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Self-Variation Theory-Part II

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Abstract In this article we present the second part of the Self-Variation Theory. The article includes three sections; the gravitational interaction, the justification of the cosmological data, and the system of equations for the structure of matter and quantum phenomena.

Keywords: *electromagnetism, gravity, particle interactions, origin of universe, evolution of universe, structure of matter, quantum phenomena*

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1. Introduction

In this section we formulate the gravitational field equations. The Self-Variation Theory formulates gravity and electromagnetism with the same equations. These Equations concern the field created by the rest mass / electric charge of a particle. The central equation of the Theory relates three physical quantities, the rest mass / electric charge-source of the field, the relative velocity of the field source to the observer, and the propagation velocity of the field relative to the observer. These velocities are directly related to the potential and intensity of the field measured by an observer. The first calculations give consistency of the Theory at the distance scales that we have observational data. Theory predicts increased stellar velocities on the outskirts of galaxies. It also predicts increased velocities of galaxies on the outskirts of galaxy clusters.

The Field Equations for gravity, as given by Self-Variation Theory, predict that near the rest mass-source of the field, gravity is repulsive. Above a value of distance, gravity becomes attractive.

In the context of the Theory, the equations we present in this section do not only apply to gravity and electromagnetism. Three other interactions with traits that do not correspond to gravity resulted from the investigation of the original equation. Like the gravitational interaction, the other three interactions are either attractive or repulsive, depending on the distance from the source of the field. None of the interactions are only attractive or only repulsive. Further investigation of the equations will yield the complete, accurate prediction of the Theory.

1.1. Gravitational Potential

Through a series of mathematical calculations, the self-variation principle necessarily involves a modification of the electromagnetic potential. For comparison the classical electromagnetic Liénard–Wiechert potentials are,

$$V_{LW}=\frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}$$

$$\mathbf{A}_{LW}=V\frac{\mathbf{u}}{c^2}$$

whereas the corresponding self-variation potentials are,

$$V=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}+\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})q}{4\pi\varepsilon_0 c^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

$$\mathbf{A}=V\frac{\boldsymbol{\upsilon}}{c^2}$$

The difference lies in the potential of Equations (3.38), which are not present in Equations (3.31).

The self-variation potential for the gravitational interaction is derived from the above Equations of the electromagnetic self-variation potential by substituting the charge q, with the rest mass M, of the source of the gravitational field hence, $\frac{q}{4\pi\varepsilon_0}\to -GM$, where G, is the constant of gravity and by substituting the acceleration $\boldsymbol{\alpha}$ of the particle in the electromagnetic field, with the intensity $\mathbf{g}$ of the gravitational field, hence, $\boldsymbol{\alpha}\to\mathbf{g}$. Also notice that now $\boldsymbol{\upsilon}$, represents the speed of propagation of the gravitational field, hence we must substitute the speed of light in vacuum c, with the speed of propagation of the gravitational field υ, hence, $c\to\upsilon$. These substitutions lead to the corresponding gravitational potentials of the self-variation,

$$V=-\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}-\frac{GM}{\upsilon^3}\frac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}$$

$$\mathbf{A}=V\frac{\boldsymbol{\upsilon}}{\upsilon^2} \quad .(4.1)$$

where $\mathbf{u}$, is the velocity of the rest mass M relative to the observer, and r, is the distance from the rest mass M. Deriving the gravitational potentials in this way, implies that there is a gravitational analog to the magnetic field $\mathbf{B}$, and has units s^{-1} (see Equations (3.30) – (3.33)). Notice that in the limit case where the speed of propagation of the gravitational field approaches infinity, $\upsilon\to\infty$, we get the limit potential

$$V=-\frac{GM}{r},$$

which is no other than the one of classical mechanics which assumed instant action of gravity at distance r.

Like the corresponding Equation for electromagnetism, Equation (4.1) refers to the gravitational field created by the rest mass of a particle. By taking into account the distribution of particles in spacetime we get the gravitational field on a macroscopic scale.

1.2. Potential, Propagation Speed and Intensity of the Gravitational Field Caused by a Single Rest Mass M

The differential equations we get from Equation (4.1) depend on the direction of the vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$. We study two of these cases in detail. In the first case, we assume that the vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have opposite directions,

$\boldsymbol{\upsilon}=\upsilon\frac{\mathbf{r}}{r}$ and $\mathbf{g}=-g\frac{\mathbf{r}}{r}$, $\boldsymbol{\upsilon}\cdot\mathbf{g}=-\upsilon g$, where $\upsilon=\|\boldsymbol{\upsilon}\|$ and $g=\|\mathbf{g}\|$. Then we have,

$$\frac{d\boldsymbol{\upsilon}}{dt}=\frac{d\upsilon}{dt}\frac{\mathbf{r}}{r}+\upsilon\frac{d}{dt}\left(\frac{\mathbf{r}}{r}\right)=\mathbf{g}=-g\frac{\mathbf{r}}{r} \quad (4.2)$$

Then from Equation (4.1) we have,

$$V=-\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}+\frac{GM}{\upsilon^2}\frac{g}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} \quad (4.3)$$

The gravitational field intensity $\mathbf{g}(r)$ is given by

$$\mathbf{g}(r)=-\nabla V(r)=-\frac{dV}{dr}\frac{\mathbf{r}}{r} \quad (4.4)$$

In Equation (4.2) (see section 3, Figure 3.1) we have that, $dr=-cdw$ (see, section 3). However using the symbols of the current section this Equation is written as, $dr=-\upsilon dt$. From Equation (4.2) we get $\frac{d\upsilon}{dt}=-g$. Combining these Equations we get

$$\upsilon\frac{d\upsilon}{dr}=g=\frac{dV}{dr} \quad (4.5)$$

where with t, we have denoted the time of the observer. From Equation (4.5) we have,

$$\upsilon^2=\sigma+2V \quad (4.6)$$

where σ is a constant parameter. In this section we study the case $\sigma\neq 0$.

From Equations (4.3) and (4.5) we get,

$$V=-\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}+\frac{1}{\upsilon^2}\frac{GM}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{dV}{dr} \quad (4.7)$$

Then, from Equations (4.6) and (4.7) we obtain the differential equation for the potential,

$$V=-\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}+\frac{1}{\sigma+2V}\frac{GM}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{dV}{dr} \quad (4.8)$$

and the differential equation for the speed of propagation of the gravitational field,

$$\upsilon^2-\sigma=-\frac{2GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}+\frac{1}{\upsilon^2}\frac{GM}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{d\upsilon^2}{dr} \quad (4.9)$$

Thus we obtain the system of Equations (4.9) and (2.6),

$$\upsilon^2-\sigma=-\frac{2GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}+\frac{1}{\upsilon^2}\frac{GM}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{d\upsilon^2}{dr} \quad (4.10)$$

$$V=\frac{1}{2}\left(\upsilon^2-\sigma\right)$$

Equation (4.9) relates three physical quantities, the rest mass M of the field source, the velocity $\mathbf{u}$ of the field source relative to the observer, and the speed of propagation $\boldsymbol{\upsilon}$ of the field relative to the observer. The fact that this equation relates only these three physical quantities makes it fundamental to the gravitational interaction. For the observer, the properties of spacetime depend on the rest mass M .

In the electromagnetic interaction the Self-Variation Theory predicts two independent pairs of potentials. One gives the electromagnetic field of the moving electric charge and depends on the velocity $\mathbf{u}$ of the charge (see Equations (3.36)),

$$V_u = \frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

$$\mathbf{A}_u = V_u\frac{\boldsymbol{\upsilon}}{c^2}.$$

The other gives the electromagnetic radiation emitted by the electric charge and depends on the acceleration $\mathbf{a}$ of the charge (see Equations (3.38)),

$$V_\alpha = \frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})q}{4\pi\varepsilon_0 c^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

$$\mathbf{A}_a = V_a\frac{\boldsymbol{\upsilon}}{c^2}.$$

As a consequence of the substitution $\mathbf{a}\rightarrow\mathbf{g}$, this separation cannot be made in the gravitational interaction. The intensity of the gravitational field, the acceleration of gravity $\mathbf{g}$ is related to both the

$$-\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}$$

term and the

$$-\frac{GM}{\upsilon^3}\frac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}$$

term in the second part of Equation (4.1). In addition to Equation (45.4), the potential and the gravitational field strength are also related to each other through Equation (4.1).

We made the substitution $\frac{q}{4\pi\varepsilon_0}\rightarrow -GM$ in order for the interaction resulting from the potentials (4.1) to be attractive. However, this is not achieved. Depending on the value of r , the interaction is either attractive or repulsive. Therefore, in the study we are doing, it is required to investigate from the beginning the case where the intensity $\mathbf{g}$ of the field or the direction of the velocity $\boldsymbol{\upsilon}$ is reversed. In this case the equation $dr = -\upsilon dt$ becomes $dr = \upsilon dt$, and finally Equation (4.6) becomes,

$$\upsilon^2 + 2V = \sigma \tag{4.11}$$

Then, from Equations (4.9) and (4.11) we obtain,

$$\begin{aligned}\upsilon^2 - \sigma = &-\frac{2GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\\ &+\frac{1}{\upsilon^2}\frac{GM}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{d\upsilon^2}{dr}\\ V = &\frac{1}{2}\left(\sigma-\upsilon^2\right)\end{aligned} \tag{4.12}$$

Equations (4.10) and (4.12) give the possible interactions resulting from the potentials (4.1). There are interactions in which the exponential integral $E_i(x)$ appears which we have omitted.

1.3. Gravitational Interaction of Two Bodies

We study the case where a body of rest mass m moves in the gravitational field of a stationary body of rest mass $M >> m$.

In polar coordinates (r,φ) , the orbits $r = r(\varphi)$ of the body of rest mass m is given by the solution of the system of equations,

$$L = mr^2\frac{d\varphi}{dt} = mr^2\dot{\varphi} = constant \tag{4.13}$$

$$\ddot{r} - r(\dot{\varphi})^2 = -g(r) \tag{4.14}$$

and

$$mV(r) + K = E = constant \tag{4.15}$$

In these Equations t is the time of the observer, L and K the angular momentum and the kinetic energy of the body of rest mass m , $\ddot{r} = \frac{d^2r}{dt^2}$, $\dot{\varphi} = \frac{d\varphi}{dt}$ and E the mechanical energy of the system of the two bodies.

From the solution of the differential Equation (4.8) we get the potential $V(r)$. Then, from Equation (4.5) we get the intensity $g(r)$ of the field. Alternatively, from the solution of the differential Equation (4.9) we obtain the speed of propagation of the field $\upsilon(r)$. Then, from Equation (4.6) we get the potential $V(r)$. The solutions given by differential equations (4.8) and (4.9) depend on the inner product $\mathbf{u}\cdot\frac{\boldsymbol{\upsilon}}{\upsilon^2}$. If $\mathbf{u}\cdot\frac{\boldsymbol{\upsilon}}{\upsilon^2} = 0$ we get the simplest possible solutions.

1.4. The Equations of Interaction

In this subsection we present the interaction equations resulting from the potentials (4.1). There are four interactions with different characteristics. One of them has the characteristics of gravity.

Let

$$x = \frac{\sigma}{GM} r \tag{4.16}$$

From the first of Equations (4.10) and transformation (4.13) we get the following equation,

$$\begin{aligned} &\left(\frac{\upsilon^2}{\sigma} - 1\right)\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2 x \\ &= -2 + 2\frac{u^2}{\upsilon^2} + \frac{1}{\upsilon^2}\frac{x d\upsilon^2}{dx} \end{aligned} \tag{4.17}$$

In Equation (4.17), $\boldsymbol{\upsilon}$ is the speed of propagation of the field relative to the observer, and $\mathbf{u}$ is the relative velocity of the observer to the source (rest mass M) of the field.

