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Methods for Evaluating Some Integrations Involving Inverse Trigonometric Functions

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Abstract This paper uses the mathematical software Maple as the auxiliary tool to study the integral problems. The infinite series expressions of some types of indefinite integrals involving the powers of inverse trigonometric functions can be obtained by using power series expansions, binomial series and integration term by term theorem. Moreover, some examples are proposed to demonstrate the calculations. The research approach adopted in this study is to get the answers through manual calculations and then use Maple to verify the answers. This kind of research method not only allows us to find the calculation errors, but also helps us to modify the direction of original thinking from manual calculation and Maple calculation. Therefore, Maple provides us insights into the problems.

Keywords: *infinite series expressions, indefinite integrals, inverse trigonometric functions, power series expansions, binomial series, integration term by term theorem, Maple*

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1. Introduction

This paper describes how to use Maple to study mathematics. The main reasons of using Maple in this article are its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. Using the powerful computing capabilities of Maple, difficult problems can be easily solved. Even when Maple cannot determine the solution, problem-solving hints can be identified and inferred from the approximate values calculated and solutions to similar problems, as determined by Maple. For this reason, Maple can provide insights into scientific research. The following is a brief chart to demonstrate our research method:

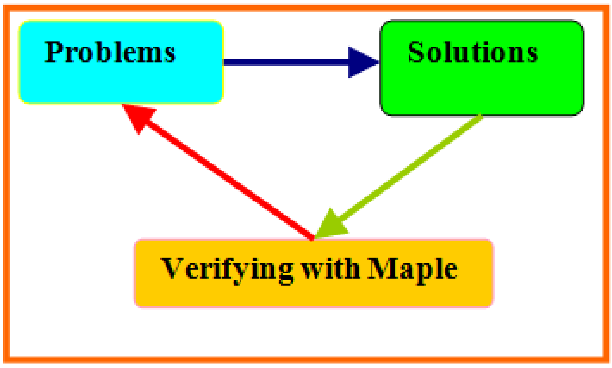

Figure 1. A chart of the research method used in this paper

In calculus and engineering mathematics, there are many methods to solve the integral problems including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, etc. This article considers the following six types of indefinite integrals involving the powers of inverse trigonometric functions which are not easy to obtain their answers using the methods mentioned above.

$$\int \left(\sin^{-1} x\right)^r dx, \tag{1}$$

$$\int \left(\cos^{-1} x\right)^r dx, \tag{2}$$

$$\int \left(\tan^{-1} x\right)^r dx, \tag{3}$$

$$\int \left(\cot^{-1} x\right)^r dx, \tag{4}$$

$$\int \left(\sec^{-1} x\right)^r dx, \tag{5}$$

$$\int \left(\csc^{-1} x\right)^r dx, \tag{6}$$

where r, x are real numbers. We can obtain the infinite series expressions of these types of indefinite integrals by using power series expansions, binomial series and integration term by term theorem; these are the major results of this paper (i.e., Theorems 1-6). Adams et al. [1], Nyblom [2], and Oster [3] provided some methods to solve some integral problems. Yu [4-27], Yu and Huang [28], Yu and Chen [29], and Yu and Sheu [30,31,32] used some techniques which include complex power series expansions, integration term by term theorem, differentiation with respect to a parameter, Parseval's theorem, and area mean value theorem to evaluate some types of integrals. In this article, some examples are used to demonstrate the proposed calculations, and the manual calculations are verified using Maple.

2. Preliminaries and Results

Firstly, the power series expansions of six basic trigonometric functions are introduced below, which can be found in [[33], p44].

2.1. Formulas

2.1.1. $\sin\theta = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}\theta^{2k+1}$ for any real number θ.

2.1.2. $\cos\theta = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\theta^{2k}$ for any real number θ.

2.1.3. $\tan\theta = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}2^{2k}(2^{2k}-1)B_{2k}}{(2k)!}\theta^{2k-1}$

for $|\theta| < \frac{\pi}{2}$, where B_k is the k -th Bernoulli number.

