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The Real Removal of the Moon from the Earth. The Age of the Universe

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Abstract Now we know that the constant Moon removal of 3.82+-0.07 "(Nasa information)" cm, per year, from the Earth has a measurement error due to the variation of time to the expansion of the Universe. We already know that time varies in the inverse proportion of the square root of Universal Density of Potential Energy in time "Ref. [1]", and we also know that the gravitational radius increases because G increases and this increases in the reverse proportion of density of potential energy in time. With this information we will be able to calculate the true value of the removal of the Moon from Earth and thus calculate how many years it took for the Moon to be at a distance that is now.

Keywords: *universe, gravitation, potential, gravity, velocity, mass, physics, variable*

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1. The Relativity of the Time with the Universal Density of Potential Energy at Different Stationary Reference Frame

1.1. Methods Used for the Analysis

1.1.1. Method 1- Schwarzschild Geometry, [1]

Through the analysis in GR, of Einstein's relativity throw the metric derived from the Schwarzschild geometry for a static field with spherical symmetry it is proposed to vary the time between the reference frame A, within the gravitational field of the mass M, with a radius R_A , located on its surface and another reference frame C on the limit of the gravitational field, $R_C = \infty$.

This expression is the one usually used to calculate time on satellites.

The relativity of time between A and B.

$$\frac{t_A}{t_B} = \sqrt{1 - \frac{2GM}{R_A C^2} + \frac{2GM}{R_B C^2}} \qquad (1.1)$$

At C, on the limit of the gravitational field, with $R_B = \infty$.

$$\frac{t_A}{t_B} = \sqrt{1 - \frac{2GM}{R_A C^2}}. \qquad (2.1)$$

1.1.2. Method 2- The Variation of Time between Different Locations belonging to the Perpendicular Path

We will study the difference between the times at different reference frame belonging to a gravitational field, created by a mass M with a radius R_A. We will study the time difference found along the path of the escape of object launched at a speed with a potential U from the reference frame A located on the surface of mass M. We will measure the differential time between reference frame A, the reference frame C located on the limit of the gravitational field and the reference frame B located at a distance R_B from the center of mass M. Mediation will be based on the observer's referential O the observer's referential.

$$V_A{}^2 = U$$

$$V_C{}^2 = U - \frac{2GM}{R_A}$$

$$V_B{}^2 = U - \frac{2GM}{R_A} + \frac{2GM}{R_B}$$

$$\left(\frac{t_A}{t_0}\right)^2 = \frac{1 - \frac{U}{C^2}}{1 - \frac{0^2}{C^2}} = 1 - \frac{U}{C^2}. \qquad (3.1)$$

$$\left(\frac{t_B}{t_0}\right)^2 = \frac{1 - \frac{U - \frac{2GM}{R_A} + \frac{2GM}{R_B}}{C^2}}{1 - \frac{0^2}{C^2}} \qquad (4.1)$$

$$\left(\frac{t_B}{t_0}\right)^2 = 1 - \frac{U}{C^2} + \frac{2GM}{R_A C^2} - \frac{2GM}{R_B C^2}. \tag{5.1}$$

For t_B unitary, we have:

$$\left(\frac{t_A}{t_B}\right)^2 = \left(\frac{t_B}{t_B}\right)^2 = -\frac{2GM}{R_A C^2} + \frac{2GM}{R_B C^2} \tag{6.1}$$

This differential is independent of U. Whatever, the time difference between A and B will always be the same.

When subtracting the time of reference frame B from the time of reference frame A, we take the time of B

$$\frac{t_A{}^2 - t_B{}^2}{t_B{}^2} = -\frac{2GM}{R_A C^2} + \frac{2GM}{R_B C^2} \tag{7.1}$$

$$t_A{}^2 = \left(1 - \frac{2GM}{R_A C^2} + \frac{2GM}{R_B C^2}\right) t_B{}^2. \tag{8.1}$$

$$\frac{t_A}{t_B} = \sqrt{1 - \frac{2GM}{R_A C^2} + \frac{2GM}{R_B C^2}}. \tag{9.1}$$

At C, on the limit of the gravitational field, with $R_B = \infty$

$$\frac{t_A}{t_C} = \sqrt{1 - \frac{2GM}{R_A C^2}}. \tag{10.1}$$

1.2. The Relativity of Time with the Universal Density of Potential Energy at Local

We learned from Einstein that the speed of light is a structuring element of the theory of relativity it makes its potential, $C^2 = 2G\rho_u$, depend on G and ρ_u, which in turn will also be structuring elements of physics. When analyzing the expression Eq. (1.1, 2.1, 9.1 and 10.1), we senses the importance of ρ_u.