We made the substitution $\frac{q}{4\pi\varepsilon_0} \to -GM$ (and not $\frac{q}{4\pi\varepsilon_0} \to GM$) in order for the gravitational interaction to be attractive. However, this is not achieved. From Equation (4.17) it follows that there are values of σ and x for which gravity is repulsive. Consequently, the general case of the gravitational interaction is obtained by substituting $\frac{q}{4\pi\varepsilon_0} \to \pm GM$. Through the transformations

$\frac{q}{4\pi\varepsilon_0} \leftrightarrow \pm GM$, $\mathbf{a} \leftrightarrow \mathbf{g}$ and $c \leftrightarrow \upsilon$

we pass from one interaction to another. As a consequence of the equality $-\frac{GM}{x} = \frac{GM}{-x}$, these interactions are related through the $x \leftrightarrow -x$ transformation. Through this transformation one interaction arises from the other. In this sense, the two solutions are 'complementary'. Therefore, for each solution $\upsilon(x)$, $V(x)$, $g(x)$ of the Equations of the field we also get its complementary solution through the transformation $x \to -x$.

From Equation (4.17) and the transformation $x \to -x$ we get the complementary equation,

$$\begin{aligned} &-\left(\frac{\upsilon^2}{\sigma} - 1\right)\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2 x \\ &= -2 + 2\frac{u^2}{\upsilon^2} + \frac{1}{\upsilon^2}\frac{x d\upsilon^2}{dx} \end{aligned} \tag{4.18}$$

For the case we studied ($\boldsymbol{\upsilon} = \upsilon\frac{\mathbf{r}}{r}$ and $\mathbf{g} = -g\frac{\mathbf{r}}{r}$), (4.14) and (4.15) are the general Equations of the gravitational field. Corresponding equations are obtained for all possible combinations in the directions of vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$. The resulting solutions of these differential equations depend on the inner product $\mathbf{u}\cdot\frac{\boldsymbol{\upsilon}}{\upsilon^2}$.

In the differential equations (4.17) and (4.18) the unknown function is the υ . Thus we make the transformation

$$\upsilon(x) = cF(x) \tag{4.19}$$

and we get,

$$\begin{aligned} &\left(\frac{c^2}{\sigma}F^2 - 1\right)\left(1 - \frac{u\cos\theta}{c}\frac{1}{F}\right)^2 x \\ &= -2 + 2\frac{u^2}{c^2}\frac{1}{F^2} + \frac{2}{F}\frac{xdF}{dx} \end{aligned} \tag{4.20}$$

$$\begin{aligned} &-\left(\frac{c^2}{\sigma}F^2 - 1\right)\left(1 - \frac{u\cos\theta}{c}\frac{1}{F}\right)^2 x \\ &= -2 + 2\frac{u^2}{c^2}\frac{1}{F^2} + \frac{2}{F}\frac{xdF}{dx} \end{aligned} \tag{4.21}$$

where θ is the angle of the vectors $\mathbf{u}$ and $\boldsymbol{\upsilon}$.

After finding the function $F(x)$, the velocity $\upsilon(x)$ is given by equation (4.19). The field potential $V(x)$ is given by the second of the Equations (4.10),

$$\begin{aligned} V(x) &= \frac{1}{2}\left(\upsilon^2(x) - \sigma\right) \\ &= -\frac{\sigma}{2}\left(1 - \frac{c^2}{\sigma}F^2(x)\right) \end{aligned} \tag{4.22}$$

The field strength is given by equation (4.4) written in the form,

$$\begin{aligned} \mathbf{g}(x) &= -\frac{\sigma}{GM}\frac{dV}{dx}\frac{\mathbf{r}}{r} \\ &= -\frac{\sigma c^2}{GM}F(x)\frac{dF(x)}{dx}\frac{\mathbf{r}}{r} \end{aligned} \tag{4.23}$$

Thus we obtain the two interactions of the following equations,

$$\begin{aligned} &\left(\frac{c^2}{\sigma}F^2 - 1\right)\left(1 - \frac{u\cos\theta}{c}\frac{1}{F}\right)^2 x \\ &= -2 + 2\frac{u^2}{c^2}\frac{1}{F^2} + \frac{2}{F}\frac{xdF}{dx} \\ &\upsilon(x) = cF(x) \\ &V(x) = \frac{1}{2}\left(\upsilon^2(x) - \sigma\right) \\ &\mathbf{g}(x) = -\frac{\sigma}{GM}\frac{dV}{dx}\frac{\mathbf{r}}{r} \end{aligned} \tag{4.24}$$

$$-\left(\frac{c^2}{\sigma}F^2-1\right)\left(1-\frac{u\cos\theta}{c}\frac{1}{F}\right)^2 x = -2+2\frac{u^2}{c^2}\frac{1}{F^2}+\frac{2}{F}\frac{xdF}{dx}$$
$$\upsilon(x)=cF(x) \tag{4.25}$$
$$V(x)=\frac{1}{2}\left(\upsilon^2(x)-\sigma\right)$$
$$\mathbf{g}(x)=-\frac{\sigma}{GM}\frac{dV}{dx}\frac{\mathbf{r}}{r}$$

We called the interaction of Equations (4.24) "interaction I", and the interaction of Equations (4.25) "interaction II".

The differential Equations (4.20), (4.21) give triads of solutions $(F(x),u,\theta)$. For given velocities u, the solutions given by Equations (4.20) and (4.21) depend on the quotient $\frac{c^2}{\sigma}$. For a given quotient $\frac{c^2}{\sigma}$, Equations (4.20) and (4.21) give solutions for specific velocities u. There are pairs $\left(\frac{c^2}{\sigma},u\right)$ for which equations (4.20) and (4.21) give realistic solutions. A criterion for whether a solution is realistic or not arises from the values that the speed $\upsilon(x)$ takes, as given by equation (4.19). As the propagation speed of the field, it is not necessarily less than c. However, if we assume that the carrier of gravity is a particle, then we have $\upsilon(x)\le c$. In any case, we can draw conclusions about the field from the possible values of its propagation speed.

From Equations (4.12) we obtain the corresponding Equations (4.24) and (4.25),

$$\left(\frac{c^2}{\sigma}F^2-1\right)\left(1-\frac{u\cos\theta}{c}\frac{1}{F}\right)^2 x = -2+2\frac{u^2}{c^2}\frac{1}{F^2}+\frac{2}{F}\frac{xdF}{dx}$$
$$\upsilon(x)=cF(x) \tag{4.26}$$
$$V(x)=\frac{1}{2}\left(\sigma-\upsilon^2(x)\right)$$
$$\mathbf{g}(x)=-\frac{\sigma}{GM}\frac{dV}{dx}\frac{\mathbf{r}}{r}$$

$$-\left(\frac{c^2}{\sigma}F^2-1\right)\left(1-\frac{u\cos\theta}{c}\frac{1}{F}\right)^2 x = -2+2\frac{u^2}{c^2}\frac{1}{F^2}+\frac{2}{F}\frac{xdF}{dx}$$
$$\upsilon(x)=cF(x) \tag{4.27}$$
$$V(x)=\frac{1}{2}\left(\sigma-\upsilon^2(x)\right)$$
$$\mathbf{g}(x)=-\frac{\sigma}{GM}\frac{dV}{dx}\frac{\mathbf{r}}{r}$$

We called the interaction of Equations (4.26) "interaction III", and the interaction of Equations (4.27) "interaction IV". Interaction IV, Equations (4.27), has the characteristics of gravity. In the following subsections we study the main features and consequences for each of the interactions (4.24) - (4.27).

The simplest case of interactions occurs when the source of the field (M) is stationary with respect to the observer, $\mathbf{u}=\mathbf{0}$. Furthermore, from Equations (4.24) - (4.27) we obtain exact solutions for the propagation speed, potential, and interaction strength. In the following subsections we present these solutions.

1.5. Interaction I. Potential, Propagation Speed, and Intensity of the Field Induced by a Stationary Rest Mass Relative to An Observer

We apply Equations (4.24) if $u=0$. From the differential Equation we get,

$$\left(\frac{c^2}{\sigma}F^2-1\right)x=-2+\frac{2}{F}\frac{xdF}{dx} \tag{4.28}$$

Solving (4.28) for F, we have

$$F^2(x)=\frac{\sigma}{c^2}\frac{x^2}{ke^x+x^2+2x+2} \tag{4.29}$$

where k, is the integration constant. Then from (4.29), (4.19) we have

$$\upsilon^2(x)=\sigma\frac{x^2}{ke^x+x^2+2x+2} \tag{4.30}$$

Finally applying the Transformation (4.16) to (4.30) we get the speed of propagation of the gravitational field, as derived from the self-variation gravitational potential, with respect to r,

$$\upsilon^2(r)=\sigma\frac{a^2r^2}{ke^{ax}+a^2r^2+2ar+2} \tag{4.31}$$

where $a=\frac{\sigma}{GM}$. (4.32)

From Equations (4.24) we obtain the gravitational potential with respect to x,

$$V(x)=-\sigma\frac{ke^x+2x+2}{2\left(ke^x+x^2+2x+2\right)} \tag{4.33}$$

Then from Equations (4.33) and Transformation (4.16) we obtain the gravitational potential with respect to r,

$$V(r)=-\sigma\frac{ke^{ar}+2ar+2}{2\left(ke^{ar}+a^2r^2+2ar+2\right)} \tag{4.34}$$

The gravitational field intensity $\mathbf{g}$ is calculated as follows. From Equation (4.4) and Transformation (4.16) we get,

$$\mathbf{g}(x) = -\frac{\sigma}{GM}\frac{dV(x)}{dx}\frac{\mathbf{r}}{r}$$

and from the third of Equations (4.24) we get,

$$\mathbf{g}(x) = -\frac{\sigma}{2GM}\frac{d\upsilon^2(x)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (4.30) we obtain,

$$\mathbf{g}(x) = -\frac{\sigma^2}{2GM}\frac{2kxe^x - kx^2e^x + 2x^2 + 4x}{\left(ke^x + x^2 + 2x + 2\right)^2}\frac{\mathbf{r}}{r} \quad (4.35)$$

We now make a first comparison of the complementary solutions. From Equation (4.30) we have $\sigma > 0$, if $x > 0$. From Equation (4.27) we have that gravity is repulsive ($V(x) < 0$) for the values of k and x for which,

$$ke^x + 2x + 2 > 0.$$

Therefore if $k > 0$, the interaction is attractive for all $x > 0$.

From Equations (4.30), (4.33) and (4.35) we get the limit values,

$$\upsilon(0) = 0 \text{ and } \lim_{x\to\infty}\upsilon(x) = 0, \quad (4.36)$$

$$V(0) = -\frac{\sigma}{2} \text{ and } \lim_{x\to\infty}V(x) = -\frac{\sigma}{2} \quad (4.37)$$

$$g(0) = 0 \text{ and } \lim_{x\to\infty}g(x) = 0. \quad (4.38)$$

From these Equations we get the following conclusions when x takes small, $x \to 0$ and large, $x \to \infty$ values. These limit values are independent of the mass M. At these limiting values, the potential takes the same constant value (Equation (4.37)) with the consequence that the intensity of the field becomes zero (Equation (4.30)). From equation (4.36) it follows that the interaction has a limited range. The range of the interaction depends on the value of the parameter k. When its value increases, the range of the interaction decreases. Therefore, the interaction (4.24) does not have the characteristics of gravity.

Equations (4.30), (4.33) and (4.35) suggest that rest mass may be responsible for interactions that we do not "recognize" as gravity. Considering that charge contributes to the rest mass of a particle, we will "recognize" these interactions as a consequence of a self-variating charge.

