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The Impact of Longevity Risk on the Price of Life Insurance with the Accidental Option (Type AI and ADI)

Magdalena Homa*

Institute of Economic Sciences, University of Wroclaw, Wroclaw, Poland
*Corresponding author: magdalena.homa@uwr.edu.pl

Abstract Regardless of the type of insurance the basic part of the insurance premium is the pure premium, often referred to as net premium, which is based on the risk of death of the insured, averaged over the entire insurance period. The net premium is therefore the fund amount that the insured should pay in order for the insurer to possess sufficient funds to cover all benefits during the contract period. In accordance with the principles of actuarial mathematics the valuation of standard life insurance is based on the equivalence principle and takes into account the risk of death and also the change in time value of money, i.e. actuarial risk, which is estimated by the insurer at the time of signing the contract. Such method of calculating the net premium does not ensure the protection of collected funds against aggregated and individual longevity risk, which may negatively affect long-term financial stability of insurers as well as the level of financial security of the insured. Hence it is necessary to, on the one hand precisely valuate the trend of further life expectancy, and on the other there is a need to modify the methods of calculating premium rates and their adjustment in the duration of the contract. Therefore the article includes premium calculations that take into account the extended actuarial risk that encompasses longevity risk and its effect on their amount. This problem is the aim of the analysis carried out in the article. That is to say, taking into account the extended actuarial risk that encompasses longevity risk, a fair premium formula including the need for a risk adjustment was presented, followed by a premium calculation based on the example of life insurance with the AI and ADI option, as well as a look into the effect of change in longevity risk on their amount.

Keywords: *multi-state life insurance, longevity risk, premium*

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1. Introduction

According to the International Association of Insurance Supervisors (IAIS) those who bear the risk of longevity will have to pay extra for each year of longevity underestimation. IAIS emphasizes that life expectancy is rising faster than expected, thus posing a challenge to those taking responsibility for the risk of longevity (life expectancy). This risk is a potential risk related to the rising life expectancy of life insurance beneficiaries. In IAIS' opinion the market situation should be monitored and those institutions that take the longevity risk should counteract the problems resulting from rise of life expectancy [1].

In the insurance industry challenges associated with the risk of longevity are related to future changes in the longevity trend and the fact that it will be different than assumed in the original calculations, i.a. an insurance premium. In other words, it is the risk of inaccurate valuation of the future trend of the mortality rate [2], which significantly determines the amount of due premium and causes the premium to no longer be a “fair price”. As a consequence of risk, insurance companies cannot fully rely on official national indicators and have to manage the risk of longevity at their own risk. Because the observed changes in mortality rates imply unexpected increases in life expectancy, the corresponding actuarial values underlying the calculation of premiums also increase. Therefore, in order to handle this long-term risk, insurers should ultimately create higher accumulated capital to secure future payments [3]. Thus, it has on the one hand become essential to manage the longevity risk using life expectancy tables containing the longevity trend prognoses, which will act as the basis for estimating the projected life expectancy, and on the other insurance companies will be forced to carry out an appropriate premium adjustment during the insurance period. Therefore the aim of this work is to first study the impact of the mortality rate risk on the amount of due premiums and the resulting financial shortages during the insurance period as well as the proposal to calculate the net premium taking into account the extended actuarial risk granting the insurance company the possibility of compensation ensuring its solvency. Thus, a necessity to adjust the risk of longevity was proposed, and the associated cost (level of the adjustment required) was assessed by comparing the fair price of the insurance while using a standard approach based on national mortality rates and taking into consideration the longevity risk estimated on the basis of trend prognoses. This aspect was analyzed from the point of view of an insurer.

2. Literature Review

Longevity risk is one of the largest, though the least understandable, types of risk to which both insurance companies and annuity and pension-paying entities are exposed [4]. Particularly the risk associated with paying out pensions and annuities, which turned out to be longer than expected, has become the main cause for concern in terms of stability of the existing savings products for retirement purposes [5]. Therefore, literature on the subject has considered many ways to correctly model mortality processes and also to protect against the risk of longevity. To reflect the future evolution of the mortality rate deterministic modeling traditionally used and appropriate analytical models were applied. In recent years, the ability of insurance companies to correctly read demographic trends has improved significantly, but annuity and pension-paying agents and those offering insurance products are still very much at risk of longevity. In reality, even if the mortality rates are correctly predicted an uncertainty in regards to their future tendencies still remains. Thus, applying stochastic models is necessary to correctly measure the systematic part of the risk. In regards to this, numerous works refer to using the stochastic approach in modeling mortality dynamics [5-14]. The application of stochastic models is currently even more needed in relation to the development of the Solvency II system. As a way of protection against the risk of longevity, securities linked with the longevity of the base population were proposed, which was assessed [15,16]. Dahl et al. [17] were the first ones to study hedging against the risk of systematic longevity with the use of longevity-linked securities. The existing research considers benefits arising from the application of hedging strategies by using financial risk measures. In addition to the problems widely discussed in literature regarding the correct longevity risk modeling or dealing with it, the correct valuation of products related to longevity is equally important. However this problem is given attention to in literature to a much lesser extent and refers to annuity and pension contracts. Traditionally, valuation of annuity contracts was based on consolidated actuarial techniques, which mitigated the mortality risk, however those methods are ineffective in the context of the growing longevity risk problem. However this problem is given attention to in literature to a much lesser extent and refers to annuity and pension contracts. Traditionally, valuation of annuity contracts was based on consolidated actuarial techniques, which mitigated the mortality risk, however those methods are ineffective in the context of the growing longevity risk problem. Denuit et al. [18] as well as Richter and Weber [19] thus proposed entering contracts according to which benefits would by adjusted over time in accordance with the observed value of the longevity index. In such types of indexed contracts annuity providers reduce the risk resulting from longevity to a "safe" level (with the allowed probability of a shortage) through creating an additional emergency fund. This fund is financed by the insured in the form of an additional fee, which either increases the contractual premium or decreases benefits. In Alho et al. [20] the authors study consequences of those changes.