Multiplying the numerator and denominator in Eq. (1.1) or in Eq. (8.1) by $\frac{C^2}{2G}$, we have:

$$\frac{t_A}{t_B} = \sqrt{\frac{\frac{C^2}{2G} - \frac{M}{R_A} + \frac{M}{R_B}}{\frac{C^2}{2G}}} \tag{11.1}$$

$$\frac{t_A}{t_B} = \sqrt{\frac{\rho_A - \frac{M}{R_A} + \frac{M}{R_B}}{\rho_A}} \tag{12.1}$$

$$\frac{t_A}{t_B} = \sqrt{\frac{\rho_B}{\rho_A}} \tag{13.1}$$

ρ_u - Universal density of potential enegy, generated by all the universal masses, at A, ρ_A.

$\rho_u - \frac{M}{R}$ - Universal density of potential enegy, generated by all the universal masses, we have to subtract the influence of the Earth on its surface $\left(\frac{M}{R}\right)$, at C, ρ_C.

If we managed to remove the $\frac{M}{R_A}$ part of the whole, ρ_u, it is a sign that this part is part of that whole.

The density of potential energy created by the Earth on its surface $\frac{M}{R_A}$ is part of the ρ_u.

Yes, the time is inversely proportional to the square root of (ρ) the Universal Density of Potential Energy at local.

The universal density of potential energy varies from one location to another location. If this didn't happen we would have the same time in all referential frames.

It's not just the speed variation that makes time vary, now we know that (ρ) the Universal Density of Potential Energy at local also makes varies the time.

Now, we have the relativity between two reference frames, A and B, given by:

$$\frac{t_A}{t_B} = \sqrt{\frac{C^2 - V_A{}^2}{C^2 - V_B{}^2} \frac{\rho_B}{\rho_A}}. \tag{14.1}$$

Expression deduced from:

$$\frac{t_A}{t_B} = \sqrt{\frac{\frac{C^2}{C^2 - V_B{}^2}}{\frac{C^2}{C^2 - V_A{}^2}} \frac{\rho_B}{\rho_A}}. \tag{15.1}$$

Time is inversely proportional to the square root of $\boldsymbol{\rho_u}$ traversed by the moving object.

1.3. The Variation of Time with the Universal Expansion

With universal expansion, the universal density of potential energy at local decreases, because the masses will be increasingly apart, and will cause the contraction of the time, too in the Earth reference.

t_t - Unit of time in the future

$$\frac{t_t}{t_0} = \sqrt{\frac{\rho_0}{\rho_t}}. \tag{16.1}$$

The contraction of time will have an implication in the values read from the universe over time. The reading of radiation from space will also, be altered due to the contraction of time. The radiation is, pushed to the red, giving the apparent reading that the source is moving away with increasing speed. A constant removal will appear to be occurring at an accelerated rate.

1.4. The Time on Satellites in a Gravitational Field

To do the calculation we need:

M - Generating mass of the gravitational field.

$\boldsymbol{R_M}$ - Mass radius

$\boldsymbol{R_S}$ - Satellite gravitational radius

$\boldsymbol{V_M}$ - The rotational velocity of the mass

$$\frac{t_{Mass}}{t_{Satel}} = \sqrt{\frac{C^2 - V_M{}^2}{C^2 - V_{Satel}{}^2} \frac{\rho_{Satell}}{\rho_{Mass}}} \quad (17.1)$$

$$\frac{t_{Mass}}{t_{Satel}} = \sqrt{\frac{C^2 - V_M{}^2}{C^2 - \frac{GM}{R_{Sat}}} \frac{\frac{C^2}{2G} - \frac{M}{R_{Earth}} + \frac{M}{R_{Sat}}}{\frac{C^2}{2G}}}. \quad (18.1)$$