We now give the graphical representations of the functions $y = \frac{\upsilon^2(x)}{\sigma}$, $y = \frac{V(x)}{\sigma}$ and $y = \frac{GM}{\sigma^2}g(x)$, if $k \neq 0$. We give the graphs of functions if $k = -0.001$. Figure 4.1 – Figure 4.9 show the main characteristics of the interaction.

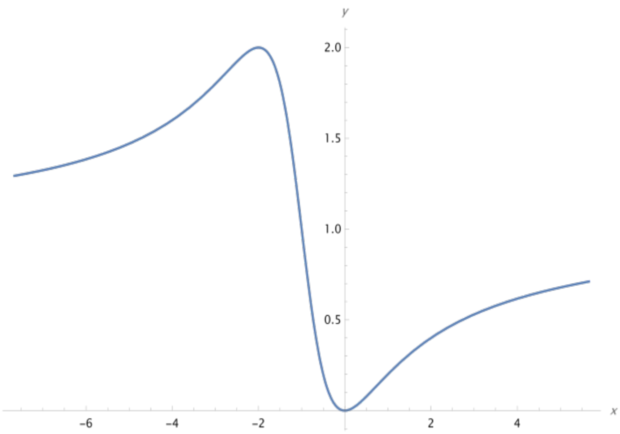

Figure 4.1. The function $y = \frac{\upsilon^2(x)}{\sigma}$, if $k = -0.001$.

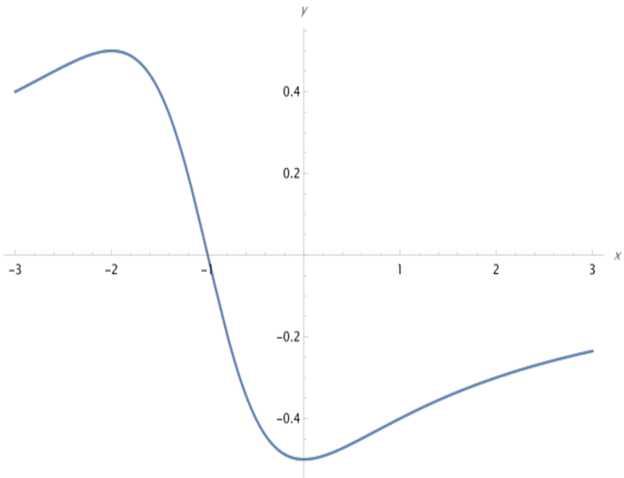

Figure 4.2. The function $y = \frac{V(x)}{\sigma}$, if $k = -0.001$, for small values of x.

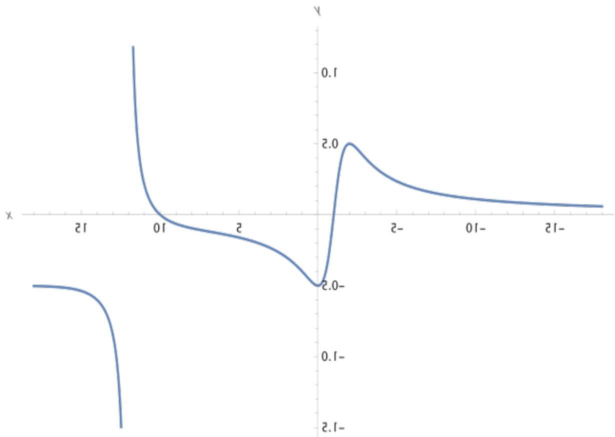

Figure 4.3. The function $y = \frac{V(x)}{\sigma}$, if $k = -0.001$, for large values of x. The graph shows the values of x for which the interaction is attractive ($V(x) < 0$), and the values of x for which the interaction is repulsive ($V(x) > 0$).

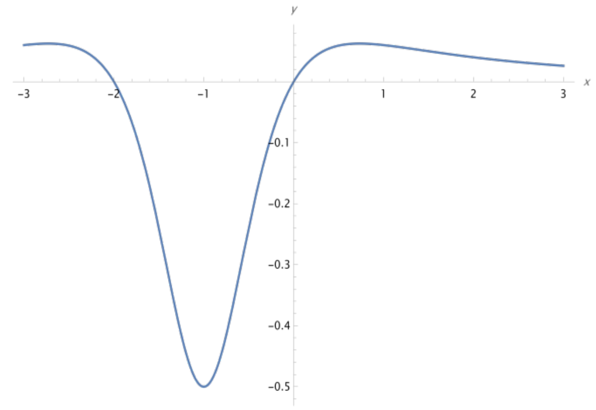

Figure 4.4. The function $y = \frac{GM}{\sigma^2} g(x)$, if $k = -0.001$, for small values of x.

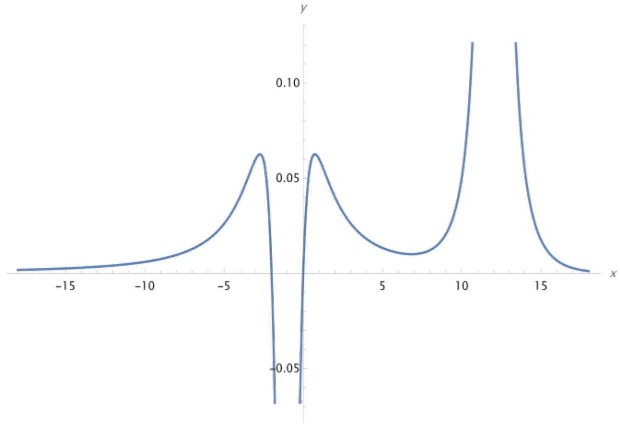

Figure 4.5. The function $y = \frac{GM}{\sigma^2} g(x)$, if $k = -0.001$, for large values of x.

If $k > 0$ the potential is negative for every $x > 0$. In Fig. 4.6 we give the graphical representation of the function $y = \frac{V(x)}{\sigma}$ if $k = 0.01$.

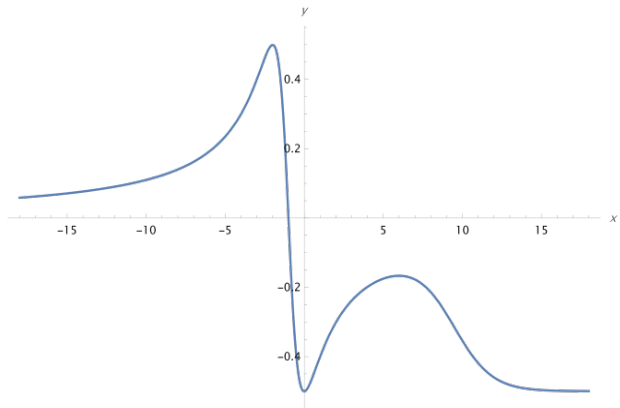

Figure 4.6. The function $y = \frac{V(x)}{\sigma}$, if $k = 0.01$. For positive values of k ($k > 0$) the potential remains negative ($V(x) < 0$) for every value of x.

We give the graphical representations of the functions $y = \frac{\upsilon^2(x)}{\sigma}$, $y = \frac{V(x)}{\sigma}$ and $y = \frac{GM}{\sigma^2} g(x)$ for large values of $|k|$. We give the graphs of functions if $k = 1$.

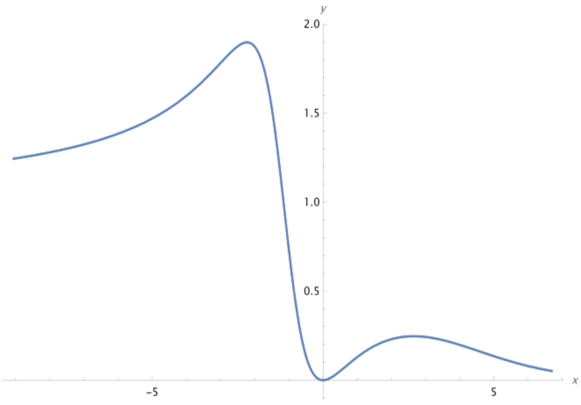

Figure 4.7. The function $y = \frac{\upsilon^2(x)}{\sigma}$, if $k = 1$

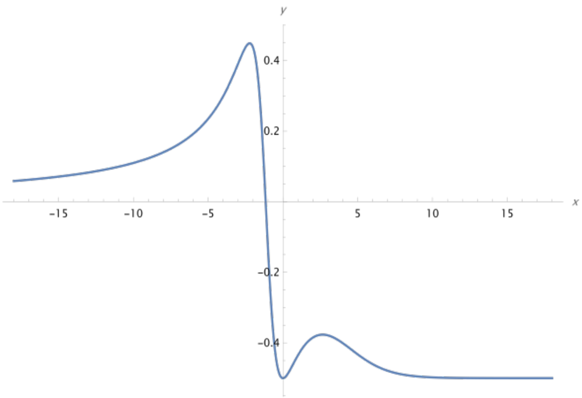

Figure 4.8. The function $y = \frac{V(x)}{\sigma}$, if $k = 1$

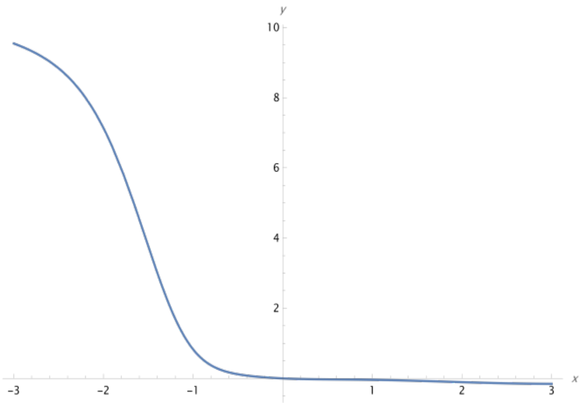

Figure 4.9. The function $y = \frac{GM}{\sigma^2} g(x)$, if $k = 1$

The graphs 4.1 - 4.9 show the dependence of the interaction I on x, and equivalently the distance r. Also, the role of the parameter k is shown.

1.6. Interaction II. Potential, Propagation Speed, and Intensity of the Field Induced by a Stationary Rest Mass Relative to An Observer

We apply Equations (4.25), if $u=0$. From the differential Equation we get,

$$-\left(\frac{c^2}{\sigma}F^2-1\right)x=-2+\frac{2}{F}\frac{xdF}{dx} \qquad (4.39)$$

From Equations (4.39) and (4.25) we get the solution,

$$\upsilon^2(x)=\sigma\frac{x^2}{ke^{-x}+x^2-2x+2} \qquad (4.40)$$

$$V(x)=-\sigma\frac{ke^{-x}-2x+2}{2\left(ke^{-x}+x^2-2x+2\right)} \qquad (4.41)$$

and

$$\mathbf{g}(x)=\frac{\sigma^2}{2GM}\frac{-2kxe^{-x}-kx^2e^{-x}+2x^2-4x}{\left(ke^{-x}+x^2-2x+2\right)^2}\frac{\mathbf{r}}{r} \qquad (4.42)$$

We now make a first comparison of the complementary solutions.

Considering the inequality

$$x^2-2x+2=(x-1)^2+1>0,$$

from Equation (4.40) we have $\sigma>0$.

From Equation (4.41) we have that gravity is repulsive ($V(x)<0$) for the values of k and x for which,

$$ke^{-x}-2x+2>0 \qquad (4.43)$$

From this inequality it follows that for large values of x, the interaction is attractive for $x<1$, and equivalently $r<\frac{GM}{\sigma}$ (see, Transformation (4.16)). Therefore the interaction (4.25) does not have the characteristics of gravity.

