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Self-Variation Theory-Part I

Emmanuil Manousos*

APM Institute for the Advancement of Physics and Mathematics, Athens, Greece
*Corresponding author: emanoussos@phys.uoa.gr

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Abstract We present the principles and main consequences of Self-Variation Theory. The Theory is based on three principles, the principle of self-variation, principle of conservation of energy-momentum and a definition of the rest mass of a fundamental particle. The main conclusions of the Theory are the following; it predicts a structure of the particles, predicts and justifies the particle interactions, predicts and justifies the cosmological data and it shows that quantum phenomena are implicit in the Self-Variation Theory. The Self-Variation Theory provides a mathematically consistent paradigm for nature. The origin, evolution and current form of the universe are consistent with the theoretical prediction. At all distance scales, from the microcosm to observations billions of light years away, the Theory is remarkably consistent with experimental and observational data.

Keywords: *electromagnetism, gravity, particle interactions, origin of universe, evolution of universe, structure of matter, quantum phenomena*

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1. Introduction

A principle absent from the current theories of physics that is introduced by the Self-Variation Theory is the principle of self-variation. It is a simple, though obviously a somewhat unexpected principle. With the term "self-variation principle" we mean an exactly determined increase of the rest mass of fundamental particles and generally of the "self-variating charge" q.

Taking into account the energy-momentum conservation principle, the self-variation of the rest mass of the material particle can only take place with the simultaneous emission of energy-momentum into the surrounding spacetime of the particle. The combination of the principle of self-variation with the conservation of energy-momentum has as a consequence the presence of energy-momentum in the surrounding spacetime of the material particle. The introduction of the principle of the rest mass self-variation was made with the expectation that this energy-momentum in spacetime could provide a cause for the interaction of material particles. In retrospect, this expectation was confirmed. Taking into account the existence of the gravitational interaction, we introduce the self-variation of the rest mass. Similarly, due to the existence of the electromagnetic interaction we introduce the principle of self-variation of the electric charge. Generally each interaction results from a "self-variating charge" q.

The Self-Variation Theory is based on three principles; the principle of self-variation, the principle of conservation of energy-momentum, a definition of the rest mass of fundamental particles.

1.1. Axiomatic foundation of Self-Variation Theory

In a N-dimensional Riemannian spacetime (see, [1]) the Self-Variation Theory is based on three principles, the principle of self-variation, the principle of conservation of energy-momentum and a definition of the rest mass of fundamental particles. We present the three principles of the Theory.

A. The self-variation principle

With the term "self-variation principle" we mean an exactly determined increase of the rest mass of material particles. Moreover the self-variation principle generally applies to all kind of charges of the fundamental particles. Direct consequence of the principle of self-variation is that energy, momentum, angular momentum and charge (if the particle is charged) are distributed in the surrounding spacetime. For example, to compensate for the increase, in absolute value, of the negative electric charge of the electron, the particle emits a corresponding positive electric charge into the surrounding spacetime. As a consequence of this emission the total electric charge is conserved. Similarly, the increase of the rest mass of the material particle involves the "emission" of negative energy as well as momentum in the spacetime surrounding the material particle (spacetime energy-momentum) P.

We generally denote the rest mass or charge of particle with q. The principle of self-variation quantitatively describes the interaction of the 'self-variation charges'. Let

$$q = q\left(x^0, x^1, x^2, ..., x^{N-1}\right) = q\left(x^k\right),$$
$$N \in \mathbb{N}^* = \{1, 2, 3, ...\}$$

be the self-variating charge and let P be the energy-momentum the particle emit in spacetime as a consequence of the self-variation of the charge q. The self-variation principle asserts that valid

$$\frac{\partial q}{\partial x^k} = \frac{b}{\hbar} P_k q \qquad (1.1)$$
$$k = 0, 1, 2, ..., N-1$$

in every system of reference

$$\mathrm{O}\left(x^0, x^1, x^2, ..., x^{N-1}\right) = \mathrm{O}\left(x^k\right)$$

where $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant and $b \in \mathbb{C}$, $b \neq 0$ is a constant. $P_0 = \frac{iE}{c}$ denotes the energy and $x^0 = ict$ the time measured by an observer, where c is vacuum velocity of light and i is the imaginary unit, $i^2 = -1$. If $q = m_0$ Equation (1) becomes

$$\frac{\partial m_0}{\partial x^k} = \frac{b}{\hbar} P_k m_0 \qquad (1.2)$$

The principle of self-variation quantitatively describes the interaction of material particles with the spacetime energy-momentum. For the formulation of the equations the following symbolism is used,

W is the energy of the particle,

J is the momentum of the particle,

m_0 is the rest mass of the particle,

E is the energy of the spacetime energy-momentum related to the particle,

P is the momentum of the spacetime energy-momentum related to the particle,

E_0 is the rest energy of the spacetime energy-momentum related to the particle. We define the N-vectors,

$$X = \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ . \\ . \\ . \\ x^{N-1} \end{bmatrix} = \begin{bmatrix} ict \\ x^1 \\ x^2 \\ . \\ . \\ . \\ x^{N-1} \end{bmatrix} \qquad (1.3)$$

$$J = \begin{bmatrix} J_0 \\ J_1 \\ J_2 \\ . \\ . \\ . \\ J_{N-1} \end{bmatrix} = \begin{bmatrix} \frac{iW}{c} \\ J_1 \\ J_2 \\ . \\ . \\ . \\ J_{N-1} \end{bmatrix} \qquad (1.4)$$

$$P = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ . \\ . \\ . \\ P_{N-1} \end{bmatrix} = \begin{bmatrix} \frac{iE}{c} \\ P_1 \\ P_2 \\ . \\ . \\ . \\ P_{N-1} \end{bmatrix} \qquad (1.5)$$

$$C = \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ . \\ . \\ . \\ C_{N-1} \end{bmatrix} \qquad (1.6)$$

where, $C = J + P$.

In Equation (1.1), the momentum J of the particle is due to the charge q. The momentum P arises as a consequence of the self-variation of the charge q. The physical quantities q, J, P are determined at the same point X of spacetime.

B. The principle of conservation of energy-momentum

The material particle and the spacetime energy-momentum with which the material particle interacts comprise a dynamic system, which we call "generalized particle". We consider the covariant (see, [2,3]) momentum of the particle J, the momentum of spacetime P and the total momentum C of generalized particle,

$$C_n = J_n + P_n, \quad n = 0, 1, 2, ..., N-1 \qquad (1.7)$$

As a consequence of Equation (1.1), the N-vectors C, J and P are covariant. Equation (1.7) expresses the energy-momentum conservation of the generalized particle in a N-dimensional spacetime.

C. The rest mass of the material particles

As invariant physical quantities, the rest masses corresponding to the N-vectors J, P, C are given by the following equations,

$$J_n J^n = m_0^2 c^2 \qquad (1.8)$$

$$P_n P^n = \frac{E_0^2}{c^2} \quad (1.9)$$

$$C_n C^n = M_0^2 c^2 \quad (1.10)$$

For the contravariant N-vectors we have $C^n = g^{nk} C_k$, $P^n = g^{nk} P_k$, $J^n = g^{nk} J_k$ where g^{ij} is the metric tensor. The N-vector C is constant, therefore rest mass M_0 is also constant. In Equations (1.8), (1.9), (1.10) we follow Einstein's summation convention for terms where an index appears twice.

The goal of Self-Variation Theory is to find the functions $J = J(X)$, $m_0 = m_0(X)$, $P = P(X)$ and $E_0 = E_0(X)$. The differential equations resulting from the axiomatic foundation of the Theory give specific solutions for these functions. These solutions have a common feature. The material particle has structure, even if we assume it to be a point. In the context of the Self-Variation Theory, the generalized particle replaces the concept of the material particle.

Concluding first section, we present three direct consequences of the principles of the Theory. The first of these is given by the following equations,

$$\frac{\partial P_k}{\partial x^a} = \frac{\partial P_a}{\partial x^k} \quad (1.11)$$

$$\frac{\partial J_k}{\partial x^a} = \frac{\partial J_a}{\partial x^k} \quad (1.12)$$

Proof. From Equation (1.1) we get,

$$\left(\frac{\partial q}{\partial x^k}\right)_{;a} = \frac{b}{\hbar}\left(P_k q\right)_{;a}$$

where with $;a$ we denote the covariant derivative with respect to x^a. Then we get,

$$\frac{\partial^2 q}{\partial x^a \partial x^k} - \Gamma^l_{ka}\frac{\partial q}{\partial x^l} = \frac{b^2}{\hbar^2}P_k P_a q + \frac{b}{\hbar}q\left(\frac{\partial P_k}{\partial x^a} - \Gamma^l_{ka}P_l\right)$$

and equivalently we get,

$$\frac{\partial^2 q}{\partial x^a \partial x^k} - \Gamma^l_{ka}\frac{\partial q}{\partial x^l} = \frac{b^2}{\hbar^2}P_k P_a q + \frac{b}{\hbar}q\frac{\partial P_k}{\partial x^a} - \frac{b}{\hbar}q\Gamma^l_{ka}P_l$$

and with Equation (1.1) we get,

$$\frac{\partial^2 q}{\partial x^a \partial x^k} - \frac{b}{\hbar}q\Gamma^l_{ka}P_l = \frac{b^2}{\hbar^2}P_k P_a q + \frac{b}{\hbar}q\frac{\partial P_k}{\partial x^a} - \frac{b}{\hbar}q\Gamma^l_{ka}P_l$$

and finally we obtain,

$$\frac{\partial^2 q}{\partial x^a \partial x^k} - \frac{b^2}{\hbar^2}P_k P_a q = \frac{b}{\hbar}q\frac{\partial P_k}{\partial x^a}.$$

Similarly, from the equation

$$\frac{\partial q}{\partial x^\alpha} = \frac{b}{\hbar}P_\alpha q$$

we get,

$$\frac{\partial^2 q}{\partial x^k \partial x^\alpha} - \frac{b^2}{\hbar^2}P_\alpha P_k q = \frac{b}{\hbar}q\frac{\partial P_\alpha}{\partial x^k}.$$

Therefore we have

$$q\frac{\partial P_k}{\partial x^a} = q\frac{\partial P_a}{\partial x^k}$$

and taking into consideration that $q \neq 0$ we get Equation (1.11). From Equations (1.11) and (1.7) we get Equation (1.12). In the proof process we used the symbols of Christoffel,

$$\Gamma^k_{an} = \frac{1}{2}g^{k\nu}\left(\frac{\partial g_{a\nu}}{\partial x^n} + \frac{\partial g_{\nu n}}{\partial x^a} - \frac{\partial g_{an}}{\partial x^\nu}\right)$$

1.2. Self-variation of the Rest Mass $\frac{E_0}{c^2}$

If $E_0 \neq 0$, the rest mass $\frac{E_0}{c^2}$ is self-variating. Therefore, for each solution $J = J(X)$, $m_0 = m_0(X)$, $P = P(X)$ and $E_0 = E_0(X)$ that we get from the differential equations of the Theory, one of the following equations holds.

$$\frac{\partial E_0}{\partial x^k} = \frac{b}{\hbar}J_k E_0 \quad (1.13)$$

or

$$\frac{\partial E_0}{\partial x^k} = -\frac{b}{\hbar}J_k E_0 \quad (1.14)$$

1.3. The Relative Position of N-vectors J and P

The relative position of N-vectors J and P in spacetime can be given by the following equations,

$$P_0 = \Phi_{00}J_0 + \Phi_{01}J_1 + \Phi_{02}J_2 + \ldots + \Phi_{0(N-1)}J_{N-1}$$
$$P_1 = \Phi_{10}J_0 + \Phi_{11}J_1 + \Phi_{12}J_2 + \ldots + \Phi_{1(N-1)}J_{N-1}$$
$$P_2 = \Phi_{20}J_0 + \Phi_{21}J_1 + \Phi_{22}J_2 + \ldots + \Phi_{2(N-1)}J_{N-1}$$

.

