoning they gave less attention on their solutions when answering.

Some challenges students faced when working with rational algebraic formulas were determined by the researchers. Three sub-themes were identified, including: students' weak on the foundation of the concept, errors in cancellation, and dependence on cues.

3.3. Weak Foundation of the Concept

The topic rational algebraic expression are challenging for students with a weak grasp of its concept [34]. Research has been done in the Eastern Cape Province of South Africa on these misunderstandings and mistakes that students make when simplifying algebraic fractions [33]. The findings showed that poor conceptual understanding and incorrect application of algebraic fractions by learners are hampered by their lack of understanding of the mathematical concept and inappropriate application to the problems. The conceptual understanding appeared to permeate students' attempts to simplify algebraic fractions [33]. Similarly, in the study of Baidoo (2019) [33] revealed that students have difficulties in solving problems involving rational expressions due to their insufficient understanding of mathematical concepts; such as division, variables, equations, perfect squares, exponent, factorization, rational numbers, and algebraic fraction. Moreover, Baker, S. *et al.* (2002) [35] emphasized that rational algebraic expressions are governed by the principles and ideas of arithmetic functions, thus the need for students to master and grasp the concept of arithmetic functions is very essential for them to understand rational algebraic expressions. [35]. This further indicates that it will be difficult for students to solve algebraic fraction problems when these concepts are not sufficiently emphasized and understood by them [33].

3.4. Cancellation Errors

It was hinted that rational algebraic fractions contain all the difficulties seen in fractions as well as additional challenges often encountered in algebra. In the study of Canobi, K. H. (2009) [26] when students were given task to solve problems involving rational expressions, it revealed that students find it hard to perform factoring of the numerator and denominator of two expressions, which also make it more difficult for them when they try to cancel any common or like terms to both a numerator and a denominator. Similarly, a study made by Makonye JP, et al. (2014) [11] on errors and mistakes on operations made

by Grade 10 students when simplifying rational algebraic expressions was also carried out in South Africa. The research showed that students in grade 10 make several mistakes and exhibit a variety of misconceptions when simplifying rational expressions such errors involving cancellation, partial cancellation, and like term error [12]. More explorations by Ruhl *et al*, (2011) [36] were made on attempt to analyze the errors in the simplification of a rational expression of learners at Australia. The analysis of the errors across the whole data set produced three error categories simple cancellation error, cancellation by subtraction error, cancellation by division of coefficients retaining the variable error that were defined precisely. The results provide teachers with these difficulties students have in solving rational expression. Due to improper factoring and cancellation of common terms techniques, students have trouble in correctly solving rational expressions.

3.5. Dependence on Cues

Additionally, researches on how students complete rational algebra lessons have been conducted in several nations. At the University of Western Cape in South Africa, a study on grade 9 students' approaches to dealing with rational algebraic expression was conducted. The study showed that students rely more on cues when working with rational algebraic fractions, but they do not have a sufficient conceptual understanding or relational grasp of what they accomplish in each step. [12]. Similarly, in the study of Otten S (2008) [37] about errors that students make when simplifying mathematical formulas, it revealed that their solutions are significantly related to their past grasp of common fractions and are brought on by the fact that students rely and depend more on cues than they do understanding [37]. Moreover, according to Figuaeras *et al.* (2008) [37], students tend to rely and depend on cues when simplifying rational algebraic expressions since it only requires learners to follow certain procedures without understanding the reasoning behind as they simplify algebraic expressions. These studies have stressed that excessive reliance and dependence on cues without a thorough knowledge of the underlying idea will make students find it difficult to solve rational expressions, especially the complex ones.

The aforementioned challenges faced by the students when learning rational algebraic expressions greatly affect their understanding on the concept of the lesson. It is necessary to provide solutions to these concerns to be able for the students to grasp its idea. If these challenges are left uncorrected, it will be harder for learners to advance in their ability to solve problems involving rational algebraic expressions and other higher mathematical concepts.

3.6. Strategies in Teaching Rational Algebraic Expressions

After examining the elements that contribute to students' problems when learning algebra, it is critical to consider strategies options that can be utilized by mathematics teachers to clear up students difficulties in simplifying rational algebraic expressions [26]. These strategies in teaching the concept will serve as a plan or series of steps designed to assist and help students improve abilities they find challenging [35].

The authors were able to determine common strategies that were utilized by mathematics teachers in teaching the lesson. These includes: probing, collaborative learning, differentiated learning and small group instruction.