From Equations (4.40), (4.41) and (4.42) we get the limit values,

$\upsilon(0)=0$ and $\lim_{x\to\infty}\upsilon(x)=\sqrt{\sigma}$,

$V(0)=-\frac{\sigma}{2}$ and $\lim_{x\to\infty}V(x)=0$,

$g(0)=0$ and $\lim_{x\to\infty}g(x)=0$.

These limit values do not depend on the rest mass M.

We now give the graphical representations of the functions $y=\frac{\upsilon^2(x)}{\sigma}$, $y=\frac{V(x)}{\sigma}$ and $y=\frac{GM}{\sigma^2}g(x)$, if $k\neq 0$. We give the graphs of functions if $k=1$. Figure 4.2 – Figure 4.10 show the main characteristics of the interaction.

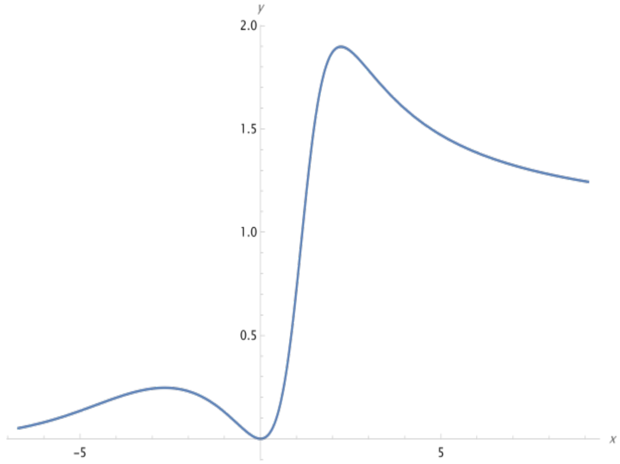

Figure 4.10. The function $y=\frac{\upsilon^2(x)}{\sigma}$, if $k=1$.

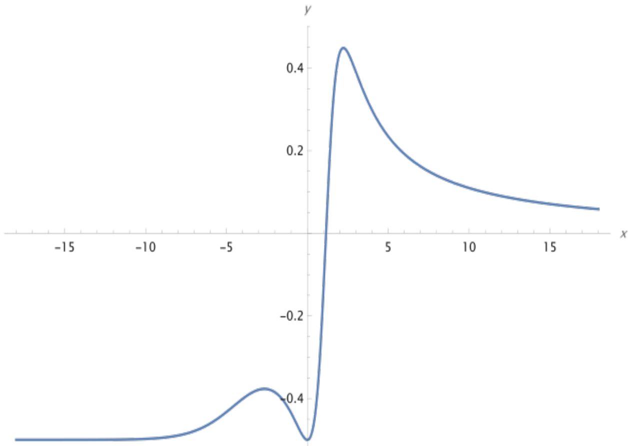

Figure 4.11. The function $y=\frac{V(x)}{\sigma}$, if $k=1$. For small values of x the interaction is attractive ($V(x)<0$) and then becomes repulsive ($V(x)>0$).

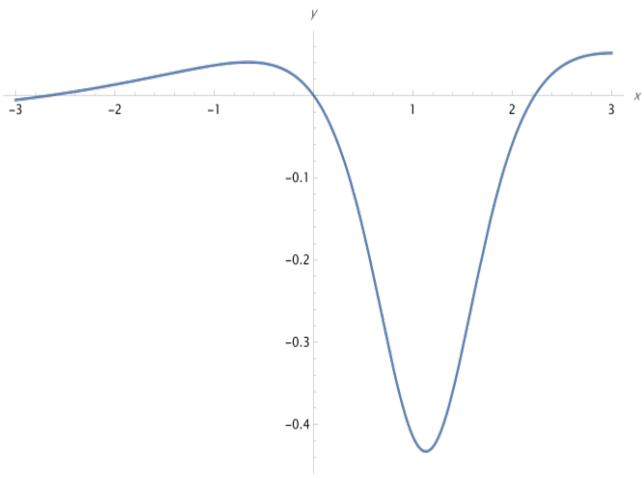

Figure 4.12. The function $y=\frac{GM}{\sigma^2}g(x)$, if $k=1$, for small values of x.

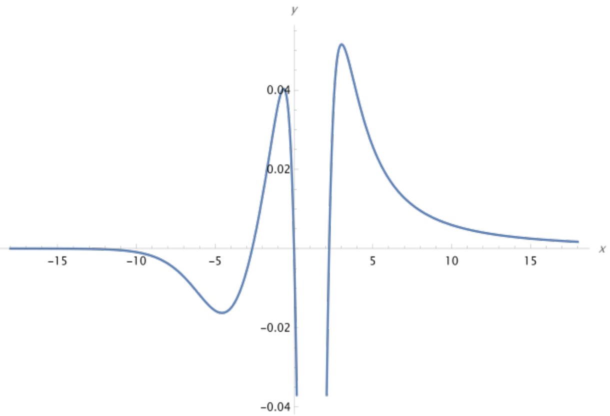

Figure 4.13. The function $y = \frac{GM}{\sigma^2} g(x)$, if $k = 1$, for large values of x.

We now give the graphs of the functions $y = \frac{\upsilon^2(x)}{\sigma}$, $y = \frac{V(x)}{\sigma}$ and $y = \frac{GM}{\sigma^2} g(x)$, if $k = 0$.

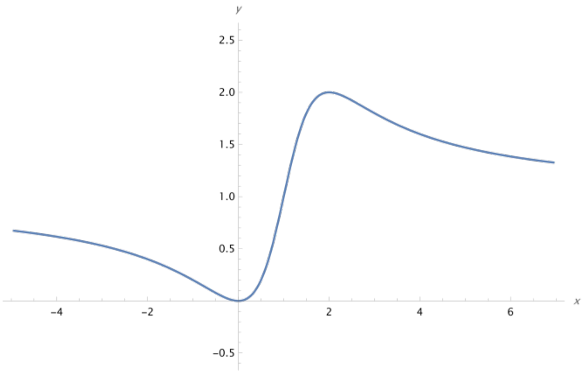

Figure 4.14. The function $y = \frac{\upsilon^2(x)}{\sigma}$, if $k = 0$

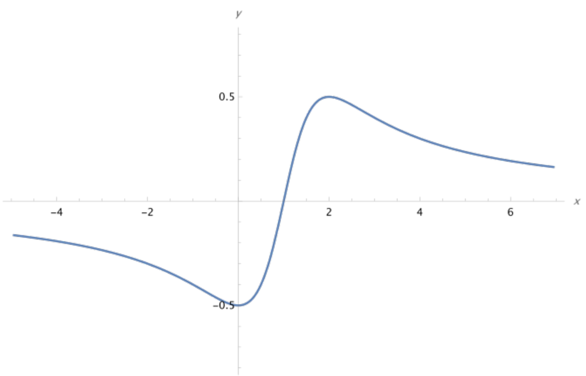

Figure 4.15. The function $y = \frac{V(x)}{\sigma}$, if $k = 0$. For small values of x the interaction is attractive ($V(x) < 0$) and then becomes repulsive ($V(x) > 0$).

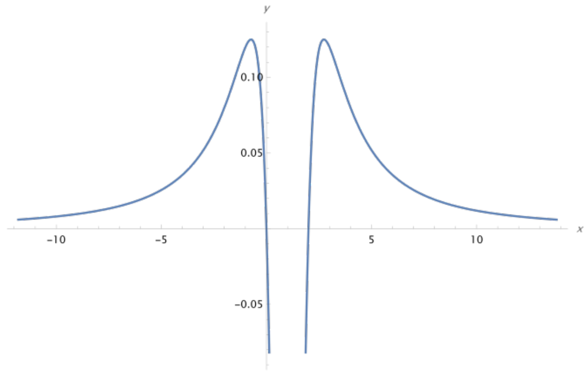

Figure 4.16. The function $y = \frac{GM}{\sigma^2} g(x)$, if $k = 0$.

Equations (4.40), (4.41), and (4.42) suggest that rest mass may be responsible for interactions that we do not "recognize" as gravity in the laboratory. The characteristics of the interaction depend on the parameters σ and κ. The values of these parameters enter into the measurements we make in the laboratory, and not the value of the mass M. Therefore, the interaction II will be "recognized" as a consequence of a "self-variating charge".

1.7. Interaction III. Potential, Propagation Speed, and Intensity of the Field Induced by a Stationary Rest Mass Relative to An Observer

If $u = 0$, from Equations (4.26) we get the following equations,

$$\upsilon^2(x) = \sigma \frac{x^2}{ke^x + x^2 + 2x + 2} \qquad (4.44)$$

$$V(x) = \sigma \frac{ke^x + 2x + 2}{2\left(ke^x + x^2 + 2x + 2\right)} \qquad (4.45)$$

$$\mathbf{g}(x) = \frac{\sigma^2}{2GM} \frac{2kxe^x - kx^2e^x + 2x^2 + 4x}{\left(ke^x + x^2 + 2x + 2\right)^2} \frac{\mathbf{r}}{r} \qquad (4.46)$$

From Equation (4.44) we get that the parameter σ is positive, $\sigma > 0$. Then, from Equation (4.45) we have that for $x > 0$ the potential is positive ($V(x) > 0$). So, if $x > 0$ the interaction is repulsive. Hence the interaction does not have the characteristics of gravity.

We give the graphs of the functions $y = \frac{\upsilon^2(x)}{\sigma}$, $y = \frac{V(x)}{\sigma}$ and $y = \frac{GM}{\sigma^2} g(x)$, if $k = 0$. Figure 4.18, Figure 4.19 and Figure 4.20 show the main characteristics of the interaction.

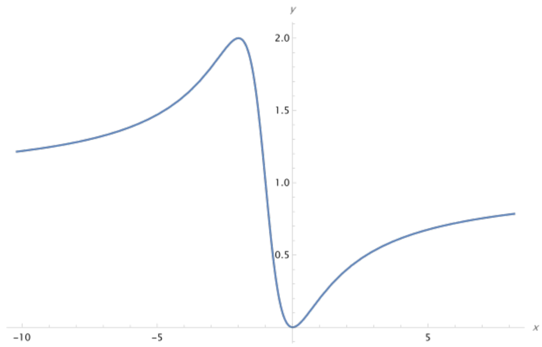

Figure 4.17. The function $y = \dfrac{\upsilon^2(x)}{\sigma}$, if $k = 0$

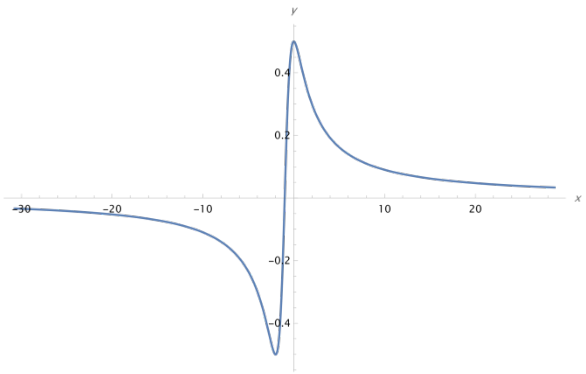

Figure 4.18. The function $y = \dfrac{V(x)}{\sigma}$, if $k = 0$. If $x > 0$, the interaction is repulsive ($V(x) > 0$).

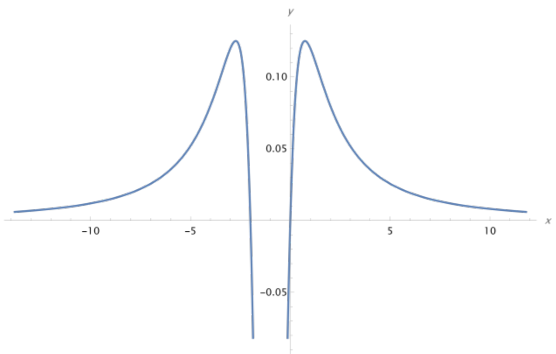

Figure 4.19. The function $y = \dfrac{GM}{\sigma^2} g(x)$, if $k = 0$

From equation (4.44) it follows that the range of the interaction is limited, and its value decreases as the value of k increases. As with interactions I and II, interaction III can be attributed to a self-variating charge.

