

Electric-field-tuned Binding Energies of Trions in Silicene, Germanene, and Stanene Monolayers

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Abstract We predict the formation of intravalley controllable trions in buckled two-dimensional (2D) materials such as silicene, germanene, and stanene monolayers in an external electric field. Performing a study within the framework of a nonrelativistic potential model using the method of hyperspherical harmonics (HH), the three-body Schrödinger equation is solved with the Rytova-Keldysh potential by expanding the wave functions of a trion in terms of the HH. Then, we numerically solve a resultant system of coupled differential equations. The ground state energies of intravalley trions controlled by the external electric field are presented. The dependencies of the binding energy (BE) of trions in silicene, germanene, and stanene as a function of the electric field are shown to be qualitatively similar. BEs of trions formed by *A* and *B* excitons have a non-negligible difference that increases slightly as the electric field increases. We demonstrate that trion BEs can be controlled by the external electric field.

Keywords: trion, binding energy, Xene monolayer

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1. Introduction

The prediction of the existence of trions [1] consisting of an exciton and an electron or a hole, known as negatively or positively charged excitons (X^{\mp}), gave rise to many theoretical and experimental studies of trions in bulk materials, quantum-well systems, and two-dimensional (2D) materials. Atomically thin transition-metal dichalcogenides (TMDCs) are a class of 2D materials that have remarkable optical and electronic properties [2,3]. Since the observation of trions in two-dimensional MoS_2 monolayers [4] in 2013, trions have been the subject of intense studies, both experimentally and theoretically, in TMDCs monolayer. In the past decade, different experimental groups have observed and reported the signature of a trion in TMDCs monolayers.

Theoretical studies of trions have integrated a wide variety of techniques and carried them out to calculate the binding energies (BEs) of excitonic complexes in monolayer TMDCs (see the reviews [5,6,7,8,9]). Results for BEs of trions in TMDC monolayers yielded impressively accurate results consistent with experimental data. In the framework of the few-body physics approaches such as the hyperspherical harmonic method and three-body Faddeev equations in configuration or momentum spaces trions in TMDC are investigated in [10,11,12,13].

Another category of 2D semiconductors is the buckled

2D allotropes of silicon, germanium, and tin, known as silicene, germanene, and stanene, and collectively referred to as Xenes [14,15]. Experimental studies revealed one of the most crucial differences between Xenes and graphene and TMDC that Xenes monolayer is not a perfectly flat sheet, but instead it is slightly buckled [14,16]. As a result, this unique structure of Xenes makes them sensitive to the external electric field applied perpendicular to the monolayer, allowing the band gap to be opened and controlled. The tunable band gap of Xenes gives researchers, among other things, extraordinary in situ control over binding energies and optical properties of excitons in these materials.

In contrast to TMDCs, there is no extensive research on excitonic complexes in Xenes monolayers. The reason is that the synthesis of Xenes monolayers has not been as successful and extensive, as for example, TMDCs monolayers because Xenes monolayers are unstable in the air [17,18]. In contrast to graphene, silicene and other Xenes monolayers do not occur in nature. Nevertheless, silicene nanoribbons were experimentally synthesized on a metal substrate [19,20]. This opened the way for silicene, germanene, and stanene monolayers to be transferred on metal such as Au [21,22,23,24,25] and substrates such as MoS_2 , Ir, ZrB_2 [24,26] and hexagonal boron nitride (hBN) [27,28] and synthesized as freestanding monolayers [29]. Working with a metallic substrate is easier. For example, silicene grown on Ag (111) [18,30] and germanene synthesized by dry deposition on Au (111) surface [23]

have been thoroughly investigated. However, depositing Xene on a metal leads to a significant alteration of properties of the Xene monolayer. Depending on a substrate, the properties of Xenon monolayers vary; see Refs. [14,27] for the list of Xenon properties on different substrates. In contrast to deposition on a metal, depositing Xenon on hBN is harder. However, Xene deposited on hBN preserves its properties because Xene and hBN weakly interact [27]. Xenon monolayers deposited on hBN present a particular interest for studying magnetoexcitons in monolayers and vdWHs [31].

