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Electrodynamics in Noninertial Metrices Predicts Entirely New Origin of 4-Dimensional Electromagnetic Wave

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Abstract: Electrodynamics in noninertial metrices based on unified transformation law (UTL) for 4-vectors and tensors inherit new origin of 4D electromagnetic (EM) wave, complete form of 4D Lorentz force that is not possible in usual Lorentz Transformation (LT) and Galilean transformation (GT). Noninertial metrices consist of vacuum metric in terms of numbers, velocity and acceleration. Minkoskian metric is a special case of vacuum metric. As a consequence of transformation, a new symmetry appeared along the diagonal of electromagnetic field (EMF), Maxwell's equations (ME) and conservation law in terms of tensors resulted in the complete symmetrization of Maxwell's equations. The most beautiful consequence is the emergence of these singularities as complete 4D EM wave along the diagonal in the conservation law of electrodynamics for the first time. In noninertial metric transformation of Gauss's law gives the combination of Gauss's and Ampere's law. Similarly, Ampere's law gives the similar result. As a whole, electrodynamic quantities are doubled implies that noninertial metric is causing pair production. Similarly, electrodynamics in noninertial metric in terms of velocity incorporate 4D Lorentz force as a natural part of electrodynamics. For consistency with the classical electrodynamics, usual Lorentz transformation is generalized from 2D to 4D that gives the same results as that of velocity metric with a difference of Lorentz factor γ given in the Table 3. The results of electrodynamics in accelerating metric are very exciting predicts the possibility of electromagnetic energy driven engine. Matrix method and Einstein's summation convention method are applied. Both methods agree in the transformation of EMF and ME but differ in the case of conservation law. Theory of inertia and gravitation possess the same formulation. Thomas precession relation is obtained in dual of Lorentz force of inertia as special case. It is also present in generalized LT in the same manner but with Lorentz factor. SR in noninertial frame is presented shortly.

Keywords: *Noninertial metrices, new origin of 4D EM wave, New gauge, Origin of 4d Lorentz force, generalized Lorentz transformation, Thomass precession, wonders of accelerating metric, electromagnetic energy driven engine, SR in Noninertial frame*

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1. Introduction

The most beautiful structure of relativistic electrodynamics is emerged when EMF, ME and EM conservation law is transformed in the language of tensor components. In the contemporary literature, physicists consider only transformation of EMF in tensor form and don't bother about ME and EM conservation law in tensor form perhaps due to complex calculations. Generally, wave equations are derived by taking the curl of Faraday's law and Ampere's law and similarly EM conservation law is defined by taking the divergence of Ampere's law. These relations must appear as an integral part of electrodynamics.

It is well-known that EM waves travel with the speed of light predicted by Maxwell and experimentally confirmed by Hertz. What is the trajectory of EM wave in the structure of electrodynamics emerging as a natural part of the theory? How does 4D Lorentz force appear as an integral part of electrodynamics through noninertial metric? The new symmetry terms appearing in EM conservation law in universal Lorentz transformation matrix (ULTM) based on UTL provided a fractional form of 4D EM wave [1]. Electrodynamics in noninertial vacuum metric of which Minkouskian metric is a special case, provided complete form of 4D EM wave corresponding to set of singularities in conservation law. As a consequence of this transformation law in noninertial metric, a new symmetry of EMF, ME and electromagnetic conservation law written in tensor components is emerged along their diagonals. These new symmetry terms are the representatives of set of singularities viz. $\{F^{00}, F^{11}, F^{22}, F^{33}\}, \{F^{00}{}_{,0}, F^{11}{}_{,1}, F^{22}{}_{,2}, F^{33}{}_{,3}\}$ and $\{F^{11}{}_{,11}, F^{22}{}_{,22}, F^{33}{}_{,33}\}$ which are definite physical quantities. Newton's view of action and reaction appears naturally to sustain the balance. Temporal and spatial singularities play the role of action and reaction. These models have outlandish predictions at each step of development and having no counter example in the contemporary world of spacetime physics. The noninertial metric excites these singularities and arrange them in the spacetime fabric such that the upcoming entities of EMF find their natural place. The home of singularities is the diagonal and their expansion is above and below the diagonal. The action of metric demystifies the origin of electric field, time varying electric field, second order time varying electric field and similar results for magnetic field in terms of physical singularities. When spacetime operator acts on these fields in the presence of noninertial metric then Maxwell's equations are generated. Here we get surprising results again in the form of singularities of Maxwell's equations where they represent the origin of time varying electric field and Gauss's Law. The transformation of Gauss's law appears as the combination of Gauss's law and Ampere's law in their complete form. Similar behavior is observed for Ampere's Law. In other words; electromagnetic laws are doubled. When the same transformation is applied for dual of EMF then the origin of magnetic sources appears in terms of singularities such that temporal singularity $*F^{0'0'} = \frac{\partial B}{\partial t}$ and Spatial singularity consists of Gauss's law for magnetism $*F^{i'i'}{}_{,i'} = -\nabla . B$. This discovery has symmetrized Maxwell's equations completely. This is the marvelous prediction of this model. Gauss's law for magnetism and Faraday's law appear together. Now, we come to the case of conservation law. In, Einstein's summation method, singularities do form 4D wave of electric field and similar wave of 4D wave for magnetism. In terms of potentials, it represents 4D wave of Lorenz gauge. This shows that the model is completely consistent with EPR and Noether's principle of symmetry. A few models are available on electrodynamics in noninertial frame related to Galilean metric [2,3]. Other advance references will be discussed after the development of the models.