1.4.1. The Time on Satellites in the Earth Gravitational Field

To do the calculation we need:
M - Generating mass of the gravitational field.
$\boldsymbol{R_M}$ - Mass radius
$\boldsymbol{R_S}$ - Satellite gravitational radius
$\boldsymbol{V_M}$ - The rotational velocity of the mass
To do the calculation we need:

$$\frac{t_{Sat}}{tEarth} = \sqrt{\frac{C^2 - \frac{GM}{R_{Sat}}}{C^2 - 355.313^2} \frac{\frac{C^2}{2G}}{\frac{C^2}{2G} - \frac{M}{R_{Earth}} + \frac{M}{R_{Sat}}}}. \quad (19.1)$$

$$\frac{t_{Sat}}{tEarth} = 1,00000000051126. \quad (20.1)$$

Within a day, we will have:
$t_{Satel} - tEarth$= 0,00004417311 s
$t_{Satel} - tEarth$= 44 173 nanoseconds
Or:
$\boldsymbol{R_S}$ - GPS system - 20200000 m
To do the calculation we need:

$$\frac{t_{Sat}}{tEarth} = 1,00000000044585 \quad (21.1)$$

Within a day, we will have:
$t_{Satel} - tEarth$= 0,00003852161 s
$t_{Satel} - tEarth$= 38 522 nanoseconds.

2. The Universal Gravitational Variable [2]

2.1. The Constant Velocity of Light

There is a data in the local universe that has come to us through Einstein, the constancy of the "speed of light" C in all directions. This is the maximum speed allowed in any direction of space. We are in the presence of local maximum escape potential, given by:

$$C^2 = 2G\rho_u$$

Where:
ρ_u - Universal density of potential energy at in place, generated at the locality by all the universal masses.
G - "Universal gravitational constant":
As postulated, C constant.

$$G\rho_u = C^2/2 \quad (1.2)$$

$$G\rho_u = W(\text{constant}) \quad (2.2)$$

In an expanding Universe, with the removal of all masses from the location, we will have in time:

$$G_t \rho_t = G_0 \rho_0 \quad (3.2)$$

$$\frac{Gt}{G0} = \frac{\rho_0}{\rho_t} \quad (4.2)$$

G is inversely proportional to ρ_u.
G is no more than the coefficient of the gravity radiation capacity through the vacuum, through the ρ_u.

The lower ρ_u, the lower the resistance to radiation propagation through the void, causing G to increase in inverse proportion.

We have a Universal Gravitational Variable and not a Constant.

But why will G be variable and increasing?

The contraction of time will have an implication in the values read from the universe over time. The reading of radiation from space will also, be altered due to the contraction of time. The radiation is, pushed to the red, giving the apparent reading that the source is moving away with increasing speed. A constant removal will appear to be occurring at an accelerated rate

2.2. G and ρ_u

In universal terms:

Einstein characterized the speed of light as result of the escape potential anywhere and in all directions.

The same happens in the local escape potential where the escape velocity to abandon a mass is the same in all directions.

In gravitational potential, M/R is the density of potential energy created by mass M at distance r. Since we are facing a universal escape potential, C^2, then ρ_u can only be the density of potential energy created by all the universal mass, in the place.

$$C_i{}^2 = 2\, G_i\, \rho_{ui} \quad (5.2)$$

$$R_{umi} = \frac{\sum_1^n M_{u\,ji} R_{e\,ji}}{\sum_1^n M_{uji}} \quad (6.2)$$

Where:
R_{umi} - The Universal average distance that creates density of potential energy in location.

$$\rho_{ui} = \frac{M_{ui}}{R_{umi}} = \frac{\sum_1^n M_{uji}}{R_{umi}}. \quad (7.2)$$

The amount of universal mass / energy will always be constant.

$$\rho_{ui} = Y\,(\text{constant})/\,R_{umi}. \quad (8.2)$$

In an expanding universe, all the universal masses will be more and more distant from location i, so the average radius of universal mass emission to the site will be increasing.

