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A Behavioral Portfolio Model with Interval Return and Investor's Sentiment

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Abstract This paper proposes a behavioral portfolio decision model with interval returns and investor's sentiment. A sentiment-adjusted mean model for behavioral portfolio selection is presented by taking into account investor's sentiment return and multiple mental accounts. The proposed behavioral model maximizes the sentimental mean value of portfolio interval return and ensures the portfolio interval return of each mental account exceeding the given minimum return level with a given possibility degree. Then, multiple programming models are designed to solve the optimal behavioral portfolio strategy. Finally, a numerical example is given to illustrate the validity of the proposed approach.

Keywords: *behavioral portfolio decision, interval return, possibility degree, mental account*

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1. Introduction

Portfolio selection as a field of study began with the Markowitz model [1] in which return is quantified as the mean and risk as the variance. Portfolio selection discusses the problem of how to allocate a certain amount of investor's wealth among different assets and form a satisfying portfolio. In 1952 Markowitz proposed the mean-variance portfolio decision model which provides a fundamental basis for modern portfolio selection theory by maximizing the expected return for a given level of risk or minimizing the expected risk for a given level of expected return. After that, many scholars have studied the portfolio selection models in fuzzy uncertain environment [2,3,4]. Moreover, in real world the portfolio selection problem are usually involved by decision-maker's pshchology [5,6] and mental accounts, since the investor decision-makers tend to segregate different types of gambles into separate accounts, and then apply the prospect theory [7,8] to each account by ignoring possible interactions. It refers to the tendency for people to separate their money into separate accounts based on a variety of subjective criteria. Mental account proposed by Thaler [9] is a foundation for the way decision-makers set reference points for the accounts that determine profits and losses. Behavioral portfolio theory proposed by Shefrin and Statman [10] is a positive theory of asset choice under uncertainty. They developed a model of multi-layered portfolio construction in which each layer is associated with a particular aspiration level and the covariances between the layers are overlooked. Thus, each portfolio layer resembles a separate mental account. Shefrin and Statman suggest that investors have varied aims and create an investment portfolio that meets a broad range of goals. After that, Ma [11] proposed a practical decision making method for behavioral portfolio choice, Muradoglu Yaz [12] studied a behavioral approach to efficient portfolio formation, Mehlawat [13] and Amelia [14] developed multi-criteria behavioral portfolio decision models. Jin [15] developed multi-period and multi-objective behavioral portfolio approach. Also, Xie [16] studied the behavioral assets portfolio method based on sentiment recognition.

Recently, interval number or interval value fuzzy set have been generally used in handling and describing imprecise and complex phenomena that often rise in business, financial and managerial systems [17,18,19,20]. In uncertain portfolio decision scenario, the return of financial asset is conveniently evaluated by interval. Inspired by the idea of Markowitz's M-V model, a lot of interval portfolio model extensions have been proposed to deal with portfolio decision with interval return and risk under interval uncertain environment [21-36]. For example, Giove [21], Zhang [22], Wu [23], Chen [24] studied on interval portfolio selection problem, and Mohagheghi [25,26] studied the portfolio selection problem under interval fuzzy environment. Rupak Bhattacharyya [27], Li [28], Guo [29], Xu [30], MASAAKI [31], Feng [32] studied the fuzzy portfolio selection models by interval analysis. Also Liu [33] discussed a multi-period portfolio selection optimization model by using interval analysis. Zhao [34], Lu [35], Lai [36] developed the multiple objective interval number linear programming model for the portfolio investment.

However, the above mentioned interval-valued portfolio decision models have not consider the investor's sentiment and investor's behavioral interacting factors as well as the socio-psychological nature. Therefore, we will provide a new methodology to build portfolios for behavioral investors that follow ethical, environmental and social considerations in their investment decisions. To do so, we construct interval behavioral portfolio theory with mental accounts, multiple-objective programming models and sentiment factors. In this work, we propose the interval behavioral portfolio models for determining the allocation between the different mental accounts according to the investor's profile and the expert knowledge provided by the financial manager.

2. Preliminaries

Let us first review some basic concepts and preliminary theory about interval value, which will be utilized in the following sections about interval behavioral portfolio decision model construction.

Definition 1 [17]. $a=[a_L,a_R]$ is named an interval value, if $a_L \le a_R$. And the length of this interval is defined as length($\bar{a}$) $= a_R - a_L$.