Xenon optical and magneto-optical properties have been addressed in Refs. [16] [32,33,34] and [35,36], respectively. Different physical phenomena, such as the Hall effect [37], the valley-locked spin-dependent Seebeck effect [22], the anomalous quantum Hall effect [38], the quantum spin Hall effect [38] and the Landau levels [30,35,39] are studied because of their essential role in applications of Xenon monolayers in nanodevices and quantum devices [38] [40,41,42,43]

Because of the band inversion, these honeycomb materials are also topological insulators [44,45,46,47,48,49]. The existence of an excitonic insulator phase in silicene, germanene, and stanene was first studied in [50,51] in the framework of the effective-mass approximation. The influence of the screening, band dispersion, and external electric field on transitions in Xenon between excitonic, topological, and trivial insulator phases was investigated in [52].

Currently, there is a shortage of research on exciton complexes in Xenon. In particular, there has not been a study of the formation of three quasiparticle states trions in Xenon monolayers. In this paper, we address this gap and focus on a theoretical investigation of trions in Xenon within the method of hyperspherical harmonics (HH).

The paper is organized as follows. In Sec. 2, we present nonrelativistic potential model for a system of three interacting electrons and holes, and we employ the three-body Schrödinger equation in the effective-mass approximation. Following [13] in the framework of the HH method, the Schrödinger equation with the Rytova-Keldysh potential [53,54] is reduced to a system of coupled differential equations for hyperradial functions. A numerical solution of this system provides the BE and wave function for trions in Xenon. In Sec. 3, we discuss intravalley X^- and X^+ trions in Xenon monolayers and present results of calculations of controllable ground state energies of intravalley trions by an external electric field. Here, we analyze the dependence of the BEs and a probability distribution of three bound particles on the external electric field. Conclusions follow in Sec. 4.

2. Effective-mass Approach for Trions in Buckled 2D Materials

The starting point of the discussion needs to be the framework of the low-energy model for excitons in Xenon monolayer and heterostructure when the external electric field perpendicular to the structure is present. The detailed overview of the low-energy model for excitons in the buckled 2D material can be found in Refs. [31,51]. The

most stable form of Xenon monolayers has the honeycomb structure where sublattices A and B are offset with respect to each other. The offset is denoted by d_0 and is called the buckling constant. When there is no external electric field, Xenon monolayers resemble graphene in the vicinity of the K/K' points. However, the application of the perpendicular electric field leads to on-site potential difference between sublattices, that results in the variable band gap at the K/K' points that changes electron and hole effective masses. Considering the electric-field-dependent band gap at $p = 0$ [55], the effective mass can be written as:

$$m = \frac{|\xi\sigma\Delta_{gap} - ed_0E_{\perp}|}{v_F^2}, \quad (1)$$

where v_F is the Fermi velocity, $\xi, \sigma = \pm 1$ are the valley and spin indices, respectively, $2\Delta_{gap}$ is the intrinsic band gap, E_{\perp} is the external electric field in z -direction. When, $\xi = -\sigma$ the gap is large, since conduction and valence bands are the furthest from the Fermi energy level, and the electron and hole form A excitons. When $\xi = \sigma$, the gap is small and the electron and hole form B exciton. According to Eq. (1) the effective mass of the electron or hole depends on the bandgap, Fermi velocity, buckling constant and is a function of E_{\perp} .

The promotion of an electron from the filled valence band to the empty conduction band leaves an empty electron state in the valence band. The description of such a many-body system can be reduced to the two-particle problem of a negatively charged conduction electron interacting with a positively charged valence hole that forms an exciton or other excitonic complexes such as charged excitons or trions. The trions are formed when an exciton binds another electron or hole to form a negatively or positively charged three-particle system: X^- or X^+ , respectively. A description of the properties of trions requires a solution of a three-particle problem. In buckled two-dimensional monolayers, the resulting trions are considered as Wannier–Mott trions since the correlation between an electron and a hole extends over many lattice periods. The representation of the electron-hole pair bound in a Wannier–Mott exciton shows the strong spatial correlation of these two constituents. Therefore, we are assuming that the interaction of the exciton with the third particle (electron or hole) leads to a large-radius-type system.