If the average radius of radiation increases, then the local density of universal potential energy decreases.

On the other hand, as we have seen, and according to Eq. (3.2):

$$G = \frac{\frac{W}{Y}}{R_{umi}} \quad (9.2)$$

$$P = \frac{W}{Y} \quad (10.2)$$

$$G = PR_{umi}. \quad (11.2)$$

G increases in proportion to the increase of the average universal emission radius, then G will also grow at the ratio of the average radius of universal mass emission to the local. Because we are in a homogeneous Universe, then we can say that G increases in proportion to the expansion of the Universe.

$$G = QR_u \quad (12.2)$$

$$\frac{Gt}{G0} = \frac{R_{ut}}{R_{u0}} \quad (13.2)$$

We have a Universal Gravitational Variable and not a Constant.

3.2. Impact of the Universal Gravitational Variable on Local Gravitational Fields

With the available information, that the Moon is moving away from Earth, at a rate of a constant 3.82 ±0.07 cm per year [11] obtained as measured since 1969, i.e. measurements taken for over 51 years, through the Apollo Laser Ranging Experiments Yield Results and "Jupiter has more than 60 natural satellites, but only the top four deserve particular attention: Io, Europe, Ganymede and Calisto. They have nearly circular orbits, and exhibit the same face toward Jupiter. They are also slowly moving away from the planet. Saturn has over forty satellites, except for two, always run with the same face toward the planet, and they are slowly moving away."(Extracted from the book "Discovering the Universe", PhD Teresa Lago from the Astrophysics Center of the University of Porto), [3].

These phenomena require an analysis of the local universe and the laws that govern it.

It´s known by all the expression that allows us to calculate the gravitational potential of a body around the other:

$$U = G\frac{M}{R}. \quad (14.2)$$

The gravitational potential will always be constant. (Constant velocity of the gravitational bodies in void U=V^2): A body in the void, in a null working system is always constant. U= Constant.

The mass generanting the gravitational field will also remain constant, solving Eq.(1.1): M= Constant

$$\frac{r}{G} = \frac{M}{U} \quad (15.2)$$

$$\frac{r}{G} = K(constant) \quad (16.2)$$

$$r = K\,G. \quad (17.2)$$

In an expanding Universe, with the removal of all masses from the location, we will have in time:

$$\frac{R_t}{R_0} = \frac{Gt}{G0}. \quad (18.2)$$

As we have seen before, bodies belonging to a gravitational field are moving away from the masses that generate these fields. In the Earth / Moon System, the Moon is moving away from the Earth every year, that is, the radius of gravitation increases, r_1>r_0. The same happens with the moons of Jupiter and Saturn. From Eq. (5.3), we know that the gravitational radius increases indicating that G increases proportionally to the radius.

Increasing G, we have the same gravitational potential, generated the largest distance of mass M and hence the Moon to adjust to the location of this potential, moving away.

3. The Removal of the Moon from the Earth

There are values already known such as:

Dc - Actual distance between center of the Earth and center of theMoon, 385 000 600 meters.

Rt - Radius of the Earth, 6 378 100 m

Rl - Radius of the Moon, 1 737 400 m

D= 385 000 600

D_c= 376 885 100 meters (385 000 600-6 378 100-1 737 400)

Dm - Aparent removal, actualy calculated, 3.82+-0.07 cm per year.

d - Real annual average removal of the Moon.

t_0 - Value of the measure of the time of the light beam.

t_t - Value of the measure of the time of the light beam, conseding the variation of time.

Let's consider D, the distance between the faces of the Earth and the Moon, because at the time of reading the emitter of the signal will be located on the face facing the moon and the reflecting mirrors will always face the Earth.

$$t_0 = \frac{D_c + 0.0382 \pm 0.0007}{C}. \quad (1.3)$$

Taking into account the shrinking of the Earth and the Moon, we will have:

$$t_t = \frac{D_c + d}{C}. \quad (2.3)$$

From, (18.1), then ρ is inversely proportional to the expansion of the Universe and this proportional to the increase in the radius of gravitation, we will have:

$$\frac{\rho_t}{\rho_o} = \frac{D}{D+d} \quad (3.3)$$

$$\frac{t_t}{t_0} = \sqrt{\frac{\rho_0}{\rho_t}} = \sqrt{\frac{D+d}{D}} \quad (4.3)$$

$$t_t = t_0\sqrt{\frac{D+d}{D}}. \quad (5.3)$$

The reading made will be conditioned by the contraction of time on the watch.

