

Application of Complex Rotation Method for Calculate to Correlation Factor and the $2sns^{+1,3}S^e$, $2snp^{+1,3}P^0$, $2pnp^{+1,3}D^e$, $3d4d^{+1,3}G^e$ Autoionizing States of He-like Atoms

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Abstract Calculations of the energy levels of atoms and ions with $Z \leq 10$ are carried out in this paper using the complex rotation method. Using a new wave function with two terms including a new method to calculate the correlation factor taking into account spherical harmonics through hypergeometric functions to calculate excited $2sns^{+1,3}S^e$, $2snp^{+1,3}P^0$, $2pnp^{+1,3}D^e$, $3d4d^{+1,3}G^e$ states. The results obtained show quantitatively the great significance of electron correlation effects in the doubly excited states. These results are in compliance with some experimental and theoretical data.

Keywords: wave function correlated, two terms, complex rotation method, excited states, correlation factor

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1. Introduction

The study of atomic resonances in plasma environment has gained tremendous attention in the recent years and is one of the interesting topics of current research [1-4]. The complex rotation method was used in atomic physics by Doolen [5] to determine the resonance parameters in the system $e^- + H$ and by Lonnie W. Manning and Frank C. Sanders [6] to study the resonance parameters ($^1P^0$ and $^3P^0$) of He-like systems. Subsequently other calculations were made; using the variational wave functions built on a Hylleraas basis, by Ho [7-10], Gning et al [11] used the Hylleraas wave functions to compute resonance parameters. Some of his work [7] focused on the determination of the energies and widths of the first two resonances $^1S^e$ and $^1,3P^0$ of helium and the H^- and Li^+ ions under the excitation threshold $n = 4$. Similarly the complex rotation method allows calculating positions and widths of resonances in atomic systems [12,13,14]. To mention just a few applications, the method has been used in nonrelativistic quantum-mechanical calculations of resonance parameters of doubly excited states [15,16] or Stark resonances [17] in such systems as He, H^- , or Ps^- . The above mentioned applications of the complex rotation procedure neglected relativistic effects. Recent developments of the experimental technique perfected experimental data to a degree where relativistic effects become observable.

A number of methods based on close-coupling approximation [18] and the Feshbach projection operator formalism [19,20] have been used to calculate positions and widths of doubly excited states. In the latter method, resonance positions and widths are calculated separately and therefore need not be of the same accuracy.

In this work we report a resonance parameters calculation for some lower-lying doubly excited states for a He-like systems below the $n = 2$ hydrogenic thresholds. Hylleraas-type wave functions are used and $2sns^{+1,3}S^e$, $2snp^{+1,3}P^0$, $2pnp^{+1,3}D^e$, $3d4d^{+1,3}G^e$ states are calculated.

2. Wave Functions and Calculations

The existence of this term makes impossible to solve exact the fundamental equation of quantum mechanics (Schrödinger equation):

$$H\psi_{jkm} = E\psi_{jkm} \quad (1)$$

In this equation, ψ_{jkm} is the wave function of the quantum state of the system (core-electrons) and E is the total energy of the same quantum state. The exact solution of the last equation is impossible because of the correlation term involved in the expression of total Hamiltonian. Hence it is necessary to implement a method of approximate calculation using a correlated wave function, which is given by:

$$\psi_{jkm} = \sum_{\nu=0}^{\nu=N-\ell_1-1} (N^2 r_0^2)^\nu \sum_{\nu'=0}^{\nu'=N-\ell_2-1} (N^2 r_0^2)^{\nu'} \times (C_{jkm} + (-1)^S C_{jkm} z r_{12}^m) e^{-\alpha(r_1+r_2)} Y_\ell(\Omega_1) Y_\ell(\Omega_2) \quad (2)$$

Where $j+k+m \leq \Omega$, Ω is a positive integer number which determines the expansion length N ; α is a nonlinear parameter and S the total spin of atomic system, r_1 and r_2 are the coordinates of electrons with respect to the nucleus; N is the principal number ; ℓ_1 and ℓ_2 are orbital angular, Z the nuclear charge. C_{jkm} are parameters to be determined, r_0 is Bohr radius.