In the contemporary literature, the origin of 4D Lorentz force [4,5,6,7,8,9,10] is the consequence of transformation of EMF under usual Lorentz transformation but the fact is that 4D Lorentz force is the consequence of transformation of EMF in noninertial frame where three components of velocity are involved. Usual Lorentz transformation (LT) and Galilean transformation (GT) can't provide complete structure of 4D Lorentz force as one component of velocity is utilized. Electromagnetic field constitute magnetic force, a very strange outcome. Here, set of singularities along the diagonal of EM conservation law represent 4D wave of electric power. Similarity based transformation of electrodynamics in noninertial velocity metric, predicts such results. Some results match with the contemporary models. Here Lorentz force appear as an integral part of electrodynamics not as a separate entity. In order to synchronize the classical theory with these models, usual Lorentz transformation matrix is generalized from 2D to 4D such that all of the results are true with the difference of Lorentz factor. This development will help the physicists to understand new theory without confusion. The results of electrodynamics in accelerating metric predict the existence of magnetic sources that are not static. Acceleration and rotation base model of electrodynamics will be presented separately as involves long details but ULTM for noninertial frame in 2D and 4D are presented just for comparison with the contemporary models. Teleportation technology strongly needs universal spacetime laws of physics to retain the state of the objects same before and after transformation. For this purpose, a set of universal transformation matrices is constructed in terms of numerical and physical terms. Each matrix is an usual identity matrix collected in table-5. These matrices play the role of guidance system. Basic structure and equations for the theory of inertia and gravitation are presented in tables 1 to 4 as a comparison with electrodynamics. The origin of a well-known concept of Thomas Precession relation exists as a special case of dual Lorentz force of inertia. Dual of Inertia field tensor in velocity metric directly represent Thomas precession relation. It is also discovered in generalized LT in the same way but with Lorentz factor. Lorentz transformation matrix for accelerating and rotating context is presented with basic concepts. Theory of inertia and gravitation in detail will be presented in a separate paper.

In sec-I, electrodynamics is developed in noninertial metric consisting of numbers only but contains Minkouskian metric as a special case. Sec-II, consists of electrodynamics in noninertial velocity metric by employing UTL. Sec-III contains electrodynamics in noninertial velocity metric based on similarity transformation. In sec-4, electrodynamics in an accelerating metric is developed. In sec-5, a comparison of theory of electrodynamics, inertia and gravitation is presented in the form of tables from 1to 4. Universal transformation matrices are given in table-5. Sec-6, consists of discussion and comparison. Sec-7 on conclusion. References are given at the last.