Definition 2 [18]. Let $\bar{a}=[a_L,a_R]$, $\bar{b}=[b_L,b_R]$ be two interval values, the addition and the scalar multiplication operators between them are defined as

(1) $\bar{a}+\bar{b}=[a_L+b_L, a_R+b_R]$.

(2) $x\bar{a}=[xa_L, xa_R], \forall x \ge 0.$.

Definition 3 [19]. Let $\bar{a}=[a_L,a_R]$, $\bar{b}=[b_L,b_R]$ be any two interval values, the degree of possibility of $\bar{a} \ge \bar{b}$ is defined as

$$P(\bar{a} \ge \bar{b}) = \max\{1-\max\{\frac{b_R - a_L}{length(\bar{a})+length(\bar{b})},0\},0\},$$

where $length(\bar{a}) = a_R - a_L$, $length(\bar{b}) = b_R - b_L$ denote the length of interval values $\bar{a},\bar{b}$, respectively.

Theorem 1 [20]. Let $\bar{a}=[a_L,a_R]$, $\bar{b}=[b_L,b_R]$, $\bar{c}=[c_L,c_R]$ be any two interval values, then

(1) $0 \le P(\bar{a} \ge \bar{b}) \le 1$.

(2) $P(\bar{a} \ge \bar{b}) + P(\bar{b} \ge \bar{a}) = 1$; $P(\bar{a} \ge \bar{a}) = 1/2$.

(3) $P([a_L,a_R] \ge [b_L,b_R]) = 1$ if and only if $b_R \le a_L$.

(4) $P([a_L,a_R] \ge [b_L,b_R]) = 0$ if and only if $a_R \le b_L$.

(5) $P(\bar{a} \ge \bar{b}) \ge 0.5$ if and only if $a_L + a_R \ge b_L + b_R$;

Especially, $P(\bar{a} \ge \bar{b}) = 0.5$ if and only if

$$a_L + a_R = b_L + b_R;$$

(6) If $P(\bar{a} \ge \bar{b}) \ge 1/2$, $P(\bar{b} \ge \bar{c}) \ge 1/2$, then

$$P(\bar{a} \ge \bar{c}) \ge 1/2.$$

Definition 4. Let $\bar{a}=[a_L,a_R]$ be an interval value, then the mean value of interval value $\bar{a}$ is defined as

$$E(\bar{a}) = (a_L + a_R)/2.$$

Definition 5 [16] Let s_j be the sentiment of investor on financial asset j, the sentiment influential function $f(s_j)$ and the sentiment-adjusted return $\hat{R}_j$ of asset j , respectively, are defined as the following forms.

$$f(s_j) = e^{as_j}, (a > 0), \hat{R}_j = f(s_j)R_j.$$

Obviously, $f(s_j) > 0$ and is a increasing function, that is to say, the greater is the investor 's sentiment, the higher is the estimated return of asset .

(1) If $s_j > 0$, then $f(s_j) > 1$, i.e., if the investor is optimistic on asset j , the return of asset j will be over-estimated;

(2) If $s_j = 0$, then $f(s_j) = 1$, i.e., if the investor is rational on asset j , the return of asset j is estimated properly;

(3) If $s_j < 0$, then $f(s_j) < 1$, i.e., if the investor is pessimistic on asset j , the return of asset j is under-estimated.

3. The Formulation of Interval Behavioral Portfolio Decision Problem

In this section, we discuss the behavioral portfolio selection problem with interval returns and sentiment. We first introduce the problem description and notations used in the following section. Then, we formulate the interval behavioral portfolio model by maximizing the sentimental mean of return of portfolio.

3.1. Problem Description and Notations

Let us consider a behavioral portfolio selection problem with T mental accounts. Each mental account consists n_i risky assets. The return rates of risky assets are evaluated by interval values. Assume that the investor intends to allocate his/her wealth among the n_i risky assets for making accounting investment plan in T mental accounts. To make it easier to follow our exposition, we put together all the notations that will be used hereafter.

x_{ij} : the investment proportion of risky asset $j(j=1,2,\cdots,n_i)$ in mental account $i(i=1,2,\cdots,T)$;

w_i : the importance degree of the holding mental account $i(i=1,2,\cdots,T)$;

l_{ij} : the lower boundary of investment proportion of risky asset j at mental account $i(i=1,2,\cdots,T)$;

u_{ij} : the upper boundary of investment proportion of risky asset j at mental account $i(i=1,2,\cdots,T)$;

s_{ij} : the sentiment of investor on asset j in mental account $i(i=1,2,\cdots,T)$;

$f(s_{ij}) = e^{0.2s_{ij}}$: the sentiment function of investor on asset j in mental account $i(i=1,2,\cdots,T)$.