.

.

$$P_{(N-1)} = \Phi_{(N-1)0}J_0 + \Phi_{(N-1)1}J_1 + \Phi_{(N-1)2}J_2 + \ldots + \Phi_{(N-1)(N-1)}J_{N-1} \quad (1.15)$$

where $\Phi_{ij} = \Phi_{ij}\left(X\right)$. Denoting T the $N \times N$ matrix $\left\{\Phi_{ij}\right\}$,

$$T = \begin{bmatrix} \Phi_{00} & \Phi_{01} & \Phi_{02} & \cdot & \cdot & \cdot & \Phi_{0(N-1)} \\ \Phi_{10} & \Phi_{11} & \Phi_{12} & \cdot & \cdot & \cdot & \Phi_{1(N-1)} \\ \Phi_{20} & \Phi_{21} & \Phi_{22} & \cdot & \cdot & \cdot & \Phi_{2(N-1)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \Phi_{(N-1)0} & \Phi_{(N-1)1} & \Phi_{(N-1)2} & \cdot & \cdot & \cdot & \Phi_{(N-1)(N-1)} \end{bmatrix} \quad (1.16)$$

Equation (1.13) is written in the form,

$$P = TJ \quad (1.17)$$

1.4. Main Conclusions of the Self-Variation Theory

We present the main conclusions of Self-Variation Theory. In section 2 we study the generalized particle in the flat 4-dimensional spacetime of Special Relativity. This study is fundamental, since it highlights the basic consequences of the self-variation of material particles. Moreover, this study is a model for the study of the generalized particle in curved spacetime.

The main conclusion of the section is the Internal Symmetry Theorem. This Theorem gives the rest mass and in general the charge of a particle as a function of spacetime. It also gives the relation of the energy-momentum and rest mass of a particle to the energy-momentum and rest mass in the surrounding space-time of the particle. If the particle is charged, the Theorem gives a distribution of charge in the surrounding spacetime of the particle.

The Internal Symmetry Theorem justifies the so far known cosmological facts in a flat and static universe. The analytical justification of the cosmological data is done in section 5.

In section 3 we present the potentials that are compatible with the self-variation principle and replace the Liénard-Wiechert potentials. We study the electromagnetic field generated by an electric point charge moving arbitrarily in an inertial frame of reference. This study results in the replacement of Liénard-Wiechert potentials by self-variation potentials. Liénard-Wiechert potentials and self-variation potentials give the same electromagnetic field. However, self-variation potentials are compatible with Lorentz-Einstein transformations and, obviously, with the self-variation principle. The Liénard-Wiechert potentials are compatible with Lorentz-Einstein transformations, but it are not compatible with the self-variation principle. Maxwell's Equations are obviously compatible with Lorentz-Einstein transformations. We prove that they are also compatible with self-variance.

If we denote by L the set of equations that are compatible with the Lorentz-Einstein transformations and by S the set of equations that are compatible with then it is $S \subset L$. Regarding the mathematical formalism of the laws of physics, the Self-Variation Theory imposes additional constraints than those imposed by Special Relativity.

In this section we have a precise calculation for the consequences of self-variation in the surrounding spacetime of q. As a consequence of the self-variation, an electric charge of opposite sign on q is distributed in the surrounding spacetime of the electric charge q. We calculate the electric charge density and the current density in the surrounding spacetime of q.

Another consequence on the surrounding spacetime of q is given by the Orbit Representation Theorem. For each direction in space, the curve C_p of orbit of q is mapped to a curve C in the surrounding spacetime.

In section 4 we formulate the gravitational field equations. The Self-Variation Theory formulates gravity and electromagnetism with the same equations. These Equations concern the field created by the rest mass / electric charge of a particle. The central equation of the Theory relates three physical quantities, the rest mass or charge of the field source, the relative velocity of the field source to the observer, and the propagation velocity of the field relative to the observer. These velocities are directly related to the potential and intensity of the field measured by an observer. The first calculations give consistency of the Theory at the distance scales that we have observational data. Theory predicts increased stellar velocities on the outskirts of galaxies. It also predicts increased velocities of galaxies on the outskirts of galaxy clusters.

The Field Equations for gravity, as given by Self-Variation Theory, predict that near the rest mass-source of the field, gravity is repulsive. Above a value of distance, gravity becomes attractive.

In the context of the Theory, the equations we present in this section do not only apply to gravity and electromagnetism. Three other interactions with traits that do not correspond to gravity resulted from the investigation of the original equation. Like the gravitational interaction, the other three interactions are either attractive or repulsive, depending on the distance from the source of the field. None of the interactions are only attractive or only repulsive.

In section 5 we present the implications of self-variation on the cosmological scale. As a consequence of self-variation, in our cosmological-scale observations, the rest mass and electric charge (generally the self-variating charge) of a particle have a smaller value than the corresponding values of the same particle in the laboratory, on earth. This fact has consequences for all physical phenomena occurring in distant astronomical objects, which depend on rest mass and electrical charge. These consequences are recorded in the cosmological data. The redshift of distant astronomical objects is one such consequence.

Fundamental quantities of astrophysics depend on redshift. We calculate as a function of the redshift the mass of the electron and in general the mass of the fundamental particles, the charge of the electron and in general the charge of the fundamental particles, the ionization energy and the degree of ionization of the atoms, the Thomson and Klein-Nishina scattering coefficients, the position-momentum uncertainty and the Bohr radius, and the energy produced in nuclear reactions and hydrogen fusion.

As a consequence of the self-variation of the rest mass of particles, gravity has no consequences on the cosmological scale, it cannot drive the universe into collapse or expansion. Its consequences are limited to smaller distance scales.

In the last subsection of the section we compare the Standard Cosmological Model with the predictions of Self-Variation Theory on the cosmological scale. The reasons why the Standard Cosmological Model has been forced into a series of assumptions to come to terms with the cosmological data are highlighted. However, there are now data, such as the two measured values of Hubble's constant, for which there is no plausible hypothesis that could bring them into agreement with the model. The origin and evolution of the universe as predicted by the Self-Variation Theory presents a remarkable compatibility with the cosmological data.

In section 6 we present the structure of the generalized particle. As a consequence of the principles of Self-Variation Theory, this structure is given by a System of two Equations. The first of these Equations is a linear system of NN equations. The second is a differential equation. Self-variation propagates as a "disturbance" in space-time, i.e. it creates a wave. The mathematical formalism of this wave is given by the same System of Equations.

2. Self-variation in the Spacetime of Special Relativity

In this section we study the generalized particle in the flat 4-dimensional spacetime of Special Relativity. This study is fundamental, since it highlights the basic consequences of the self-variation of material particles. Moreover, this study is a model for the study of the generalized particle in curved spacetime.

The main conclusion of the section is the Internal Symmetry Theorem. This Theorem gives the rest mass and in general the charge of a particle as a function of spacetime. It also gives the relation of the energy-momentum and rest mass of a particle to the energy-momentum and rest mass in the surrounding space-time of the particle. If the particle is charged, the Theorem gives a distribution of charge in the surrounding spacetime of the particle.

The Internal Symmetry Theorem justifies the so far known cosmological facts in a flat and static universe. The analytical justification of the cosmological data is done in section 5.

2.1. The Basic Equations of the Theory in Flat 4-dimensional Spacetime

In the flat 4-dimensional spacetime (Minkowski spacetime) of Special Relativity (see, [4,5,6,7]) Equations (1.3) - (1.6) and (1.8) - (1.10) take the form,

$$X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ict \\ x \\ y \\ z \end{bmatrix} \tag{2.1}$$

$$J = \begin{bmatrix} J_0 \\ J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \frac{iW}{c} \\ J_x \\ J_y \\ J_z \end{bmatrix} \tag{2.2}$$

$$P = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} \frac{iE}{c} \\ P_x \\ P_y \\ P_z \end{bmatrix} \tag{2.3}$$

$$C = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} \tag{2.4}$$

$$J_0^2 + J_1^2 + J_2^2 + J_3^2 + m_0{}^2 c^2 = 0 \tag{2.5}$$

$$P_0^2 + P_1^2 + P_2^2 + P_3^2 + \frac{E_0^2}{c^2} = 0 \tag{2.6}$$

$$c_0^2 + c_1^2 + c_2^2 + c_3^2 + M_0{}^2 c^2 = 0 \tag{2.7}$$

respectively.