The problem of the connection between paid benefits and mortality rates is also undertaken in a lot of works [21,22,23]. Denuit et al. [24] suggests an alternative (or supplementary) way of sharing the longevity risk, which encompasses the adjustment of the deferred period but all the same maintaining payments at a constant level. This approach to the problem does not include analysis from the point of view of life insurance and fair premiums. The article therefore proposes the use of an analogical mechanism in life and endowment insurance contracts. This refers to the insurance premium adjustment, the basis of which will be the corrected estimates of the future life expectancy of the insured.

3. Longevity Risk

The risk of longevity applies to both individuals as well as entire demographic age groups (cohort). Individual risk of longevity (sometimes called a specific longevity risk) means that a given person lives longer than expected [25]. Individual risk of longevity, the implementation of which may yield intensely negative consequences for particular entities, does not, however, pose a threat to the financial stability of the insurer. Yet there is still the aggregated longevity risk, sometimes referred to as the risk of trend, which applies to whole populations. It means that in an entire age group (cohort) the average life expectancy will be longer than expected. In other words, it is the risk of inaccurate estimation of the mortality rate trend. Together, specific and aggregated longevity risk amounts to the total longevity risk [2,26].

The only thing closely related to the risk of longevity is the mortality risk, in an analogous way defined as the risk that a person or a group of people will live shorter than expected. Certainly the rise in longevity risk is equivalent to the fall of the risk of mortality and vice versa. Hence the interchangeable usage of the terms in literature. The risk of longevity can be dissected into the following components:

- the risk that someone will die earlier or later than expected (volatility risk),
- the risk of inaccurate estimation of the current level of mortality for a given population (the risk of mortality rate),
- the risk of inaccurate estimation of the future trend in mortality rate (the risk of mortality rate trend).

The risk of volatility and mortality rate are by nature a specific risk, their influence can therefore be reduced by diversification. It means that the people whose life expectancy exceeds the average may be offset by those who do not live to the average. The risk of trend on the other hand, to a large extent remains a systematic risk, therefore a non-diversifiable one. The risk of trend arises from the possibility of unexpected changes in average life expectancy resulting from lifestyle changes, diet, or technological advances. What is particularly important for insurance companies is the aggregated longevity risk, which plays a crucial role in the insurance valuation and the calculations conducted. Because an incorrect estimation of further life expectancy of the insured may cause the collected funds to be insufficient to cover future benefits and may thus disturb the financial fluidity of the insurance company [27].

4. The Concept and Probabilistic Structure of Insurance with the Option

Life and endowment insurance with the possibility of purchasing an additional option is a contract concluded between the insured and the insurer, under which the insurer undertakes to pay the following benefits:

- standard contract (covers basic risk),
- additional contracts (covers extended insurance risk).

The standard life and endowment insurance contract applies to two main cases: life and death.

On the other hand, additional contracts concern life cases distinguished within the first one of them, such as: loss of health, disability, unfortunate accident, serious illness, inability to work, and others. From the point of view of the insured the occurrence of a fortuitous event covered by the insurance contract means a life event, however from the point of view of a company it means a life event as well as the appropriate active policy option referred to as its state. Thus, at a specified time of the insurance contract only one insurance option remains active and it depends on the life event of the insured. Any change in the life situation of the insured (life event change) causes the implementation of the appropriate policy option and involves changing its status so that the model describing the dynamics of life event changes of the insured is simultaneously the model that describes the dynamic character of activation of possible policy options. The simplest probabilistic insurance model with an additional option encompasses three states correspond to the following life events [28]:

H – the insured is healthy,

S – the insured is ill,

D – the insured has died.

In the presented work the probabilistic model has been generalized and used to describe those policies that cover the following accident options specified within the S state:

AI - accident insurance is an insurance option in the case of an unfortunate accident called casualty insurance and includes a single payment as well as annuity paid to the insured.

ADI - accidental death insurance is an insurance in the case of death as a result of an unfortunate accident. Implementing this option involves payment and ends the insurance.

There is a function used to describe the policy option activation process X(t) and it indicates the process status and the active insurance option at the time $t \in T$ of the contract. Life events occurring in the life of the insured are correlated with the particular implementation of the option activation process $\{X(t)\}_{t\in T}$ Because every life event is a random, then for every moment t, X(t) is a random variable and $\{X(t)\}_{t\in T}$ a stochastic process taking values from the finite state space S. The stochastic processes used in the description of the insurance process are the Markov processes. In this case the Markov process is a family of random variables $\{X(t)\}_{t\in T}$ defined on a common probabilistic space $(\Omega, \mathcal{F}, \mathrm{P})$ [29,30]. In the case of Markov processes, conditional probability is not dependent on the entire history of the process $\{X(t)\}_{t\in T}$, but only on its current state, then the probability of transition is marked by the following formula:

$$ {}_t p_x^{jk} = P\big(X(x+t)=k \mid X(x)=j\big) \tag{1}$$

where x- age of the insured at the moment of entering into the contract.

Taking into account the actuarial notation, the following symbols of probability of changing the multi-option policy status with the options AI and ADI have been adopted in the article:

${}_t p_x^{HH}$ - probability that a healthy person of x years of age will also be healthy at x+t years of age, i.e. at the t moment of the contract,

${}_t p_x^{AI}$, ${}_t p_x^{ADI}$ - probabilities that a healthy person of x years of age will fall victim of an accident and use the AI or ADI option at the age of x+t,

${}_t p_x^{AI|AI}$ - probabilities that a sick person using the AI option at age x will use the same option at x+t years of age,

${}_t p_x^{AId}$ - probability of death at x+t years of age of a person who has used the AI option,

${}_t p_x^{Hd}$ - probability of death at x+t years of age of a healthy person of x years of age.