We follow the approach in which one assumes that the electron and hole bands are isotropic and parabolic, which is a good approximation for the low-energy spectrum of 2D materials. This form of the Hamiltonian implies that both the electron and hole single particle states form a single parabolic band. The corresponding eigenproblem equation reduces to the Schrödinger equation in the effective mass approximation. This approach is commonly used in the literature to describe excitons and trions in 2D materials. See, for example, Refs. [2,6,8,13] [56,57,58,59,60]. Below, we follow the effective-mass approximation.

To obtain the eigenfunctions and eigenenergies of a 2D trion in Xenon when the electric field is perpendicular to

the Xenon monolayer, we write the Schrödinger equation for an interacting three-particle electron-hole system. Because we are considering the varying electric field E_{\perp} , which is directed along the z -axis, the corresponding term in the 2D Schrödinger equation vanishes. However, the effect of the electric field action is present through the effective mass as follows from Eq. (1). Thus, one can write the 2D Schrödinger equation for the interacting three-particle electron-hole system within the effective-mass approximation in the following form

$$\left[-\frac{\hbar^2}{2} \sum_{i=1}^3 \frac{1}{m_i} \nabla_i^2 + \sum_{i < j}^3 V_{ij}(|\mathbf{r}_i - \mathbf{r}_j|) \right] \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = E \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3), \quad (2)$$

where m_i is the effective mass of the electron or hole defined by Eq.(1), and \mathbf{r}_i is the i th particle position. We assume only two types of charge carriers: an electron and hole with the corresponding effective masses. In Eq. (2), $V_{ij}(|\mathbf{r}_i - \mathbf{r}_j|)$ is the interaction potential between q_i and q_j charges in a 2D material that was first derived in Ref. [53] and was independently obtained by Keldysh [54]. We refer to it as the Rytova-Keldysh (RK) potential. The Rytova-Keldysh potential describes the Coulomb interaction screened by the polarization of the electron orbitals in the 2D lattice and has the following form

$$V_{ij}(r) = \frac{\pi k q_i q_j}{2\kappa \rho_0} \left[H_0\left(\frac{r}{\rho_0}\right) - Y_0\left(\frac{r}{\rho_0}\right) \right], \quad (3)$$

where $r = |\mathbf{r}_i - \mathbf{r}_j|$ is the relative coordinate between two charge carriers q_i and q_j . In Eq. (3) $k = 9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$, κ is the dielectric constant of the environment that is defined as $\kappa = (\epsilon_1 + \epsilon_2)/2$, where ϵ_1 and ϵ_2 are the dielectric constants of two materials that the Xenon layer is surrounded by, ρ_0 is the screening length, which sets the boundary between two different behaviors of the potential due to a nonlocal macroscopic screening, and $H_0\left(\frac{r}{\rho_0}\right)$ and $Y_0\left(\frac{r}{\rho_0}\right)$ are the Struve function and Bessel function of the second kind, respectively. The screening length ρ_0 can be written as $\rho_0 = (2\pi\chi_{2D})/\kappa$ [56]. where χ_{2D} is the 2D polarizability, which in turn is given by $\chi_{2D} = \epsilon/4\pi$ [54], where ϵ is the bulk dielectric constant of the Xenon monolayer. It is worth noting that in Ref. [61] a very good approximation to the RK potential that is simpler to use, fairly precise in both limits, and remarkably accurate for all distances was introduced.