2.1.4. $\cot\theta = \sum_{k=0}^{\infty}\frac{(-1)^k 2^{2k} B_{2k}}{(2k)!}\theta^{2k-1}$, $0 < |\theta| < \pi$.

2.1.5. $\sec\theta = \sum_{k=0}^{\infty}\frac{(-1)^k E_{2k}}{(2k)!}\theta^{2k}$ for $|\theta| < \frac{\pi}{2}$, where E_k is the k -th Euler number.

2.1.6. $\csc\theta = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}2(2^{2k-1}-1)B_{2k}}{(2k)!}\theta^{2k-1}$

for $0 < |\theta| < \pi$.

Next, we introduce two important theorems used in this study, which can be found in ([[34], p269]) and ([[34], p244]) respectively.

2.2. Integration Term By Term Theorem

Let $\{g_n\}_{n=0}^{\infty}$ be a sequence of Lebesgue integrable functions defined on I. If $\sum_{n=0}^{\infty}\int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n = \sum_{n=0}^{\infty}\int_I g_n$.

2.3. Binomial Series

$(1+y)^r = \sum_{k=0}^{\infty}\frac{(r)_k}{k!}y^k$, where r, y are real numbers, $|y| < 1$, $(r)_k = r(r-1)\cdots(r-k+1)$ for any positive integer k, and $(r)_0 = 1$.

In the following, we determine the infinite series expressions of the indefinite integrals (1) and (2) respectively.

Theorem 1 *Suppose that r, x are real numbers, $\frac{-r-1}{2}$ is not a non-negative integer, $|x| \le 1$, C is a constant, and $(\sin^{-1}x)^r$ exists, then*

$$\int\left(\sin^{-1}x\right)^r dx = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!(2k+r+1)}\left(\sin^{-1}x\right)^{2k+r+1} + C. \tag{7}$$

Proof

$\int\left(\sin^{-1}x\right)^r dx$

$= \int \theta^r \cos\theta d\theta\left(let\ \theta = \sin^{-1}x\right)$

$= \int\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\theta^{2k+r}d\theta\left(\text{by Formula 2.1.2}\right)$

$= \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!(2k+r+1)}\theta^{2k+r+1} + C$

(by integration term by term theorem)

$= \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!(2k+r+1)}(\sin^{-1}x)^{2k+r+1} + C.$

q.e.d.

Theorem 2 *If r, x are real numbers, $\frac{-r-2}{2}$ is not a non-negative integer, $|x| \le 1$, C is a constant, and $(\cos^{-1}x)^r$ exists, then*

$$\int\left(\cos^{-1}x\right)^r dx = -\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!(2k+r+2)}\left(\cos^{-1}x\right)^{2k+r+2} + C. \tag{8}$$

Proof $\int(\cos^{-1}x)^r dx$

$= -\int\theta^r \sin\theta d\theta$ (where $\theta = \cos^{-1}x$)

$= -\int\theta^r\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}\theta^{2k+r+1}d\theta$

(by Formula 2.1.1)

$= -\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!(2k+r+2)}\theta^{2k+r+2} + C$

(by integration term by term theorem)

$= -\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!(2k+r+2)}(\cos^{-1}x)^{2k+r+2} + C.$

q.e.d.

Next, the indefinite integrals (3) and (4) can be obtained as the following two theorems respectively.

Theorem 3 *If r, x are real numbers, $\frac{1-r}{2}$ is not a positive integer, and $(\tan^{-1}x)^r$ exists, then*

$$\int\left(\tan^{-1}x\right)^r dx = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}(2k-1)2^{2k}\left(2^{2k}-1\right)B_{2k}}{(2k+1)!(2k+r-1)}\left(\tan^{-1}x\right)^{2k+r-1} + C. \tag{9}$$