Such descriptions of probabilities of transition fulfill the Kolmogorov differential equations for each one [31].

5. Cash Flow and Its Actuarial Value

Every insurance contract involves specific payments made by the insured (premiums) and the insurer (benefits). These payments create cash flow between the insurer and the insured and determination of their amount is essential to conducting multi-option policy valuation and establishing a fair price. The main role in these calculations is played by the updated cash flow value, i.e. The current equivalent of future payments. The valuation of financial streams B(t) i.e. the function of payment stream values updated in t time made in the T time period is determined by the following formula:

$$ ZB_t(T) = \frac{1}{\upsilon(t)} \int_{\mathrm{T}} \upsilon(\tau)\, dB(\tau) \tag{2}$$

When calculating insurance the payment stream marked by a symbol should acknowledge all payments resulting from the concluded contract. These payments can be divided into two groups. The first one relates to the duration of the option activation process in a particular state. Such types of payments include [32]:

- insurance premiums paid by the insured and the annuities paid by the insurer,
- endowment benefits paid by the insured until the end of the insurance period.

The second group of payments involves changing the state through the option activation process. This group includes benefits paid out in case of health damage or death of the insured. They are paid out at the time of a fortuitous event, therefore at any moment of the insurance duration. The following payment indications have been adopted in this work:

- the premium amount paid by the insured at the t moment is $\pi(t)$,
- the annuity amount paid out at the t moment is $r(t)$,
- the amount of a single endowment benefit paid out by the insurer specified in the contract at the t moment is $d(t)$,
- a single benefit related to the death of the insured at the t moment is equal $c_d(t)$ or $c_{ADI}(t)$ dependent on the active option.

Thus the updated value of accumulated payments is a random process of the following form:

$$\begin{aligned} ZB_t(T \wedge n) &= \frac{\upsilon(n)}{\upsilon(t)} d(n) \sum_{j\in\{AI,H\}} I_{\{X(n)=j\}} \\ &+ \sum_{j\in\{AI,H\}} \int_T \frac{\upsilon(\tau)}{\upsilon(t)} c_d(\tau) dN_{jd}(\tau) \\ &+ \sum_{j\in\{AI,H\}} \int_T \frac{\upsilon(\tau)}{\upsilon(t)} c_{ADI}(\tau) dN_{jADI}(\tau) \\ &+ \int_T \frac{\upsilon(\tau)}{\upsilon(t)} r(\tau) I_{\{X(\tau)=AI\}} d\tau \\ &- \int_T \frac{\upsilon(\tau)}{\upsilon(t)} \pi(\tau) I_{\{X(\tau)=H\}} d\tau \end{aligned} \qquad (3)$$

where

$$N_{jk}(t) = \#\{\tau \in (0,t] : X(\tau - 0) = j, X(\tau) = k\}.$$

The actuarial value of the benefit or premium in standard life insurance is the expected value of the updated benefit or premium amount. In life and endowment insurance with additional options the actuarial value of the payment stream is calculated as the conditional expectation of discounted payments under the condition of the entire history of the process and defined by a general formula [33]:

$$E\left[ZB_t(T \wedge n) | \mathcal{F}_t\right] \qquad (4)$$

where $\{\mathcal{F}_t\}_{0 \le t \le T}$ – filtration describing the process history in the t moment.

In case of standard insurance the filtration is generated by the insurance portfolio and based on the mortality process, i.e.:

$$\mathcal{F}_t = \sigma\{I(T \le t), 0 \le t \le T_x\} \qquad (5)$$

where $T_x \equiv T$– future lifespan of the insured at the age of x.

Taking into consideration the above filtration an actuarial value of payment has been established on the account of:

- living until the end of the insurance period,
- death during while the insurance is still active, constituting the basis for further calculation of the net premium.

Should the insured live to the established moment (the end of the insurance period) the insurer will pay the insured a benefit whose updated value for the moment t is $ZD_t(n)$ and after taking into account the risk of death that value is equal to:

$$I\{T > n\} ZD_t(n) = I\{T > n\} e^{-\delta(n-t)} d(n). \qquad (6)$$

Therefore the actuarial value of the payment is equal to:

$$\begin{aligned} &E\left(ZD_t(n) | \mathcal{F}_t\right) \\ &= E\left(I\{T > n\} | \ T > n\right) E\left(e^{-\delta(n-t)} d(n) | \mathcal{F}_t\right) \\ &= {}_{n-t}p_{x+t}^{HH} \bullet d(n) E\left(e^{-\delta(n-t)} | \mathcal{F}_t\right). \end{aligned} \qquad (7)$$

In case of life insurance on the other hand, the death benefit is paid out to the insured for death (i.e. during the insurance period) and similarly as in the case of endowment insurance portfolio (endowment insurance - EI) also for the life insurance portfolio (life insurance - LI) the actuarial value is equal to:

$$\begin{aligned} &E\left(ZC_t(T \wedge n) | \mathcal{F}_t\right) \\ &= E\left(\int_t^{T\wedge n} e^{-\delta(\tau - t)} c(\tau) dI\{\tau \le n\} | \mathcal{F}_t\right) \\ &= \int_t^{T\wedge n} E\left[e^{-\delta(\tau-t)} c(\tau) | \mathcal{F}_t\right] E\left[dI\{\tau \le n\} | \{T > t\}\right] \\ &= \int_t^{T\wedge n} e^{-\delta(\tau-t)} c(\tau) E\left[I\{\tau \le n\} \mu(x + | \{T > t\}\right] d\tau. \end{aligned} \qquad (8)$$