Where

$$Y_\ell(\Omega_1) Y_\ell(\Omega_2) = \sum_{m=-\ell}^{\ell} Y_{\ell,m}^*(\Omega_1) Y_{\ell,m}(\Omega_2) \quad (3)$$

represent the spherical harmonics of electron 1 or 2.

Which $\Omega_i = (\theta_i, \varphi_i)$ (4) and $i=1, 2$.

The resonance parameters are determined by finding a rate of change that is stable with respect to the nonlinear parameters α , and the complex value of the energy.

$$E = E_r - i \frac{\Gamma}{2} = \frac{\langle \psi_{jkm} / H / \psi_{jkm} \rangle}{\langle \psi_{jkm} / \psi_{jkm} \rangle} \quad (5)$$

where E_r gives the resonance position and Γ gives the width.

And the Hamiltonian can be written as:

$$H = -(\Delta_1 + \Delta_2) e^{-2i\theta} - 2Ze^{-i\theta} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) + \frac{2e^{-i\theta}}{r_{12}} \quad (6)$$

$$= T + C + W$$

With

$$T = -(\Delta_1 + \Delta_2) e^{-2i\theta} = -e^{-2i\theta} \left\langle \psi_{jkm}^* / \Delta_1 + \Delta_2 / \psi_{jkm} \right\rangle \quad (7)$$

$$C = -2Ze^{-i\theta} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) = -2Ze^{-i\theta} \left\langle \psi_{jkm}^* / \frac{1}{r_1} + \frac{1}{r_2} / \psi_{jkm} \right\rangle \quad (8)$$

$$W = \frac{2e^{-i\theta}}{r_{12}} = 2e^{-i\theta} \left\langle \psi_{jkm}^* / \frac{1}{r_{12}} / \psi_{jkm} \right\rangle. \quad (9)$$

In equation (6), T is the kinetic energy; C is the Coulomb interaction between the atomic nucleus and the two electrons and W is the Coulomb interaction between electrons. In the study of atomic resonances, the application of this theorem consists in the transformation of real Hamiltonian system $H(r)$ to a complex one $H(r,\theta)$ where r is the inter-particle radial coordinate and θ is a positive real number denoting the rotation angle. This processing is executed by a change of the real variable r to complex variable $r \exp(i\theta)$.

The matrix element of the normalization constant N_{jkm} is defined by:

$$N_{jkm} = \iiint d\tau \psi_{jkm}^* \psi_{jkm} \quad (10)$$

Where $d\tau$ is elementary volume element. The existence of the disturbance term (Equation 9) makes impossible the exact resolution of equation (1). This is how we used the complex rotation method by introducing a new method of calculating the correlation coefficient, which is one of the calculation techniques for providing the resonance parameters to two-electron atomic systems. According to the calculation of correlation coefficients, proposed by Varshalovich D.A et al. [32], we found a new formula that is proposed by Gning et al. [11]

$$r_{12}^m = 4\pi \sum_{\ell=0}^{\infty} \frac{1}{2\ell+1} a_\ell^m(r_1, r_2) (Y_\ell(\Omega_1) Y_\ell(\Omega_2)) \quad (11)$$

for $r_1 < r_2$.

With

$$a_\ell^m(r_1, r_2) = \frac{\left(\frac{-m}{2}\right)_\ell}{\left(\frac{1}{2}\right)_\ell} r_2^m \left(\frac{r_1}{r_2}\right)^\ell F\left(\ell - \frac{m}{2}, -\frac{1}{2} - \frac{m}{2}, \ell + \frac{3}{2}, \frac{r_1^2}{r_2^2}\right) \quad (12)$$

the hypergeometric wave function defined by Samarskogo [33]

$$F\left(\ell - \frac{m}{2}, -\frac{1}{2} - \frac{m}{2}, \ell + \frac{3}{2}, \frac{r_1^2}{r_2^2}\right) = 1 - \frac{(2\ell - m)(1 + m)}{2(2\ell + 3)} \times \frac{r_1^2}{r_2^2}. \quad (13)$$

Replacing equation (13) with its value in equation (12)

$$a_\ell^m(r_1, r_2) = \frac{\left(\frac{-m}{2}\right)_\ell}{\left(\frac{1}{2}\right)_\ell} r_2^m \left(\frac{r_1}{r_2}\right)^\ell \left(1 - \frac{(2\ell - m)(1 + m)}{2(2\ell + 3)} \frac{r_1^2}{r_2^2}\right). \quad (14)$$