3.7. Probing

Probing is effective as a strategy for one-to-one interventions, as well as whole-class discussions. Well-managed, effective probing should secure the attention of all students, allowing the teacher to direct the dialogue from student to student, developing ideas through repeated exchanges and deeper thinking [38]. This type of strategy entails a process where the practice is intentionally guided by thought, motivating teachers to impart knowledge more effectively [39]. A study focusing on probing have been employed as a teaching strategy for the students to identify their mistakes made when they tried to simplify algebraic expressions, this revealed that when probing is applied, teachers had the chance to ask students about their mistakes and encouraged them to find solutions on their own [12]. Similarly, in the study of Baker, S. *et al.* (2002) [35] about strategies in teaching mathematics lessons, it revealed that most teachers favour probing over focused ones for they have the opportunity homing in on a point in each answer, and asking more and more detail at each level [35]. One consideration here in applying probing is that teacher needs to pay attention to what the students say since it can help clear up misconceptions. The manner a teacher questions students in class affects how well they learn mathematics and can be used as a strategy to help students better understand challenging mathematical concepts like fractions. As emphasized by Ntuli, E. *et al.* (2010) [39]., while questioning is a crucial component of the mathematical discourse, if done incorrectly, it can negatively impact learner conversations and stunt their growth.

3.8. Collaborative Learning

Another strategy for enhancing rational algebraic expression teaching and learning is collaborative learning [40]. Collaborative learning is based on the theory of Constructivism. This theory emphasizes learner-centred teaching and acknowledges that knowledge is created by learners. The constructivist viewpoint contends that genuine learning requires collaboration between students as well as between the teacher and students [41]. They learn to solve their own problems and come to a conclusion that everyone agrees within the group. In the study of Van Steenbrugge, H., et al. (2015) [42] about teaching and learning algebraic expressions in elementary, it showed that exposing students in a collaborative environment and by making the material relevant to students' everyday life can have a significant impact on students' capacity to acquire and learn abstract concepts of algebraic functions. Similarly, Martin-Stanley, C. R. (2007) [40] highlighted that students will learn, create new knowledge and gain ideas through collaborating with one another. Further, teachers need a culture that values every

student's strengths and a school community that believes everyone can learn from each other.

3.9. Differentiated Learning

Additionally, one strategy being applied also in teaching rational expressions is differentiated learning. It is a predominant instructional strategy that teachers employ to facilitate the diverse needs of students. Differentiation provides one method by which teachers can provide appropriate challenge at appropriate levels for all learners in a heterogeneously grouped mathematics classroom where the range of abilities and interests can be wide Reed, (2004) [43]. In the study of Tomlinson, *et al.* (2008) [44] about learners' diverse needs in the classroom, it showed that employing differentiated learning inside the classroom was able to meet student's various needs because the teacher is attending to the challenges and strengths of the students. Similarly, in the study of Tom, (2013) [45] about differentiated mathematics instruction, it was found that students in a differentiated classroom have wider opportunity to utilize their strengths and are motivated to persevere even when tasks become more difficult. Furthermore, Lane (2008) [46] also found that differentiated learning leads to students being engaged in tasks that are based on their individual level most especially in mathematics lessons. According to these findings, differentiated learning technique will help students overcome their own unique problems, particularly when it comes to problems involving rational algebraic expressions.

3.10. Small Group Instruction

Lastly, small group instruction is one of the instructional strategies that is utilized in mathematics classroom. Kameenui (2004) [47] stated that the identification of children as diverse learners itself suggests that multiple perspectives and approaches will be necessary to accommodate the needs of children who possess differences in abilities and learning histories, and one of these approaches is employing small group instruction. In the study of Vula, E., *et al.* (2015) [48] related to mathematics word problem solving strategies through collaborative action, it was seen that small group instruction serves as a structure that offers opportunities to meet with a student or students to support them as they work and to support them as they transition to their own independence. Students have the chance to share and to acquire new learning. Similarly, in the study of Serravallo (2010) related to small group instruction and discussion, it was found that engaging students in small-group provide them opportunities to watch the teacher demonstrate, opportunities for the student go practice with teacher support, and opportunities to practice independently, offering a bridge to independence. Here, students are allowed to explore their thoughts and seize chances to better understand the subject.

These strategies are just a few of the commonly utilized strategies by math teachers to teach rational algebraic expressions. These strategies were applied by teachers to help students' learning, comprehension, and overcoming of challenges associated with learning rational algebraic formulas.

4. Conclusion

Based on the analysis and findings of the study the researchers concluded:

Mathematics teachers placed a greater emphasis on conceptual understanding than on procedural fluency in solving rational algebraic expressions. They believed that emphasizing the concept first may help students to better grasp the idea.

Students faced challenges in learning rational algebraic expressions includes; students' weak conceptual foundation in algebra, cancellation of like terms errors, and too much dependence on cues without understanding the idea behind the procedure.