Taking into account the fact that the payout of the death benefit can apply to a standard contract and thus at the t moment a lump insurance sum equal to $c_D(t)$ is paid out or from an additional agreement and thus due to implementation of the ADI option a sum $c_{ADI}(t)$ the actuarial value paid out is equal to:

$$\begin{aligned} &E\left(ZC_t(T \wedge n) | \mathcal{F}_t\right) \\ &= \sum_{j\in\{H,AI\}} \int_t^{T\wedge n} e^{-\delta(\tau-t)} c_d(\tau) \, {}_{\tau-t}p_{x+t}^{Hj} \mu_{jd}(x+\tau) d\tau \\ &\sum_{j\in\{H,AI\}} \int_t^{T\wedge n} e^{-\delta(\tau-t)} c_{ADI}(\tau) \, {}_{\tau-t}p_{x+t}^{Hj} \mu_{jADI}(x+\tau) d\tau. \end{aligned} \qquad (9)$$

The last type of benefits foreseen by the contract covers the disability annuity as a result of an unfortunate accident. The actuarial value of such annuity is equal to:

$$E\left(ZR_t(T \wedge n) | \mathcal{F}_t\right) == \int_t^{T\wedge n} e^{-\delta(\tau-t)} r(\tau) \, {}_{\tau-t}p_{x+t}^{HAI} d\tau. \qquad (10)$$

6. The Equivalence Principle and Fair Price

The calculation of an insurance premium is one of the tasks underlying an insurance business. The premium should be established in a way to ensure funds that will guarantee the payout of compensations, creation of insurance technical reserves, and covering costs related to operating an insurance business. Defined in such way, it is called the gross premium and constitutes the price that the insurer pays the social insurance institution for guaranteed insurance security. That part of the premium which is supposed to be sufficient to pay compensation is called the net premium and forms the basis for insurance valuation.

In life and endowment insurance the value of the net premium is determined based on the expected value of future discounted cash flows, that is their actuarial values. The rule of premium accrual is founded on the principle of pure risk referred to as the classic equivalence principle, according to which the actuarial value of premiums and benefits under the insurance contract throughout the duration of the insurance should balance. Based on the actuarial value in accordance with the equivalence principle the calculation of an insurance premium is done in the following way [33,34]:

$$E(\textit{present value of the future premiums}) \\ = E\left(\textit{present value of the future payment}\right).$$

Thus while conducting a valuation at $t_0 = 0$ moment the equivalence principle takes the following form [29]:

$$\begin{aligned} &E\left(Z\Pi_0(n)|\mathcal{F}_0\right)=E\left(ZB_0(T\wedge n)|\mathcal{F}_0\right) \\ &=E\left(ZD_0(n)+ZC_0(T\wedge n)+ZR_0(T\wedge n)|\mathcal{F}_0\right). \end{aligned} \quad (11)$$

The net premium arising from the above principle is called a fair price and is determined based on actuarial values considering the risk-free rate and the mortality risk. If an insurance company possesses a large portfolio then in consonance with the law of large numbers the risk of death is diversified, however an underestimated longevity risk will result in different premium amounts. In accordance with the equivalence value, using the derived formulas for the actuarial value we can indicate:

- a single fixed amount net premium paid in advance for the entire duration of the insurance, that is at the moment of concluding the contract (Π_0),
- a periodic premium paid throughout the duration of the insurance period.

By substituting the above formula with actuarial values of the benefits linked to life and endowment insurance with the AI+ADI options the actuarial value (at the moment the insurance contract is concluded) of the sum of all benefits is obtained. A single net premium for such policy determined according to the equivalence principle is given by the following formula:

$$\begin{aligned} &\Pi_0 = {}_n p_x^{HH} d(n) E\left(e^{-\delta n}|\mathcal{F}_0\right) \\ &+\sum_{j\in\{H,AI\}}\int_t^{T\wedge n} e^{-\delta\tau}\, {}_\tau p_x^{Hj}\begin{pmatrix} c_d(\tau)\mu_{jd}(x+\tau) \\ +c_{ADI}(\tau)\mu_{jADI}(x+\tau)\end{pmatrix} d\tau \quad (12) \\ &+\int_t^{T\wedge n} e^{-\delta\tau} r(\tau)\, {}_\tau p_x^{HAI}\, d\tau. \end{aligned}$$

The single net premium is therefore the sum of part of the premium intended to cover the risk resulting from the basic contract as well as the additional contracts that constitute additional options to the basic insurance.

Regardless of the type of contract the net premium is intended to cover current and future benefits, meaning it is entirely intended to cover death (or endowment) benefits to the insured by the company. Thus, in every case an inaccurate estimation of the probability of death means that a premium determined in such a way is not fair and, depending on the portfolio structure, can lead to insufficiency in funds to be paid out.

The insured can pay the premiums not only in the form of a single payment but also for any period of the insurance term. The insured pays the premiums when he or she is healthy and this corresponds to the situation when no option is active, that is, when the process $\{X(t)\}$ is in the H state. Then the equivalence principle marked by a formula takes the following form:

$$\begin{aligned} &\int_t^{T\wedge n} e^{-\delta\tau}\pi(\tau)\, {}_\tau p_x^{HH}\, d\tau \\ &== E\left(ZD_0(n)+ZC_0(T\wedge n)+ZR_0(T\wedge n)|\mathcal{F}_0\right). \end{aligned} \quad (13)$$

By replacing the formula with actuarial values of benefits and assuming that the premiums are being paid with a constant intensity, that is $\pi(\tau) = \pi$. Its amount is calculated according to the following formula:

$$\begin{aligned} \pi =& \frac{{}_n p_x^{HH} d(n) E\left(e^{-\delta\tau}|\mathcal{F}_0\right)}{\int_t^{T\wedge n} e^{-\delta\tau}\, {}_\tau p_x^{HH}\, d\tau} + \\ &+\frac{\sum_{j\in\{H,AI\}}\int_t^{T\wedge n} e^{-\delta\tau}\, {}_\tau p_x^{Hj} c_d(\tau)\mu_{jd}(x+\tau)d\tau}{\int_t^{T\wedge n} e^{-\delta\tau}\, {}_\tau p_x^{HH}\, d\tau} \\ &+\frac{\sum_{j\in\{H,AI\}}\int_t^{T\wedge n} e^{-\delta\tau}\, {}_\tau p_x^{Hj} c_{ADI}(\tau)\mu_{jADI}(x+\tau)d\tau}{\int_t^{T\wedge n} e^{-\delta\tau}\, {}_\tau p_x^{HH}\, d\tau} \\ &+\frac{\int_t^{T\wedge n} e^{-\delta\tau} r(\tau)\, {}_\tau p_x^{HAI}\, d\tau}{\int_t^{T\wedge n} e^{-\delta\tau}\, {}_\tau p_x^{HH}\, d\tau}. \end{aligned} \quad (14)$$