3.2. Sentiment-adjusted Mean of Interval Return for Behavioral Portfolio

Assume that the whole investment process is self-financing, that is, the investor does not invest the additional capital during the portfolio selection process. Let $\overline{R}_{ij}=[R_{ijL},R_{ijU}]$ be the interval return of asset j at mental account i. According to the previous section, the sentimental mean value of the return rate of the portfolio $x_i=(x_{i1},x_{i2},\cdots,x_{in_i})$ at mental account i is determined by

$$E(R_{pi})=\sum_{j=1}^{n_i}x_{ij}E(\overline{R}_{ij})f(s_{ij})$$
$$=\sum_{j=1}^{n_i}f(s_{ij})x_{ij}(R_{ijL}+R_{ijU})/2.$$

3.3. Construction of Behavioral Portfolio Selection Model with Interval Return and Investor's Sentiment

Assume that the objective of the investor wants to maximize the expected sentimental return of portfolio over the whole T mental accounts. At the same time, the portfolio return at each mental account must achieve or exceed the given minimum wealth return level. Thus, the multi-mental accounting behavioral portfolio selection problem can be formulated as the following programming model denoted by (P1):

$$\text{(P1)}\ \max\sum_{i=1}^{T}w_i(\sum_{j=1}^{n_i}x_{ij}E(\overline{R}_{ij})f(s_{ij}))$$

s.t.

$$P(\sum_{j=1}^{n_i}x_{ij}\overline{R}_{ij}\geq\overline{r}_i)$$
$$=P([\sum_{j=1}^{n_i}x_{ij}R_{ijL},\sum_{j=1}^{n_i}x_{ij}R_{ijU}]\geq[r_{i1},r_{i2}])\geq\alpha_i,$$
$$i=1,2,\cdots,T$$
$$\sum_{i=1}^{T}\sum_{j=1}^{n_i}x_{ij}=1,\ 0\leq l_{ij}\leq x_{ij}\leq u_{ij}\leq 1$$

where $W=(w_1,w_2,\cdots,w_T)$ is the weight vector of all the mental accounts, $\sum_{i=1}^{T}w_i=1, w_i\in[0,1]$, w_i is the importance degree of mental account i. And $\overline{r}_i=[r_{i1},r_{i2}]$ represents the given minimum aspiration interval return level of the portfolio wealth regarding the i-th mental account; α_i is the given possibility degree level assuring that the interval return of i-th mental account greater than the given minimum aspiration interval return level $\overline{r}_i$. In general, the higher level is the account mental, the greater is the parameter α_i.

If we suppose $f(s_{ij})=e^{0.2s_{ij}}$ is the sentiment function of investor on asset j at mental account i, then the above programming model (P1) can be transformed to the following optimization models (P11-P13) according to Definition 3 of possibility of interval returns and Theorem 1.

$$\text{(P11)}\ \max\sum_{i=1}^{T}w_i\ \sum_{j=1}^{n_i}\frac{x_{ij}(R_{ijL}+R_{ijU})}{2}e^{0.2s_{ij}}$$

s.t.

$$\sum_{j=1}^{n_i}x_{ij}R_{ijL}\geq r_{i2},\ i=1,2,\cdots,T$$
$$\sum_{i=1}^{T}\sum_{j=1}^{n_i}x_{ij}=1,\ 0\leq l_{ij}\leq x_{ij}\leq u_{ij}\leq 1.$$

$$\text{(P12)}\ \max\sum_{i=1}^{T}w_i\ \sum_{j=1}^{n_i}\frac{x_{ij}(R_{ijL}+R_{ijU})}{2}e^{0.2s_{ij}}$$

s.t.

$$1-\frac{r_{i2}-\sum_{j=1}^{2}x_{ij}R_{ijL}}{(r_{i2}-r_{i1})+\sum_{j=1}^{2}x_{ij}(R_{ijU}-R_{ijL})}\geq\alpha_i,i=1,2,\cdots,T$$
$$\sum_{i=1}^{T}\sum_{j=1}^{n_i}x_{ij}=1,\ 0\leq l_{ij}\leq x_{ij}\leq u_{ij}\leq 1.$$

$$\text{(P13)}\ \max\sum_{i=1}^{T}w_i\ \sum_{j=1}^{n_i}\frac{x_{ij}(R_{ijL}+R_{ijU})}{2}e^{0.2s_{ij}}$$

s.t.