So equation (14) becomes

$$a_\ell^m(r_1, r_2) = \frac{\left(\frac{-m}{2}\right)_\ell}{\left(\frac{1}{2}\right)_\ell} \frac{r_1^\ell}{r_2^{\ell-m+2}} \left(r_2^2 - \frac{(2\ell - m)(1 + m)}{2(2\ell + 3)} r_1^2\right). \quad (15)$$

And replacing equation (15) in equation (11)

$$r_{12}^m = 4\pi \sum_{\ell=0}^{\infty} \frac{1}{2\ell+1} \times \frac{\left(\frac{-m}{2}\right)_\ell}{\left(\frac{1}{2}\right)_\ell} \times \frac{r_1^\ell}{r_2^{\ell-m+2}} \times \left(r_2^2 - \frac{(2\ell - m)(1 + m)}{2(2\ell + 3)} r_1^2\right) (Y_\ell(\Omega_1) Y_\ell(\Omega_2)). \quad (16)$$

for $2sns^{+1,3}S^e$ states ($\ell=0; S=0$), $Y_{0,0}(\Omega_1) Y_{0,0}(\Omega_2) = \frac{1}{4\pi}$

(17) so equation (2) becomes:

$$\psi_{jkm} = \left(\frac{1}{4\pi}\right) \sum_{\nu=0}^{\nu=N_1-\ell_1-1} (N^2 r_0^2)^\nu \sum_{\nu'=0}^{\nu'=N_2-\ell_2-1} (N^2 r_0^2)^{\nu'} \times (C_{jkm} + (-1)^S C_{jkm} z r_{12}^m) e^{-\alpha(r_1+r_2)} \quad (18)$$

For $2snp\ ^{+1,3}P^0$, $2pnp\ ^{+1,3}D^e$ states corresponding to $\ell=1$,

$$\begin{aligned}
 & Y_\ell(\Omega_1)Y_\ell(\Omega_2) \\
 &= Y_{1,-1}^*(\Omega_1)Y_{1,-1}(\Omega_2) \\
 &+ Y_{1,0}^*(\Omega_1)Y_{1,0}(\Omega_2) + Y_{1,1}^*(\Omega_1)Y_{1,1}(\Omega_2)
 \end{aligned} \tag{19}$$

by replacing the values of the spherical harmonics of $\ell=1$ or 2 , we obtain:

$$\begin{aligned}
 & Y_\ell(\Omega_1)Y_\ell(\Omega_2) \\
 &= \frac{3}{4\pi} \left(\begin{array}{l} \sin\theta_1 \sin\theta_2 \cos\varphi_{12} \\ + \cos\theta_1 \cos\theta_2 \end{array} \right).
 \end{aligned} \tag{20}$$

For $\varphi_1 = \varphi_2$ we have $\cos\varphi_{12}=1$

$$Y_\ell(\Omega_1)Y_\ell(\Omega_2) = \frac{3}{4\pi} (\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2). \tag{21}$$

For $3d4d\ ^{+1,3}G^e$ states

$$\begin{aligned}
 & Y_\ell(\Omega_1)Y_\ell(\Omega_2) \\
 &= \sum_{m=-\ell}^{\ell} Y_{\ell,m}^*(\Omega_1)Y_{\ell,m}(\Omega_2) \\
 &= \frac{15 \sin^2\theta_1 \sin^2\theta_2}{16\pi} - \frac{15 \cos\theta_1 \sin\theta_1 \sin\theta_2}{8\pi} \\
 &= \frac{\sqrt{75} (3 \cos^2\theta_1 - 1) \sin\theta_2}{2^{9/2} \pi}
 \end{aligned} \tag{22}$$

3. Results and Discussions

In our calculations, we set the dimension Ω to 4 for the $2s3s\ ^{1,3}S^e$, $2s4s\ ^{1,3}S^e$ states and $\beta=0.25$ rad. $\Omega=3$ for the $2s2p\ ^{1,3}P^0$, $2s3p\ ^{1,3}P^0$ states and $\beta=0.2$ rad. Similarly $\beta=0.2$ rad for the $2p3p\ ^{1,3}D^e$, $2p4p\ ^{1,3}D^e$, $3d4d\ ^{1,3}G^e$ states and $\Omega=4$. After having fixed these parameters, we have varied the nonlinear parameter α . The minimum value of the energy is obtained following a stabilization of the latter with a variation of the nonlinear parameter. At each minimum value of the energy corresponds a minimum value of the given resonance width 10^{-3} . Based on the processing of the calculations, energy conversion of 1 a.u. = $2Ry = 27.211385$ eV was used.