We consider an inertial frame of reference $O'(x_0', x_1', x_2', x_3')$ moving with velocity $(u, 0, 0)$ with respect to another inertial frame of reference $O(x_0, x_1, x_2, x_3)$, with their origins O' and O coinciding at $x_0' = x_0$. With this symbolism the Lorentz-Einstein transformations have the following form,

$$\begin{aligned} x_0' &= \gamma\left(x_0 - i\frac{u}{c}x_1\right) \\ x_1' &= \gamma\left(x_1 + i\frac{u}{c}x_0\right) \\ x_2' &= x_2 \\ x_3' &= x_3 \\ \frac{\partial}{\partial x_0'} &= \gamma\left(\frac{\partial}{\partial x_0} - i\frac{u}{c}\frac{\partial}{\partial x_1}\right) \\ \frac{\partial}{\partial x_1'} &= \gamma\left(\frac{\partial}{\partial x_1} + i\frac{u}{c}\frac{\partial}{\partial x_0}\right) \\ \frac{\partial}{\partial x_2'} &= \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_3'} &= \frac{\partial}{\partial x_3} \end{aligned} \tag{2.8}$$

$$\begin{aligned}
J_0' &= \gamma\left(J_0 - i\frac{u}{c}J_1\right)\\
J_1' &= \gamma\left(J_1 + i\frac{u}{c}J_0\right)\\
J_2' &= J_2\\
J_3' &= J_3\\
P_0' &= \gamma\left(P_0 - i\frac{u}{c}P_1\right)\\
P_1' &= \gamma\left(P_1 + i\frac{u}{c}P_0\right)\\
P_2' &= P_2\\
P_3' &= P_3
\end{aligned} \tag{2.9}$$

where $\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$.

From these transformations and Equation (1.15) (see, Appendix A) we get the following equations,

$$\begin{aligned}
\Phi_{ij} &= -\Phi_{ji}, i \neq j\\
\Phi_{00} &= \Phi_{11} = \Phi_{22} = \Phi_{33} = \Phi
\end{aligned} \tag{2.10}$$

and transformations,

$$\Phi' = \Phi \tag{2.11}$$

$$\begin{aligned}
\Phi_{01}' &= \Phi_{01}\\
\Phi_{02}' &= \gamma\left(\Phi_{02} + i\frac{u}{c}\Phi_{21}\right)\\
\Phi_{03}' &= \gamma\left(\Phi_{03} - i\frac{u}{c}\Phi_{13}\right)
\end{aligned} \tag{2.12}$$

$$\begin{aligned}
\Phi_{32}' &= \Phi_{32}\\
\Phi_{13}' &= \gamma\left(\Phi_{13} + i\frac{u}{c}\Phi_{03}\right)\\
\Phi_{21}' &= \gamma\left(\Phi_{21} - i\frac{u}{c}\Phi_{02}\right)
\end{aligned} \tag{2.13}$$

Actually, in the 4-dimensional spacetime of Special Relativity, from Equations (1.15) we get,

$$\begin{aligned}
P_0 &= \Phi_{00}J_0 + \Phi_{01}J_1 + \Phi_{02}J_2 + \Phi_{03}J_3\\
P_1 &= \Phi_{10}J_0 + \Phi_{11}J_1 + \Phi_{12}J_2 + \Phi_{13}J_3\\
P_2 &= \Phi_{20}J_0 + \Phi_{21}J_1 + \Phi_{22}J_2 + \ldots + \Phi_{23}J_3\\
P_3 &= \Phi_{30}J_0 + \Phi_{31}J_1 + \Phi_{32}J_2 + \ldots + \Phi_{33}J_3
\end{aligned}.$$

For the inertial reference system $O'\left(x_0', x_1', x_2', x_3'\right)$, from the first of this Equations (A.1) we get,

$$P_0' = \Phi_{00}'J_0' + \Phi_{01}'J_1' + \Phi_{02}'J_2' + \Phi_{03}'J_3'$$

and by Transformations (2.9) we get,

$$\begin{aligned}
\gamma\left(P_0 - \frac{iu}{c}P_1\right) &= \Phi_{00}'\gamma\left(J_0 - \frac{iu}{c}J_1\right)\\
&+\Phi_{01}'\gamma\left(J_1 + \frac{iu}{c}J_0\right) + \Phi_{02}'J_2 + \Phi_{03}'J_3
\end{aligned}$$

and with the first two Equations (A.1) we get,

$$\begin{aligned}
&\gamma\left(\Phi_{00}J_0 + \Phi_{01}J_1 + \Phi_{02}J_2 + \Phi_{03}J_3\right)\\
&-\gamma\frac{iu}{c}\left(\Phi_{10}J_0 + \Phi_{11}J_1 + \Phi_{12}J_2 + \Phi_{13}J_3\right)\\
&= \Phi_{00}'\gamma\left(J_0 - \frac{iu}{c}J_1\right) + \Phi_{01}'\gamma\left(J_1 + \frac{iu}{c}J_0\right)\\
&+\Phi_{02}'J_2 + \Phi_{03}'J_3
\end{aligned}$$

and taking into account that this Equation is valid for every tetrad $\left(J_0, J_1, J_2, J_3\right)$ we get,

$$\begin{aligned}
\gamma\Phi_{00} - \gamma\frac{iu}{c}\Phi_{10} &= \gamma\Phi_{00}' + \gamma\frac{iu}{c}\Phi_{01}'\\
\gamma\Phi_{01} - \gamma\frac{iu}{c}\Phi_{11} &= -\gamma\frac{iu}{c}\Phi_{00}' + \gamma\Phi_{01}'\\
\gamma\Phi_{02} - \gamma\frac{iu}{c}\Phi_{12} &= \Phi_{02}'\\
\gamma\Phi_{03} - \gamma\frac{iu}{c}\Phi_{13} &= \Phi_{03}'
\end{aligned}$$

and equivalently we get,

$$\begin{aligned}
\Phi_{00}' &= \Phi_{00}\\
\Phi_{01}' &= -\Phi_{10}\\
\Phi_{00}' &= \Phi_{11}\\
\Phi_{01}' &= \Phi_{01}\\
\Phi_{02}' &= \gamma\left(\Phi_{02} - \frac{iu}{c}\Phi_{12}\right)\\
\Phi_{03}' &= \gamma\left(\Phi_{03} - \frac{iu}{c}\Phi_{13}\right)
\end{aligned}$$

and equivalently we get,

$$\begin{aligned}
\Phi_{00}' &= \Phi_{00} = \Phi_{11}\\
\Phi_{01} &= -\Phi_{10}\\
\Phi_{01}' &= \Phi_{01}\\
\Phi_{02}' &= \gamma\left(\Phi_{02} - \frac{iu}{c}\Phi_{12}\right)\\
\Phi_{03}' &= \gamma\left(\Phi_{03} - \frac{iu}{c}\Phi_{13}\right)
\end{aligned}.$$

Working in the same way for the other three Equations, we finally get Equations (2.10) - (2.13).

It follows from our study that as we move from one frame of reference to another through Lorentz-Einstein transformations, we get equations that apply to the same frame of reference. They are Equations (2.10). Also, the vectors $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$,

$$\boldsymbol{\alpha} = \begin{pmatrix} ic\Phi_{01} \\ ic\Phi_{02} \\ ic\Phi_{03} \end{pmatrix} \quad (2.14)$$

$$\boldsymbol{\beta} = \begin{pmatrix} \Phi_{32} \\ \Phi_{13} \\ \Phi_{21} \end{pmatrix} \quad (2.15)$$

are transformed like the electromagnetic field. The vector $\boldsymbol{\alpha}$ corresponds to the electric field and the vector $\boldsymbol{\beta}$ to the magnetic one.

From Equations (1.15) and (2.10) we get,

$$\begin{aligned} P_0 &= \Phi J_0 + \Phi_{01} J_1 + \Phi_{02} J_2 + \Phi_{03} J_3 \\ P_1 &= -\Phi_{01} J_0 + \Phi J_1 - \Phi_{21} J_2 + \Phi_{13} J_3 \\ P_2 &= -\Phi_{02} J_0 + \Phi_{21} J_1 + \Phi J_2 - \Phi_{32} J_3 \\ P_3 &= -\Phi_{03} J_0 - \Phi_{13} J_1 + \Phi_{32} J_2 + \Phi J_3 \end{aligned} \quad (2.16)$$

The determinant $|T|$ of the system of Equations (2.16) is given by the following equation,

$$|T| = \Phi^2 \left(\Phi^2 + \Phi_{01}^2 + \Phi_{02}^2 + \Phi_{03}^2 + \Phi_{32}^2 + \Phi_{13}^2 + \Phi_{21}^2 \right) + \left(\Phi_{01}\Phi_{32} + \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{21} \right)^2$$

as obtained after the necessary calculations. If $P \neq 0$, the system of equations (2.16) is non-homogeneous its determinant is non-zero,

$$\Phi^2 \left(\Phi^2 + \Phi_{01}^2 + \Phi_{02}^2 + \Phi_{03}^2 + \Phi_{32}^2 + \Phi_{13}^2 + \Phi_{21}^2 \right) + \left(\Phi_{01}\Phi_{32} + \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{21} \right)^2 \neq 0 \quad (2.17)$$

From the inequality (2.17) it follows that if $\Phi_{ij} = 0$ for every $i \neq j$, then $\Phi \neq 0$. If $\Phi = 0$ then $\Phi_{01}\Phi_{32} + \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{21} \neq 0$. One of the conclusions derived from the study we did is given by the following Internal Symmetry Theorem.

2.2. Internal Symmetry Theorem

In flat spacetime the following applies.

A. *If* $\Phi = 0$ *them* $\Phi_{01}\Phi_{32} + \Phi_{02}\Phi_{13} + \Phi_{03}\Phi_{21} \neq 0$.

B. *If* $\Phi_{ij} = 0$ *for each* $i \neq j$ *then the following applies.*

1. *The 4-vectors* P *and* J *are parallel,*
$P = \Phi J$.(2.18)
2. Exactly one of the following applies,
$E_0 = m_0 c^2$ and $M_0 = 0$ (2.19)

or

$$\Phi = K \exp\left[-\frac{b}{\hbar}\left(c_0 x_0 + c_1 x_1 + c_2 x_2 + c_3 x_3 \right) \right] \quad (2.20)$$

$$m_0 = \pm \frac{M_0}{1+\Phi} \quad (2.21)$$

$$E_0 = \pm \frac{\Phi M_0 c^2}{1+\Phi} \quad (2.22)$$

$$J_i = \frac{c_i}{1+\Phi}, i = 0,1,2,3 \quad (2.23)$$

$$P_i = \frac{\Phi c_i}{1+\Phi}, i = 0,1,2,3 \quad (2.24)$$

where $K \neq 0$ *is a dimensionless constant.*

Proof. A. A has already been proven, following Inequality (2.17). As a consequence of self-variation principle, $P \neq 0$ and the system of Equations (2.16) is non-homogeneous.