To obtain a solution of the Schrödinger equation (2) for the negatively and positively charged trions, we use the method of hyperspherical harmonics (HH) [62]. The main idea of this method is the expansion of the wave function of the trion in terms of HH that are the eigenfunctions of the angular part of the Laplace operator in the four-

dimensional (4D) space. As the first step, let us separate the center-of-mass (c.m.) and the relative motion of three particles and introduce sets of mass-scaled Jacobi coordinates [62,63]. There are three equivalent sets of Jacobi coordinates and there is the orthogonal transformation between these sets [64,65]. For three non-identical particles that have different masses the mass-scaled Jacobi coordinates for the partition i read as follows [62,63,64]:

$$\begin{aligned} x_i &= \sqrt{\frac{m_j m_k}{(m_j + m_k)\mu}} (\mathbf{r}_j - \mathbf{r}_k), \\ y_i &= \sqrt{\frac{m_i (m_j + m_k)}{(m_i + m_j + m_k)\mu}} \left(\frac{m_j \mathbf{r}_j + m_k \mathbf{r}_k}{m_j + m_k} - \mathbf{r}_i \right), \quad i \neq j \neq k = 1, 2, 3, \end{aligned} \quad (4)$$

where

$$\mu = \sqrt{\frac{m_i m_j m_k}{m_i + m_j + m_k}} \quad (5)$$

is the three-particle effective mass. In Eq. (4) the subscripts i , j , and k are a cyclic permutation of the particle numbers.

The transformation (4) allows the separation of c.m. and relative motions of three particles with Hamiltonian (2), and the Schrödinger equation for the relative motion of the three-body system reads

$$\left[-\frac{\hbar^2}{2\mu} (\nabla_{x_i}^2 + \nabla_{y_i}^2) + V(x_1) + V(x_2) + V(x_3) \right] \Psi(x_i, y_i) = E \Psi(x_i, y_i). \quad (6)$$

In Eq. (6), $V(x_i)$ is the interaction potential between two particles at the relative distance x_1 , x_2 , and x_3 , respectively, where x_i is the modulus of the Jacobi vector x_i (4), and (6) is written for any of the set $i = 1, 2, 3$ of the Jacobi coordinates (4). The orthogonal transformation between three equivalent sets of the Jacobi coordinates simplifies calculations of matrix elements involving $V(x_i)$ potentials.

This method is presented in detail in Ref. [13] and is briefly outlined here. We introduce in the 4D space the hyperradius $\rho = \sqrt{x_i^2 + y_i^2}$ and a set of three angles $\Omega_i \equiv (\alpha_i, \varphi_{x_i}, \varphi_{y_i})$ where φ_{x_i} and φ_{y_i} are the polar angles for the Jacobi vectors x_i and y_i respectively, and α_i is an angle defined as $x_i = \rho \cos \alpha_i$, $y_i = \rho \sin \alpha_i$. Next, we rewrite the Schrödinger equation (6) for the trion using hyperspherical coordinates in the 4D configuration space [13]. This transformation allows us to reduce the solution of the problem for the three particles in the 2D configuration space to the motion of one particle in the 4D configuration space. Then we introduce the hyperspherical harmonics $\Phi_{K\lambda}(\Omega)$ in the 4D configuration space, which are the eigenfunctions of the angular part of the generalized Laplace operator $\hat{K}^2(\Omega_i)$ in the 4D configuration space $\hat{K}^2(\Omega_i)\Phi_{K\lambda}(\Omega) = K(K+2)\Phi_{K\lambda}(\Omega)$ [62],

where K is a grand angular momentum. Here we are using the short-hand notation $\lambda = \{l_x, l_y, L, M\}$, where L is the total orbital angular momentum of the trion, M is its projection, and the grand angular momentum $K = 2n + l_x + l_y$, l_x , where l_x and l_y are the angular momentum corresponding to x and y Jacobi coordinates, respectively, and $n \geq 0$ is an integer number.

The functions $\Phi_{K\lambda}(\Omega)$ present a complete set of orthonormal bases, and one can expand the wave function of the trion $\Psi(\rho, \Omega)$ in terms of the HH $\Phi_{K\lambda}(\Omega)$ as

$$\Psi(\rho, \Omega_i) = \rho^{-3/2} \sum_{K\lambda} u_{K\lambda}(\rho) \Phi_{K\lambda}(\Omega_i). \quad (7)$$