Table 1, Table 2, Table 3, Table 4 show the results of targeted energy $2s3s\ ^{1,3}S^e$, $2s4s\ ^{1,3}S^e$, $2s2p\ ^{1,3}P^0$, $2s3p\ ^{1,3}P^0$, $2p3p\ ^{1,3}D^e$, $2p4p\ ^{1,3}D^e$, $3d4d\ ^{1,3}G^e$ respectively.

Based on the processing of the calculations, energy conversion of 1 a.u. = $2Ry = 27.211385$ eV was used. For the calculation of doubly excited states for Helium-like ions, several authors have used different methods. Thus, Ho [21,29] used the method of complex rotation convoluted with a wave function of Hylleraas type, Dieng [22] used Special Forms of Hylleraas-Type Wave Functions, Ivanov and Safronova [24] performed calculation using double sum method over the complete hydrogen spectrum, Sakho [23,25] with the Screening Constant by Unit Nuclear Charge method, Ivanov and Safronova [24] using computing double sum method, Bachau [30] using the Fechbach projection formalism method.

Table 1. Energy doubly excited $2sns\ ^{+1,3}S^e$ states of He-like systems (2-10). Both Energy and widths are expressed in rydbergs

$2s3s\ ^1S^e$			$2s3s\ ^3S^e$			$2s4s\ ^1S^e$			$2s4s\ ^3S^e$			
Z	α	-E	Γ	α	-E	Γ	α	-E	Γ	α	-E	Γ
2	0.963	1.174857	0.001110	0.955	1.222736	0.0004957	0.970	1.078567	0.000161	0.935	1.089823	0.000556
3	1.150	2.837435	0.002682	1.135	2.885916	0.0011700	1.125	2.549938	0.000241	1.110	2.559737	0.001307
4	1.300	5.215416	0.004930	1.280	5.245563	0.0021266	1.284	4.912449	0.000464	1.250	4.622176	0.002360
5	1.427	8.283079	0.007831	1.440	8.398197	0.0091759	1.390	7.697545	0.000245	1.374	7.395109	0.003776
6	1.541	12.12459	0.011463	1.555	12.289478	0.0134275	1.510	10.99787	0.000103	1.480	10.699627	0.005464
7	1.643	16.65804	0.015749	1.657	16.837186	0.0183963	1.600	14.67166	0.000138	1.578	14.710585	0.007512
8	1.736	21.88285	0.020689	1.750	22.067967	0.0241115	1.700	19.80088	0.001872	1.666	19.261070	0.009836
9	1.823	27.87674	0.026356	1.838	28.130958	0.0307360	1.790	25.55697	0.000241	1.750	24.583568	0.012554
10	1.904	34.56911	0.032683	1.918	34.738038	0.0379549	1.850	30.15211	0.000285	1.825	30.279830	0.015463

Table 2. Energy doubly excited $2snp\ ^{+1,3}P^0$ states of He-like systems (2-10). Both Energy and widths are expressed in rydbergs