B. 1. If $\Phi_{ij} = 0$ for each $i \neq j$, Equation (2.18) results from the system of Equations (2.16).

2. From Equation (2.18) we have $P_i = \Phi J_i$ and with Equation (1.7) we get $c_i - J_i = \Phi J_i$ and equivalently we obtain,

$$\left(\Phi + 1 \right) J_i = c_i \quad (2.25)$$

If $\Phi = -1$ we have $c_i = 0$ and $P_i = -J_i$. Then, from Equation (2.7) we obtain $M_0 = 0$ and from Equations (2.5), (2.6) we obtain, $E_0 = m_0 c^2$.

If $\Phi \neq -1$, from Equation (2.25) we get,

$$J_i = \frac{c_i}{1+\Phi} \quad (2.26)$$

From Equations (2.5) and (1.2) we get,

$$2J_0 \frac{\partial J_0}{\partial x_k} + 2J_1 \frac{\partial J_1}{\partial x_k} + 2J_2 \frac{\partial J_2}{\partial x_k} + 2J_3 \frac{\partial J_3}{\partial x_k} + 2\frac{b}{\hbar} P_k m_0^2 c^2 = 0$$

and with Equation (2.5) we get,

$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} - \frac{b}{\hbar} P_k \left(J_0^2 + J_1^2 + J_2^2 + J_3^2 \right) = 0$$

and with Equation (1.7) we get,

$$J_0 \frac{\partial J_0}{\partial x_k} + J_1 \frac{\partial J_1}{\partial x_k} + J_2 \frac{\partial J_2}{\partial x_k} + J_3 \frac{\partial J_3}{\partial x_k} - \frac{b}{\hbar}\left(c_k - J_k \right)\left(J_0^2 + J_1^2 + J_2^2 + J_3^2 \right) = 0$$

and with Equation (2.26) we get,

$$\frac{c_0}{1+\Phi}\frac{\partial}{\partial x_k}\left(\frac{c_0}{1+\Phi}\right)+\frac{c_1}{1+\Phi}\frac{\partial}{\partial x_k}\left(\frac{c_1}{1+\Phi}\right)$$
$$+\frac{c_2}{1+\Phi}\frac{\partial}{\partial x_k}\left(\frac{c_2}{1+\Phi}\right)+\frac{c_3}{1+\Phi}\frac{\partial}{\partial x_k}\left(\frac{c_3}{1+\Phi}\right)$$
$$-\frac{b}{\hbar}\left(c_k-\frac{c_k}{1+\Phi}\right)\left(\frac{c_0^2+c_1^2+c_2^2+c_3^2}{\left(1+\Phi\right)^2}\right)=0$$

and after the calculations we get,

$$\frac{\partial\Phi}{\partial x_k}=-\frac{bc_k}{\hbar}\Phi \tag{2.27}$$

From Equation (2.27) we obtain,

$$\Phi=K\exp\left[-\frac{b}{\hbar}\left(c_0x_0+c_1x_1+c_2x_2+c_3x_3\right)\right]$$

where K is a dimensionless constant physical quantity.

From Equations (2.18) and (2.26) we obtain,

$$P_i=\frac{\Phi c_i}{1+\Phi}.$$

From this Equation and (2.6) we obtain,

$$E_0=\pm\frac{\Phi M_0c^2}{1+\Phi}.$$

Similarly, from Equations (2.26) and (2.5) we obtain,

$$m_0=\pm\frac{M_0c^2}{1+\Phi}.$$

The proof is completed by confirming the self-variation of the rest energy E_0. For Equations (2.19) we have,

$$\frac{\partial E_0}{\partial x_k}=\frac{\partial m_0c^2}{\partial x_k}$$

and with Equation (1.2) we get,

$$\frac{\partial E_0}{\partial x_k}=\frac{b}{\hbar}P_km_0c^2$$

and with Equation (2.19) we get,

$$\frac{\partial E_0}{\partial x_k}=\frac{b}{\hbar}P_kE_0$$

and considering that it is $P_k=-J_k$ we obtain,

$$\frac{\partial E_0}{\partial x_k}=-\frac{b}{\hbar}J_kE_0.$$

From Equations (2.21) and (2.22) we get,

$$E_0^2=\Phi^2m_0^2c^4 \tag{2.28}$$

Then we have

$$2E_0\frac{\partial E_0}{\partial x_k}=2\Phi\frac{\partial\Phi}{\partial x_k}m_0^2c^4+2\Phi^2m_0c^4\frac{\partial m_0}{\partial x_k}$$

and with the Equations $\frac{\partial\Phi}{\partial x_k}=-\frac{bc_k}{\hbar}\Phi$ and (1.2) we get,

$$E_0\frac{\partial E_0}{\partial x_k}=-\Phi\frac{bc_k}{\hbar}\Phi m_0^2c^4+\Phi^2\frac{bP_k}{\hbar}m_0^2c^4$$

and with the Equation (2.28) we get,

$$E_0\frac{\partial E_0}{\partial x_k}=-\frac{bc_k}{\hbar}E_0^2+\frac{bP_k}{\hbar}E_0^2$$

and considering that it is $E_0\neq 0$ we get,

$$\frac{\partial E_0}{\partial x_k}=-\frac{bc_k}{\hbar}E_0+\frac{bP_k}{\hbar}E_0$$

and equivalently we get,

$$\frac{\partial E_0}{\partial x_k}=\frac{b}{\hbar}\left(P_k-c_k\right)E_0$$

and with the Equation (1.7) we obtain,

$$\frac{\partial E_0}{\partial x_k}=-\frac{b}{\hbar}J_kE_0$$

The Internal Symmetry Theorem is generally valid for any self-variating charge, since in equation (1.1), the momentum J of the particle is due to the charge Q.

Equations (2.19) predict a generalized particle with zero total rest mass, $M_0=0$. In addition, the Equation $E_0=m_0c^2$ applies. In this case, the Internal Symmetry Theorem does not give the relative position of the 4-vectors J and P.

For the generalized particle of Equations (2.20) - (2.24), the Internal Symmetry Theorem gives a remarkable set of information. From Equations (2.23) and (2.24) it follows that the 4-vectors J, P and C are parallel in 4-dimensional spacetime. Equations (2.21) and (2.22) give the distribution of the total rest mass M_0 in m_0 and $\frac{E_0}{c^2}$. Similarly, Equations (2.23) and (2.24) give the distribution of the total momentum c_k along the x_k axis. That is, we have energy-momentum and rest-mass distribution in space-time. This distribution is determined by the function Φ. If $b=i$ in Equation (2.20) the distribution is periodic. In general, if the constant b is not a real number, $b\in\mathbb{C}-\mathbb{R}$ the distribution has wave characteristics. If it is a real number, $b\in\mathbb{R}$ the distribution is non-periodic.

From Equation (2.23) we have

$$\frac{\partial J_i}{\partial x_k}=-\frac{c_i}{\left(1+\Phi\right)^2}\frac{\partial\Phi}{\partial x_k}$$

and with Equation (2.27) we get,

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} \frac{c_i c_k}{(1+\Phi)^2} \Phi$$

and with Equations (2.23), (2.24) we obtain,

$$\frac{\partial J_i}{\partial x_k} = \frac{b}{\hbar} P_k J_i \tag{2.29}$$

From Equations (2.29) and (1.7) we obtain,

$$\frac{\partial P_i}{\partial x_k} = -\frac{b}{\hbar} P_k J_i \tag{2.30}$$

From equations (2.29) and (2.30) it follows that the Internal Symmetry Theorem gives the rates of change of the 4-vectors J and P.

The rest mass m_0 is considered "positive" and the rest energy E_0 "negative". Therefore, if $\Phi \in \mathbb{R}$, then the product

$$\frac{m_0}{M_0} \frac{E_0}{M_0 c^0}$$

is negative,

$$\frac{m_0}{M_0} \frac{E_0}{M_0 c^0} < 0$$

and with Equations (2.21), (2.22) we get

$$\frac{\Phi}{(1+\Phi)^2} < 0$$

and equivalently we get $\Phi < 0$.

The function Φ also depends on the 4-vector C. If $c_1 = c_2 = c_3 = 0$ we have

$$\Phi = K \exp\left(-\frac{b}{\hbar} c_0 x_0 \right).$$

Then, from Equation (2.7) we get $c_0 = \pm M_0 c$ and

$$\Phi = K \exp\left(-\frac{b}{\hbar} c_0 x_0 \right) = K \exp\left(\pm \frac{b M_0 c^2}{\hbar} t \right).$$

Then, from Equation (2.21) we obtain,

$$m_0 = \pm \frac{M_0}{1 + K \exp\left(\pm \frac{b M_0 c^2}{\hbar} t \right)} \tag{2.31}$$

Equation (2.31) gives the rest mass m_0 as a function of time in an inertial frame of reference in which is $c_1 = c_2 = c_3 = 0$.

2.3. Internal Symmetry Theorem for Charge

We consider the 4-vector J' of the momentum of the particle and the corresponding rest mass m_0',

$$J_0'^2 + J_1'^2 + J_2'^2 + J_3'^2 + m_0'^2 c^2 = 0.$$

Repeating the proof process of the Internal Symmetry Theorem we get,

$$P_k' = \frac{\Phi c_k'}{1+\Phi},$$

where

$$\Phi = K' \exp\left[-\frac{b}{\hbar} \left(c_0' x_0 + c_1' x_1 + c_2' x_2 + c_3' x_3 \right) \right]$$

and

$$c_0'^2 + c_1'^2 + c_2'^2 + c_3'^2 + M_0'^2 c^2 = 0.$$

Then, from Equation (1.1) we get

$$\frac{\partial q}{\partial x_k} = \frac{b}{\hbar} P_k' q = \frac{b}{\hbar} \frac{\Phi c_k'}{1+\Phi} q$$

and equivalently we get

$$q = \pm \frac{Q}{1+\Phi} \tag{2.32}$$

where Q is a constant.

From Equation (2.32), if $c_1' = c_2' = c_3' = 0$ we get,

$$q = \pm \frac{Q}{1 + K \exp\left(\pm \frac{b M_0' c^2}{\hbar} t \right)} \tag{2.33}$$

Therefore, by substituting $m_0 \to q$ in Equations (2.18) - (2.31) we obtain the Internal Symmetry Theorem for the charge. The rest mass M_0' is due to the energy-momentum that the particle has due to the charge q.

If the rest mass due to the charge is at the same point in space-time as the rest mass of the particle, then the charge results in an increase in the rest mass of the particle. If the rest energy due to the charge is at the same point in space-time as the rest mass of the particle, then the charge results in a decrease in the rest mass of the particle.