Proof $\int(\tan^{-1}x)^r dx$

$$= \int \theta^r \sec^2 \theta d\theta \text{ (where } \theta = \tan^{-1} x\text{)}$$

$$= \int \theta^r \left(\frac{d}{d\theta} \tan \theta \right) d\theta$$

$$= \int \sum_{k=1}^{\infty} \frac{(-1)^{k-1}(2k-1)2^{2k}(2^{2k}-1)B_{2k}}{(2k)!} \theta^{2k+r-2} d\theta$$

(by Formula 2.1.3)

$$= \sum_{k=1}^{\infty} \frac{(-1)^{k-1}(2k-1)2^{2k}(2^{2k}-1)B_{2k}}{(2k)!(2k+r-1)} \theta^{2k+r-1} + C$$

$$= \sum_{k=1}^{\infty} \frac{(-1)^{k-1}(2k-1)2^{2k}(2^{2k}-1)B_{2k}}{(2k)!(2k+r-1)} (\tan^{-1} x)^{2k+r-1} + C.$$

q.e.d.

Theorem 4 *If r, x are real numbers, $\frac{1-r}{2}$ is not a non-negative integer, and $(\cot^{-1} x)^{r-1}$ exists, then*

$$\int \left(\cot^{-1} x\right)^r dx = \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)2^{2k} B_{2k}}{(2k)!(2k+r-1)} \left(\cot^{-1} x\right)^{2k+r-1} + C. \quad (10)$$

Proof $\int (\cot^{-1} x)^r dx$

$$= -\int \theta^r \csc^2 \theta d\theta \text{ (where } \theta = \cot^{-1} x\text{)}$$

$$= \int \theta^r \left(\frac{d}{d\theta} \cot \theta \right) d\theta$$

$$= \int \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)2^{2k} B_{2k}}{(2k)!} \theta^{2k+r-2} d\theta$$

(by Formula 2.1.4)

$$= \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)2^{2k} B_{2k}}{(2k)!(2k+r-1)} \theta^{2k+r-1} + C$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)2^{2k} B_{2k}}{(2k)!(2k+r-1)} (\cot^{-1} x)^{2k+r-1} + C.$$

q.e.d.

Finally, we evaluate the indefinite integrals (5) and (6) below.

Theorem 5 *Assume that r, x are real numbers, $|x| \geq 1$, and $(\sec^{-1} x)^r$ exists.*

Case 1. If $x \geq 1$ and $\frac{-r}{2}$ is not a non-negative integer, then

$$\int \left(\sec^{-1} x\right)^r dx = \sum_{k=0}^{\infty} \frac{(-1)^k (2k) E_{2k}}{(2k)!(2k+r)} \left(\sec^{-1} x\right)^{2k+r} + C. \quad (11)$$

Case 2. If $x \leq -1$, then

$$\int \left(\sec^{-1} x\right)^r dx = \pi^r \left[x - \sum_{m=1}^{\infty} \left[\sum_{k=0}^{\infty} \frac{(-1)^k (2k) E_{2k} (r)_m}{m!(2k)!(2k+m)} \left(-\frac{1}{\pi}\right)^m \left[\sec^{-1}(-x)\right]^{2k+m} \right] \right] + C. \quad (12)$$

Proof *Case 1*. If $x \geq 1$, then

$$\int (\sec^{-1} x)^r dx$$

$$= \int \theta^r \sec \theta \tan \theta d\theta \text{ (where } \theta = \sec^{-1} x\text{)}$$

$$= \int \theta^r \left(\frac{d}{d\theta} \sec \theta \right) d\theta$$

$$= \int \sum_{k=0}^{\infty} \frac{(-1)^k (2k) E_{2k}}{(2k)!} \theta^{2k+r-1} d\theta$$

(by Formula 2.1.5)

$$= \sum_{k=0}^{\infty} \frac{(-1)^k (2k) E_{2k}}{(2k)!(2k+r)} \theta^{2k+r} + C$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k (2k) E_{2k}}{(2k)!(2k+r)} (\sec^{-1} x)^{2k+r} + C.$$