If an insurance company possesses a large portfolio then, in consonance with the law of large numbers the risk of death is diversified, however when calculating the premium the insurer relies on historical knowledge regarding death risk, therefore its change in the duration of the insurance period cannot be taken into account in the given premium. Thus, the premium will not cover the actual risk taken by the insurer, and the underestimated longevity risk will result in other premium amounts. Such way of calculating the net premium does not ensure the protection of collected funds against aggregated and individual risk of longevity, which can negatively affect the long-term financial stability of insurers. Therefore in the case of life insurance not only is a precise dynamic estimation of the trend of a further life expectancy necessary, but there is also a need for an appropriate adjustment of premium amounts during the insurance period. The solution is an appropriate modification of the insurance premium adjusted to change in longevity risk in the duration of the insurance period. This is why it is suggested to, taking into account the specifics of longevity risk, use methods based on the equivalence principle that consider appropriate risk allowance. An example of such solution is the application of the expected value principle, according to which:

$$E\left(Z\Pi_0(n)|\mathcal{F}_0\right)=(1+\alpha)E\left(ZB_0(T\wedge n)|\mathcal{F}_0\right) \quad (15)$$

where α ∈ (0,1).

Parameter α is the adjustment adapting the "fair premium" amount to the structure of actuarial risk, particularly encompassing the risk of the mortality trend and change in the distribution. Clearly from the point of view of the insurance company it is crucial to appropriately establish the adjustment, because both an overestimate as well as an underestimate of a premium can be detrimental. Setting it at a too-low level can lead to collecting insufficient funds to cover compensations and, as a consequence, waver the financial stability of the insurance company. Whereas setting the premium too high, while assuming a protective stance, can lead to losses related to a lack of market competition and the outflow of policies. Therefore it is suggested that the insurer set the premium adjustment based on financial shortages arising during the insurance period as a result of changes in mortality rate trend in time.

7. Examples of Application – Results

As an example a futures contract of an n-year standard life insurance LI, endowment insurance EI or mixed insurance ELI (depending on the variant) was analyzed, along with additional contracts acknowledging the accident options: ADI and AI.

The subject of the insurance is therefore the death of the insured or a total disability being the result of an accident. Under such basic contract an insurance company pays out a single benefit in the amount of c m.u. (monetary unit) in case of death of the insured or in the amount of d m.u. in case of the insured living to the end of the insurance period depending on the variation adopted. If the insured suffers permanent health damage as a result of an accident, the insurance company according to the additional contract at the moment of recognizing the disability pays sum insured or disability pension. Cash flows characteristic of this type of insurance are presented in Table 1.

The subject of insurance of the basic contract is the life of the insured who will receive a death insurance sum depending on the adopted variation in the duration of the insurance or living to the end of the insurance period. The subject of the additional contract is the health of the insured, and the AI and ADI options cover full or partial permanent disability caused by an accident. It is particularly related to the payout of a benefit for permanent disability caused by an accident and the benefit amount for death caused by an accident (which is usually double the insurance sum) established within the General Terms and Conditions of Insurance. Such insurance involves the following payment streams:

1. Premiums paid in the duration of the insurance period.
2. Annuity paid out for permanent disability.
3. Benefit for death as a result of an accident.
4. Benefit for the death of the insured within the life insurance or mixed insurance.
5. Endowment benefit within the life insurance or endowment insurance.

In order to determine the premium amount it is necessary to determine the updated and actuarial value of the payment streams arising from the contract. The actuarial value of total cash flows for the multi-option insurance with the AI+ADI option is presented in Table 2.

Table 1. Payment streams for insurance with additional options: AI and ADI

status	premiums	Accident annuity	Benefits in case of living to the end of insurance period	Single accident benefit	Single death benefit
H	$\pi(t)=\pi$	-	$d(t)=d$	-	$c_{Hd}(t)=c$
AI	-	$r_{AI}(t)=r$	$d(t)=d$	-	$c_{AId}(t)=c$
ADI	-	-	-	$c_{ADI}(t)=\beta c$	-

Table 2. Actuarial value of future cash flows for insurance with additional options AI and ADI

Policy status	payment	Actuarial value
premiums	$\pi_H(t)\approx\pi$	$E(Z\Pi_t\|H)=\frac{\pi}{v(t)}\int_t^n v(\tau)\left({}_{\tau-t}p_{x+t}^{HH}\right)d\tau$
accident annuity	$r_{AI}(t)\approx r$	$E(ZR_t\|H)=\frac{r}{v(t)}\int_t^n v(\tau)\left({}_{\tau-t}p_{x+t}^{AI}\right)d\tau$ $E(Z\Pi_t\|AI)=\frac{r}{v(t)}\int_t^n v(\tau)\left({}_{\tau-t}p_{x+t}^{AI\|AI}\right)d\tau$
benefits in case of living to the end of insurance period	$d_H(t)\approx d$ $d_{AI}(t)\approx d$	$E(ZD_t\|H)=d\frac{v(n)}{v(t)}\left({}_{\tau-t}p_{x+t}^{HH}+{}_{\tau-t}p_{x+t}^{AI}\right)$ $E(ZD_t\|AI)=d\frac{v(n)}{v(t)}{}_{\tau-t}p_{x+t}^{AI\|AI}$
single death benefit	$c_{Hd}(t)\approx c$ $c_{AId}(t)\approx c$	$E(ZC_t\|H)=\frac{c}{v(t)}\int_t^n v(\tau)\mu(x+\tau)\left({}_{\tau-t}p_{x+t}^{HH}+{}_{\tau-t}p_{x+t}^{AI}\right)d\tau$ $E(ZC_t\|AI)=\frac{c}{v(t)}\int_t^n v(\tau)\,{}_{\tau-t}p_{x+t}^{AI\|AI}\mu(x+\tau)d\tau$
single accident benefit	$c_{ADI}(t)\approx 2c$	$E(ZC_t\|H)=\frac{2c}{v(t)}\int_t^n v(\tau)\sigma(x+\tau)\,{}_{\tau-t}p_{x+t}^{HH}d\tau$