$$1-\frac{r_{i2}-\sum_{j=1}^{2}x_{ij}R_{ijL}}{(r_{i2}-r_{i1})+\sum_{j=1}^{2}x_{ij}(R_{ijU}-R_{ijL})}\geq\alpha_i,i=1,2,\cdots,m,$$
$$\sum_{j=1}^{n_i}x_{ij}R_{ijL}\geq r_{i2},\ i=m+1,m+2,\cdots,T$$
$$\sum_{i=1}^{T}\sum_{j=1}^{n_i}x_{ij}=1,$$
$$0\leq l_{ij}\leq x_{ij}\leq u_{ij}\leq 1\forall m=1,2\cdots,T-1.$$

Hence, we can get the optimal portfolio solution that achieves the maximum objective function value by solving the above multiple-account programming models (P11-P13) with Lingo or Matlab programming software.

4. Numerical Example

Example 1. In order to express the idea of our model and the effectiveness of the proposed interval behavioral portfolio algorithm, we give an example for simulating the real transaction. For simplicity, in the example we consider two-mental accounting behavioral portfolio decision problem with interval-valued returns. Assume that the financial market has two mental accounts, i.e. T = 2. The lower-level mental account has two alternative financial assets A_{11},A_{12}. The high-level mental account has two alternative financial assets A_{21},A_{22}. All the financial assets in the above mental accounts of this example are selected from Shanghai Stock Exchange. To simulate the transaction, we collect the weekly closing pricing of assets from September 2018 to September 2019, with 1 yearly observations. By analyzing the stock historical data, the corresponding corporations' financial reports and the future information, we can utilize the simple statistical frequency method to assess the interval return of assets in the above two mental accounts, which are shown in Table 1.

Table 1. The assessed interval return of the selected stock assets

Mental account 1	Interval return of assets	Mental account 2	Interval return of assets
Asset 1 (Stock A_{11})	[0.23, 0.50]	Asset 3 (Stock A_{21})	[0.62, 0.86]
Asset 2 (Stock A_{12})	[0.17, 0.46]	Asset 4 (Stock A_{22})	[0.71, 0.93]

Suppose that the investor's initial sentiment vector on the selected four financial assets is

$$S = (s_{11}, s_{12}, s_{21}, s_{22}) = (0, -1, 1, 2).$$

If we choose $f(s) = e^{0.2s}$ as the sentiment influential function, we can compute the sentiment influential function value vector as

$$f(S) = (f(s_{11}), f(s_{12}), f(s_{21}), f(s_{22}))$$
$$= (e^{0.2s_{11}}, e^{0.2s_{12}}, e^{0.2s_{21}}, e^{0.2s_{22}})$$
$$= (1, 0.8187, 1.2214, 1.4918).$$

In this example we let the lower boundary l_{ij} and upper boundary u_{ij} of investment proportion of risky asset j at mental account i is 0 and 1, respectively. Assume $\overline{r}_1 = [0.02, 0.08]$, $\overline{r}_2 = [0.2, 0.3]$ are the given minimum expected interval return of the portfolio for mental account 1 and 2, respectively. And we let $\alpha_1 = 0.8$ be the possibility degree that the interval return of the first mental account MA_1 is greater than $\overline{r}_1 = [0.02, 0.08]$, and $\alpha_2 = 0.1$ is the given possibility degree that the interval return of the second mental account MA_2 exceeds the minimum return aspiration $\overline{r}_2 = [0.2, 0.3]$. In order to obtain the corresponding portfolio solution $x = (x_{11}, x_{12}, x_{21}, x_{22})$, we construct the following sentiment-adjusted interval behavioral portfolio model (P2).

$$\text{(P2)} \ \max \sum_{i=1}^{2} w_i \sum_{j=1}^{2} \frac{x_{ij}(R_{ijL} + R_{ijU})}{2} f(s_{ij})$$

s.t.

$$P([\sum_{j=1}^{2} x_{1j} R_{1jL}, \sum_{j=1}^{2} x_{1j} R_{ijU}]$$
$$\geq \overline{r}_1 = [0.02, 0.08]) \geq \alpha_1 = 0.8,$$

$$P([\sum_{j=1}^{2} x_{2j} R_{2jL}, \sum_{j=1}^{2} x_{2j} R_{2jU}]$$
$$\geq \overline{r}_2 = [0.2.0.3]) \geq \alpha_2 = 0.1,$$
$$x_{11} + x_{12} + x_{21} + x_{22} = 1,$$
$$0 \leq x_{11}, x_{12}, x_{21}, x_{22} \leq 1.$$

In fact, the above interval portfolio model is equivalent to the following four linear programming models (**P21-24**).