Common strategies were also utilized by mathematics teachers for students to have a full facilities in learning the topic and thoroughly learn the concept of rational expressions.

One question that comes up in addressing students' challenges in learning rational algebraic expressions is whether mathematics teachers should place a higher emphasis on conceptual understanding or on procedural fluency, or if they should emphasize both.

Recommendations

This brief review illustrated the need for more in-depth investigation and thorough research on a number of fronts, including additional investigations about mathematics teachers' emphasis on conceptual understanding and procedural fluency in teaching rational expressions, the introduction of new strategies to help students who have challenges with rational algebraic expressions, and consideration of a wider scope using various perspectives for better generalization.

References

[1] Elbaz, F.L. (1983). *Teacher thinking: A study of practical knowledge*. London: Croom Helm.

[2] Shulman, L.S. (1986). Paradigms and research programs in the study of teaching: A contemporary perspective. In M.C. Wittrock (Ed.), *Handbook of research on teaching* (pp. 3–36). New York: Macmillan.

[3] Schon, D.A. (1983). *The reflective practitioner: How professionals think in action*. New York: Basic Books.

[4] Baykul, Y. (1999): *Primary mathematics education*. Ankara, Turkey, Ani Printing Press.

[5] Crooks, Noelle & Alibali, Martha. (2014). Defining and measuring conceptual knowledge in mathematics. Developmental Review. 34. 10.1016/j.dr.2014.10.001.

[6] Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). An alternative re-conceptualization of procedural and conceptual knowledge. *Journal for Research in Mathematics Education, 38*, 115-131.

[7] McGehee, J. (1990). Prospective secondary teachers' knowledge of the function concept. Unpublished Doctoral Dissertation, University of Texas.

[8] Khashan, K.H. (2014). Conceptual and procedural knowledge of rational numbers for Riyadh elementary school teachers. *Journal of Education and Human development, 3(4), 181-197.*

[9] Doyle, A. B., & Markiewicz, D. (2003). Associations between parenting style and attachment to mother in middle childhood and adolescence. *International Journal of Behavioral Development, 27*(2), 153–164., G. Donald (2007), *Student Thinking*, Available: http://www.math.tamu.edu/~snite/MisMath.pdf

[10] Makonye JP, Hantibi N 2014. Exploration of Grade 9 learners' errors on operations with directed numbers. *Mediterranean Journal of Social Sciences, 5(20): 1564-1572*.

[11] Maphini, N.V. (2018). Implementing An Intentional Teaching Model to Investigate Grade 9 Learners' Way of Working with Rational Algebraic Fractions. University of the Western Cape.

[12] Kieran, C. (2007). Research on the learning and teaching of algebra. In A. Gutiérrez & P. Boero (Eds.), Handbook of research on the psychology of mathematics education (pp. 11–50). Sense.

[13] Crooks, T. (2007, April). Key factors in the effectiveness of assessment for learning. Paper presented at the meeting of the American Educational Research Association, Chicago, IL.

[14] Hirsh, H., Coen, M.H., Mozer, M.C., Hasha, R. and Flanagan, J.L, "Room service, AI-style," *IEEE intelligent systems*, 14 (2). 8-19. Jul.2002.

[15] Gough, D., Oliver, S., & Thomas, J. (2012). *An introduction to systematic reviews. London: Sage.*

[16] Frykholm, J.A. (1999). The impact of reform: Challenges for mathematics teacher preparation. *Journal of Mathematics Teacher Education*, 2, 79–105.

[17] Vula, E., & Kurshumlia, R. (2015).Mathematics Word Problem Solving Through Collaborative Action Research. *Journal of Teacher Action Research, 1(2), 34-46*.

[18] Given, L. M. (2008). *The Sage encyclopedia of qualitative research methods*. Los Angeles, CA: Sage.

[19] Ibrahim, Noorashikin Noor. 2003. Pedagogical content knowledge of mathematics teacher in algebra. Master of Education, Research Project. Bangi: Universiti Kebasngaan Malaysia.

[20] Bryan, L., & Atwater, M. (2002), Teacher beliefs and cultural models: A challenge for science teacher preparation programs. Science Education, 86, 821-839.

[21] Schoenfeld, A.H 2002. "Making mathematics work for all children: Issues of standards, testing, and equity". Educational researcher, 31(1), 13-25.

[22] Rittle-Johnson, B., & Schneider, M. (2015). Developing conceptual and procedural knowledge in mathematics. In R. Cohen Kadosh & A. Dowker (Eds.), Oxford handbook of numerical cognition (pp. 1102-1118). Oxford, UK: Oxford University Press. https://.org/10.1093/oxfordhb/9780199642342.013.014

[23] Star, J.R. 2002. "Developing conceptual understanding and procedural skill in mathematics: An interactive process". *Journal of Educational psyhology, 93(2), 346-362*.