1.8. Interaction IV. Potential, Propagation Speed, and Intensity of the Field Induced by a Stationary Rest Mass Relative to An Observer

In the context of Self-Variation theory, the gravitational interaction is given by Equations (4.27). From these Equations, for a stationary ($\mathbf{u} = \mathbf{0}$) rest mass M we get the following equations,

$$\upsilon^2(x) = \sigma \frac{x^2}{ke^{-x} + x^2 - 2x + 2} \tag{4.47}$$

$$V(x) = \sigma \frac{ke^{-x} - 2x + 2}{2\left(ke^{-x} + x^2 - 2x + 2\right)} \tag{4.48}$$

$$\mathbf{g}(x) = -\frac{\sigma^2}{2GM} \frac{-2kxe^{-x} - kx^2e^{-x} + 2x^2 - 4x}{\left(ke^{-x} + x^2 - 2x + 2\right)^2} \frac{\mathbf{r}}{r} \tag{4.49}$$

Taking into account the inequality

$$ke^{-x} + x^2 - 2x + 2 = ke^{-x} + (x-1)^2 + 1 > 0,$$

from Equation (4.47) we get that the parameter σ is positive, $\sigma > 0$. Then, from Equation (4.48) it follows that the gravity is attractive ($V(x) < 0$) if

$$ke^{-x} - 2x + 2 < 0 \tag{4.50}$$

and repulsive ($V(x) > 0$) if

$$ke^{-x} - 2x + 2 > 0.$$

From inequality (4.50) it follows that for large values of x, gravity is attractive if $x > 1$. Equivalently (see, Transformation (4.16)), gravity is attractive if $r > \dfrac{GM}{\sigma}$.

If $0 < x < 1$, equivalently if $0 < r < \dfrac{GM}{\sigma}$, gravity is repulsive. Therefore, close to the mass M, up to the distance

$$r_0 = \frac{GM}{\sigma} \tag{4.51}$$

the gravity is repulsive and for larger distances it is attractive. From Equation (4.51) and Transformation (4.16) we get,

$$r = x\frac{GM}{\sigma} = xr_0 \tag{4.52}$$

The parameter $x \in \mathbb{R}$ is the value of r, if we use r_0 as a unit.

For the application of Equations (4.47), (4.48), (4.49) two factors must be taken into account. The first concerns the value of the parameter k. In the following study we applied the Equations for $k = -1$ and $k = 0$. These Equations apply to a point mass M. Thus, the second factor concerns the application of the Equations to the cases of astronomical objects in which matter is distributed in space. We first present the consequences of Equations of the field for a point mass.

For large values of x, from Equation (4.48) we get in a first approximation

$$V(x)=\sigma\frac{-2x+2}{2\left(x^2-2x+2\right)}$$

$$=\sigma\frac{-x+1}{x^2-2x+2}=-\sigma\frac{x-1}{x^2-2x+2}.$$

For large values of x, to a first approximation the potential is independent of the value of k.

In a second approach we have,

$$V(x)=-\sigma\frac{x}{x^2}=-\frac{\sigma}{x},$$

and with Transformation (4.16) we get,

$$V(r)=-\frac{GM}{r}$$

After two approximations the self-variation gravitational potential gives the Newtonian potential,

$$V(x)=-\frac{\sigma}{x} \tag{4.53}$$

We repeat the approximations for the gravitational field strength. For large values of x, from Equation (4.49) we get in a first approximation

$$\mathbf{g}(x)=-\frac{\sigma^2}{GM}\frac{x^2-2x}{\left(x^2-2x+2\right)^2}\frac{\mathbf{r}}{r}$$

and in a second approximation we have,

$$\mathbf{g}(x)=-\frac{\sigma^2}{GM}\frac{x^2}{x^4}\frac{\mathbf{r}}{r}=-\frac{\sigma^2}{GM}\frac{1}{x^2}\frac{\mathbf{r}}{r}$$

and with the Transformation (4.16) we get,

$$\mathbf{g}(x)=-\frac{GM}{r^2}\frac{\mathbf{r}}{r}.$$

For large values of x, the self-variation gravitational field equations yield the Newtonian field.

The simplest orbits for the movement of bodies within a gravitational field are circular ones. We study this case for the gravitational field of mass M. For circular orbits, in Equation (4.14) we have $\ddot{r}=0$ and then we get,

$$r\left(\dot{\varphi}\right)^2=g(r)$$

and equivalently we get,

$$r^2\left(\dot{\varphi}\right)^2=rg(r) \tag{4.54}$$

Taking into consideration that $\dot{r}=0$, the velocity v of the body of mass m is $v=r\dot{\varphi}$ and with Equation (4.54) we get,

$$v(r)=\sqrt{rg(r)} \tag{4.55}$$

From Equation (4.55) and Transformation (4.16) we get,

$$v(x)=\sqrt{\frac{GM}{\sigma}xg(x)} \tag{4.56}$$

From Equations (4.56) and (4.44) we obtain,

$$v(x)=\sqrt{\sigma}\frac{x\sqrt{-ke^{-x}-0.5kxe^{-x}+x-2}}{ke^{-x}+x^2-2x+2} \tag{4.57}$$

For a specific value of the parameter k, Equation (4.57) holds if

$$-ke^{-x}-0.5kxe^{-x}+x-2>0 \tag{4.58}$$

We now give the graph of the function $y=\frac{v(x)}{\sqrt{\sigma}}$, if $k=-1$. From Equation (4.57) we get

$$y=\frac{v(x)}{\sqrt{\sigma}}=\frac{x\sqrt{e^{-x}+0.5e^{-x}+x-2}}{-e^{-x}+x^2-2x+2} \tag{4.59}$$

and from inequality (4.58) we get $x>1.735$. In Figure 4.20 we have the graph of function, if $k=-1$. In Figure 4.21 we have the graph of the function for small values of x, and in Figure 4.22 for large values of x, if $k=0$. If $k=0$, from inequality (4.58) we get $x>2$.

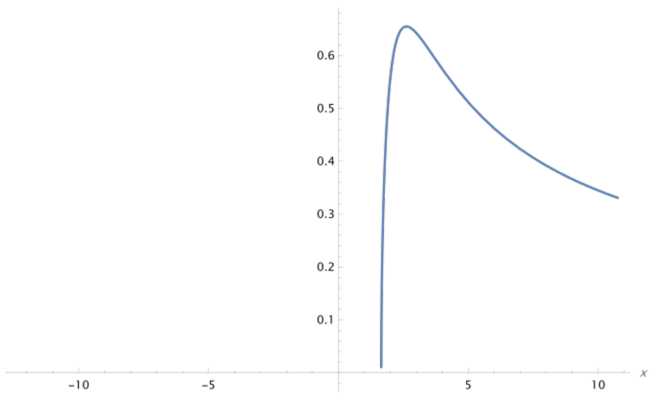

Figure 4.20. The graph of the function $y=\frac{v(x)}{\sqrt{\sigma}}$, if $k=-1$

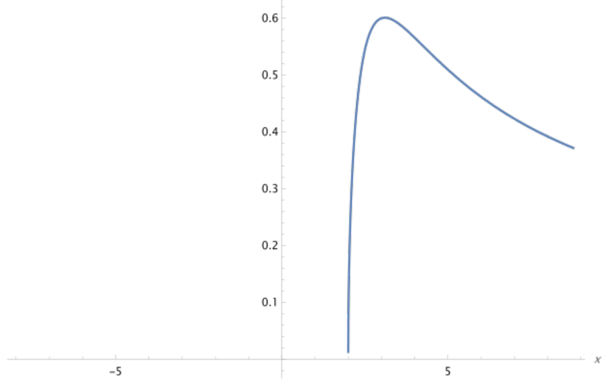

Figure 4.21. The graph of the function $y=\frac{v(x)}{\sqrt{\sigma}}$, if $k=0$, for small values of x.

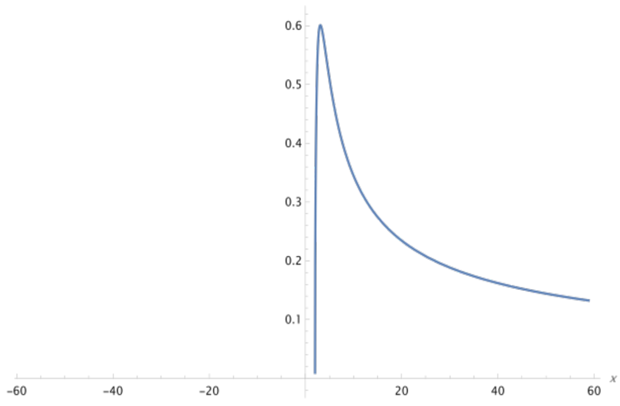

Figure 4.22. The graph of the function $y = \frac{v(x)}{\sqrt{\sigma}}$, if $k = 0$, for large values of x.

For Newtonian gravity, from Equation (4.55) we get

$$v(r) = \sqrt{r\frac{GM}{r^2}} = \sqrt{\frac{GM}{r}}$$

and with Transformation (4.16) we get,

$$v(x) = \sqrt{\frac{\sigma}{x}}$$

so we have,

$$y = \frac{v(x)}{\sqrt{\sigma}} = \frac{1}{\sqrt{x}} \qquad (4.60)$$

In Figure 4.23 we have the graph of Equation (4.60). This graph is different from the graphs in Figure 4.20, Figure 4.21 and Figure 4.22.

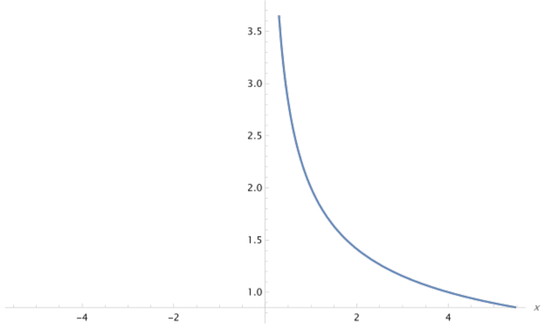

Figure 4.23. The graph of the function $y = \frac{v(x)}{\sqrt{\sigma}} = \frac{1}{\sqrt{x}}$.

Diagrams 4.21, 4.22 and 4.23 already appear in the observational data (see, [1-14]). For large values of x, the graph of the function $y = \frac{v(x)}{\sqrt{\sigma}}$ is measured as given in figure 4.20, Figure 4.21, Figure 4.22 and not as given in figure 4.23. To get the exact prediction of the Theory in objects such as galaxies and galaxy clusters, requires taking into account the distribution of matter in these astronomical objects. A more accurate prediction of the Theory for the velocities of stars in the outskirts of galaxies and the velocities of galaxies in the outskirts of galaxy clusters can be made using appropriate mathematical models on the corresponding distance scales, based on Equation (4.57). Already existing models can be applied by replacing Equation (4.60) with Equation (4.57).

Another difference of the gravitational field equations of Self-Variation Theory with Newtonian gravity exists for the field near the mass M. From Equations (4.47), (4.48) and (4.409) we get the limit values,

$$\upsilon(0) = 0 \text{ and } \lim_{x\to\infty} \upsilon(x) = \sigma, \quad (4.61)$$

$$V(0) = \frac{\sigma}{2} > 0 \text{ and } \lim_{x\to\infty} V(x) = 0, \quad (4.62)$$

$$g(0) = 0 \text{ and } \lim_{x\to\infty} g(x) = 0. \quad (4.63)$$

In Newtonian gravity the field is not defined if $x = 0$. On the contrary, in the self-variation gravitational field, from Equations (4.62) and (4.63) we get $V(0) = \frac{\sigma}{2} > 0$ and $g(0) = 0$. As a consequence of the inequality $V(0) = \frac{\sigma}{2} > 0$, gravity is repulsive in a region near the mass M.