(P21)

$$\max w_1(0.365x_{11} + 0.2579x_{12}) + w_2(0.9038x_{21} + 1.2233x_{22})$$

s.t.

$$x_{11}R_{11L} + x_{12}R_{12L} \geq 0.08,$$

i.e., $0.23x_{11} + 0.17x_{12} \geq 0.08$

and

$$x_{21}R_{21L} + x_{22}R_{22L} \geq 0.3,$$

i.e.,

and $0.62x_{21} + 0.71x_{22} \geq 0.3,$

$$x_{11} + x_{12} + x_{21} + x_{22} = 1,$$
$$0 \leq x_{11}, x_{12}, x_{21}, x_{22} \leq 1.$$

(P22)

$$\max w_1(0.365x_{11} + 0.2579x_{12}) + w_2(0.9038x_{21} + 1.2233x_{22})$$

s.t.

$$1 - \frac{0.08 - \sum_{j=1}^{2} x_{1j} R_{1jL}}{(0.08 - 0.02) + \sum_{j=1}^{2} x_{1j}(R_{1jU} - R_{1jL})} \geq 0.8,$$

$$1 - \frac{0.3 - \sum_{j=1}^{2} x_{2j} R_{2jL}}{(0.3 - 0.2) + \sum_{j=1}^{2} x_{2j}(R_{2jU} - R_{2jL})} \geq 0.1,$$
$$x_{11} + x_{12} + x_{21} + x_{22} = 1,$$
$$0 \leq x_{11}, x_{12}, x_{21}, x_{22} \leq 1.$$

The above model (P22) can also be transformed into the following linear programming model.

(P'_{22})

$$\max w_1(0.365x_{11} + 0.2579x_{12}) + w_2(0.9038x_{21} + 1.2233x_{22})$$

s.t.

$$0.08 - (0.23x_{11} + 0.17x_{12})$$
$$\leq 0.2 \times [0.06 + 0.27x_{11} + 0.29x_{12}],$$

i.e., $0.284x_{11} + 0.228x_{12} \geq 0.068$

and

$$0.3 - (0.62x_{21} + 0.71x_{22}) \leq 0.9 \times [0.1 + 0.24x_{21} + 0.22x_{22}],$$

i.e.,

and $0.836x_{21} + 0.908x_{22} \geq 0.21,$

$$x_{11} + x_{12} + x_{21} + x_{22} = 1,$$
$$0 \leq x_{11}, x_{12}, x_{21}, x_{22} \leq 1.$$

(P23)

$$\max w_1(0.365x_{11} + 0.2579x_{12}) + w_2(0.9038x_{21} + 1.2233x_{22})$$

s.t. $1 - \dfrac{0.08 - \sum_{j=1}^{2} x_{1j} R_{1jL}}{(0.08 - 0.02) + \sum_{j=1}^{2} x_{1j}(R_{1jU} - R_{1jL})} \geq 0.8$

i.e.,

and $0.284x_{11} + 0.228x_{12} \geq 0.068,$

$$x_{21}R_{21L} + x_{22}R_{22L} \geq 0.3$$

i.e.,

$$0.62x_{21} + 0.71x_{22} \geq 0.3$$

and $x_{11} + x_{12} + x_{21} + x_{22} = 1,$

$$0 \leq x_{11}, x_{12}, x_{21}, x_{22} \leq 1.$$

(P24)

$$\max w_1(0.365x_{11} + 0.2579x_{12}) + w_2(0.9038x_{21} + 1.2233x_{22})$$

s.t. $x_{11}R_{11L} + x_{12}R_{12L} \geq 0.08$,

i.e., $0.23x_{11} + 0.17x_{12} \geq 0.08$,

and $1 - \dfrac{0.3 - \sum_{j=1}^{2} x_{2j} R_{2jL}}{(0.3-0.2) + \sum_{j=1}^{2} x_{2j}(R_{2jU} - R_{2jL})} \geq 0.1,$