$2s2p\ ^1P^0$			$2s2p\ ^3P^0$			$2s3p\ ^1P^0$			$2s3p\ ^3P^0$			
z	α	-E	Γ	α	-E	Γ	α	-E	Γ	α	-E	Γ
2	0.411	1.384520	0.00277	0.435	1.518125	0.000583	0.374	1.128617	0.0004575	0.380	1.157603	0.000169
3	0.535	3.529412	0.00494	0.555	3.754265	0.000753	0.473	2.718729	0.0009894	0.488	2.790356	0.000172
4	0.635	6.635136	0.00797	0.655	6.976910	0.000653	0.558	5.026144	0.0002037	0.575	5.164385	0.000195
5	0.740	10.74939	0.00688	0.760	11.257263	0.0006964	0.633	8.085786	0.0003278	0.645	8.238113	0.000207
6	0.810	15.87203	0.00676	0.828	16.450448	0.0006669	0.699	11.832375	0.0004797	0.712	12.045357	0.000130
7	0.825	21.979827	0.00891	0.902	22.975692	0.0006944	0.760	16.319597	0.0006616	0.785	16.558325	0.000120
8	0.945	29.123500	0.00880	0.962	29.990312	0.0007232	0.816	21.501381	0.0008717	0.843	21.822549	0.000159
9	1.0055	37.245801	0.00898	1.085	38.079336	0.0009183	0.869	27.434005	0.0011122	0.895	27.672276	0.000179
10	1.0630	46.356087	0.00930	1.080	47.395576	0.0008554	0.919	34.091587	0.0013821	0.947	34.422331	0.000197

Table 3. Energy doubly excited 2pnp ^{+1,3}D^e states of He-like systems (2-10). Both Energy and widths are expressed in rydbergs

z	2p3p ¹ D ^e			2p3p ³ D ^e			2p4p ¹ D ^e			2p4p ³ D ^e		
	α	-E	Γ	α	-E	Γ	α	-E	Γ	α	-E	Γ
2	0.680	1.137185	0.000461	0.684	1.170925	0.0004747	0.670	1.076210	0.000428	0.651	1.096103	0.000444
3	0.812	2.757372	0.001117	0.815	2.810212	0.0011393	0.798	2.530060	0.001025	0.800	2.561816	0.001038
4	0.918	5.092810	0.002064	0.921	5.176195	0.0020985	0.900	4.614617	0.001870	0.899	4.589138	0.001860
5	1.008	8.125539	0.003294	1.012	8.2675704	0.0033518	0.986	7.280419	0.002951	0.987	7.317252	0.002966
6	1.090	12.008379	0.004868	1.091	12.063260	0.0048906	1.063	10.599764	0.004297	1.065	10.699378	0.004337
7	1.161	16.415429	0.006655	1.163	16.599058	0.0067295	1.133	14.576301	0.005909	1.134	14.640411	0.005935
8	1.227	21.691387	0.008794	1.229	21.867623	0.0088655	1.197	19.180034	0.007775	1.198	19.229848	0.007808
9	1.288	27.638463	0.011205	1.289	27.842274	0.0112918	1.255	24.294058	0.009849	1.256	24.439044	0.009888
10	1.344	34.183866	0.013858	1.347	34.534397	0.0140131	1.310	30.098139	0.012202	1.287	30.673410	0.012435

Table 4. Energy doubly excited 3d4d ^{+1,3}G^e states of He-like systems (2-10). Both Energy and widths are expressed in rydbergs

z	3d4d ¹ G ^e			3d4d ³ G ^e		
	α	-E	Γ	α	-E	Γ
2	0.102	0.525520	0.002130	0.103	0.535886	0.00001170
3	0.130	1.280462	0.005191	0.132	1.320193	0.00002352
4	0.154	2.395852	0.016132	0.155	2.427124	0.00002440
5	0.175	3.857269	0.017678	0.176	3.911686	0.00002585
6	0.193	5.644482	0.022889	0.194	5.713264	0.00002812
7	0.210	7.796414	0.025608	0.211	7.871034	0.00003191
8	0.226	10.319652	0.029183	0.226	10.319906	0.00004183
9	0.240	13.092518	0.030079	0.242	13.241968	0.00004396
10	0.254	16.309918	0.036605	0.256	16.441943	0.00004498

In fact, a comparison is made in Table 5 based on the theoretical results from Ho [21], Dieng [22], Sakho et al. [23,25] and Ivanov and Safronova [24] for the 2s3s ^{1,3}S^e, 2s4s ^{1,3}S^e states. For these state, best consistency of the results was found and a little discrepancies regard Dieng results.

Table 6 compares results with those from Ho [8,21], Seminario and Sanders [26], Drake et al [27], Sakho et al. [23,28], Ivanov and Ho [29], Bachau [30] for the 2s2p ^{1,3}P⁰, 2s3p ^{1,3}P⁰. For these states, we note generally a quite good agreement.