The above formulas for the actuarial value of particular flows constitute the basis for the conducted premium calculations. The amount of a single net premium for life insurance with the AI+ADI option is therefore expressed by the following formula:

$$\Pi_0 = de^{-\delta n}\left({}_n p_x^{HH} + {}_\tau p_x^{AI}\right) + c\int_t^n e^{-\delta\tau}\left({}_\tau p_x^{HH} + {}_\tau p_x^{AI}\right)\mu(x+\tau) + 2c\int_t^n e^{-\delta\tau}\,{}_\tau p_x^{HH}\,\sigma_{ADI}(x+\tau) + r\int_t^{T\wedge n} e^{-\delta\tau}\,{}_\tau p_x^{AI}\,d\tau. \tag{16}$$

In order to consider the changes in mortality risk in the further analysis and conducted calculations to set the probability of survival and death, the following were used:

- current lifetime tables (variant I),
- prognoses considering the risk of longevity, ie. changes in the mortality rate trend (variant II).

Based on this, parameters in the mortality function $\mu(x+\tau)$ were estimated in two variations and the intensity of an unfortunate event happening $(x+\tau)$, which were based on the Gompertz-Makeham law. The Gompertz model quite accurately describes the actual force of mortality in the 80+ age group, however in regards to a younger population this accurancy is much worse. Thus, an assumption has emerged that besides the two mentioned reasons there are also others, which were not taken into consideration while constructing the model and which cause the Gompertz function to not describe the empirical intensity well. The Gompertz-Makeham model describes the actual intensity of deaths while considering factors dependent on age, which Gompertz had considered, who claimed that mortality (force of mortality) is the result of two types of causes. The first cause are illnesses which affect both young and old people equally. The second cause of deaths is the decline in human's ability to oppose death. Makehan, however, noticed that certain factors also affect this force in an additive way regardless of age. These factors are aggregated and appear in the model in a constant form. The received model is called the Gompertz-Makeham law, and the distribution has a following form:

$$F(x) = \begin{cases} 0 & x \le 0 \\ 1 - exp\left(-Ax - \frac{B}{C}\left(e^{Cx} - 1\right)\right) & x > 0 \end{cases}.$$

The estimators of maximum likelihood of parameters used in the hazard function are presented.

Table 3. Maximum likelihood estimation of parameters Gompertza-Makehama's function

Parameters of the hazard function	variant I $\mu_1(x+t)$	variant II $\mu_2(x+t)$	$\sigma(x+t)$
A	0,0000100	0,000475	0,0004
B	0,000143493	0,000068272	3,4674E-06
C	1,0869039	1,091756	1,148153621

Source: own elaboration based on [3,35].

Based on this, the appropriate probabilities were determined by solving Chapman-Kolmogorov's differential equations with the application of a standard approach based on national mortality rates (variant I) and while taking into account the longevity risk estimated by companies based on trend prognoses (variant II).

In consonance with the presented formulas an accurately calculated premium is the function of the insurance period (n), age of the insured (x) and it is also dependent on the active insurance option and the adopted interest rate. Thus in the further part of the work, using the derived formulas and solving relevant integrals with the help of the Mathematica program, the influence of longevity risk on the net premium amount in life insurance (LI), endowment insurance (EI), and mixed insurance was studied (ELI). The Figure 1 below represent the net premium amount depending on age and the insurance duration for major type of insurance.

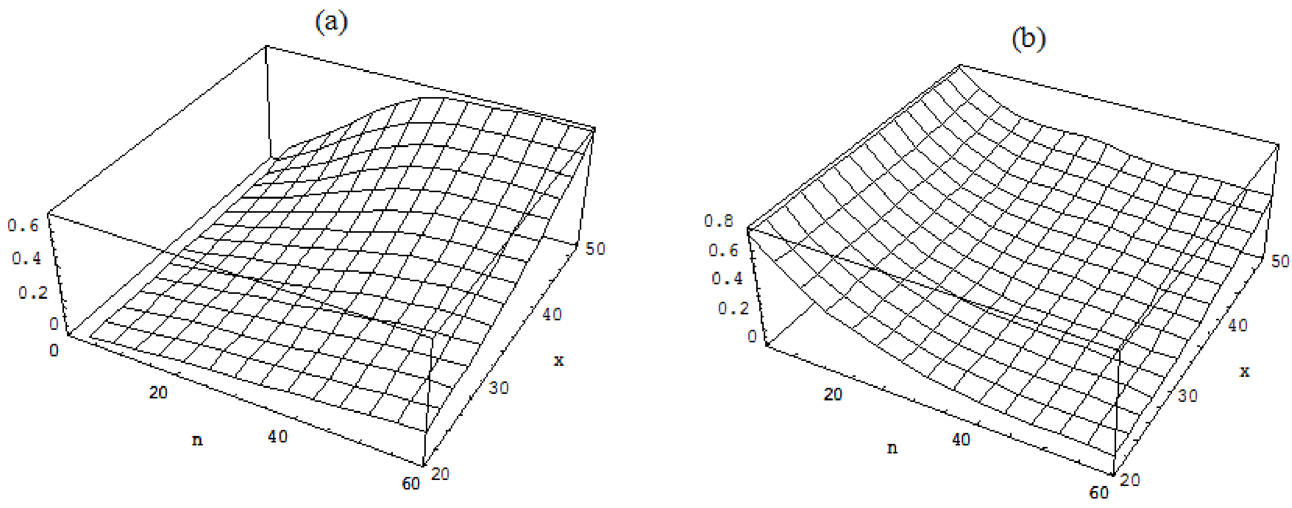

Figure 1. Net premium as the function x and n for life insurance and mixed insurance LI (a) and ELI (b) with the AI +AD