Case 2. If $x \leq -1$, then

$$\int (\sec^{-1} x)^r dx$$

$$= \int [\pi - \sec^{-1}(-x)]^r dx$$

$$= \pi^r \int \left[1 - \frac{1}{\pi} \sec^{-1}(-x) \right]^r dx$$

$$= \pi^r \int \sum_{m=0}^{\infty} \frac{(r)_m}{m!} \left(-\frac{1}{\pi}\right)^m [\sec^{-1}(-x)]^m dx$$

(by binomial series)

$$= \pi^r \left[x + \sum_{m=1}^{\infty} \frac{(r)_m}{m!} \left(-\frac{1}{\pi}\right)^m \int [\sec^{-1}(-x)]^m dx \right]$$

$$= \pi^r \left[x - \sum_{m=1}^{\infty} \sum_{k=0}^{\infty} \left[\frac{(-1)^k (2k) E_{2k} (r)_m}{m!(2k)!(2k+m)} \left(-\frac{1}{\pi}\right)^m \left[\sec^{-1}(-x)\right]^{2k+m} \right] \right] + C.$$

(by Case 1)

q.e.d.

Theorem 6 *Suppose that r, x are real numbers, $\frac{1-r}{2}$ is not a non-negative integer, $|x| \geq 1$, and $(\csc^{-1} x)^r$ exists.*

Case 1. If $x \geq 1$, then

$$\int \left(\csc^{-1} x\right)^r dx = \sum_{k=0}^{\infty} \frac{(-1)^{k+1} 2(2k-1)\left(2^{2k}-1\right) B_{2k}}{(2k)!(2k+r-1)} \left(\csc^{-1} x\right)^{2k+r-1} + C. \quad (13)$$

Case 2. If $x \leq -1$ and $(-1)^r$ exists, then

$$\int \left(\csc^{-1} x\right)^r dx = (-1)^{k+1} \sum_{k=0}^{\infty} \left[\frac{(-1)^{k+1} 2(2k-1)\left(2^{2k}-1\right) B_{2k}}{(2k)!(2k+r-1)} \left[\csc^{-1}(-x)\right]^{2k+r-1} \right] + C. \quad (14)$$

Proof *Case 1.* If $x \geq 1$, then

$$\int (\csc^{-1} x)^r dx$$

$$= -\int \theta^r \csc\theta \cot\theta d\theta \text{ (where } \theta = \csc^{-1} x\text{)}$$

$$= \int \theta^r \left(\frac{d}{d\theta}\csc\theta\right) d\theta$$

$$= \int \sum_{k=0}^{\infty} \frac{(-1)^{k+1} 2(2k-1)(2^{2k-1}-1)B_{2k}}{(2k)!} \theta^{2k+r-2} d\theta$$

(by Formula 2.1.6)

$$= \sum_{k=0}^{\infty} \frac{(-1)^{k+1} 2(2k-1)(2^{2k-1}-1)B_{2k}}{(2k)!(2k+r-1)} \theta^{2k+r-1} + C$$

(by integration term by term theorem)

$$= \sum_{k=0}^{\infty} \frac{(-1)^{k+1} 2(2k-1)(2^{2k-1}-1)B_{2k}}{(2k)!(2k+r-1)} (\csc^{-1} x)^{2k+r-1} + C.$$

Case 2. If $x \leq -1$ and $(-1)^r$ exists, then

$$\int (\csc^{-1} x)^r dx$$

$$= \int [-\csc^{-1}(-x)]^r dx$$

$$= (-1)^r \int [\csc^{-1}(-x)]^r dx$$

$$= (-1)^{r+1} \sum_{k=0}^{\infty} \frac{(-1)^{k+1} 2(2k-1)(2^{2k-1}-1)B_{2k}}{(2k)!(2k+r-1)} [\csc^{-1}(-x)]^{2k+r-1} + C.$$

(by Case 1)

q.e.d.

3. Examples

In the following, for the six types of indefinite integrals in this article, some examples are proposed and we use Theorems 1-6 to determine their infinite series expressions. In addition, Maple is used to calculate the approximations of some definite integrals and their solutions for verifying our answers.