In Eq. (7), $u_{K\lambda}(\rho)$ are the hyperradial functions, and by substituting (7) into the Schrödinger equation written in the hyperspherical coordinates [13], one can separate the radial and angular variables and get a set of coupled differential equations for the hyperradial functions $u_{K\lambda}(\rho)$:

$$\left[\frac{d^2}{d\rho^2} - \frac{(K+1)^2 - 1/4}{\rho^2} + \kappa^2 \right] u_{K\lambda}(\rho) = \frac{2\mu}{\hbar^2} \sum_{K'\lambda'} \mathcal{W}_{K\lambda K'\lambda'}(\rho) u_{K'\lambda'}(\rho). \quad (8)$$

In Eq. (8) $\kappa^2 = 2mB/\hbar^2$, where B is trion BE. The coupling effective potential energy $\mathcal{W}_{K\lambda K'\lambda'}(\rho)$ is

$$\mathcal{W}_{K\lambda K'\lambda'}(\rho) = \int \Phi_{K\lambda}^*(\Omega_i) \sum_{i < j}^3 V_{ij}(|\mathbf{r}_i - \mathbf{r}_j|) \Phi_{K'\lambda'}(\Omega_i) d\Omega_i. \quad (9)$$

The coupling effective interaction (9) is defined via the RK potential (3). Substituting (3) into Eq. (9), one obtains the matrix elements of the effective potential energies. The method of calculations of the effective potential energies is given in [13]. Calculations of matrix elements $\mathcal{W}_{K\lambda K'\lambda'}(\rho)$ of the two-body $V_{ij}(|\mathbf{r}_i - \mathbf{r}_j|)$ interactions in the hyperspherical harmonics expansion method for a three-body system are greatly simplified by using the HH basis states appropriate for the partition corresponding to the interacting pair. Using the matrix elements $\mathcal{W}_{K\lambda K'\lambda'}(\rho)$ in Eq. (8), one can solve the system of coupled differential equations numerically. Results of numerical solutions of (9) for trions in Xenes are presented in the next section.

3. Intravalley Trions in Xenes

We apply the present theoretical approach for calculations of the trion BEs in the following freestanding Xenes monolayers: silicene (Si), germanene (Ge), and stanene (Sn) in the external electric field $E_z = E_\perp$ perpendicular to the Xene layer, as shown in Figure 1. The form of the trion wave function (7) is the most general, not restricted to any particular mass ratio of electrons and

holes and describes the three-particle relative motion. The splitting of the conduction and valence bands in Xenes due to spin-orbit coupling at non-zero electric fields leads to the formation of A and B excitons in the larger or smaller band gaps, with corresponding larger or smaller masses of the electron and hole. Two of the three particles constituting a positive or a negative trion in the Xene monolayer have the same masses due to the equity of the mass of electron and hole that form an exciton. However, they are not identical because they have different charges. Below we consider trions formed by a singlet bright A or B exciton and an electron (X^-) or hole (X^+).

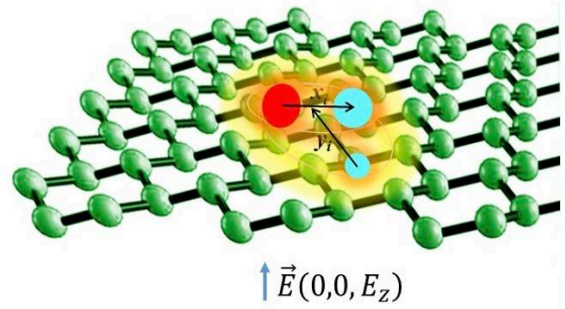


Figure 1. (Color online) Schematic representation of the X^- trion in the external electric field perpendicular to the freestanding Xenes layer. x_i and y_i are Jacobi coordinates for the partition i .