Based on the above graphs we can unequivocally conclude that the premium development is characterized by different trends in the case of LI and EI insurance, which is directly related to the subject of the insured. Therefore influence of longevity risk is of different nature in case of life insurance and endowment insurance contracts. Clearly it stems from the fact that the increase in longevity risk is equivalent with the decrease in mortality risk. So the risk of inaccurate estimation of the future trend of the mortality rate translates into an overestimation of the premium amount in life insurance and its underestimation in case of endowment insurance. The financial shortfalls relate to EI and mixed insurance, which will be analyzed later in the paper. In order to precisely study these dependencies, amounts of a single net premium for particular types of basic contracts in both variations entered into a period of 20 years are presented in Table 4 - Table 7.

Table 4. The fixed net premium amount for a standard 20-year Endowment insurance - variant I i II

x	Variant I	Variant II	changes%
20	0,362346	0,371685	2,6%
25	0,355087	0,369506	4,1%
30	0,344353	0,365197	6,1%
35	0,328685	0,356750	8,5%
40	0,306270	0,340474	11,2%
45	0,275151	0,310186	12,7%
50	0,233877	0,257566	10,1%
55	0,182775	0,187748	2,7%

Table 5. The fixed net premium amount for a standard 20-year Mixed insurance insurance - variant I i II

x	Variant I	Variant II	changes%
20	0,379804	0,384005	1,1%
25	0,380624	0,387600	1,8%
30	0,382253	0,392981	2,8%
35	0,385476	0,400980	4,0%
40	0,391797	0,412754	5,3%
45	0,403992	0,429822	6,4%
50	0,426770	0,454020	6,4%
55	0,466809	0,487234	4,4%

Table 6. The fixed net premium amount for a standard 20-year Endowment insurance with the AI+AD option - variant I i II

x	Variant I	Variant II	changes%
20	0,348905371	0,357488795	2,46%
25	0,350204132	0,363552566	3,81%
30	0,35461384	0,374114562	5,50%
35	0,36586302	0,392525295	7,29%
40	0,3908738	0,424100037	8,50%
45	0,44082823	0,475976732	7,97%
50	0,52926696	0,584164137	10,37%
55	0,6615479	0,767135871	15,96%

Table 7. The fixed net premium amount for a standard 20-year Mixed insurance with the AI+AD option - variant I i II

x	Variant I	Variant II	changes%
20	0,422096931	0,426590137	1,06%
25	0,429846212	0,437330643	1,74%
30	0,44410994	0,455663183	2,60%
35	0,47029582	0,487055248	3,56%
40	0,5176853	0,540382279	4,38%
45	0,60057843	0,628441662	4,64%
50	0,73618416	0,775091259	5,28%
55	0,9332816	0,994157359	6,52%

By analyzing the presented tables we can notice that in case of the insurance types discussed, the premium amount increases with the age of the insured in every case with additional option, which is different from standard endowment insurance. It has been shown that in case of a complex product (with an additional option) the net premium is the sum of the part covering the risk resulting from the basic contract as well as as the part intended to cover the risk related to additional contracts. At the same time it results in larger capital shortages as a consequence of an inaccurate estimation of the trend in future mortality rates. This difference also deepens with the age of the insured which is illustrated by the figures below.

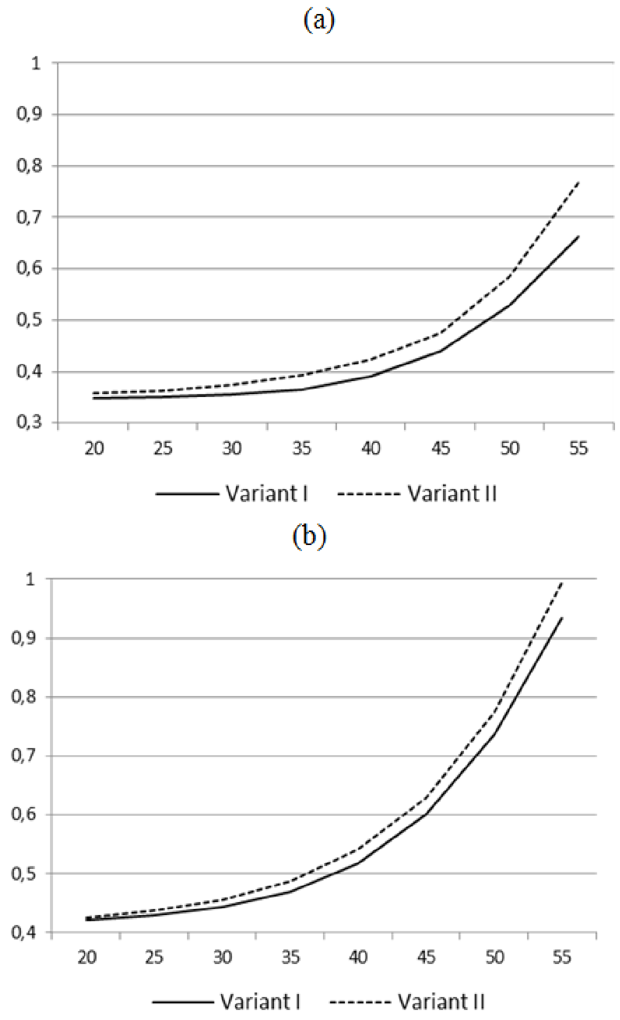

Figure 2. Comparison of the premium amount for a 20-year endowment insurance (a) and mixed insurance (b) estimated in two variations regarding the mortality rates