2. Spacetime Electrodynamics in Noninertial Vacuum Metric

Notations in this model are adopted according to modern approach of relativity. Greek alphabets $\mu, \nu, \alpha, \beta, \ldots$ runs from 0 to 3 and Latin letters i, j, k, from 1 to 3. Comma (,) denote partial differentiation e. g. $E_{,0} = \frac{\partial E}{\partial t}$ Partial derivative of electric field w. r. t. time, $E_{,1} = \frac{\partial E}{\partial x}$ Partial derivative of electric field w. r. t. x-axis,

$E,_2 = \frac{\partial E}{\partial y}$ partial derivative of electric field w. r. t. y-axis,

$E,_0 = \frac{\partial E}{\partial x}$ partial derivative of electric field w. r. t. z-axis,

$F^{\mu\nu},_\nu$ means 4-dimensional or spacetime partial derivative of EMF tensor. 4-dimensional Coordinates $x^\mu = x^0, x^1, x^2, x^3$ =(ct, x, y, z)= (ct, x^i) with x^0 =ct and x^i =(x, y, z). Time component ct is scalar while space components xi is vector such that x^μ is the unification of time and space. The dimensions of all components are that of length.

2.1. Noninertial Vacuum Metric in 4-Dimensions

Minkouskian metric is the representative of flat spacetime without gravity but it doesn't provide complete structure of electromagnetic field. Its limited structure is obvious from its 4-components along the diagonal in terms of numbers. Its generalization is the metric for noninertial frame that contains this metric as a special case.

The components of the noninertial metric are.

$$g_0^0 = 1, g_1^1, g_2^2, g_3^3 = -1, g_i^0 = g_0^i = -1$$

$$[g_\nu^\mu] = \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

This matrix is the home of wonders that provides the secrets of nature in terms of equations.

The UTL for 4-vectors and tensors is

$$F^{\mu'\nu'} = g_\alpha^{\mu'} F^{\alpha\nu'}$$

employed for the development of the model. All the calculations are performed by using Einstein summation convention method. The results are presented in matrix form.

2.2. Transformation of Electromagnetic Field

EMFT $F^{\mu\nu}$ is related to its components electric field E and magnetic field B as follows.

$$F^{0i} = E^i \text{ and } F^{ij} = \varepsilon^{ijk} B_k$$

EMF tensor in component form is needed to get new terms along the diagonal of EMF. It is represented as 4 by 4 antisymmetric matrix.

$$[F^{\mu\nu}] = \begin{bmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & B^1 & -B^1 \\ -E^1 & B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{bmatrix},$$

$$F^{\mu'\nu'} = g_0^{\mu'} F^0\nu' + g_1^{\mu'} F^1\nu' + g_2^{\mu'} F^2\nu' + g_3^{\mu'} F^3\nu'$$

2.2.1. Electromagnetic Field in Matrix Form

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \\ . \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & B^1 & -B^1 \\ -E^1 & B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'0'} & F^{0'1'} & F^{0'2'} & F^{0'3'} \\ F^{1'0'} & F^{1'1'} & F^{1'2'} & F^{1'3'} \\ F^{2'0'} & F^{2'1'} & F^{2'2'} & F^{2'3'} \\ F^{3'0'} & F^{3'1'} & F^{3'2'} & F^{3'3'} \end{bmatrix} = \begin{bmatrix} E^1 + E^2 + E^3 & [E^1 + (B^3 - B^2)] & [E^2 + (B^1 - B^3)] & [E^3 + (B^2 - B^1)] \\ E^1 & -E^1 & -E^1 - B^3 & -E^3 + B^2 \\ E^2 & -E^1 + B^3 & -E^2 & -E^3 - B^1 \\ E^3 & -E^1 - B^2 & -E^2 + B^1 & -E^3 \end{bmatrix}$$

EMF is transformed into a new symmetry which is neither symmetric nor anti-symmetric. Its diagonal consists set of singularities of EMF. The temporal part of singularity tells us about the origin of electric field and spatial singularities also represent electric field but with opposite sign that shows the balance in nature.

$$F^{0'0'} = E$$

This equation tells us about the origin of electric field. It is resulted due to the action of noninertial metric on EMF. The metric field has excited the temporal singularity of EMF.

$$F^{i'i'} = -E$$

$$F^{\mu'\mu'} = E - E$$

Here electric field is the result of transformation of spatial singularities but with opposite sign such that its trace is zero. In this way, EMF is balanced by action and reaction of kinematical electric forces.