From Equation (4.61) it follows that gravity has a range up to infinity, with the propagation speed of the field tending to the constant value σ. Also, the limiting values (4.47), (4.48) and (4.49) are independent of rest mass M.

1.9. The Potentials V, $\mathbf{A}$

The complete investigation of the Theory of Gravitation is done through Equation (4.1) and the pair of potentials $(V, \mathbf{A})$, where

$$\mathbf{A} = V\frac{\boldsymbol{\upsilon}}{\upsilon}.$$

These potentials relate the gravitational field to spacetime, through the equations,

$$\mathbf{g} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t},$$

$$\mathbf{B} = \nabla \times \mathbf{A},$$

where $\mathbf{B}$ is a gravitational proportional to the magnetic field and has units s^{-1} (see Equations (3.30) – (3.33)).

1.10. Comments

As a consequence of self-variation, at time t, the rest mass / electric charge located at point P acts on point A with the value it had at another point E (see section 3, Figure 3.1). The intensity of the gravitational field given by the potentials $(V, \mathbf{A})$ is the same whether we consider the rest mass M to be constant or consider it to vary according to the principle of self-variation (see section 3, Electromagnetic interaction). This property of the potentials of the Self-Variation Theory allows us to solve the differential Equations of the gravitational field by considering the rest mass M as constant.

We have presented the Equations for Gravity as predicted by the Self-Variation Theory. The substitution $\boldsymbol{\alpha} \to \mathbf{g}$, by which we get the Gravitational potential of self-variation from the corresponding Electromagnetic potential, is an idea belonging to Einstein. Without this substitution the Gravitational field of self-variation cannot arise. The Equations we have presented include Einstein's proposal for the equivalence of acceleration and gravity.

Through the substitutions $\frac{q}{4\pi\varepsilon_0} \leftrightarrow \pm GM$, $\mathbf{a} \leftrightarrow \mathbf{g}$, $c \leftrightarrow \upsilon$ Equation (4.1) is common to gravity and electromagnetism. By doing all the combinations we get the possible equations of the gravitational interaction. A common characteristic of the resulting cases is that gravity is attractive or repulsive as the distance from the rest mass changes. The correlation of attraction / repulsion with the distance from the rest mass is a direct consequence of the Equations of this section.

As a consequence of the self-variation of rest masses, spacetime contains negative energy (see, Equations (3.47) for electric charge). As distance scales increase spacetime contains a large number of particles distributed over a negative energy background. This background of negative energy arises from the self-variation of the remaining masses of the material particles of the universe. At the macroscopic scale the gravitational interaction depends on the distribution of particles and negative energy. On the cosmological scale this combination makes the universe flat (see, [4]). A mathematical model for the Theory's predictions at the macrocosmic and cosmological scales is necessary.

One way to obtain all interactions predicted by Equation (4.1) is to make all combinations for the substitutions

$$\frac{q}{4\pi\varepsilon_0} \leftrightarrow \pm GM ,$$

$$\boldsymbol{\upsilon} \cdot \mathbf{g} = \pm \upsilon g ,$$

and $dr = \pm \upsilon dt$.

A total of eight interactions result. We studied four of them, (4.24), (4.25), (4.26) and (4.27). In the other four the exponential integral $E_i(x)$ appears. The study of these interactions is not included in this publication.

2. The Cosmological Data as a Consequence of the Self-Variation of the Material Particles

In this section we present the implications of self-variation on the cosmological scale. As a consequence of self-variation, in our cosmological-scale observations, the rest mass and electric charge (generally the self-variating charge) of a particle have a smaller value than the corresponding values of the same particle in the laboratory, on earth. This fact has consequences for all physical phenomena occurring in distant astronomical objects, which depend on rest mass and electrical charge. These consequences are recorded in the cosmological data. The redshift of distant astronomical objects is one such consequence.

Fundamental quantities of astrophysics depend on redshift. We calculate as a function of the redshift the mass of the electron and in general the mass of the fundamental particles, the charge of the electron and in general the charge of the fundamental particles, the ionization energy and the degree of ionization of the atoms, the Thomson and Klein-Nishina scattering coefficients, the position-momentum uncertainty and the Bohr radius, and the energy produced in nuclear reactions and hydrogen fusion.

As a consequence of the self-variation of the rest mass of particles, gravity has no consequences on the cosmological scale, it cannot drive the universe into collapse or expansion. Its consequences are limited to smaller distance scales.

In the last subsection of the section we compare the Standard Cosmological Model with the predictions of Self-Variation Theory on the cosmological scale. The reasons why the Standard Cosmological Model has been forced into a series of assumptions to come to terms with the cosmological data are highlighted. However, there are now data, such as the two measured values of Hubble's constant, for which there is no plausible hypothesis that could bring them into agreement with the model. The origin and evolution of the universe as predicted by the Self-Variation Theory presents a remarkable compatibility with the cosmological data.

2.1. Rest Mass and Electric Charge on the Cosmological Scale

The reduced values of rest mass in the past time result in the weakening of gravity, compared to its strength on earth and nearby galaxies. This attenuation is extremely large at cosmological-scale distances. The equations given by the Self-Variation Theory predict that gravity cannot cause the universe to collapse or expand. The consequences of gravity are limited to other distance scales, much smaller than the cosmological one.

The Standard Cosmological Model is based on General Relativity. However, it has repeatedly had to introduce additional assumptions in order to bring the Model into agreement with the observational data. From the hypothesis of Dark Matter (see, [14]) and inflation (see [15]), to the more recent hypothesis of Dark Energy (see, [16,17]). The Standard Cosmological Model is inconsistent with recent measurements from the early twenty-first century to the present (see [4,18,19,20,21]). There is no hypothesis that could bring the Standard Cosmological Model into agreement with the two measured values for the Hubble constant (see, [22,23]).

The Internal Symmetry Theorem justifies the so far known cosmological data, in a flat and static universe. The increased velocities of stars on the outskirts of galaxies, and the increased velocities of galaxies on the outskirts of galaxy clusters are justified by the conclusions of section 4 on gravity.

In a flat and static universe, from Equation (2.31) for $b, K \in \mathbb{R}$, $b > 0$ we get the following equation for the increase in rest mass to over time t,

$$m_0 = m_0(t) = \frac{M_0}{1 + K\exp\left(\frac{bM_0c^2}{\hbar}t\right)} \tag{5.1}$$

From equations (2.21) and (2.22), requiring

$$m_0 E_0 = \frac{\Phi M_0^2 c^2}{(1+\Phi)^2} < 0$$

we get $\Phi < 0$ and equivalently we get,

$$K < 0 \tag{5.2}$$

In Equation (5.1) the constant K is negative.

We consider an astronomical object at distance r from Earth. The emission of the electromagnetic spectrum of the far-distant astronomical object we observe "now" on Earth has taken place before a time interval $\delta t = \frac{r}{c}$. From equation (5.1) we have that the rest mass $m_0(r)$ on the distant astronomical object at the moment of emission was,

$$m_0(r) = \frac{M_0}{1 + K\exp\left(\frac{bM_0c^2}{\hbar}\left(t - \frac{r}{c}\right)\right)} \tag{5.3}$$

From Equations (5.1) and (5.3) we obtain,

$$m_0(r) = m_0 \frac{1 + K\exp\left(\frac{bM_0c^2}{\hbar}t\right)}{1 + K\exp\left(\frac{bM_0c^2}{\hbar}t\right)\exp\left(-\frac{bM_0c}{\hbar}r\right)} \tag{5.4}$$

Different particles have different rest mass M_0. Furthermore, in different particles the self-variation can evolve in a different way, which can be expressed by a different value of the constant b. Thus in Equation (5.4) we denote,

$$\frac{bM_0c^2}{\hbar} = \frac{b_k M_0 c^2}{\hbar} = k_p > 0 \tag{5.5}$$

With the index 'p' we denote the particle to which the constant k_p refers. With this symbolism, Equation (5.4) is written in the form,

$$m_0(r) = m_0 \frac{1 + Ke^{k_p t}}{1 + Ke^{k_p t}e^{-k_p \frac{r}{c}}} \tag{5.6}$$

We now denote by A the time-dependent function,

$$A = A(t) = -\Phi(t) = -Ke^{k_p t} > 0 \tag{5.7}$$

From Equations (5.6) and (5.7) we obtain,

$$m_0(r) = m_0 \frac{1 - A}{1 - Ae^{-k_p \frac{r}{c}}} \tag{5.8}$$

From Equation (5.7) we obtain,

$$\frac{dA}{dt} = \dot{A} = k_p A > 0 \tag{5.9}$$

Similarly, starting from Equation (2.33) we get the following equations,

$$\frac{bM_0'c^2}{\hbar} = \frac{b_k' M_0 c^2}{\hbar} = k_p' > 0 \tag{5.10}$$

$$q(r) = q\frac{1 - B}{1 - Be^{-k_p' \frac{r}{c}}} \tag{5.11}$$

$$B = -K'e^{k_p' t} > 0 \tag{5.12}$$

$$\frac{dB}{dt} = \dot{B} = k_p' B > 0 \tag{5.13}$$

The replacement of the function $\Phi(t)$ with the functions $A(t)$ and $B(t)$ was done to preserve the notation of the cosmological scale equations as they are in already published articles (see, [24]).

2.2. The Redshift of the Distant Astronomical Objects

The fine structure constant a is defined as

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0 c\hbar} \tag{5.14}$$

where q_e is the electric charge of the electron. From Equation (5.11) if $q = q_e$ the charge of electron we obtain,

$$\alpha(r) = \alpha\left(\frac{1 - B}{1 - Be^{-k_e' \frac{r}{c}}}\right)^2 \tag{5.15}$$

The energy of the electron in the atom is

$$E_n = -\frac{1}{n^2}\frac{Z^2K^2m_e q_e^4}{2\hbar^2}$$
$$n = 1, 2, 3, \ldots$$

where m_e is the rest mass and q_e is the electric charge of the electron, Z is the atomic number and K is Coulomb's constant (see [12,25]). The wavelength λ inversely proportional to the photon energy E, $\lambda = \frac{2c\pi\hbar}{E}$ (see, [26]). Therefore, the wave length λ of the linear spectrum is inversely proportional to the factor $m_e q_e^4$. If we denote by λ_0 the wavelength of a photon emitted by an atom "now" on Earth, in the laboratory and by λ the same wavelength of the same atom received "now" on Earth from the far-distant astronomical object, the following relation holds,

$$\frac{\lambda}{\lambda_0} = \frac{m_e q_e^4}{m_e(r) q_e^4(r)}$$

and from equations (5.8) and (5.11) if $q = q_e$ the charge and $m_0 = m_e$ the rest mass of electron we obtain,

$$\frac{\lambda}{\lambda_0} = \frac{1 - Ae^{-k_e \frac{r}{c}}}{1 - A} \left(\frac{1 - Be^{-k_e' \frac{r}{c}}}{1 - B} \right)^4 \quad (5.16)$$

From equation (5.16) we have for the redshift z,

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1$$

of the astronomical object that

$$z = \frac{1 - Ae^{-k_e \frac{r}{c}}}{1 - A} \left(\frac{1 - Be^{-k_e' \frac{r}{c}}}{1 - B} \right)^4 - 1 \quad (5.17)$$

From Equations (5.15) and (5.17) we get,

$$z = \frac{1 - Ae^{-k_e \frac{r}{c}}}{1 - A} \left(\frac{a}{a(r)} \right)^2 - 1 \quad (5.18)$$