[24] Conley, D. T., & French, E. M. (2014). Student ownership of learning as a key component of college readiness. *American Behavioral Scientist, 58*(8), 1018–1034.

[25] Canobi, K. H. (2009). Concept-procedure interactions in children's addition and subtraction. Journal of Experimental Child Psychology, 102, 131-149.

[26] Eisenhart, Margaret & Borko, Hilda & Underhill, Robert & Brown, Catherine & Jones, Doug & Agard, Patricia. (1993). Conceptual Knowledge Falls through the Cracks: Complexities of Learning to Teach Mathematics for Understanding. *Journal for Research in Mathematics Education*. 24. 8. 10.2307/749384.

[27] Tirosh, D. 2000. "Enhancing prospective teachers' knowledge of children conceptions: The case of division of fractions" *Journal for Research in Mathematics Education, 30(1), 5-25*.

[28] Torbeyns, J., Schneider, M, Xin, Z. & Siegler, R. S. 2015. Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different continents.Learning and Instruction,37, 5-13.

[29] Star, J.R. 2002. "Developing conceptual understanding and procedural skill in mathematics: An interactive process". *Journal of Educational psyhology, 93(2), 346-362*.

[30] Leung, F., & Park, K. (2002). Competent students, competent teachers? *International Journal of Educational Research, 37, 113-129.*

[31] Capraro, M. M., & Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? Reading Psychology, 27(2-3), 147-164.

[32] Baidoo, J. (2019). Dealing with Grade 10 Learners' Misconceptions and Errors When Simplifying Algebraic Fractions. University of Johannesburg, South Africa.

[33] Thomas, C. 2010. Fraction competency and algebra success. Thesis Submitted in partial fulfilment of the Requirements for the degree of Master of Natural Sciences, Louisiana State University.

[34] Baker, S., Gersten, R. & Lee, D.-S. 2002. A synthesis of empirical research on teaching mathematics to low-achieving students.*The Elementary School Journal,* 103, 51-73.

[35] Ruhl J.B, Stephen M. Posner, and Taylor H. Ricketts, *Engaging Policy in Science Writing: Patterns and Strategies*, (2011) Available at: https://scholarship.law.vanderbilt.edu/faculty-publications/1107

[36] Figuaeras H, Males H, Otten S 2008. Algebra Students' Simplification of Rational Expressions. Michigan: Michigan State University Press.

[37] Ntuli, E. & Godfrey, A. Wearable technology: Improving mathematical classroom discourse using Pivot-head Eyeglasses. 2018. Association for the Advancement of Computing in Education (AACE), 1113-1118

[38] Chen, X. 2013. Meta-teaching: Meaning and strategy.Africa Education Review, 10, S63- S74.

[39] Martin-Stanley, B. L. & Martin-Stanley, C. R. 2007. Constructivism and technology: Strategies for increasing student learning outcomes. National Social Science Association. Retrieved on January, 25, 2012.

[40] Vygotsky, L. S. 1978. The prehistory of written language. Mind in society: The development of higher psychological processes, 105-119.

[41] Van Steenbrugge, H., Remillard, J., Valcke, M. & Desoete, A. 2015.Teaching fractions in elementary school: An observational study. *the elementary school journal*, 116, 49-75

[42] Reed, J. G., & Baxter, P. M. (2004). The author/citation approach to searching. In J. G. Reed & P. M. Baxter, *Library use: A handbook for psychology* (pp. 75–80). American Psychological Association.

[43] Tomlinson, C. A., Brighton, C., Hertberg, H., Callahan, C. M., Moon, T. R., Brimijoin, K., Conover, L. A., & Reynolds, T. (2008). Differentiating instruction in response to student readiness, interest, and learning profile in academically diverse classrooms: A review of literature. Journal for the Education of the Gifted, 27(2-3), 119-145.

[44] Tom, C. A. (2013). The differentiated classroom: Responding to the needs of all learners (2nd ed.). ASCD

[45] Lane, M. (2008), Subsequent educational attainment in a US national sample. Journal of Psychiatric Research, 42, 708-716.

[46] Kame'enui, E.J. (Eds.). (2004). Vocabulary instruction: Research to practice (pp. 14-17, 29-63, 160). New York: Guilford.

[47] Vula, E., & Kurshumlia, R. (2015).Mathematics Word Problem Solving Through Collaborative Action Research. *Journal of Teacher Action Research, 1(2),* 34-46.

[48] Serravallo, J. & Goldberg, G. (2010) Conferring with readers: Supporting each student's growth and independence. Portsmouth, NH: Heinemann.