In 2D monolayer of Xenes intravalley and intervalley trions can be formed. Here we consider only intravalley trions. An interaction of bright the A or B exciton with another charged carrier in the same valley, which can either be an electron or a hole, forms X^- or X^+ intravalley trion. Schematics in Figure 2 show the possible formations of the X^- (Figure 2a and Figure 2b) and X^+ (Figure 2c and Figure 2d). As follows from Eq. (1), the electron and hole effective masses in A exciton are the same. The B excitons are composed of electron and hole which also have equal masses but smaller than masses of the electron and hole in A excitons. We denoted these masses as m_A and m_B , respectively. The intravalley X^- and X^+ trions have the same two particles' masses. As follows from Eq. (5), the effective masses of the X^- and

X^+ trions formed by A or B exciton are $\mu_A = \sqrt{\frac{m_B m_A^2}{m_B + 2m_A}}$ and $\mu_B = \sqrt{\frac{m_A m_B^2}{m_A + 2m_B}}$, respectively [13]. Because

$m_A > m_B$, it follows that $\mu_A > \mu_B$. Therefore, the effective mass of the X^- and X^+ trions formed by the A exciton is larger than mass of the X^- and X^+ trions formed by the B exciton. Due to the proportionality of the BE of trions to the three-particle effective mass μ [13], a BE of X^- and X^+ trions formed by the A exciton is larger than BE of trions formed by the B exciton. In the ground state, both intravalley trions formed by charge carriers from the same valley are spin-singlet trions. The intervalley trions can exist in either the singlet or triplet states. They are not considered in this work.

The intravalley X^- and X^+ trions in Xenes monolayers have two particles (the electron and hole) that have the same masses, and the third particle (the electron or hole) has a different mass, as seen in Figures 2a–2d. Therefore, we have to deal with three non-identical particles because two particles with the same masses have different charges.

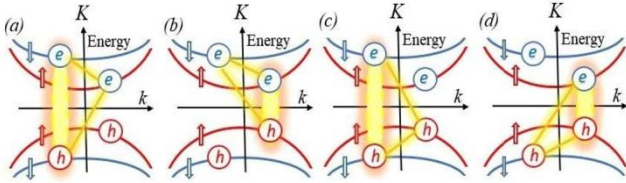


Figure 2. (Color online) Schematic representation of the low-energy band structure for 2D Xenes material and formation of the intravalley X^- and X^+ trions. Panels (a), (b) and (c), (d) represent the intravalley X^- and X^+ , respectively.

In calculations of the BEs of trions in Xenes monolayers, we use the RK potential. We solve the system of coupled differential equations (8) for the hyperradial functions $u_{K\lambda}^L(\rho)$ numerically. By solving the system of equations (8), one finds the binding energy as well as the corresponding hyperradial functions. The latter allows one to construct the wave function (7). Numerical solution of the coupled differential equations requires the control of the convergence of the BEs for trions with respect to the grand angular momentum K for each value of the external electric field. The relative convergence of the BE is checked as $\Delta B/B = [B(K+2) - B(K)]/B(K)$, where $B(K)$, is the BE for the given K . The analysis of the results for the BEs at different values of the electric field shows that the reasonable convergence is reached for $K_{max} = 14$, so we limit our considerations to this value.

The input parameters for calculations of BEs of trions in the freestanding are given in Ref. [31]. The formation of Wannier–Mott excitons due to the electron-hole interaction via the RK potential in semiconducting phases in Xenes monolayers occurs when the external electric field exceed some critical value which is unique to each material [50]. A value of the critical electric field is slightly different for A excitons and for B excitons. Following *ab initio* calculations [21] which determined that the crystal structure of silicene becomes unstable around 2.6 V/Å, we consider in our calculations electric fields up to 2.7 V/Å and study the formations of trions in Xenes at the range of the external electric field from the critical value up to 2.7 V/Å. Results of calculations of dependencies of BE of intravalley trions in a singlet state on the external electric field are presented in Figure 3.

According to Figure 3, the BE increases for all materials as E_{\perp} increases. In addition, in FS Si, Ge, and Sn, we observe a non-negligible difference in the BE of trions formed by A and B excitons. These differences slightly increase as the electric field increases. The trion BEs for FS Ge and FS Sn are qualitatively similar to FS Si, but they are smaller than freestanding silicene. The curves for FS Ge and FS Sn qualitatively resemble that of FS silicene, but at 2.7 V/Å FS germanene reaches a maximum trions

BE of 24.8 (24.3) meV, and the maximum BE for FS stanene is roughly 21.1 (20.5) meV, significantly smaller than for FS silicene—30.1 (29.8) meV. In parentheses the BEs of trions formed by B excitons are given. The percent differences between the trion BE of FS Si and FS Ge and FS Si and FS Sn at the largest electric field considered, are 82% (81%), and 70% (69%), respectively.