The increase of rest mass to over time is given by equation (2.31). Considering that the electric charge of the electron contributes a small percentage to its total rest mass, we conclude that in Equations (2.31) and (2.33) is, $M_0' << M_0$. If we assume that in the same particle the constant b is the same for rest mass and electric charge, then then we conclude that the charge q of the electron increases at a much lower rate, compared to the rate of increase of its rest mass m_0. This conclusion is confirmed by the cosmological data (see, [27,28]). Considering that the electric charge of the electron increases at a much slower rate than its rest mass, from equation (5.18) we get,

$$z = \frac{1 - Ae^{-k_e \frac{r}{c}}}{1 - A} - 1$$

and equivalently we obtain,

$$z = \frac{A}{1 - A} \left(1 - e^{-k_e \frac{r}{c}} \right) \quad (5.19)$$

For small distances r, from Equation (5.19) we get,

$$z = \frac{A}{1 - A} \left(1 - 1 + k_e \frac{r}{c} \right)$$

and equivalently we get,

$$z = \frac{k_e A}{c(1 - A)} r$$

and comparing this Equation with Hubble's law $cz = Hr$ (see, [29]) we get,

$$H = H_e = \frac{k_e A}{1 - A} \quad (5.20)$$

where $H = H_e$ is the Hubble constant for the linear spectrum of atoms. If the electromagnetic radiation we measure depends on ′′heavy′′ particles, such as the proton and the neutron, Equation (20) becomes,

$$H_p = \frac{k_p A}{1 - A} \quad (5.21)$$

The Self-Variation Theory predicts the measurement of at least two values of the Hubble constant (see, [22,23]). On the cosmological scale, the self-variation of the electron and the heavy particles correspond to different values of Hubble's constant.

Taking into account that $H > 0$, $A > 0$ and $k_e > 0$, from Equation (5.20) we get,

$A < 1$.

From Equation (5.10) we get,

$$z < \frac{A}{1 - A}$$

and taking into account that $A > 0$ we obtain,

$$\frac{z}{1 + z} < A < 1 \quad (5.22)$$

From this inequality and considering the possible values of redshift we conclude that,

$$A \to 1^- \quad (5.23)$$

From equations (5.21) and (5.9) we have,

$$\frac{dH}{dt} = \dot{H} = \frac{H^2}{A} \quad (5.24)$$

From this Equation and relation (5.23) we conclude that Hubble's constant increases slightly with time.

2.3. The Rest Mass of the Electron As a Function of Redshift

From equations (5.8) and (5.19) we obtain,

$$m_e(z) = \frac{m_e}{1 + z} \quad (5.25)$$

The redshift is measured with great precision. Therefore, Equation (5.25) gives the relation between $m_e(z) = m_e(r)$ and m_e very precisely.

A large set of physical phenomena and mechanisms depend on the rest mass of the electron. Therefore it is important to know precisely its value in distant astronomical objects. This accuracy is given by Equation (5.25).

2.4. The Diminished Energies of Distant Astronomical Objects

From Equation (5.25) we get,

$$E(z)=\frac{E}{1+z} \tag{5.26}$$

for the energy $E=m_e c^2$ of the electron.

Taking into account the two measured values of the Hubble constant (see [22,23]) we conclude that equation (5.26) is also approximately valid for heavy particles. This equation predicts that the energy resulting from hydrogen fusion and nuclear reactions is reduced in distant astronomical objects.

2.5. The Thomson and Klein-Nishina Scattering Coefficients As a Function of Redshift of the Distant Astronomical Objects

The laboratory value of the Thomson scattering coefficient is given by equation,

$$\sigma_T=\frac{8\pi}{3}\frac{q_e^2}{m_e^2 c^4} \tag{5.27}$$

where m_e the rest mass and q_e the electric charge of the electron. Thus we have,

$$\frac{\sigma_T(z)}{\sigma_T}=\left(\frac{m_e}{m_e(z)}\right)^2\left(\frac{\alpha}{\alpha(z)}\right)^2$$

and taking into account the very slow rate of change of the fine structure constant ($\alpha(z)\approx\alpha$) we get,

$$\frac{\sigma_T(z)}{\sigma_T}=\left(\frac{m_e}{m_e(z)}\right)^2$$

and with Equation (5.25) we obtain,

$$\frac{\sigma_T(z)}{\sigma_T}=(1+z)^2 \tag{5.28}$$

The Thomson coefficient concerns the scattering of photons with low energy E. For photons with high energy E the photon scattering is determined from the Klein-Nishina coefficient,

$$\sigma=\frac{3}{8}\sigma_T\frac{m_0c^2}{E}\left(\ln\left(\frac{2E}{m_0c^2}\right)+\frac{1}{2}\right) \tag{5.29}$$

in the laboratory and,

$$\sigma(z)=\frac{3}{8}\sigma_T(z)\frac{m_0(z)c^2}{E(z)}\left(\ln\left(\frac{2E(z)}{m_0(z)c^2}\right)+\frac{1}{2}\right) \tag{5.30}$$

in an astronomical object with redshift z. From Equations $E(z)=m(z)c^2$ and (5.26) we get,

$$\frac{m_0(z)}{E(z)}=\frac{m_0}{E}$$

and Equation (5.30) becomes,

$$\sigma(z)=\frac{3}{8}\sigma_T(z)\frac{m_0c^2}{E}\left(\ln\left(\frac{2E}{m_0c^2}\right)+\frac{1}{2}\right)$$

and with Equation (5.29) we get,

$$\frac{\sigma(z)}{\sigma}=\frac{\sigma_T(z)}{\sigma_T}$$

and with Equation (5.28) we obtain,

$$\frac{\sigma(z)}{\sigma}=\frac{\sigma_T(z)}{\sigma_T}=(1+z)^2 \tag{5.31}$$

From Equation (5.31) we conclude that the Thomson and Klein-Nishina scattering coefficients increase with redshift and in the same way.

From Equation (5.19) we obtain,

$$\lim_{r\to\infty} z=\frac{A}{1-A} \tag{5.32}$$

Then from Equations (5.31) and (5.32) we get,

$$\frac{\sigma(r\to\infty)}{\sigma}=\frac{\sigma_T(r\to\infty)}{\sigma_T}=\frac{1}{(1-A)^2} \tag{5.33}$$

Considering the limit (5.23) and Equation (5.33) we conclude that the Thomson and Klein-Nishina scattering coefficients had enormous values in the very early universe. In its initial phase the universe was totally opaque. From this initial phase stems the Cosmic Microwave Background Radiation (see, [30,31]) we observe today.

2.6. The Ionization and Excitation Energies of Atoms As a Function of Redshift of the Distant Astronomical Objects

The ionization energy as well as the excitation energy of atoms X_n is proportional to the factor $m_e q_e^4$, where m_e is the rest mass and q_e the electric charge of the electron. Therefore we have,

$$\frac{X_n(r)}{X_n}=\frac{m_e(r)}{m_e}\left(\frac{q_e(r)}{q_e}\right)^4$$

and considering that the electric charge of the electron increases at a much slower rate than its rest mass we get,

$$\frac{X_n(r)}{X_n}=\frac{m_e(r)}{m_e}$$

and with equation (5.25) we have,

$$\frac{X_n(r)}{X_n}=\frac{X_n(z)}{X_n}=\frac{1}{1+z}$$

and equivalently we obtain,

$$X_n(r) = X_n(z) = \frac{X_n}{1+z} \qquad (5.34)$$

From Equation (5.34) we conclude that the ionization and excitation energies of atoms decrease with increasing redshift. This fact has consequences on the degree of ionization of atoms in the distant astronomical objects.

The number of excited atoms in a gas in a state of thermodynamic equilibrium is given by Boltzmann's equation,

$$\frac{N_n}{N_1} = \frac{g_n}{g_1}\exp\left(-\frac{X_n}{KT}\right) \qquad (5.35)$$

where N_n is the number of atoms at energy level n, X_n the excitation energy from the the 1st to the n^{th} energy level, $K = 1\cdot 38\times 10^{-23} JK^{-1}$ Boltzmann's constant, T the temperature in degrees Kelvin, and g_n the multiplicity of level n, i.e. the number of levels into which level n is split apart inside a magnetic field.

From Equations (5.34) and (5.35) we obtain,

$$\frac{N_n}{N_1} = \frac{g_n}{g_1}\exp\left(-\frac{X_n}{KT(1+z)}\right) \qquad (5.36)$$

For the hydrogen atom for $n=2$, $X_2 = 10.5eV = 16.4\times 10^{-10} J$, $g_1 = 2$, $g_2 = 8$ and at the surface of the Sun where $T \approx 6000K$ equation (5.36) implies that just one in 10^8 atoms is at state $n=2$. Correspondingly from equation (5.36) and for $z=1$ wee have $z=1$, for $z=2$ we have $\frac{N_2}{N_1} = 5.8\times 10^{-3}$, and for $z=5$ we have $\frac{N_2}{N_1} = 0.15$.

From Equations (5.34) and (5.36) we conclude that in the past, the universe went through an ionization phase of possibly long duration.

2.7. The Position-Momentum Uncertainty As a Function of Redshift of the Distant Astronomical Objects

Combining equations (2.23) and (5.7) we have,

$$J_i = \frac{c_i}{1-A(t)}$$

in the laboratory, and

$$J_i = \frac{c_i}{1-A\left(t-\frac{r}{c}\right)} = J_i = \frac{c_i}{1-A(t)\exp\left(-k_p\frac{r}{c}\right)}$$

for an astronomical object at distance r, and combining these two equations with equation (5.8) we get,

$$\frac{J_i(r)}{J_i} = \frac{m_0(r)}{m_0}$$

and with equation (5.25) we obtain,

$$\frac{J_i(r)}{J_i} = \frac{1}{1+z} \qquad (5.37)$$

From the position-momentum uncertainty, for the axis x_i we have,

$$J_i\Delta x_i \sim \hbar$$

in the laboratory, and

$$J_i(z)\Delta x_i(z) \sim \hbar$$

for the astronomical object, and combining these two relations we get,

$$J_i(z)\Delta x_i(z) = J_i\Delta x_i$$

and with equation (5.37) we have,

$$\Delta x_i(z) = (1+z)\Delta x_i \qquad (5.38)$$

From equation (5.38) we conclude that the uncertainty (see, [32]) $\Delta x_i(z)$ of position of a material particle increases with the redshift. Moreover, as the universe evolved towards the state we observe today, the uncertainty of position of material particles was decreasing.

From equations (5.38) and (5.32) we have,

$$\Delta x_i(r\to\infty) = \frac{\Delta x_i}{1-A} \qquad (5.39)$$

Considering the limit (5.23) and Equation (5.39) we conclude that in the very early universe there existed great uncertainty of position of material particles. The same conclusions arise for the Bohr radius,

$$R_{Bohr}(z) = (1+z)R_{Bohr} \qquad (5.40)$$

$$R_{Bohr}(r\to\infty) = \frac{R_{Bohr}}{1-A} \qquad (5.41)$$

2.8. On the type Ia Supernovae

The production of energy in the universe is mainly through hydrogen fusion and nuclear reactions. Therefore, the energy produced in the past at distant astronomical objects was smaller than the corresponding energy produced today in our galaxy (see, Equation (5.26)). Furthermore, the self-variation of the electron's rest mass played a defining role in the energy produced in the past at distant cosmological objects. This is due to the fact that the fundamental astrophysical parameters depend on the rest mass of the electron, which depends on the redshift.