Example 1 By Theorem 1, we obtain

$$\int \left(\sin^{-1} x^2\right)^{2/3} dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!(2k+5/3)} \left[\sin^{-1}(-x)\right]^{2k+5/3} + C \tag{15}$$

for all $|x| \leq 1$.

Hence, the definite integral

$$\int_{1/2}^{4/5} \left(\sin^{-1} x\right)^{2/3} dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!(2k+5/3)} \left\{ \begin{array}{l} \left[\sin^{-1}(4/5)\right]^{2k+5/3} \\ -\left[\sin^{-1}(1/2)\right]^{2k+5/3} \end{array} \right\}. \tag{16}$$

Using Maple to verify the correctness of Eq. (16) as follows:

```
>evalf(int((arcsin(x))^(2/3),x=1/2..4/5),18);
                 0.238813256307957296
>evalf(sum((-1)^k/((2*k)!*(2*k+5/3))*((arcsin(4/5))^(2*
k+5/3)-(arcsin(1/2))^(2*k+5/3)),k=0..infinity),18);
                 0.238813256307957295
```

On the other hand, using Theorem 2 yields

$$\int \left(\cos^{-1} x\right)^{4/7} dx = -\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!(2k+18/7)} \left(\cos^{-1} x\right)^{2k+18/7} + C, \tag{17}$$

for all $|x| \leq 1$.

Thus,

$$\int_{3/8}^{5/6} \left(\cos^{-1} x\right)^{4/7} dx = -\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!(2k+18/7)} \left\{ \begin{array}{l} \left[\cos^{-1}(5/6)\right]^{2k+18/7} \\ -\left[\cos^{-1}(3/8)\right]^{2k+18/7} \end{array} \right\}. \tag{18}$$

Next, we use Maple to verify the correctness of Eq. (18).

```
>evalf(int((arccos(x))^(4/7),x=3/8..5/6),18);
                 0.432557196501370483
>evalf(-sum((-1)^k/((2*k+1)!*(2*k+18/7))*((arccos(5/6))
^(2*k+18/7)-(arccos(3/8))^(2*k+18/7)),k=0..infinity),18);
                 0.432557196501370486
```

Example 2 We can obtain the following result by Theorem 3,

$$\int \left(\tan^{-1} x\right)^{7/6} dx = \sum_{k=1}^{\infty} \frac{(-1)^k (2k-1) 2^{2k} \left(2^{2k}-1\right) B_{2k}}{(2k)!(2k+1/6)} \left(\tan^{-1} x\right)^{2k+1/6} + C, \tag{19}$$

for all real number x.

Therefore,

$$\int_4^9 \left(\tan^{-1} x\right)^{7/6} dx = \sum_{k=1}^{\infty} \frac{(-1)^{k-1} (2k-1) 2^{2k} \left(2^{2k}-1\right) B_{2k}}{(2k)!(2k+1/6)} \left[\left(\tan^{-1} 9\right)^{2k+1/6} - \left(\tan^{-1} 4\right)^{2k+1/6}\right]. \tag{20}$$

Verifying the correctness of Eq. (20) using Maple.

```
>evalf(int((arctan(x))^(7/6),x=4..9),18);
                 7.46752708175224704
>evalf(sum((-1)^(k-1)*(2*k-1)*2^(2*k)*(2^(2*k)-1)*
bernoulli(2*k)/((2*k)!*(2*k+1/6))*((arctan(9))^(2*k+1/6)
-(arctan(4))^(2*k+1/6)),k=1..infinity),18);
                 7.46752708175224684
```

In addition, by Theorem 4 we have

$$\int \left(\cot^{-1} x\right)^{5/9} dx = \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1) 2^{2k} B_{2k}}{(2k)!(2k-4/9)} \left(\cot^{-1} x\right)^{2k-4/9} + C, \tag{21}$$

for all real number x.