Insurer does not consider the aggregated longevity risk in terms of increase in longevity risk and the decrease in mortality risk when calculated premiums. Examining insured persons under 45 years of age, a significant part of the net premium in case of all types of insurance contracts (endowment insurance, and mixed insurance) is comprised of the part designated to cover the risk arising from the basic contract. In this case the shortages resulting from the underestimation of longevity risk for complex products is identical with basic products, however due to the long-term nature of contracts entered into with younger individuals the prognoses are more vulnerable and projecting the risk remains is a challenge for insurance companies. In case of contracts entered into with elderly people the insurance period is shortened, nonetheless in this case the vital part of the net premium is the risk related to the payout of additional benefits, which results in an increasing capital deficit. These differences are illustrated below.

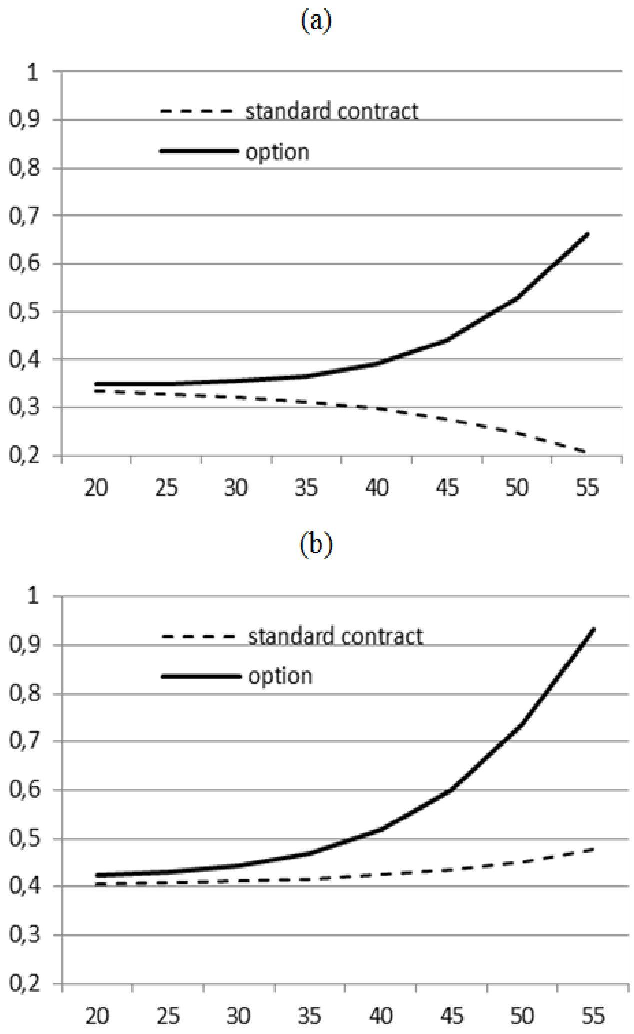

Figure 3. Comparison of the premium amount for a 20-year endowment insurance (a) and mixed insurance (b) estimated in two variations regarding the mortality rates

It is worth noticing that a premium determined in accordance with the equivalence principle characteristic to premium calculations in life insurance is based on the current mortality rates arising from lifetime tables. A solution to this might be the adjustment of premium in the duration of the insurance period, however it is difficult in practice, because in life insurances the insured pays the fixed premium amounts throughout the duration of the insurance period, overpaying it in the first years and therefore the shortages cannot be leveled. Thus in a situation of the underestimation of a net premium resulting in shortages of funds guaranteeing a payout, a premium calculation involving a risk allowance might be the solution. The amount of a net premium for each of the insurance types discussed for minimum security levels equal α(EI)=0,1 i α(ELI)=0,05 is presented in the table below.

Table 8. The single net premium amount for a 20-year insurance with the AI+AD option for minimal various security levels

x	Endowment		Mixed	
	Variant II	min α =0,1	Variant II	min α =0,05
20	0,3574887	0,38379591	0,384005	0,443201778
25	0,3635525	0,38522455	0,387600	0,451338523
30	0,3741145	0,39007522	0,392981	0,466315437
35	0,3925252	0,40244932	0,400980	0,493810611
40	0,4241000	0,42996118	0,412754	0,543569565
45	0,4759767	0,48491105	0,429822	0,630607352
50	0,5841641	0,58219366	0,454020	0,772993368
55	0,7671358	0,72770269	0,487234	0,97994568

The above results prove that the modification of the net premium calculation method gives the possibility of compensation and establishing the funds at a level that will ensure the solvency of the insurer.

Acknowledgements

In this work the classic equivalence principle has been presented according to which insurers valuate the insurance value and establish the net premium amount as the fair price (variant I). In this work the classic equivalence principle has been presented according to which insurers valuate the insurance value and establish the net premium amount as the fair price (variant I). But because the aggregate longevity risk, called to trend risk, is a threat to the financial stability of the insurer and may have severe negative consequences for individual policyholders, a proposal emerged for the necessity to use an appropriate adjustment of the premium amount during the insurance period, adapted to the change in longevity risk (variant II).

The presented results illustrate the influence of lifetime tables on the conducted net premium calculations in insurance and prove the significant influence of the risk of longevity on overestimation or underestimation of premiums depending on the type of contract as well as the age of the insured. The conducted research shows that first, insurance companies cannot rely solely on official national indicators constituting the basis for life insurance valuation only at the moment of entering this type of insurance. It is necessary to incorporate dynamic lifetime tables instead of the traditional ones in the valuation, which will allow taking into account, in premium calculations, adjustment of the trend risk, all the same allowing to eliminate a the detrimental influence of the longevity risk.

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