i.e.,

and $0.836x_{21} + 0.908x_{22} \geq 0.21,$

$$x_{11} + x_{12} + x_{21} + x_{22} = 1,$$

$$0 \leq x_{11}, x_{12}, x_{21}, x_{22} \leq 1.$$

According to the different importance agree the investor regarding each mental account, we consider three different types of investment importance vectors for the two mental accounts as W1=(0.1,0.9), W2=(0.2,0.8), W3=(0.5,0.5), W4=(0.8,0.2), W5=(0.9,0.1). Then, we apply the optimization software package Lingo to solve the above-mentioned four linear programming models. Finally, we get the optimal behavioral portfolio strategy, which is the solver corresponding to maximum objective function value of portfolio. The optimal behavioral investment portfolio solution corresponding to the maximum objective function value or maximum sentimental mean regarding the different weight of mental accounts are easily computed as listed in the following Table 2.

Table 2. The Optimal behavioral portfolio strategy regarding different weight vector of mental accounts

Weight vector of mental accounts	Maximum objective function value of sentimental mean of portfolio	x_{11}^*	x_{12}^*	x_{21}^*	x_{22}^*
W1=(0.1,0.9)	0.8461197	0.2394	0.0001	0	0.7605
W2=(0.2,0.8)	0.7617662	0.239	0	0	0.761
W3=(0.5,0.5)	0.5089338	0.23	0.01	0	0.76
W4=(0.8,0.2)	0.2810606	0.768	0	0.001	0.231
W5=(0.9,0.1)	0.2808106	0.7687	0	0.0001	0.2312

5. Summary and Conclusion

In this paper, we consider the multi-account behavioral portfolio selection problem in interval uncertain environment. We use the sentiment-adjusted mean value to measure the interval return of the behavioral portfolio. Furthermore, based on the possibility degree of the interval return of each mental account exceeding the given minimum return aspiration we propose a sentiment-adjusted behavioral portfolio model with interval return and investor's sentiment. In order to solve the proposed model, we transform it into the equivalent linear programming models. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.

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References

[1] Markowitz H., Portfolio selection, Journal of Finance, 1952, 7, 71-93.
[2] Fang Y., Lai K.K., Wang S.Y., Portfolio rebalancing model with transaction costs based on fuzzy decision theory, European Journal of Operational Research, 2006, 175, 879-893.
[3] Gupta P., Mehlawat M.K., Saxena A., Asset portfolio optimization using fuzzy mathematical programming, Information Sciences, 2008, 178, 1734-1755.
[4] Yue W., Wang Y.P., A new fuzzy multi-objective higher order moment portfolio selection model for diversified portfolios, Physica A: Statistical Mechanics and its Applications, 2017, 465(1), 124-140.
[5] Kahneman D., Tversky A., The framing of decisions and the psychology of choice, Science, 1981, 211, 453-458.
[6] Lopes L., Between hope and fear: The psychology of risk, Advance in Experimental Social Psychology, 1987, 20: 255-295.
[7] Kahneman D., Tversky A., Prospect theory: An analysis of decision making under risk, Econometrica, 1979, 47, 263-291.
[8] Michael J. Besta, Robert R. Grauer, Prospect theory and portfolio selection, Journal of Behavioral and Experimental Finance, 2016, 11, 13-17.
[9] Thaler R., Mental accounting and consumer choice, Marketing Sciences, 1985, 4(3): 199-214.
[10] Shefrin Hersh, Meir Statman, Behavioral portfolio theory, Journal of Financial and Quantitative Analysis, 2000, 35(2): 127-151.
[11] Ma Y.K., Tang X.W., Decision making method for behavioral portfolio choice, Journal of Systems Engineering, 2003, 18(1), 71-76.
[12] Yaz Gulnur Muradoglu, Aslihan Altay-Salih, Muhammet Mercan, A behavioral approach to efficient portfolio formation, The Journal of Behavioral Finance, 2010, 6(4): 202-212.
[13] Mukesh Kumar Mehlawat, Behavioral optimization models for multicriteria portfolio selection, Yugoslav Journal on Operations Research, 2013, 23(2), 279-297.
[14] Amelia Bilbao-Terol, Mar Arenas-Parra, Veronica Canal-Fernandez, Celia Bilbao-Terol, Multi-criteria decision making for choosing socially responsible investment within a behavioral portfolio theory framework: a new way of investing into a crisis environment, Ann. Oper. Res., 2016,247, 549-580.
[15] Jin X., Chen N., Yuan Y., Multi-period and tri-objective uncertain portfolio selection model: A behavioral approach, North American Journal of Economics and Finance, 2019, 47, 492-504.
[16] Xie J., Yang C.P., Lack of diversification and investor sentiment-The behavioral assets portfolio based on sentiment recognition, Soft Sciences, 2012, 26, 8, 131-135.
[17] Bai C. Z., Zhang R., Shen S., Huang C.F., Fan X., Interval-valued probabilistic linguistic term sets in multi-criteria group decision making, International Journal of Intelligent Systems, 2018, 33(6), 1301-1321.
[18] Xu Z.S, Da Q.L., New Method for Interval multi-attribute decision-making, Journal of Southeast University, 2003, 33(4), 498-501.
[19] Xu Z.S., Da Q.L., Multi-attribute decision-making based on fuzzy linguistic assessments, Journal of Southeast University, 2002, 32(4), 656-658.
[20] Xu Z.S., A practical method for priority of interval number complementary judgment matrix, Operations Research and Management Science, 2001, 10, 16-1.
[21] Giove S., Funari S., Nardelli C., An interval portfolio selection problem based on regret function, European Journal of Operational Research 2006, 170, 253-264.
[22] Zhang P., An interval mean-average absolute deviation model for multiperiod portfolio selection with risk control and cardinality constraints, Soft Comput, 2015, 1-10.
[23] Wu M., Kong D.W., Xu J.P., Huang N.J., On interval portfolio selection problem, Fuzzy Optim Decis Making 2013, 12, 289-304.