In Table 7, results of 2p3p ^{1,3}D^e, 2p4p ^{1,3}D^e states were compared with those of Ho [21], Sakho et al. [25] and Dieng [22]. For these states, we note generally a quite good agreement and little discrepancy with Dieng results.

We have also compared in Table 8, results with those from Ivanov and Ho [29] and Bachau [30] for the 3d4d ^{1,3}G^e, 3d4d ³G^e. We note good consistency.

In Table 8 our resonance width results of the same states above are listed, correspondingly, and also compared with those found in the literature Ho [8,21], Seminario and Sanders [26], Manning and Sanders [31].

Table 5. Comparison of resonance positions (E_r) for the 2sns ^{1,3}S^e states of He-like ions with (Z=2-10)

states	Z									
		2	3	4	5	6	7	8	9	10
2s3s ¹ S ^e	-E ^a	1.174857	2.837435	5.215416	8.283079	12.12459	16.65804	21.88285	27.87674	34.56911
	-E ^b	1.179850	2.831150	5.204100	8.299000	12.11600	16.65500	21.91640	27.89980	34.60540
	-E ^c	1.179902	2.823779	5.189878	8.278200	12.08874	16.62150	21.87649	27.85370	34.55314
2s3s ³ S ^e	-E ^a	1.222736	2.885916	5.245563	8.398197	12.28947	16.83718	22.06796	28.13095	34.73803
	-E ^f	1.202960	2.876740	5.274440	8.395040	12.23816	16.80370	22.09158	28.10176	34.83420
	-E ^c	1.225489	2.903172	5.303077	8.425204	12.26955	16.83612	22.12491	28.13593	34.86917
2s4s ¹ S ^e	-E ^a	1.078567	2.549938	4.912449	7.697545	10.99787	14.67166	19.80088	25.55697	30.15211
	-E ^g	1.070599	2.525598	4.605598	7.310597	10.64059	14.59559	19.17559	24.37999	30.20998
	-E ^h	1.088999	2.552039	4.640338	7.353717	10.69217	14.65563	19.24413	24.45763	30.29612
2s4s ³ S ^e	-E ^a	1.089823	2.559737	4.622176	7.395109	10.699627	14.710585	19.261070	24.583568	30.27983
	-E ^g	1.082400	2.547000	4.636600	7.351200	10.690800	14.655200	19.244800	24.458000	30.29800
	-E ^h	1.098200	2.571120	4.669640	7.393420	10.742320	14.716280	19.315280	24.539320	30.38838

^a present work ^b Ho [21] ^c Dieng [22] ^{f, h} Sakho et al. [23,25] ^g Ivanov [24].

Table 6. Comparison of resonance positions (E_r) for the $2snp\ 1^3P^0$ states of He-like ions with ($Z=2-10$)

states	Z									
		2	3	4	5	6	7	8	9	10
$2s2p\ 1P^0$	-E ^a	1.38452	3.52941	6.63513	10.74939	15.87203	21.97982	29.123500	37.245801	46.35608
	-E ^b	1.38627	3.51512	6.63896	10.76042	15.88056	21.99993	29.11878	37.23735	46.35555
	-E ^c	1.38875	3.51124	6.63574	10.75592	15.88274	22.00624	29.12974	37.25324	46.37670
	-E ^d	1.38508	3.51402	6.63778	10.75916	15.87924	21.99854	29.11734	37.23574	46.35394
	-E ^e	1.38610	3.51472	6.63874	10.76030	15.88052	21.99990	29.11878	37.23728	46.35552
$2s2p\ 3P^0$	-E ^a	1.51812	3.754265	6.976910	11.257263	16.450448	22.975692	29.990312	38.079336	47.39557
	-E ^e	1.52099	3.75637	6.99127	11.22598	16.46057	22.69511	29.92561	38.16408	47.39853
	-E ^c	1.52765	3.75406	6.68046	11.20686	16.43326	22.36966	29.88606	38.11246	47.33886
	-E ^d	1.52294	3.75877	6.99391	11.22875	16.46345	22.69805	29.93260	38.16705	47.40155
	-E ^e	1.52114	3.75650	6.99140	11.22610	16.46069	22.69522	29.92972	38.16419	47.39862
$2s3p\ 1P^0$	-E ^a	1.12861	2.71872	5.02614	8.085786	11.83237	16.31959	21.50138	27.43400	34.09158
	-E ^b	1.12800	2.72286	5.03650	8.070700	11.82630	16.30350	21.50260	27.42380	34.06690
	-E ^f	1.12478	2.72140	5.04040	8.081290	11.84458	16.33008	21.53781	27.46775	34.11992
$2s3p\ 3P^0$	-E ^a	1.15760	2.79035	5.16438	8.238113	12.04535	16.55832	21.82254	27.67227	34.42233
	-E ^b	1.15806	2.79702	5.15851	8.242240	12.04818	16.57633	21.82669	27.79926	34.49405
	-E ^f	1.17200	2.80960	5.16942	8.251460	12.05573	16.58221	21.9669	27.80185	34.49501