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} E^1 + E^2 + E^3 & [E^1 + (B^3 - B^2)] & [E^2 + (B^1 - B^3)] & [E^3 + (B^2 - B^1)] \\ E^1 & -E^1 & -E^1 - B^3 & -E^3 + B^2 \\ E^2 & -E^1 + B^3 & -E^2 & -E^3 - B^1 \\ E^3 & -E^1 - B^2 & -E^2 + B^1 & -E^3 \end{bmatrix}$$

Spatial part of the metric field turns the electric field in to a mixture of electric and magnetic field. Similarly, metric field organizes the electric and magnetic field components in the spatial region of EMF. Its organization is so intelligent that fit naturally in to the subsequent framework of Maxwell's equations that reduces our mathematical labor. The above set up is actually infra-structure for Maxwell's equations.

On simplification, we observe that electric field has transformed into 4-dimensional form. Similarly, magnetic field.

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} E + [E^1 + (B^1 - B^2)] + [E^2 + (B^1 - B^3)] + [E^3 + (B^2 - B^1)] \\ -E - [(B^3 - B^2) - E^1] \\ -E - [(B^1 - B^3) - E^2] \\ -E - [(B^2 - B^1) - E^2] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \end{bmatrix} = \begin{bmatrix} E + [E^1 + (B^3 - B^2)] + [E^2 + (B^1 - B^3)] + [E^3 + (B^2 - B^1)] \\ -3E - [E^1 + (B^3 - B^2)] + [E^2 + (B^1 - B^3)] + [E^3 + (B^2 - B^1)] \end{bmatrix}_{\text{i}}$$

2.2.2. Electric Field in 4-dimensional Form

$$F^{0'\nu'} = E + [E^1 + (B^3 - B^2)] + [E^2 + (B^1 - B^3)] + [E^3 + (B^2 - B^1)]$$

It appears in its complete form as a combination of electric and magnetic field.

2.2.3. Magnetic Field

$$F^{1'\nu'} = -3E - [E^1 + (B^3 - B^2)] + [E^2 + (B^1 - B^3)] + [E^3 + (B^2 - B^1)]$$

Electromagnetic field is in its new symmetry becomes twice of electric field with minus sign

$$F^{\mu'\nu'} = 2E - 2E = 0$$

2.3. Transformation of Maxwell's Equations

It is to be noted that Maxwell's equations are expanded by using Einstein's summation convention and then all the components are written in matrix form

$$F^{\mu'\nu'}{}_{,\nu'} = g^{\mu'}_{\alpha} F^{\mu'\nu'}{}_{,\nu'}$$

$$F^{\mu'\nu'}{}_{,\nu'} = g^{\mu'}_{0} F^{0'}\nu'{}_{,\nu'} + g^{\mu'}_{1} F^{1'}\nu'{}_{,\nu'} + g^{\mu'}_{2} F^{2'}\nu'{}_{,\nu'} + g^{\mu'}_{3} F^{3'}\nu'{}_{,\nu'}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \\ F^{2'\nu'}{}_{,\nu'} \\ F^{3'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} E_{,0} & E^1{}_{,1} - (\nabla \times B)^1 & E^2{}_{,2} - (\nabla \times B)^2 & E^3{}_{,3} - (\nabla \times B)^3 \\ E^1{}_{,0} & -E^1{}_{,1} & -E^2{}_{,2} - B^2{}_{,2} & -E^3{}_{,3} + B^2{}_{,3} \\ E^2{}_{,0} & -E^1{}_{,1} + B^3{}_{,1} & -E^2{}_{,2} & -E^3{}_{,3} - B^1{}_{,3} \\ E^3{}_{,0} & -E^1{}_{,1} - B^2{}_{,1} & -E^2{}_{,2} + B^1{}_{,2} & E^3{}_{,3} \end{bmatrix}$$

Singularities of Maxwell's equations

The temporal singularity of Gauss's law is transformed into time varying electric field that appears in Ampere's law in usual electrodynamics. It was the term that was introduced by Maxwell in Ampere's law to establish the symmetry of Maxwell's equations. Time varying electric field is electrical jerk.