Thus, the definite integral

$$\int_3^7 \left(\cot^{-1} x\right)^{5/9} dx = \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1) 2^{2k} B_{2k}}{(2k)!(2k-4/9)} \left[\begin{array}{l} \left(\cot^{-1} 7\right)^{2k-4/9} \\ -\left(\cot^{-1} 3\right)^{2k-4/9} \end{array} \right], \tag{22}$$

>evalf(int((arccot(x))^(5/9),x=3..7),18);
1.66130019704229951
>evalf(sum((-1)^k*(2*k-1)*2^(2*k)*bernoulli(2*k)/((2*k)!*(2*k-4/9))*((arccot(7))^(2*k-4/9)-(arccot(3))^(2*k-4/9)),k=0..infinity),18);
1.66130019704229951

Example 3 The following indefinite integral can be obtained by using Eq.(12) of Theorem 5,

$$\int\left(\sec^{-1}x\right)^{8/3}dx = \pi^{8/3}\left[x-\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\left[\begin{array}{c}\frac{(-1)^k(2k)E_{2k}(8/3)_m}{m!(2k)!(2k+m)}\\ \left(-\frac{1}{\pi}\right)^m\left[\sec^{-1}(-x)\right]^{2k+m}\end{array}\right]\right]+C, \quad (23)$$

for all $x \le -1$.

Hence,

$$\int_{-5}^{-2}\left(\sec^{-1}x\right)^{8/3}dx = \pi^{8/3}\left[3-\sum_{m=1}^{\infty}\sum_{k=0}^{\infty}\left[\begin{array}{c}\frac{(-1)^k(2k)E_{2k}(8/3)_m}{m!(2k)!(2k+m)}\\ \left(-\frac{1}{\pi}\right)^m\left[\begin{array}{c}\left(\sec^{-1}2\right)^{2k+m}\\ -\left(\sec^{-1}5\right)^{2k+m}\end{array}\right]\end{array}\right]\right] \quad (24)$$

>evalf(int((arcsec(x))^(8/3),x=-5..-2),18);
16.2861270813272605
>evalf(Pi^(8/3)*(3-sum(sum((-1)^k*(2*k)*euler(2*k)*product(8/3-j,j=0..(m-1))/(m!*(2*k)!*(2*k+m))*(-1/Pi)^m*((arcsec(2))^(2*k+m)-(arcsec(5))^(2*k+m)),k=0..infinity),m=1..infinity)),18);
16.2861270813272616

On the other hand, using Eq. (14) of Theorem 6 yields

$$\int\left(\csc^{-1}x\right)^{6}dx = -\sum_{k=0}^{\infty}\left[\begin{array}{c}\frac{(-1)^{k+1}2(k-1)\left(2^{2k-1}-1\right)B_{2k}}{(2k)!(2k+5)}\\ \left[\csc^{-1}(-x)\right]^{2k+5}\end{array}\right]+C, \quad (25)$$

for all $x \le -1$.

Thus, we can determine the definite integral

$$\int_{-5}^{-2}\left(\csc^{-1}x\right)^{8/3}dx = -\sum_{k=0}^{\infty}\left[\begin{array}{c}\frac{(-1)^{k+1}2(k-1)\left(2^{2k-1}-1\right)B_{2k}}{(2k)!(2k+5)}\\ \left[\left(\csc^{-1}2\right)^{2k+5}-\left(\csc^{-1}9\right)^{2k+5}\right]\end{array}\right]+C. \quad (26)$$

>evalf(int((arccsc(x))^6,x=-9..-2),24);
0.007590619720516328629
>evalf(-sum((-1)^(k+1)*2*(2*k-1)*(2^(2*k-1)-1)*bernoulli(2*k)/((2*k)!*(2*k+5))*((arccsc(2))^(2*k+5)-(arccsc(9))^(2*k+5)),k=0..infinity),24);
0.007590619720516328635.

4. Conclusion

In this paper, we solve the indefinite integrals of the powers of inverse trigonometric functions. On the other hand, power series expansions, binomial series and integration term by term theorem play significant roles in the theoretical inferences of this article. In fact, the applications of the three methods are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other mathematical problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enrich the connotation of calculus and engineering mathematics.

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