Equations (2.31) and (2.33) give the increase in rest mass and electric charge, as required by the self-variation principle, of a particle that is stationary ($c_1 = c_2 = c_3 = 0$) with respect to an observer in the flat spacetime.

3. Electromagnetic Interaction

In this section we present the potentials that are compatible with the self-variation principle and replace the Liénard-Wiechert potentials. We study the electromagnetic field generated by an electric point charge moving arbitrarily in an inertial frame of reference. This study results in the replacement of Liénard-Wiechert potentials by self-variation potentials. Liénard-Wiechert potentials and self-variation potentials give the same electromagnetic field. However, self-variation potentials are compatible with Lorentz-Einstein transformations and, obviously, with the self-variation principle. The Liénard-Wiechert potentials are compatible with Lorentz-Einstein transformations, but it are not compatible with the self-variation principle. Maxwell's Equations are obviously compatible with Lorentz-Einstein transformations. We prove that they are also compatible with self-variance.

If we denote by L the set of equations that are compatible with the Lorentz-Einstein transformations and by S the set of equations that are compatible with then it is $S \subset L$. Regarding the mathematical formalism of the laws of physics, the Self-Variation Theory imposes additional constraints than those imposed by Special Relativity.

In this section we have a precise calculation for the consequences of self-variation in the surrounding spacetime of q. As a consequence of the self-variation, an electric charge of opposite sign on q is distributed in the surrounding spacetime of the electric charge q. We calculate the electric charge density and the current density in the surrounding spacetime of q.

Another consequence on the surrounding spacetime of q is given by the Orbit Representation Theorem. For each direction in space, the curve C_p of orbit of q is mapped to a curve C in the surrounding spacetime.

3.1. A Randomly Moving Electric Point Charge

We consider an electric point charge q moving arbitrarily in an inertial frame of reference $\mathrm{O}(x,y,z,t)$. We assume that the electromagnetic field propagates with speed $\boldsymbol{\upsilon}$, $\|\boldsymbol{\upsilon}\| = c$ where c is the speed of light in vacuum. As a consequence of self-variation, at time t, when q is at point P, it acts at point A with the value it had at point E, at the decelerating time w. We use the following symbolism, $\vec{EA} = \mathbf{r}$, $\|\mathbf{r}\| = r$, $\vec{OE} = \mathbf{r}_p(w)$, $\vec{OP} = \mathbf{r}_p(t)$, $E\left(x_p(w), y_p(w), z_p(w), w\right)$, $P\left(x_p(t), y_p(t), z_p(t), t\right)$, $A(t,x,y,z)$, where $O(0,0,0)$. The index p in the coordinates x_p, y_p, z_p indicates the position of the point particle carrying the charge q, at the corresponding moment in time w or t. At point E we denote $\mathbf{u}(w) = \mathbf{u}$ the velocity and $\mathbf{a}(w) = \mathbf{a}$ the acceleration of q, as in Figure 3.1.

With this symbolism we have,

$$\mathbf{r} = \begin{pmatrix} x - x_p(w) \\ y - y_p(w) \\ z - z_p(w) \end{pmatrix} \tag{3.1}$$

$$r = \|\mathbf{r}\| = \left(\left(x - x_p(w)\right)^2 + \left(y - y_p(w)\right)^2 + \left(z - z_p(w)\right)^2 \right)^{\frac{1}{2}} \tag{3.2}$$

$$w = t - \frac{r}{c} \tag{3.3}$$

$$\boldsymbol{\upsilon} = c\frac{\mathbf{r}}{r} = \frac{c}{r}\begin{pmatrix} x - x_p(w) \\ y - y_p(w) \\ z - z_p(w) \end{pmatrix} = \begin{pmatrix} \upsilon_x \\ \upsilon_y \\ \upsilon_z \end{pmatrix} = \begin{pmatrix} \upsilon_1 \\ \upsilon_2 \\ \upsilon_3 \end{pmatrix} \tag{3.4}$$

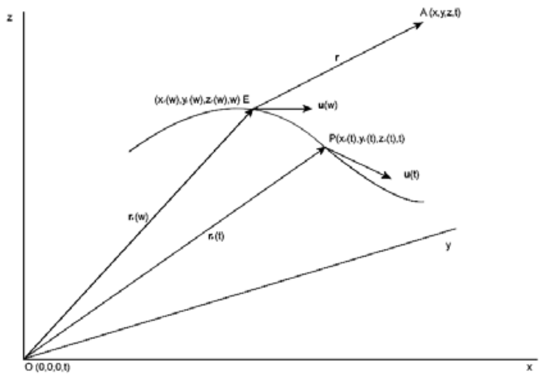

Figure 3.1. An arbitrarily moving electric point charge q at time t is at point $P\left(x_p(t), y_p(t), z_p(t), t\right)$. The source of the electromagnetic field at point $A\left(x,y,z,t\right)$ is the electric charge q positioned at $E\left(x_p(w), y_p(w), z_p(w), w\right)$ at the decelerating time $w = t - \frac{r}{c}$.

The velocity $\mathbf{u} = \mathbf{u}(w)$ of the q at point E is,

$$\mathbf{u} = \begin{pmatrix} \frac{dx_p(w)}{dw} \\ \frac{dx_p(w)}{dw} \\ \frac{dx_p(w)}{dw} \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \tag{3.5}$$

3.2. Auxiliary Equations

We prove the following list of equations which we will use next.

From Equation (3.2) we have

$$\begin{aligned} \frac{\partial r}{\partial t} &= \frac{1}{2r} 2\left(x - x_p(w)\right)\left(-\frac{dx_p(w)}{dw}\frac{\partial w}{\partial t}\right) \\ &+ \frac{1}{2r} 2\left(y - y_p(w)\right)\left(-\frac{dy_p(w)}{dw}\frac{\partial w}{\partial t}\right) \\ &+ \frac{1}{2r} 2\left(z - z_p(w)\right)\left(-\frac{dz_p(w)}{dw}\frac{\partial w}{\partial t}\right) \end{aligned}$$

and with Equations (3.4) and (3.5) we get

$$\frac{\partial r}{\partial t} = -\frac{1}{r}(\mathbf{r}\cdot\mathbf{u})\frac{\partial w}{\partial t}$$

and with Equation (3.4) we get

$$\frac{\partial r}{\partial t} = -\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c}\right)\frac{\partial w}{\partial t}$$

and with Equation (3.3) we get

$$\frac{\partial r}{\partial t} = -\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c}\right)\left(1-\frac{\partial r}{\partial t}\right)$$

and equivalently we obtain,

$$\frac{\partial r}{\partial t} = -\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)} \qquad (3.6)$$

From Equations (3.3) and (3.6) we obtain,

$$\frac{\partial w}{\partial t} = \frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} \qquad (3.7)$$

Starting again from Equation (3.2) we obtain,

$$\nabla r = \begin{pmatrix} \frac{\partial r}{\partial x} \\ \frac{\partial r}{\partial y} \\ \frac{\partial r}{\partial z} \end{pmatrix} = \frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{\boldsymbol{\upsilon}}{c} \qquad (3.8)$$

From Equations (3.3) and (3.8) we obtain,

$$\nabla w = -\frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2} \qquad (3.9)$$

From Equation (3.1) we have

$$\frac{\partial \mathbf{r}}{\partial t} = \begin{pmatrix} -\frac{dx_p(w)}{dw}\frac{\partial w}{\partial t} \\ -\frac{dy_p(w)}{dw}\frac{\partial w}{\partial t} \\ -\frac{dz_p(w)}{dw}\frac{\partial w}{\partial t} \end{pmatrix} = \frac{\partial w}{\partial t}\mathbf{u}$$

and with Equation (3.7) we obtain,

$$\frac{\partial \mathbf{r}}{\partial t} = -\frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\mathbf{u} \qquad (3.10)$$

From Equation (3.4) we have

$$\frac{\partial \boldsymbol{\upsilon}}{\partial t} = -\frac{c}{r^2}\frac{\partial r}{\partial t} + \frac{c}{r}\frac{\partial \mathbf{r}}{\partial t}$$

and with Equations (3.4), (3.6) and (3.10) we obtain,

$$\frac{\partial \boldsymbol{\upsilon}}{\partial t} = \frac{c}{r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\left(\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\boldsymbol{\upsilon}-\mathbf{u}\right) \qquad (3.11)$$

From Equation (3.4) we have

$$\upsilon_x = \frac{c}{r}\left(x - x_p(w)\right)$$

and differentiating with respect to x we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{c}{r^2}\frac{\partial r}{\partial x}\left(x-x_p(w)\right) + \frac{c}{r}\left(1-\frac{\partial x_p(w)}{\partial x}\right)$$

and equivalently we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{c}{r^2}\frac{\partial r}{\partial x}\left(x-x_p(w)\right) + \frac{c}{r}\left(1-\frac{dx_p(w)}{dw}\frac{\partial w}{\partial x}\right)$$

and with Equation (3.5) we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{c}{r^2}\frac{\partial r}{\partial x}\left(x-x_p(w)\right) + \frac{c}{r}\left(1-u_x\frac{\partial w}{\partial x}\right)$$

and with Equation (3.4) we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{1}{r}\frac{\partial r}{\partial x}\upsilon_x + \frac{c}{r}\left(1-u_x\frac{\partial w}{\partial x}\right)$$

and with Equations (3.8) and (3.9) we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{\upsilon_x^2}{cr}\frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} + \frac{c}{r}\left(1+\frac{1}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{u_x\upsilon_x}{c^2}\right)$$

and equivalently we obtain,

$$\frac{\partial \upsilon_x}{\partial x} = \frac{c}{r} + \frac{\upsilon_x(u_x-\upsilon_x)}{cr\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}.$$

Working similarly, we finally obtain,

$$\frac{\partial \upsilon_i}{\partial x_j} = \begin{cases} \frac{c}{r} + \frac{\upsilon_i(u_i-\upsilon_i)}{cr\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}, & if\ i=j \\ \frac{\upsilon_j(u_i-\upsilon_i)}{cr\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}, & if\ i\neq j \end{cases} \qquad (3.12)$$

where $i, j = 1, 2, 3$ and $(x_1, x_2, x_3) = (x, y, z)$.