^a present work ^b Ho [21] ^c Dieng [22] ^d Seminario and Sanders [26] ^e Drake et al [27], ^{f,h} Sakho et al. [23,25,28].

Table 7. Comparison of resonance positions (E_r) for the $2pnp\ 1^3D^e$ states of He-like ions with ($Z=2-10$)

states	Z									
		2	3	4	5	6	7	8	9	10
$2p3p^1D^e$	-E ^a	1.13718	2.75737	5.09281	8.12553	12.00837	16.41542	21.69138	27.63846	34.18386
	-E ⁱ	1.13844	2.74820	5.08057	8.13553	11.91292	16.41266	21.63470	27.57902	34.24560
	-E ^h	1.13813	2.74779	5.07984	8.13415	11.91073	16.40956	21.63064	27.57390	34.23942
$2p3p^3D^e$	-E ^a	1.17092	2.81021	5.17619	8.26757	12.06326	16.59905	21.86760	27.84227	34.53439
	-E ⁱ	1.16756	2.81113	5.17667	8.26426	12.07398	16.60586	21.85992	27.83617	34.53463
	-E ^h	1.16673	2.80705	5.17081	8.25728	12.06619	16.59747	21.85105	27.82692	34.52504
$2p4p^1D^e$	-E ^a	1.07621	2.53006	4.61461	7.28041	10.59976	14.57630	19.18003	24.29405	30.09813
	-E ^h	1.07289	2.52064	4.59343	7.29123	10.61405	14.56189	19.13471	24.33255	30.15540
	-E ^c	0.97594	2.37821	4.35081	7.07026	10.36590	14.29590	18.86061	24.06019	29.89473
$2p4p^3D^e$	-E ^a	1.09610	2.56181	4.58913	7.31725	10.69937	14.64041	19.22984	24.43904	30.67341
	-E ^h	1.08149	2.54134	4.62709	7.33821	10.67452	14.63593	19.22240	24.43417	30.27046
	-E ^c	1.00752	2.44490	4.51385	7.21527	10.54944	14.51651	19.11652	24.34951	30.21551

^a present work ⁱ Ho [21] ^c Dieng [22] ^d Seminario and Sanders [26] ^h Sakho et al. [28].

Table 8. Comparison of resonance positions (E_r) for the $3d4d\ 1^3G^e$ states of He-like ions with ($Z=2-10$)

states	Z									
		2	3	4	5	6	7	8	9	10
$3d4d^1G^e$	-E ^a	0.525520	1.280462	2.395852	3.857269	5.644482	7.796414	10.319652	13.092518	16.309918
	-E ^j	0.523664	1.282352	2.389035	3.843409	5.645259	7.794478	10.291009	13.134823	16.325900
	-E ^k	0.524200	1.282000	2.388000	3.840000	5.640000	7.790000	10.284000	13.128000	16.318000
$3d4d^3G^e$	-E ^a	0.535886	1.320193	2.427124	3.911686	5.713264	7.871034	10.319906	13.241968	16.441943
	-E ^j	0.539138	1.313835	2.434874	3.902706	5.717525	7.879424	10.388452	13.244640	16.448006
	-E ^k	0.539400	1.314000	2.434000	3.902000	5.716000	7.878000	10.386000	13.242000	16.444000

^a present work ^j Ivanov and Ho[29] ^k Bachau [30].