In the following year, the clock will mark more time. So it is necessary to take this into account and make the correction.

$$t_t = \frac{D_c + d}{C}\sqrt{\frac{D+d}{D}} \quad (6.3)$$

$$\frac{D_c + 0.0382}{C} = \frac{D_c + d}{C}\sqrt{\frac{D+d}{D}} \quad (7.3)$$

$$d = 0{,}0256467\text{m} \quad (8.3)$$

$$\text{d} = (0{,}025600\text{m}; 0{,}025694\text{m}).$$

4. The Age of the Universe

Now we can calculate how many years it took the mass centers to move away as far as today.

As we believe, that everything originated in the Big Bang. If we go back in time, we imagine the return of the Moon towards Earth. This meeting would only be possible at the point of the Big Bang, when the Moon, the Earth, the Way of the Milk, etc., will all remain at the point of the big bang.

The Moon is in perfect balance within the gravitational field and the constant departure from Moon from the Earth will remain constant in the future.

The main cause of this departure is the variation of G in proportion to the expansion of the universe.

Right after the Big-Bang G it was practically "null", due to the "minimal" Universal gravity field.

d - Current displacement of the Moon in relation to Earth.

Now we use the measure of D_C , because the displacement is relative to the centers of mass.

The gravitational radius will always increase in proportion to the time.

Future:

$$d = D_C \frac{I+N}{I}\frac{1}{I+N} \quad (1.4)$$

Past:

$$d = D_C \frac{I-N}{I}\frac{1}{I-N}. \quad (2.4)$$

We see that the moon's distance is always constant, depending on the constant variation of G.

We will have forever:

$$I = \frac{\text{Dc}}{d} \quad (3.4)$$

$$I = 15\ 011\ 701\ 311 \text{ years} \quad (4.4)$$

Taking into account the admitted error in the measurement of d, we will have:

$$\text{I} = (14\ 984\ 066\ 319;\ 15\ 039\ 085\ 938) \text{ years.}$$

5. The Hubble Constant

For the first time, we can evaluate the expansion of the Universe, from nearby and therefore more accurate medics.

We have the distance from the Earth to the Moon and the value of the moon's annual remoteness from the Earth. In order to obtain the equivalent value of Hubble constant, let's consider Megaparsec.

$$Mparsec = 3{,}0856775815 \times 10\hat{}22m$$

T-One year = 365.256363 dias = 31 558 150 seg

$$V = \frac{3{,}0856775815\text{x}10\hat{}22}{31558150}\frac{0{,}0256467}{385000600} \quad (1.5)$$

$$\text{V} = 65\ 134{,}20\text{m } s^{-1}\text{Mpc}^{-1} \quad (2.5)$$

$$\text{V} = 65{,}134 \text{ km } s^{-1}\text{Mpc}^{-1} \quad (3.5)$$

$$\text{V} = (65{,}016; 65{,}254) \text{ km } s^{-1}\text{Mpc}^{-1}.$$

6. Conclusion

The constant contraction of time on Earth will cause a change in the way we look at the Universe and even the Earth itself.

The value now obtained for the growth rate of the Earth's gravitational field is much lower than expected.

The Moon is moving away from Earth at 2.5466383 cm per year instead of the apparent 3.82 cm.

We also conclude that the Universe is the age of: 15 011 701 311 years.

For the first time we were able to measure Hubble Constant equal to 65,134 kms^{-1}Mpc^{-1}, through elements close to our reference, the Earth. The shorter the measurement distance, the smaller the error.

- **Hubble Constant**

The value we obtained for the Hubble Constant, with the value of 64,68$\pm$1.18 km s^{-1}Mpc^{-1} is very close to the measured value of 67,15$\pm$1.2 km s^{-1}Mpc^{-1}, measured on March 21, 2013 by Planck telescope, upper limit of the calculated value now proposed and the lower limit of measurement by the Planck telescope are almost equal. The measurement made by the telescope should contain a greater error due to the contraction of time and possible displacement by translation of the radiation source.