Now we have,

$$\nabla \cdot \boldsymbol{\upsilon} = \frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial y} + \frac{\partial \upsilon_z}{\partial z}$$

and with Equation (3.12) we get,

$$\nabla \cdot \boldsymbol{\upsilon} = \frac{3c}{r} + \frac{\upsilon_x\left(u_x - \upsilon_x\right) + \upsilon_y\left(u_y - \upsilon_y\right) + \upsilon_z\left(u_z - \upsilon_z\right)}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}$$

and equivalently we get,

$$\nabla \cdot \boldsymbol{\upsilon} = \frac{3c}{r} + \frac{\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2 - u_x\upsilon_x - u_y\upsilon_y - u_z\upsilon_z}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}$$

and taking into consideration that $\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2 = c^2$ we get,

$$\nabla \cdot \boldsymbol{\upsilon} = \frac{3c}{r} + \frac{c^2 - \mathbf{u}\cdot\boldsymbol{\upsilon}}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}$$

and equivalently we obtain,

$$\nabla \cdot \boldsymbol{\upsilon} = \frac{2c}{r} \tag{3.13}$$

Working similarly we obtain,

$$\nabla \times \boldsymbol{\upsilon} = curl\boldsymbol{\upsilon} = \frac{1}{cr\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\left(\boldsymbol{\upsilon} \times \mathbf{u}\right) \tag{3.14}$$

If a physical quantity f is defined at the point E, $f = f\left(w\right)$ then we have,

$$\frac{\partial f\left(w\right)}{\partial t} = \frac{df\left(w\right)}{dw}\frac{\partial w}{\partial t}$$

and with Equation (3.7) we obtain,

$$\frac{\partial f\left(w\right)}{\partial t} = \frac{df\left(w\right)}{dw}\frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} \tag{3.15}$$

Similarly, from Equation (3.9) we obtain,

$$\nabla f\left(w\right) = -\frac{df\left(w\right)}{dw}\frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2} \tag{3.16}$$

From Equations (3.15) and (3.16) we obtain,

$$\nabla f\left(w\right) = -\frac{\partial f\left(w\right)}{\partial t}\frac{\boldsymbol{\upsilon}}{c^2} \tag{3.17}$$

As a consequence of self-variation, at time t the electric charge acts at point A with the value it has at point E. Therefore, $q = q\left(w\right)$ and from Equations (3.15), (3.16) and (3.17) For $f = q$ we obtain,

$$\frac{\partial q}{\partial t} = \frac{dq}{dw}\frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} \tag{3.18}$$

$$\nabla q = -\frac{dq}{dw}\frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2} \tag{3.19}$$

$$\nabla q = -\frac{\partial q}{\partial t}\frac{\boldsymbol{\upsilon}}{c^2} \tag{3.20}$$

We now consider the acceleration vector $\mathbf{a} = \mathbf{a}\left(w\right)$ of q at the moment w located at point E,

$$\mathbf{a} = \mathbf{a}\left(w\right) = \frac{d\mathbf{u}\left(w\right)}{dw} \tag{3.21}$$

Applying equations (3.15) and (3.16) for the velocity components $\mathbf{u}$ we obtain,

$$\frac{\partial \mathbf{u}}{\partial t} = \frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\mathbf{a} \tag{3.22}$$

$$\frac{\partial u_i}{\partial x_j} = -\frac{\upsilon_j a_i}{c^2\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)} \tag{3.23}$$

where $i, j = 1, 2, 3$ and $\left(x_1, x_2, x_3\right) = \left(x, y, z\right)$. Applying Equations (3.15) and (3.16) for the velocity components $\mathbf{a}$ we obtain,

$$\frac{\partial \mathbf{a}}{\partial t} = \frac{1}{1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\mathbf{b} \tag{3.24}$$

$$\frac{\partial a_i}{\partial x_j} = -\frac{\upsilon_j b_i}{c^2\left(1 - \frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)} \tag{3.25}$$

where $\mathbf{b} = \mathbf{b}\left(w\right) = \frac{d\mathbf{a}\left(w\right)}{dw}$.

Using the previous Equations we obtain the following equations,

$$\frac{\partial(\mathbf{u}\cdot\boldsymbol{\upsilon})}{\partial t}=\frac{\boldsymbol{\upsilon}\cdot\mathbf{a}}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}+\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})^2-c^2u^2}{cr\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)} \tag{3.26}$$

$$\nabla(\mathbf{u}\cdot\boldsymbol{\upsilon})=\frac{c}{r}\mathbf{u}+\frac{u^2-\mathbf{u}\cdot\boldsymbol{\upsilon}}{r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{\upsilon}\cdot\mathbf{a}}{c\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c} \tag{3.27}$$

$$\frac{\partial(\boldsymbol{\upsilon}\cdot\mathbf{a})}{\partial t}=\frac{\boldsymbol{\upsilon}\cdot\mathbf{b}}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}+\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})(\boldsymbol{\upsilon}\cdot\mathbf{a})-c^2(\mathbf{u}\cdot\mathbf{a})}{cr\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)} \tag{3.28}$$

$$\nabla(\boldsymbol{\upsilon}\cdot\mathbf{a})=\frac{c}{r}\mathbf{a}-\frac{\boldsymbol{\upsilon}\cdot\mathbf{b}}{c^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\boldsymbol{\upsilon}+\frac{\mathbf{u}\cdot\mathbf{a}-\boldsymbol{\upsilon}\cdot\mathbf{a}}{cr\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\boldsymbol{\upsilon} \tag{3.29}$$

after the necessary calculations.

3.3. Liénard-Wiechert Potentials

With the notation we follow, the Liénard-Wiechert (see, [8,9,10]) scalar-vector potential pair $(V_{LW},\mathbf{A}_{LW})$ is given by the equations,

$$V_{LW}=\frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)} \tag{3.30}$$

$$\mathbf{A}_{LW}=V_{LW}\frac{\mathbf{u}}{c^2}=\frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}\frac{\mathbf{u}}{c^2} \tag{3.31}$$

The electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ at point $A(x,y,z)$ are given by the pair $(V,\mathbf{A})$ of the scalar potential V and the vector potential $\mathbf{A}$ respectively, through equations

$$\mathbf{E}=-\nabla V-\frac{\partial\mathbf{A}}{\partial t} \tag{3.32}$$

$$\mathbf{B}=\nabla\times\mathbf{A} \tag{3.33}$$

Through Equations (3.30), (3.31) and (3.32), (3.33) the Liénard-Wiechert potentials give the following equations for the electromagnetic field at point A,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)+\frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right] \tag{3.34}$$

$$\mathbf{B}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\mathbf{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right)+\frac{q}{4\pi\varepsilon_0 c^3 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\mathbf{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right)-\frac{\boldsymbol{\upsilon}}{c}\times\mathbf{a}\right] \tag{3.35}$$

The first terms in the second members of Equations (3.34), (3.35) give the electromagnetic field accompanying the electric charge in its movement, and the second terms the electromagnetic radiation.

3.4. Self-variation Potentials

As a consequence of self-variation, the electromagnetic potential splits into two pairs of potentials. One pair,

$$V_u=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

$$\mathbf{A}_u=V_u\frac{\boldsymbol{\upsilon}}{c^2}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2} \tag{3.36}$$

gives the electromagnetic field that accompanies the electric charge in its motion,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)$$

$$\mathbf{B}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\mathbf{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right) \tag{3.37}$$

The other pair,

$$V_a = \frac{(\boldsymbol{\upsilon}\cdot\mathbf{a})q}{4\pi\varepsilon_0 c^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

$$\mathbf{A}_a = V_a\frac{\boldsymbol{\upsilon}}{c^2} = \frac{(\boldsymbol{\upsilon}\cdot\mathbf{a})q}{4\pi\varepsilon_0 c^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2} \tag{3.38}$$

gives the electromagnetic radiation,

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]$$

$$\mathbf{B} = \frac{q}{4\pi\varepsilon_0 c^3 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\mathbf{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right)-\frac{\boldsymbol{\upsilon}}{c}\times\mathbf{a}\right] \tag{3.39}$$

From (3.37) and (3.39) we get Equations (3.34). The Liénard-Wiechert and self-variation potentials give the same equations for the electromagnetic field strength. From the potentials (3.36) we prove the first of Equations (3.37). Similarly, the proof of the second is done, as well as the proof of Equations (3.39) from the potentials (3.38).

Proof. From Equation (3.32) and (3.36) we have,

$$\mathbf{E} = -\nabla\left(\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right) -\frac{\partial}{\partial t}\left(\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2}\right)$$

and equivalently we get,

$$\mathbf{E} = -\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\nabla q -q\nabla\left(\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right) -\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial q}{\partial t} -q\frac{\partial}{\partial t}\left(\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2}\right) \tag{3.40}$$

and with Equation (3.20) we get,

$$\mathbf{E} = -q\nabla\left(\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right) -q\frac{\partial}{\partial t}\left(\frac{\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c^2}\right) \tag{3.41}$$

and equivalently we get,

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\nabla\left(\frac{u^2}{c^2}\right) -\left(1-\frac{u^2}{c^2}\right)q\nabla\left(\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right) +\frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\partial\left(\frac{u^2}{c^2}\right)}{\partial t}\frac{\boldsymbol{\upsilon}}{c^2} -\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c^2}\right) -\left(1-\frac{u^2}{c^2}\right)q\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial}{\partial t}\left(\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right) \tag{3.42}$$

From Equation (3.17) for $f(w) = u^2(w)$ we get,

$$\nabla\left(\frac{u^2}{c^2}\right) = -\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial}{\partial t}\left(\frac{u^2}{c^2}\right) \tag{3.43}$$

From Equations (3.42) and (3.43) we get,

$$\mathbf{E} = -\left(1-\frac{u^2}{c^2}\right)q\nabla\left(\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right) -\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c^2}\right) -\left(1-\frac{u^2}{c^2}\right)q\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial}{\partial t}\left(\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\right)$$

and equivalently we obtain,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\left(\nabla r+\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial r}{\partial t}\right)$$
$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c^2}\right)$$
$$-\frac{2\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\nabla\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)+\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial}{\partial t}\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)\right) \quad (3.44)$$

From Equations (3.8) and (3.10) we get,

$$\nabla r+\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial r}{\partial t}=\frac{\boldsymbol{\upsilon}}{c} \quad (3.45)$$

From Equations (3.26) and (3.27) we get,

$$\nabla\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)+\frac{\boldsymbol{\upsilon}}{c^2}\frac{\partial}{\partial t}\left(\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)=\frac{1}{r}\left(\frac{\mathbf{u}}{c}-\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\frac{\boldsymbol{\upsilon}}{c}\right) \quad (3.46)$$

From Equations (3.44) and (3.34), (3.45), (3.11), (3.4) we get,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c}$$
$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)$$
$$-\frac{2\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\mathbf{u}}{c}-\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\frac{\boldsymbol{\upsilon}}{c}\right)$$

and equivalently we get,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c}$$
$$+\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}(1-2)\frac{\mathbf{u}}{c}$$
$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}-2\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\right)\frac{\boldsymbol{\upsilon}}{c}$$

and equivalently we get,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\frac{\boldsymbol{\upsilon}}{c}$$
$$+\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\frac{\boldsymbol{\upsilon}}{c}$$
$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\mathbf{u}}{c}$$

and equivalently we get,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(1-\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}+\frac{(\mathbf{u}\cdot\boldsymbol{\upsilon})}{c^2}\right)\frac{\boldsymbol{\upsilon}}{c}$$
$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\mathbf{u}}{c}$$

and equivalently we get,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\boldsymbol{\upsilon}}{c}$$
$$-\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\mathbf{u}}{c}$$

and equivalently we obtain,

$$\mathbf{E}=\frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)$$

In the proof we followed, the transition from Equation (3.40) to (3.41) was made as a consequence of Equation (3.20). This Equation expresses the self-variation of the electric charge q. If we assume that the charge q does not self-variate, from the potentials (3.36) we directly obtain Equation (3.41). The self-variation potentials give the same electromagnetic field whether we consider the electric charge to vary according to the self-variation principle or to be constant.