A characteristic example concerns type Ia supernovae. The value of the rest mass of the electron, given as a function of the redshift z from Equation (5.25), plays a defining role at all phases of evolution of a star which ends up exploding as a type Ia supernova. As a consequence of equations (5.25) and (5.38) the intrinsic luminosity of supernovae of type Ia supernovae depends on redshift. The dependence of brightness on redshift is recorded at the seemingly long distances of type Ia supernovae (see, [33,34]).

2.9. The Evolution of the Universe. Vacuum State

From equation (5.34) it follows that as the universe evolved to the state we observe today the ionization energy increased. This prediction is generally valid for any kind of negative dynamical energies which bind together material particles to produce more complex particles.

From equation (5.25) we have,

$$\Delta m_0(z)c^2 = \frac{\Delta m_0 c^2}{1+z}$$

for the energy $\Delta m_0 c^2$, the mass deficiency, which ties together the particles which constitute the nuclei of the elements. According to this Equation the energy $\Delta m_0 c^2$, like the ionization energies, increased as the universe evolved towards its present state.

From Equations (5.25) and (5.32) we have,

$$m_0(r \to \infty) = m_0(1-A) \neq 0 \qquad (5.42)$$

Considering the limit (5.23) and Equation (5.42) we conclude that, as the universe tends toward its initial state, the rest masses of material particles tend to zero,

$$m_0(r \to \infty) = m_0(1-A) \to 0 \qquad (5.43)$$

With the notation we follow, from Equation (2.22) we have,

$$E_0(r \to \infty) = 0 \qquad (5.44)$$

According to the relations (5.43) and (5.44) the initial state of the universe slightly differed from vacuum. Considering the conservation of energy-momentum, we conclude that the total mass / energy of the universe asymptotically tend to zero or its is zero. We called this initial state of the universe the 'Vacuum State'.

As a consequence of the Vacuum State, the gravitational interaction cannot play the role attributed to it by the Standard Cosmological Model. Gravity cannot cause either the collapse or the expansion of the universe.

The gravitational interaction strengthens with the passage of time, as the rest masses of material particles increase. From one point and on this is in position to accumulate matter within “small” regions of space. The role of gravity is limited to the creation of the large structures of the universe.

2.10. A Comparison of the Cosmological Predictions of Self-Variation Theory Versus the Standard Cosmological Model, Based on the Cosmological Data

In this subsection we compare the predictions of the Standard Cosmological Model (SCM) and the Self-Variation Theory (SVT), based the 14 main cosmological data.

1. Origin of the universe
SCM, Big Bang.
SVT, Vacuum State.

2. Redshift
SCM, it is a direct consequence of the expansion of the universe.
SVT, it is a direct consequence of the self-variation of rest mass of the electron. In the past (large redshift) the electron transition energies inside the atom were much smaller.

3. Cosmic Microwave Background Radiation
SCM, it is a remnant of the Big Bang.
SVT, it is a consequence of the enormous values of the Thomson and Klein-Nishina scattering coefficients in the distant past.

4. Increased luminosity distances of type Ia
SCM, is forced to do the Dark Energy case.
SVT, it is a direct consequence of the self-variation of the fundamental parameters of astrophysics (mass of electron, ionization energy and degree of ionization of atoms, Thomson and Klein-Nishina scattering coefficients, Bohr radius, production of energy via hydrogen fusion and nuclear reactions). In the distant past these parameters had different values.

5. Flatness of the Universe
SCM, an attempt is made to justify it with the Inflation hypothesis.
SVT, the total energy content of the Universe is predicted to be zero, therefore the Universe on the grand scale is and was always flat.

6. Nucleosynthesis of the chemical elements
SCM, the prediction agrees with observations, for a particular decrease rate of the temperature versus the expansion rate of the universe that is adopted.
SVT, further investigation of the System of Equations (6.5), (6.6) is required.

7. Ionization of atoms in the early universe
SCM, it is predicted as a consequence of the high temperatures after the Big Bang.
SVT, it is a direct consequence of the dependence of the ionization energy from redshift. In the past (large redshift) the ionization energy was much smaller.

8. Distribution of matter on the cosmological scale.
SCM, it inconsistent with recent measurements.
SVT, it is consistent with the recent measurements.

9. Variation of the fine structure constant
SCM, is not predicted.
SVT, it is a direct consequence of the Self-variation of the electric charge.

10. The Horizon problem
SCM, an attempt is made to explain it with the hypothesis of inflation.
SVT, the self-variation model predicts that the position-momentum uncertainty is huge in the early universe, tending to infinity in the distant past. Therefore everything was connected in the early universe.

11. The larger than expected velocities of astronomical objects at the outskirts of large structures in the universe
SCM, is forced to do the Dark Matter case.
SVT, it is predicted (see, section 4).

12. Absence of magnetic monopoles in the universe
SCM, magnetic monopoles have never been observed, hence the problem for SCM, since it predicts their existence as a consequence of the Big Bang.
SVT, the detailed study of electromagnetism in section 3 rules out the existence of magnetic monopoles.

13. Olbers' paradox
SCM, it is justified by the expansion of the universe.
SVT, it is a consequence of Equation (5.26).

14. The two measured values of Hubble's constant
SCM, the two measured values of the Hubble constant are incompatible with the SCM.
SVT, on the cosmological scale, the self-variation of the electron and the heavy particles correspond to different values of Hubble's constant.

3. Self-Variation and Quantum Phenomena. the Structure of Matter

In this section we present the structure of the generalized particle. As a consequence of the principles of Self-Variation Theory, this structure is given by a System of two Equations. The first of these Equations is a linear system of NN equations. The second is a differential equation. Self-variation propagates as a "disturbance" in space-time, i.e. it creates a wave. The mathematical formalism of this wave is given by the same System of Equations.

3.1. The Momentum Distribution of the Generalized Particle

The main goal of Self-Variation Theory is to calculate the N -vectors $J = J(X)$ and $P = P(X)$. Then, from Equations (1.8) and (1.9) we obtain the functions $m_0 = m_0(X)$ and $E_0 = E_0(X)$. If the momentum P is due to a charge q , the calculation of the function $q = q(X)$ is done by solving the differential Equation (1.1) (see the proof of Equation (2.32) in section 2).

The relative position of N -vectors J and P in spacetime is given by Equations (1.15),

$$P_0 = \Phi_{00}J_0 + \Phi_{01}J_1 + \Phi_{02}J_2 + \ldots + \Phi_{0(N-1)}J_{N-1}$$

$$P_1 = \Phi_{10}J_0 + \Phi_{11}J_1 + \Phi_{12}J_2 + \ldots + \Phi_{1(N-1)}J_{N-1}$$

$$P_2 = \Phi_{20}J_0 + \Phi_{21}J_1 + \Phi_{22}J_2 + \ldots + \Phi_{2(N-1)}J_{N-1}$$

$$\vdots$$

$$P_{(N-1)} = \Phi_{(N-1)0}J_0 + \Phi_{(N-1)1}J_1 + \Phi_{(N-1)2}J_2 + \ldots + \Phi_{(N-1)(N-1)}J_{N-1} \quad (6.1)$$

where $\Phi_{ij} = \Phi_{ij}(X)$. Denoting T the $N \times N$ matrix $\{\Phi_{ij}\}$,

$$T = \begin{bmatrix} \Phi_{00} & \Phi_{01} & \Phi_{02} & \cdot\ \cdot\ \cdot & \Phi_{0(N-1)} \\ \Phi_{10} & \Phi_{11} & \Phi_{12} & \cdot\ \cdot\ \cdot & \Phi_{1(N-1)} \\ \Phi_{20} & \Phi_{21} & \Phi_{22} & \cdot\ \cdot\ \cdot & \Phi_{2(N-1)} \\ \cdot & \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ \Phi_{(N-1)0} & \Phi_{(N-1)1} & \Phi_{(N-1)2} & \cdot\ \cdot\ \cdot & \Phi_{(N-1)(N-1)} \end{bmatrix} \quad (6.2)$$

Equation (6.1) is written in the form,

$$P = TJ \quad (6.3)$$

From Equations (6.3) and (1.7),

$$J + P = C \quad (6.4)$$

We obtain,

$$(T + I)J = C \quad (6.5)$$

From Equation (6.5) it follows that the functions $\Phi_{ij} = \Phi_{ij}(X)$ determine the relationship of N -vectors J and C, and finally determine the relative position of N -vectors J , P and C in spacetime. Solving the system of Equations (6.5) we get J as a function of C. Then, from Equation (6.4) we get P as a function of C. The solution of the system of Equations (6.5) depends on whether the determinant $D = |T + I|$ of the matrix $T + I$ is different from zero or equal to zero.

From Equation (1.8) we get,

$$\left(J_n J^n\right)_{;k} = \left(m_0^2 c^2\right)_{;k}$$

and equivalently we get,

$$\left(J_n J^n\right)_{;k} = \frac{\partial}{\partial x_k}\left(m_0^2 c^2\right)$$

and with Equation (1.2) we obtain,

$$\left(J_n J^n\right)_{;k} = \frac{2b}{\hbar} P_k m_0^2 c^2$$

and with Equation (18) we obtain,

$$\left(J_n J^n\right)_{;k} = \frac{2b}{\hbar} P_k \left(J_n J^n\right)$$

and with Equation (6.4) we obtain,

$$\left(J_n J^n\right)_{;k} = \frac{2b}{\hbar} P_k \left(c_k - J_k\right)\left(J_n J^n\right) \qquad (6.6)$$

where with $;k$ we denote the covariant derivative with respect to x^k. Substituting the J_n and $J^n = g^{nk} J_k$, as derived from Equation (6.5) into Equation (6.6) we obtain a differential equation with unknown functions $\Phi_{ij} = \Phi_{ij}(X)$. Therefore, the solution of the system of Equations (6.5) and (6.6) gives the energy-momentum distribution of the generalized particle, and the rest masses m_0 and $\frac{E_0}{c^2}$. In the System of Equations (6.5), (6.6), using Equation (6.4), we can replace the N -vector J with P.

3.2. On the Wave Behavior of Matter

Self-variation propagates as a "disturbance" in space-time, i.e. it creates a wave. In the context of Self-Variation Theory, the wave created by the self-variation of the fundamental particles results in the quantum behavior of matter. Physicists have come into contact many times with the system of Equations (6.5), (6.6). Bohr's work (see, [35]), corresponds to a standing wave for which, however, we do not know the wave function. Schrödinger understands the wave behavior of matter (see, [25]), defines the homonymous operators and applies them to a dynamical system, the hydrogen atom. Heisenberg introduces matrices to quantum mechanics and formulates the uncertainty principle (see, [32]). Dirac introduces non-commutative matrices in his work on quantum mechanics (see, [36]), but does not know the self-variation of the fundamental particles. Dirac studied a dynamical system, the generalized particle of the electron. The generalized particle is always associated with an interaction. In Dirac's work, this interaction is expressed through potential, which plays a central role in his work. This was followed by the work of many distinguished physicists who brought quantum mechanics to the level we know it today.

The contribution of the Self-Variation Theory to theoretical physics is summarized in the system of Equations (6.5), (6.6), and the requirement that the mathematical form of physical laws must be compatible with the of self-variation principle. This requirement places additional restrictions, than those set by the Theory of Relativity on the mathematical form of physical laws. In section 2 we presented the simplest solution of the System of Equations (6.5), (6.6), in four-dimensional flat spacetime. The Intrinsic Symmetry Theorem relates to the observations we make on all distance scales, from the microcosm to the cosmological scale. In section 5 of Self-variation Theory we saw the predictions of the Theorem on the cosmological scale. In this article we will not present another solution of the System of Equations (6.5), (6.6).

In sections 3 and 4, we took into account that the potential is not uniquely defined and required it to be compatible with the self-variation principle. Thus we obtained the potential of the electromagnetic field, as given by the pairs of Equations (3.36) and (3.38).

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