Origin of time varying electric field

$$F^{0'0'}{}_{,0'} = \mathbf{E}_{,0} = \frac{\partial \mathbf{E}}{\partial \mathrm{t}}$$

Dual Field temporal Singularity

$$*F^{0'0'}{}_{,0'} = \mathbf{B}_{,0} = \frac{\partial \mathbf{B}}{\partial \mathrm{t}}$$

Origin of Gauss's law

$$F^{\mathrm{i}',\mathrm{i}'}{}_{,\mathrm{i}'} = -\nabla.\mathbf{E}$$

Origin of Gauss's law for magnetism

$$*F^{\mathrm{i}',\mathrm{i}'}{}_{,\mathrm{i}'} = -\nabla.\mathbf{B}$$

$$F^{\mu'\mu'}{}_{,\mu'} = \mathbf{E}_{,0} - [\nabla.\mathbf{E}]$$

Dual Field Singularity

$$*F^{\mu'\mu'}{}_{,\mu'} = \mathbf{B}_{,0} - [\nabla.\mathbf{B}]$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \\ F^{2'\nu'}{}_{,\nu'} \\ F^{3'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} \nabla.E - [(\nabla\times B)^1 - E^1{}_{,0}] - [(\nabla\times B)^2 - E^2{}_{,0}] - [(\nabla\times B)^3 - E^3{}_{,0}] \\ -[(\nabla\times B)^1 - E^1{}_{,0}] \\ -[(\nabla\times B)^2 - E^2{}_{,0}] \\ -[(\nabla\times B)^3 - E^3{}_{,0}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} \nabla.E - [(\nabla\times B)^1 - E_{,0}] \\ -3\nabla.E - [(\nabla\times B)^2 - E_{,0}] \end{bmatrix}$$

Gauss's Law

$$F^{0'\nu'}{}_{,\nu'} = \nabla.E - [(\nabla\times B)^1 - E_{,0}]$$

Duel or Gauss's Law for Magnetism

$$*F^{0'\nu'}{}_{,\nu'} = \nabla.E - [(\nabla\times E)^1 - B_{,0}]$$

It is very strange that transformation of Gauss's law appears as a combination of Gauss's law and Ampere's law. It means that Ampere's law is relativistic consequence of Gauss's law as magnetism is the relativistic consequence of motion of electric charge.

Ampere's Law

$$F^{0'\nu'}{}_{,\nu'} = \nabla.E - [(\nabla\times B) - E_{,0}]$$

Dual of Ampere's Law or Faraday's Law

$$*F^{0'\nu'}{}_{,\nu'} = \nabla.E - [(\nabla\times E) - B_{,0}]$$

Ampere's law also appears as the combination of gauss's law and Ampere's law but here Gauss's law is present three time. One time as a spatial singularity with minus sign. Similarly, dual of Gauss's law and Ampere's law as a combination.

Combined Electromagnetic Law

$$F^{\mu'\nu'}{}_{,\nu'} = -2[\nabla.E] - [(\nabla \times B) - E_{,0}]$$

Dual Maxwell's equations for magnetism

$$*F^{\mu'\nu'}{}_{,\nu'} = -2\left[\nabla.\mathbf{B}\right] + 2\left[\left(\nabla \times \mathbf{E}\right) + \mathbf{B}_{,0}\right]$$

Electromagnetic laws become double. It means that noninertial metric is responsible for scaling of laws. It also implies that pair production is happening in vacuum due to the action of noninertial metric.