Applying Maxwell's Equations for the electromagnetic field of Equations (3.34), (3.35) it follows that at point A there is an electric charge, as a consequence of self-variation, with density ρ and current density $\mathbf{j}$,

$$\mathbf{j} = \rho\boldsymbol{\upsilon} = -\frac{dq}{dw}\frac{1-\frac{u^2}{c^2}}{4\pi c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\boldsymbol{\upsilon}$$

$$= -\frac{\partial q}{\partial t}\frac{1-\frac{u^2}{c^2}}{4\pi c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\boldsymbol{\upsilon} \qquad (3.47)$$

As a consequence of self-variation, in the surrounding spacetime of q there is an electric charge of opposite sign ($\frac{dq}{dw} > 0$), as follows from Equations (3.47). We prove the first of Equations (3.47). Similarly, the proof of the second Equation is made.

Proof. From Maxwell's first law we have,

$$\rho = \varepsilon_0 \nabla\cdot\mathbf{E} \qquad (3.48)$$

We write equation (3.34) in the form

$$\mathbf{E} = q\boldsymbol{\varepsilon} \qquad (3.49)$$

where the form of the vector $\boldsymbol{\varepsilon}$ is shown in Equation (3.34).

If we ignore self-variation and consider q constant, at point A there is no electric charge. Thus from Equations (3.48) and (3.49) we get,

$$\nabla\cdot\boldsymbol{\varepsilon} = 0 \qquad (3.50)$$

From Equations (3.48) and (3.49) we get,

$$\rho = \varepsilon_0 \nabla q\cdot\boldsymbol{\varepsilon} + \varepsilon_0 q\nabla\cdot\boldsymbol{\varepsilon}$$

and with Equation (3.49) we get,

$$\rho = \varepsilon_0 \nabla q\cdot\boldsymbol{\varepsilon}$$

and with Equations (3.49) and (3.50) we get,

$$\rho = \frac{1-\frac{u^2}{c^2}}{4\pi r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\nabla q\cdot\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)$$

$$+\frac{1}{4\pi c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}\nabla q\cdot\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]$$

and with Equation (3.19) we get,

$$\rho = -\frac{dq}{cdw}\frac{1-\frac{u^2}{c^2}}{4\pi r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^4}\frac{\boldsymbol{\upsilon}}{c}\cdot\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)$$

$$-\frac{dq}{cdw}\frac{1}{4\pi c^2 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\boldsymbol{\upsilon}}{c}\cdot\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\cdot\mathbf{a}\right)}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\mathbf{u}}{c}\right)-\mathbf{a}\right]$$

and equivalently we obtain,

$$\rho = -\frac{dq}{cdw}\frac{1-\frac{u^2}{c^2}}{4\pi r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}+0 .$$

Therefore, the charge density at point A is given by the equation,

$$\rho = -\frac{dq}{cdw}\frac{1-\frac{u^2}{c^2}}{4\pi r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}$$

and with Equation (3.18) we obtain,

$$\rho = -\frac{dq}{dw}\frac{1-\frac{u^2}{c^2}}{4\pi c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3} .$$

$$= -\frac{\partial q}{\partial t}\frac{1-\frac{u^2}{c^2}}{4\pi c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

Furthermore, electromagnetic radiation does not contribute to the electric charge of spacetime.

We now prove the continuity equation at point A,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j} = 0 \qquad (3.51)$$

Proof. From Equation (3.47) we have,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j} = \frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\boldsymbol{\upsilon}\right)$$

and equivalently we get,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j} = \frac{\partial\rho}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\rho+\rho\nabla\cdot\boldsymbol{\upsilon}$$

and with Equation (3.13) we get,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j} = \frac{\partial\rho}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\rho+\frac{2c}{r}\rho \qquad (3.52)$$

The charge $q = q(w)$ and the velocity $\mathbf{u} = \mathbf{u}(w)$ are defined at point E. Then, from the first of Equations (3.47) we get the density ρ in the form,

$$\rho = -\frac{dq}{cdw}\frac{1-\frac{u^2}{c^2}}{4\pi r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3} = \frac{f(w)}{4\pi c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3} \quad (3.53)$$

From Equations (3.52) and (3.53) we get,

$$\begin{aligned}\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j} &= \frac{1}{4\pi c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\partial f(w)}{\partial t}\\ &-\frac{2f(w)}{4\pi c r^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\frac{\partial r}{\partial t}\\ &+\frac{3f(w)}{4\pi c^3 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^4}\frac{\partial(\mathbf{u}\cdot\boldsymbol{\upsilon})}{\partial t}\\ &+\frac{1}{4\pi c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\boldsymbol{\upsilon}\cdot\nabla f(w)\\ &-\frac{2f(w)}{4\pi c r^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\boldsymbol{\upsilon}\cdot\nabla r\\ &+\frac{3f(w)}{4\pi c^3 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^4}\boldsymbol{\upsilon}\cdot\nabla(\mathbf{u}\cdot\boldsymbol{\upsilon})\\ &+\frac{2f(w)}{4\pi r^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\end{aligned}$$

and equivalently we get,

$$\begin{aligned}\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j} &= \frac{1}{4\pi c r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\partial f(w)}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla f(w)\right)\\ &-\frac{2f(w)}{4\pi c r^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\left(\frac{\partial r}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla r\right)+\\ &\frac{3f(w)}{4\pi c^3 r^2\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^4}\left(\frac{\partial(\mathbf{u}\cdot\boldsymbol{\upsilon})}{\partial t}+\boldsymbol{\upsilon}\nabla(\mathbf{u}\cdot\boldsymbol{\upsilon})\right)\\ &+\frac{2f(w)}{4\pi r^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}\end{aligned} \quad (3.54)$$

From Equations (3.15) and (3.16) we get,

$$\frac{\partial f(w)}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla f(w)=0 \quad (3.55)$$

From Equations (3.6) and (3.8) we get,

$$\frac{\partial r}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla r=c \quad (3.56)$$

From Equations (3.26) and (3.27) we get,

$$\frac{\partial(\mathbf{u}\cdot\boldsymbol{\upsilon})}{\partial t}+\boldsymbol{\upsilon}\nabla(\mathbf{u}\cdot\boldsymbol{\upsilon})=0 \quad (3.57)$$

From Equations (3.54) and (3.55), (3.56), (3.57) we get,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{j} = -\frac{2cf(w)}{4\pi c r^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}+\frac{2f(w)}{4\pi r^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^3}=0$$

The continuity equation expresses the conservation of charge distributed in spacetime. This conservation of charge is equivalently expressed through the equation,

$$q(w)=q(t)+\int_V \rho dV \quad (3.58)$$

Considering the independence of velocity c (Einstein, 1905) from velocity $\mathbf{u}$ at point E, the volume V in Equation (3.58) is a sphere centered at point E and radius r. Equation (3.58) can also be proved independently of the continuity equation, by using the auxiliary Equations (3.6) – (3.29). From Equation (3.58) it follows that two observers in points E and P, for the same particle (carrying the charge q) measure a value $q(t)$ for their own particle and $q(w)$ the value with which the particle of the other acts in theirs.

To understand the physical content of Equation (3.58), let us assume that the particle at point E is an electron carrying a charge q. In the time interval from w to t, $\delta t = t - w = \frac{r}{c}$, the increase in q is balanced by the charge of spacetime, which is distributed over the sphere with center E and radius r. The charge of spacetime is due to the electromagnetic field that accompanies the electron. If we assume that this field exists in every case, the increase of q is continuous. We now assume that the electron is stationary ($\mathbf{u}=\mathbf{0}$) at point E. The increase of q to over time is given by the equation (2.33). Therefore, the constant rest mass M_0' determines the increase in q over time.

Self-variation potentials are compatible with Lorentz-Einstein transformations and, obviously, with the self-variation principle. The Liénard-Wiechert potentials were published (1899) six years before the publication of

Special Relativity (1905) by Einstein. After the formulation of Special Relativity it was shown that they are compatible with Lorentz-Einstein transformations. From Equations (3.30), (3.31) it is proven that the Liénard-Wiechert potentials are not compatible with the self-variation principle. For them to be compatible, the self-variation principle should have given the equation

$$\nabla q = -\frac{\partial q}{\partial t}\frac{\mathbf{u}}{c^2}$$

and not (3.20),

$$\nabla q = -\frac{\partial q}{\partial t}\frac{\boldsymbol{\upsilon}}{c^2}.$$

If we denote by L the set of equations that are compatible with the Lorentz-Einstein transformations and by S the set of equations that are compatible with then it is $S \subset L$. Regarding the mathematical formalism of the laws of physics, the Self-Variation Theory imposes additional constraints than those imposed by Special Relativity.

3.5. Orbit Representation Theorem

In Figure 3.2, the point electric charge q is at point $P\left(x_p(t), y_p(t), z_p(t), t\right)$. By C_p we denote the orbit in which q moved in the past time, until it is at point $P\left(x_p(t), y_p(t), z_p(t), t\right)$.