Table 9. Comparison of resonance widths (Γ) for the ($2s3s\ ^1S^e$, $2s2p\ ^1^3P^0$, $2p3p\ ^1^3D^e$, $3d4d\ ^1^3G^e$) states of helium-like ions with ($Z=2-10$)

states	Z									
		2	3	4	5	6	7	8	9	10
$2s3s\ ^1S^e$	Γ^a	0.001110	0.002682	0.004930	0.007831	0.011463	0.015749	0.020689	0.026356	0.032683
	Γ^b	0.002700	0.004300	0.005300	0.005930	0.006370	0.006700	0.006900	0.007100	0.007250
$2s2p\ ^1P^0$	Γ^a	0.00277	0.00494	0.00797	0.00688	0.00676	0.00891	0.00880	0.00898	0.00930
	Γ^b	0.00273	0.00438	0.00546	0.00619	0.00670	0.00707	0.00739	0.00762	0.00780
	Γ^d	0.00274	0.00436	0.00542	0.00614	0.00666	0.00706	0.00736	0.00760	0.00780
	Γ^L	0.00274	0.00441	0.00547	0.00692	0.00671	0.00710	0.00741	0.00765	0.00784
$2s2p\ ^3P^0$	Γ^a	0.00058	0.00075	0.00065	0.00069	0.00066	0.00069	0.00072	0.00091	0.00085
	Γ^b	0.00059	0.00062	0.00063	0.00064	0.00064	0.00065	0.00065	0.00065	0.00065
	Γ^d	0.00060	0.00064	0.00066	0.00066	0.00066	0.00066	0.00066	0.00066	0.00066
	Γ^L	0.00059	0.00063	0.00063	0.00064	0.00065	0.00065	0.00065	0.00065	0.00065
$2p3p\ ^1D^e$	Γ^a	0.000461	0.001117	0.002064	0.003294	0.004868	0.006655	0.008794	0.011205	0.013858
	Γ^i	0.001110	0.002208	0.002852	0.003246	0.003504	0.003686	0.003824	0.003928	0.004010
$2p3p\ ^3D^e$	Γ^a	0.0004747	0.0011393	0.0020985	0.0033518	0.0048906	0.0067295	0.0088655	0.0112918	0.0140131
	Γ^i	5.72×10^{-8}	3.32×10^{-8}	1.92×10^{-8}	1.26×10^{-8}	0.94×10^{-8}	0.94×10^{-8}	$< 10 \times 10^{-8}$	$< 10 \times 10^{-8}$	$< 10 \times 10^{-8}$
$3d4d\ ^1G^e$	Γ^a	0.002130	0.005191	0.016132	0.017678	0.022889	0.025608	0.029183	0.030079	0.036605
	Γ^j	0.005088	0.011441	0.015325	0.017687	0.019237	0.020324	0.021124	0.021737	0.022222
$3d4d\ ^3G^e$	Γ^a	0.000011	0.000023	0.000024	0.000025	0.000028	0.000031	0.000041	0.000043	0.000044
	Γ^j	0.000001	0.000008	0.000016	0.000024	0.000030	0.000036	0.000040	0.000044	0.000048

^a present work ^{b,i} Ho [8,21] ^d Seminario and Sanders [26] ^j Ivanov and Ho[29] ^l Manning and Sanders [31].

4. Conclusion

We have carried out calculations of the inter-shell $n\ell n'\ell'$ and $n\ell'n'\ell'$ (with $n \leq 3$, $n' \leq 4$ and $\ell = \ell'$ or $\ell \neq \ell'$) singlet and triplet doubly excited states energies of helium-like ions by using the complex rotation method and new wave functions with two terms are used to calculate resonance parameters for intrashell states. The results of the resonance parameters reported here agree better with other theoretical calculations and experiment data for low Z than those with high Z . The results that we have obtained show that it is possible by using our theoretical approach, to describe the highly doubly excited two-electrons states. However, experimental techniques using synchrotron radiation are becoming more sophisticated and can provide very accurate values of the resonance parameters of the Rydberg series of multi-electron atomic systems. We can always use the method of complex rotation and with weak wave functions of terms to validate these experimental results. So this work also provides parameters for total widths for such resonances. Since the widths are related to the inverse of the autoionisation lifetimes, the present results should be useful for future investigations of the underlying mechanism of autoionisation.

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