$$F^{\mu'\nu'}{}_{,\nu'} = -2F^{\mu'\nu'}{}_{,\nu'}$$

Dual of Maxwell's equations in tensor form

$$*F^{\mu'\nu'}{}_{,\nu'} = -2 * F^{\mu'\nu'}{}_{,\nu'}$$

2.4. Conservation Law by Matrix Method

Here electromagnetic conservation law is obtained by direct multiplication of noninertial metric with matrix of conservation law in usual form

$$\begin{bmatrix} F^{0'\nu'}{}_{\nu'0'} \\ F^{1'\nu'}{}_{\nu'1'} \\ F^{2'\nu'}{}_{\nu'2'} \\ F^{3'\nu'}{}_{\nu'3'} \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & E^1_{10} & E^2_{20} & E^3_{30} \\ -E^1_{01} & 0 & E^3_{21} & -E^2_{31} \\ -E^2_{02} & E^3_{12} & 0 & E^1_{32} \\ -E^3_{03} & E^2_{13} & -E^1_{23} & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} E^1_{01} + E^2_{02} + E^3_{03} & E^1_{10} + [B^3_{12} - B^2_{13}] & E^2_{20} + [B^1_{23} - B^3_{21}] & E^3_{30} + [B^2_{30} - B^1_{32}] \\ E^1_{01} & -E^1_{01} & -E^2_{02} - B^3_{21} & -E^3_{03} - B^2_{31} \\ E^2_{02} & -E^1_{01} + B^3_{12} & -E^2_{02} & -E^3_{03} - B^1_{32} \\ E^3_{03} & -E^1_{01} + B^2_{13} & -E^2_{02} + B^1_{23} & -E^3 \end{bmatrix}$$

We notice here the terms appearing along the diagonal do not constitute 4D wave. They appear as time varying Gauss's law with opposite sign and cancel out.

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} E^1_{01} + E^2_{02} + E^3_{03} & -[(B^3_{21} - B^2_{31})] - E^1_{01}] & [(B^3_{21} - B^2_{31}) - E^1_{01}] & -[(B^1_{32} - [B^2_{31}) - E^1_{01}] \\ E^1_{01} & -E^1_{01} & -E^2_{02} - B^3_{21} & -E^3_{03} - B^2_{31} \\ E^2_{02} & -E^1_{01} + B^3_{12} & -E^2_{02} & -E^3_{03} - B^1_{32} \\ E^3_{03} & -E^1_{01} - B^2_{13} & -E^2_{02} + B^1_{23} & E^3_{03} \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} \nabla.E_0 - \nabla.[(\nabla\times B) - E_0] \\ -\nabla.E_0 - [(\nabla\times B)_1^1 - E_{01}^1] \\ -\nabla.E_0 - [(\nabla\times B)_2^2 - E_{02}^2] \\ -\nabla.E_0 - [(\nabla\times B)_3^3 - E_{03}^3] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} \nabla.E_0 - \nabla.[(\nabla\times B) - E_0] \\ -3\nabla.E_0 - (\nabla\times B)_1^1 - E_0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} 2\nabla.E_0 \\ -2\nabla.E_0 \end{bmatrix}$$

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = 2\nabla.E_0 - 2\nabla.E_0 = 0$$

Dual Conservation Law for Magnetism

$$*F^{\mu'\nu'}{}_{,\nu'\mu'} = 2\nabla.B_0 - 2\nabla.B_0 = 0$$

In terms of sources, conservation law gives the same results as above

$$\begin{bmatrix} J^{0'}{}_{,0'} \\ J^{i'}{}_{,i'} \end{bmatrix} = \begin{bmatrix} J^0{}_{,0} - \nabla.J \\ -3J^0{}_{,0} - \nabla.J \end{bmatrix}$$

$$J^{\mu'}{}_{,\mu'} = -2J^0{}_{,0} - 2\nabla.J = -2[J^0{}_{,0} + \nabla.J]$$

$$J^{\mu'}{}_{,\mu'} = -2[J^{\mu}{}_{,\mu}]$$

It means, conservation law is doubled but it vanishes due to $J^{\mu}{}_{,\mu} = 0$ Therefore $J^{\mu'}{}_{,\mu'} = 0$

2.5. Transformation of Conservation Law by Einstein Summation Convention Method

All the components of electromagnetic conservation law are calculated by expanding the following expression using Einstein summation convention and then written in matrix form for simplicity

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = g_{\alpha}^{\mu'} F^{\mu'\nu'}{}_{,\nu'\mu'}$$