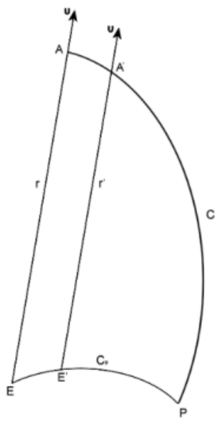

Figure 3.2. An arbitrarily moving electric point charge q at time t is at point $P\left(x_p(t), y_p(t), z_p(t), t\right)$. C_p is the orbit that q moved in the past time, until it is at point P.

The Frenet equations,

$$\frac{d\mathbf{t}}{dS} = \mathbf{k} = k\mathbf{n}$$
$$\frac{d\mathbf{n}}{dS} = -k\mathbf{t} + \tau\mathbf{b}$$
$$\frac{d\mathbf{b}}{dS} = -\tau\mathbf{n} \qquad (3.59)$$

uniquely define a curve C. $\mathbf{t}$ denotes the tangent vector, $\mathbf{k}$ the curvature vector, k and τ the curvature and torsion respectively, dS the arc length of the curve C, $\|\mathbf{n}\| = 1$ and $\mathbf{b} = \mathbf{t} \times \mathbf{n}$.

We calculate the tangent vector $\mathbf{t}_p$, the curvature k_p and the torsion τ_p of the curve C_p at the point $E\left(x_p(w), y_p(w), z_p(w), w\right)$. First we calculate the arc length dS_p. We have (see, Figure 3.1),

$$dS_p = \left\|\frac{d\overrightarrow{OE}}{dw}\right\| dw$$

and equivalently we get,

$dS_p = \|\mathbf{u}\| dw$. (3.60)

If $\mathbf{u} \neq \mathbf{0}$ we have,

$$\mathbf{t}_p = \frac{\frac{d\overrightarrow{OE}}{dS_p}}{\left\|\frac{d\overrightarrow{OE}}{dS_p}\right\|} = \frac{\frac{d\overrightarrow{OE}}{dw}}{\left\|\frac{d\overrightarrow{OE}}{dw}\right\|}$$

and equivalents we have,

$$\mathbf{t}_p = \frac{\mathbf{u}}{\|\mathbf{u}\|} \qquad (3.61)$$

The curvature vector $\mathbf{k}$ is given by equation,

$$\mathbf{k}_p = \frac{d\mathbf{t}_p}{dS_p} = \frac{d}{dS_p}\left(\frac{\mathbf{u}}{\|\mathbf{u}\|}\right) = \frac{1}{\|\mathbf{u}\|}\frac{d\mathbf{u}}{dS_p} - \frac{\mathbf{u}}{\|\mathbf{u}\|^2}\frac{d\|\mathbf{u}\|}{dS_p}$$

and with Equations (3.21) and (3.60) we get,

$$\mathbf{k}_p = \frac{1}{\|\mathbf{u}\|^2}\mathbf{a} - \frac{\mathbf{u}}{\|\mathbf{u}\|^3}\frac{d\|\mathbf{u}\|}{dw} \qquad (3.62)$$

Now we have,

$$\frac{d\|\mathbf{u}\|^2}{dw} = \frac{d(\mathbf{u})^2}{dw}$$

and equivalents we get,

$$2\|\mathbf{u}\|\frac{d\|\mathbf{u}\|}{dw} = 2\mathbf{u}\frac{d\mathbf{u}}{dw}$$

and with Equation (3.21) we get,

$$\|\mathbf{u}\|\frac{d\|\mathbf{u}\|}{dw} = \mathbf{u}\cdot\mathbf{a}$$

and equivalents we get,

$$\frac{d\|\mathbf{u}\|}{dw}=\frac{\mathbf{u}\cdot\mathbf{a}}{\|\mathbf{u}\|} \quad (3.63)$$

From Equations (3.62) and (3.63) we get,

$$\mathbf{k}_p=\frac{\|\mathbf{u}\|^2\mathbf{a}-(\mathbf{u}\cdot\mathbf{a})\mathbf{u}}{\|\mathbf{u}\|^4}$$

and equivalents we obtain,

$$\mathbf{k}_p=\frac{\mathbf{u}\times(\mathbf{a}\times\mathbf{u})}{\|\mathbf{u}\|^4} \quad (3.64)$$

From Equation (3.64) we obtain,

$$k_p=\frac{\|\mathbf{u}\times\mathbf{a}\|}{\|\mathbf{u}\|^3} \quad (3.65)$$

From Equations (3.64) and (3.65) we obtain,

$$\mathbf{n}_p=\frac{\mathbf{k}_p}{\|\mathbf{k}_p\|}=\frac{\mathbf{u}\times(\mathbf{a}\times\mathbf{u})}{\|\mathbf{u}\|\|\mathbf{a}\times\mathbf{u}\|} \quad (3.66)$$

For the vector $\mathbf{b}_p$ we have,

$$\mathbf{b}_p=\mathbf{t}_p\times\mathbf{n}_p$$

and with Equations (3.61) and (3.66) we get,

$$\mathbf{b}_p=\frac{\mathbf{u}}{\|\mathbf{u}\|}\times\frac{\mathbf{u}\times(\mathbf{a}\times\mathbf{u})}{\|\mathbf{u}\|\|\mathbf{a}\times\mathbf{u}\|}$$

and equivalents we get,

$$\mathbf{b}_p=\frac{\mathbf{u}\times\mathbf{a}}{\|\mathbf{u}\times\mathbf{a}\|} \quad (3.67)$$

From the third of Equations (3.59) we get

$$\tau_p=-\mathbf{n}_p\cdot\frac{d\mathbf{b}_p}{dS_p}$$

and after the necessary calculations we obtain,

$$\tau_p=\frac{\mathbf{a}\cdot\left(\mathbf{u}\times\frac{d\mathbf{a}}{dw}\right)}{\|\mathbf{u}\|^2\|\mathbf{a}\|^2-(\mathbf{u}\cdot\mathbf{a})^2}\|\mathbf{u}\|^2 \quad (3.68)$$

Equations (3.61), (3.65), (3.68) give the tangent vector $\mathbf{t}_p$, curvature k_p and torsion τ_p of the C_p curve respectively.

For each direction $\frac{\boldsymbol{\upsilon}}{c}$ the curve C_p is mapped onto another curve C in the surrounding space-time of the point electric charge q. This mapping is given by the following Theorem.

Orbit Representation Theorem

For each direction $\frac{\boldsymbol{\upsilon}}{c}$ the following hold.

1. The mapping $f:\mathbf{u}\to\mathbf{u}-\boldsymbol{\upsilon}$ maps the orbit C_p of point electric charge q onto the curve C in its surrounding spacetime,
$(\mathbf{t}_p,\mathbf{k}_p,\mathbf{n}_p,\mathbf{b}_p,k_p,\tau_p)\to(\mathbf{t},\mathbf{k},\mathbf{n},\mathbf{b},k,\tau)$. (3.69)

2. The mapping $f^{-1}:\mathbf{u}-\boldsymbol{\upsilon}\to\mathbf{u}$ maps the curve C onto the orbit C_p, $(\mathbf{t},\mathbf{k},\mathbf{n},\mathbf{b},k,\tau)\to(\mathbf{t}_p,\mathbf{k}_p,\mathbf{n}_p,\mathbf{b}_p,k_p,\tau_p)$. (3.70)

Proof. In Figure 3.2, in the direction of the vector $\boldsymbol{\upsilon}$ the curve C_p is depicted in the curve C. Similarly, using the auxiliary Equations (3.6) - (3.29) the components of the C curve are calculated, as given below.

The tangent vector $\mathbf{t}$ is given by the equation,

$$\mathbf{t}=\frac{\boldsymbol{\upsilon}-\mathbf{u}}{\|\boldsymbol{\upsilon}-\mathbf{u}\|} \quad (3.71)$$

The curvature vector $\mathbf{k}$ is given by the equation,

$$\mathbf{k}=\frac{(\mathbf{u}-\boldsymbol{\upsilon})\times[\mathbf{a}\times(\mathbf{u}-\boldsymbol{\upsilon})]}{\|\mathbf{u}-\boldsymbol{\upsilon}\|^4} \quad (3.72)$$

The vector $\mathbf{n}$ is given by the equation,

$$\mathbf{n}=\frac{(\mathbf{u}-\boldsymbol{\upsilon})\times[\mathbf{a}\times(\mathbf{u}-\boldsymbol{\upsilon})]}{\|\mathbf{u}-\boldsymbol{\upsilon}\|\|(\mathbf{u}-\boldsymbol{\upsilon})\times\mathbf{a}\|} \quad (3.73)$$

The vector $\mathbf{b}$ is given by the equation,

$$\mathbf{b}=\frac{(\mathbf{u}-\boldsymbol{\upsilon})\times\mathbf{a}}{\|(\mathbf{u}-\boldsymbol{\upsilon})\times\mathbf{a}\|} \quad (3.74)$$

The curvature k is given by the equation,

$$k=\|\mathbf{u}-\boldsymbol{\upsilon}\|\|(\mathbf{u}-\boldsymbol{\upsilon})\times\mathbf{a}\| \quad (3.75)$$

The torsion τ is given by the equation,

$$\tau=\frac{\mathbf{a}\cdot\left((\mathbf{u}-\boldsymbol{\upsilon})\times\frac{d\mathbf{a}}{dw}\right)}{\|\mathbf{u}-\boldsymbol{\upsilon}\|^2\|\mathbf{a}\|^2-(\mathbf{u}-\boldsymbol{\upsilon})(\cdot\mathbf{a})^2}\|(\mathbf{u}-\boldsymbol{\upsilon})\|^2 \quad (3.76)$$

From Equations (3.61), (3.64), (3.66), (3.67), (3.65) and (3.68), by substituting

$$f:\mathbf{u}\to\mathbf{u}-\boldsymbol{\upsilon}$$

we get Equations (3.71), (3.72), (3.73), (3.74), (3.75) and (3.76). From Equations(3.71), (3.72), (3.73), (3.74), (3.75) and (3.76), by substituting

$$f^{-1}:\mathbf{u}-\boldsymbol{\upsilon}\to\mathbf{u}$$

we get Equations (3.61), (3.64), (3.66), (3.67), (3.65) and (3.68).

With the proof of the Orbit Representation Theorem we have the main consequences of the self-variation in the surrounding spacetime of the point electric charge q. The first consequence concerns the geometry of spacetime. For each direction in space, curve C_p is depicted in curve C. The second consequence concerns the existence of electric charge and electric current in spacetime. The charge density and current density in space-time are given by Equations (3.47). As a consequence of self-variation, the charge q affects both the geometry and the physical quantities contained in spacetime.

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