[24] Chen G.H., Chen S., Wang S.Y., Interval Number fuzzy selection model, Systems Engineering, 2007, 25(8), 34-37.

[25] Mohagheghi Vahid , Meysam Mousavi S., Vahdani Behnam, A New Optimization Model for Project Portfolio Selection Under Interval-Valued Fuzzy Environment, Arab J Sci Eng, 2015, 8, 1-11.

[26] Mohagheghi V., Mousavi S. M., Vahdani B., Shahriari M. R., R&D project evaluation and project portfolio selection by a new interval type-2 fuzzy optimization approach, Neural Comput & Applic, 2016, 3, 1-20.

[27] Rupak Bhattacharyya , Samarjit Kar, Dwijesh Dutta Majumder, Fuzzy mean-variance-skewness portfolio selection models by interval analysis, Computers and Mathematics with Applications , 2011, 61, 126-137.

[28] Li J., Xu J.P., A class of possibilistic portfolio selection model with interval coefficients and its application, Fuzzy Optim Decis Making, 2007, 6, 123-137.

[29] Guo H.F., Sun B.Q., Karimi Hamid Reza, Ge Y.J., Jin W.Q., Fuzzy Investment Portfolio Selection Models Based on Interval Analysis Approach, Mathematical Problems in Engineering, Volume 2012, Article ID 628295, 1-15.

[30] Xu X.N., He F., Chen R., Zhang Q.Z., Solving non-linear portfolio optimization problems with interval analysis, Journal of the Operational Research Society, 2015, 66(6), 885-893.

[31] MASAAKI IDA, Solutions for the Portfolio Selection Problem with Interval and Fuzzy Coefficients, Reliable Computing, 2004, 10, 389-400.

[32] Feng B.J., Yan D.W., Chi G.T., Assets and liabilities optimization model of the risk control based on nonlinear interval number, Chinese Journal of Management Science, 2012, 20(1), 79-89.

[33] Liu Y.J., Zhang W.G., Zhang P., A multi-period portfolio selection optimization model by using interval analysis, Economic Modelling, 2013, 33, 113-119.

[34] Zhao Y.M., Chen H.Y., Multiple objective Interval number linear programming model for the portfolio investment, Operations and Managements, 2006, 15(2), 124-127.

[35] Lu Y.J., Tang X.W., Zhou Z.F., Interval number linear programming method for the portfolio investment, Journal of Systems Engineering, 2004, 19(1), 33-37.

[36] Lai K.K, Fang Y., Wang S.Y., A class of linear interval programming problems and its applications to portfolio selection, IEEE Transaction on Fuzzy Systems, 2002,10(6), 698-704.