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = g_0^{\mu'} F^{0}\nu'{}_{,\nu'\mu'} + g_1^{\mu'} F^{1}\nu'{}_{,\nu'\mu'} + g_2^{\mu'} F^{2}\nu'{}_{,\nu'\mu'} + g_3^{\mu'} F^{3}\nu'{}_{,\nu'\mu'}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu''} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu''} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} E_{,00} & -[(\nabla\times B)_0^1 - E^1{}_{,01}] & -[(\nabla\times B)_0^1 - E^2{}_{,02}] & -[(\nabla\times B)_0^3 - E^1{}_{,03}] \\ E^1{}_{,0} & -E^1{}_{,1} & -E^2{}_{,20} - B^2{}_{,21} & -E^3{}_{,30} + B^2{}_{,31} \\ E^2{}_{,0} & -E^1{}_{,10} + B^{12}{}_{,1} & -E^2{}_{,22} & -E^3{}_{,30} - B^1{}_{,2} \\ E^3{}_{,0} & -E^1{}_{,10} - B^2{}_{,13} & -E^2{}_{,20} + B^1{}_{,23} & E^3{}_{,33} \end{bmatrix}$$

The terms appearing along the diagonal of matrix represent 4D electromagnetic wave. It is possible only by using Einstein summation convention method. This is the most striking result of this model.

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} E_{,00} - \nabla.E_0 - [(\nabla\times B) - E_0] + \nabla.E_0 \\ -E^1{}_{,11} - E^2{}_{,20} - E^3{}_{,30} + E^1{}_{,01}[(\nabla\times B)_1^1] \\ -E^2{}_{,22} - E^1{}_{,10} - E^3{}_{,30} + E^2{}_{,02}[(\nabla\times B)_2^2] \\ -E^3{}_{,3} - E^1{}_{,10} - E^2{}_{,20} + E^3{}_{,03}[(\nabla\times B)_3^3] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \end{bmatrix} = \begin{bmatrix} E_{,00} - \nabla.E_0 - [(\nabla\times B) - E_0] + \nabla.E_0 \\ -\nabla^2 E - 2\nabla^2 E_{,0} + \nabla E_{,0} \end{bmatrix}$$

From Faraday's law

$$-[(\nabla\times B)]_{,0} = -\nabla^2 E$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_{,00} - \nabla^2\mathbf{E} + \nabla.\mathbf{E}_{,0} \\ -\nabla^2\mathbf{E}\nabla.\mathbf{E}_{,0} \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \end{bmatrix} = \begin{bmatrix} {}^2(\mathbf{E}) + \nabla.\mathbf{E}_{,0} \\ -\nabla^2\mathbf{E}\nabla.\mathbf{E}_{,0} \end{bmatrix}$$

Conservation of Gauss's law

$$F^{0'\nu'}{}_{,\nu'0'} = {}^2(\mathbf{E}) + \nabla.\mathbf{E}_{,0}$$

Conservation of Ampere's law

$$F^{i'\nu'}{}_{,\nu'i'} = \nabla^2(E) - \nabla.E_0$$

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = \Box^2(E) - \nabla^2 E$$

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2]E$$

This new wave operator is the inner product of $\Box, -\nabla$ and $\Box\,\nabla$ can be termed as d'Ambert Laplace wave operator

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(E)$$

The above discovery of 4D EM wave is the dream of this paper. Its source $F^{\mu'\mu'}{}_{,\mu'\mu'}$ is miraculously very strange.

In terms of potentials $A^\mu = \phi, A$

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = -\nabla^2 A^i{}_{,0} + \Box^2 A^\mu{}_{,\mu}$$

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2 A^\mu{}_{,\mu}.$$

In fact, above wave equation represents wave of Lorentz Gauge so the conservation law is gauge invariant, a well-known concept in relativistic electrodynamics.

Results of Model 1:

1. Physical significance Singularities of EMF

In this section, in the transformation of EMF, it can be seen from equations that physical quantities are created in pairs having equal magnitude but with opposite sign. The singularities of EMF are appearing as a consequence of new symmetry of EMF. Electric and magnetic field emerge as the combination of electric and magnetic field. It is clear that origin of electric field is singular. It is pair production process. Action and reaction idea of Newtonian mechanics holds good.