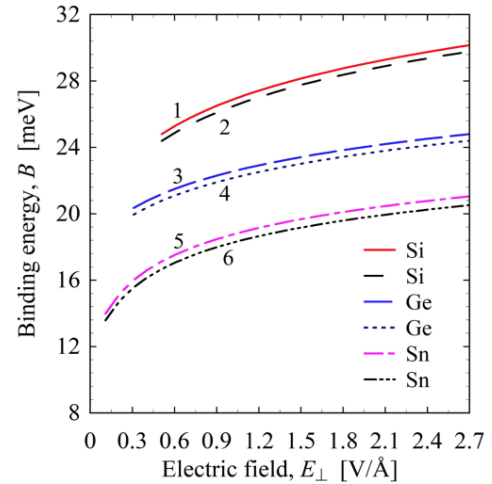


Figure 3. (Color online) Dependencies of the BE of intravalley trions in freestanding silicene, germanene, and stanene on the applied electric field. Curves 1, 3, and 5 correspond to the trions formed by A excitons coupling an electron (X^-) or hole (X^+), curves 2, 4, and 6 correspond to the trions formed by B excitons coupling an electron (X^-) or hole (X^+). The plots for FS Xenes are truncated for the external electric field E_{\perp} less than a critical field.

The following conclusions can be made: i. the increase of the BE as the external electric field increases; ii. BEs for FS silicene, germanene, and stanene are qualitatively similar; iii. a non-negligible difference in the BE of trions formed by A and B excitons.

We calculated the probability distribution for the intravalley trion for the spin-valley configuration shown in Figure 2a. In Figure 4 the interparticle radial probability distribution for intravalley trions in the silicene monolayer is shown. The difference in the probability distribution is related to the difference of the effective masses μ of intravalley trions. The analysis of the dependence of the probability distribution of three particles on the hyperradius ρ and the external electric field leads to the following conclusion: the increase of the external electric field gives the increase of the trions BE and makes trions more compact since the greater binding energy increases the trion formation probability.

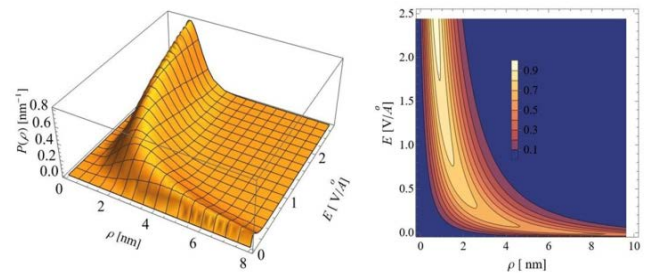


Figure 4. (Color online) Dependence of the probability distribution of three particles in freestanding silicene on hyperradius ρ and applied electric field for intravalley X^- trion

4. Conclusion

We predict the existence of electrically controlled trions in Xenes monolayers. We have applied the hyperspherical harmonics method to the calculation of BEs for the trion, and we predict the formation of trions in freestanding Xenes when the external electric field perpendicular to monolayers is present. The results of BEs calculation for trions formed by *A* and *B* excitons show a non-negligible difference in trions energies that increases slightly as the electric field increases. The BEs of the intravalley trions can be tuned in the range of 24 – 31 meV for silicene, 21 – 26 meV for germanene, and 14 – 20 meV for stanene by varying the external electric field from the critical value that is specific for each material up to 2.7 V/Å. Let us note that the trion binding energies in Xenes are of the same order as in TMDC monolayers [13]. The dependence of the BE for silicene, germanene, and stanene as a function of the electric field is qualitatively similar. Our findings pave the way toward manipulating the trion BEs by an external electric field. The results of calculations of the probability distribution show an increase in the compactness of trions with an increase of the electric field, since the greater binding energy increases the trion formation probability.

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