- **Annual removal of moons belonging to the Solar System in relation to their planets:**

D_a - Annual increase in gravitational radius.

$$D_a = \frac{R_{Gravitational}}{I}$$

Jupiter:

Io:	2,81 $\pm$ 0,005 cm
Europa:	4,48 $\pm$ 0,008 cm
Ganymede:	7,13 $\pm$ 0,013 cm
Calisto:	12,54 $\pm$ 0,023 cm
Sinope:	159,47 $\pm$ 0,29 cm

Saturn:

Titan:	8,14 $\pm$ 0,015 cm

We analyzed the increase in the gravity radius of the moons and not the increase in the distance to the planet, as this will depend on the changes caused on the planet by the increase in the G. The recently measured of 11 cm per year, to the departure of Saturn's Titan, published in

Nature Astronomy, is much better than the previous one aimed at 0.1 cm a year.

Iapetus: 23,72 ± 0,043 cm

- Annual removal of planets belonging to the Solar System in relation to the Sun:

Sun:

Earth 9,97 ± 0,018 m

Mars: 15,18 ± 0,028 m

Jupiter: 51,86 ± 0,095 m

- Annual removal of Sun in relation to the center of Milk Way:

Milk Way:

Sun (26000 al) 0,11 ± 0,0002 u.a.

-Delay in the Earth's translation period.

One of the consequences of the contraction of time and the expansion of the universe is the delay in the Earth's translation period.

-Ro -Distance Sun/Earth - 1,496x10^11 m

-Vo -Earth's translation speed - 29788,64 m/s

$$T_t = \frac{2\pi Ro \frac{I+N}{I}}{Vo}\sqrt{\frac{\rho_0}{\rho_t}}$$

$$T_t = \frac{2\pi Ro \frac{I+N}{I}}{Vo}\sqrt{\frac{I+N}{I}}$$

$$T_t = \frac{2\pi Ro}{Vo}\left(\frac{I+N}{I}\right)^{\frac{3}{2}}$$

$$T_t - T_0 = \frac{2\pi Ro}{Vo}\left[\left(\frac{I+N}{I}\right)^{\frac{3}{2}} - 1\right]$$

$$T_t - T_0 = 0{,}003131 \text{ seconds}$$

$$T_t - T_0 = 3{,}131 \text{ milliseconds.}$$

-Delay in the Earth's rotation period.

One of the consequences of the contraction of time is the delay in the Earth's rotation period.

With an interval of one year, we will have:

$$\partial T_t = +24x3600\left[\left(\frac{I+N}{I}\right)^{\frac{1}{2}} - 1\right]$$

$$\partial T_t = +24x3600\left[\left(\frac{I+N}{I}\right)^{\frac{1}{2}} - 1\right]$$

∂T_t = 2,85752E-06 seconds

∂T_t = 2857,5 nanoseconds

-Delay in the Moon's translation period.

One of the consequences of the contraction of time and the expansion of the universe is the delay in the Earth's translation period.

With an interval of one year, we will have:

-Ro -Distance Moon/Earth - 385000600m

-Vo -Moon's translation speed - 1022 m/s

$$T_t = \frac{2\pi Ro \frac{I+N}{I}}{Vo}\sqrt{\frac{\rho_0}{\rho_t}}$$

$$T_t = \frac{2\pi Ro \frac{I+N}{I}}{Vo}\sqrt{\frac{I+N}{I}}$$

$$T_t = \frac{2\pi Ro}{Vo}\left(\frac{I+N}{I}\right)^{\frac{3}{2}}$$

$$T_t - T_0 = \frac{2\pi Ro}{Vo}\left[\left(\frac{I+N}{I}\right)^{\frac{3}{2}} - 1\right]$$

$$T_t - T_0 = 0{,}00023485 \text{ seconds}$$

$$T_t - T_0 = 0{,}235 \text{ milliseconds.}$$

The constant contraction of time on Earth will cause a change in the way we view the Universe through telescopes over time.

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