2. Complete Symmetrization of Maxwell's Equations

Transformation of Maxwell's equations are also doubled

Gauss's Law

$$F^{0'\nu'}{}_{,\nu'} = \nabla.E - [(\nabla \times B) - E_{,0}]$$

very strange that transformation of Gauss's law appears as a combination of Gauss's law and Ampere's law. It is also believed that Ampere's law is relativistic consequence of Gauss's law as magnetism is the relativistic consequence of motion of electric charge.

Duel or Gauss's Law for Magnetism

$$*F^{0'\nu'}{}_{,\nu'} = \nabla.B - [(\nabla \times E) - B_{,0}]$$

Similarly, Gauss's for magnetism and Faraday's law appeared together

Ampere's Law

$$F^{i'\nu'}{}_{,\nu'} = -3\nabla.E - [(\nabla \times B) - E_{,0}]$$

Dual of Ampere's Law or Faraday's Law

$$*F^{i'\nu'}{}_{,\nu'} = -3\nabla.B - [(\nabla \times E) - B_{,0}]$$

The pairs of equations represent the complete symmetrisation of Maxwell's equations

3. Emergence of New Conservation Laws

(a) Conservation law by matrix method holds as usual

(b) Conservation law by Einstein's summation convention method consists of superposition of Laplacian wave and 4D EM wave

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2]E$$

We have got a new wave operator $[\Box^2 - \nabla^2]$, a strange consequence of this model

4. New origin of 4D EM wave

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(E)$$

3. Electrodynamics in Noninertial Velocity Metric

Velocity metric

$$g_0^0 = g_1^1 = g_2^2 = g_3^3 = 1, g_i^0 = g_0^i = -v_i$$

$$\left[g^{\mu}{}_{,\nu}\right] = \begin{bmatrix} 1 & -v_1 & -v_2 & -v_3 \\ -v_1 & 1 & 0 & 0 \\ -v_2 & 0 & 1 & 0 \\ -v_3 & 0 & 0 & 1 \end{bmatrix}$$

3.1. Transformation of EMF

$$F^{\mu'\nu'} = g_{\alpha}^{\mu'} F^{\alpha}{}_{\nu'}$$

A special case of the noninertial metric [11] is found more suitable to switch from inertial to noninertial frame to develop electrodynamics in noninertial frame where spacetime is flat.

$$\begin{bmatrix} F^{0'0'} & F^{0'1'} & F^{0'2'} & F^{0'3'} \\ F^{1'0'} & F^{1'1'} & F^{1'2'} & F^{1'3'} \\ F^{2'0'} & F^{2'1'} & F^{2'2'} & F^{2'3'} \\ F^{3'0'} & F^{3'1'} & F^{3'2'} & F^{3'3'} \end{bmatrix} = \begin{bmatrix} 1 & -v_1 & -v_2 & -v_3 \\ -v_1 & 1 & 0 & 0 \\ -v_2 & 0 & 1 & 0 \\ -v_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & B^3 & -B^2 \\ -E^2 & -B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} V.E & [E^1 + (v_2 B^3 - v_3 B^2)] & [E^2 + (v_3 B^1 - v_1 B^3)] & [E^3 + (v_1 E^1 - v_2 B^1)] \\ -E^1 & -v_1 E^1 & -v_1 E^2 + B^3 & -v_1 E^3 - B^2 \\ -E^2 & -v_2 E^1 - B^3 & -v_2 E^2 & -v_2 E^3 + B^1 \\ -E^3 & -v_3 E^1 + B^2 & -v_3 E^2 - B^1 & -v_3 E^3 \end{bmatrix}$$

Electromagnetic field is transformed in to 16 components. 4 new symmetry terms are obtained along the diagonal. Origin of electric power

$$F^{0'0'} = v.E$$

$$F^{i'i'} = -v.E$$

$$F^{\mu'\mu'} = v.E - v.E = 0$$

These new terms have physical significance as electrical power or hidden power. Temporal term and spatial terms have opposite sign so they balance the field.

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} V.E + [E^1 + (v \times B)^1] + [E^2 + (v \times B)^2] + [E^3 + (v \times B)^3] \\ -v_1 E + [(B^3 - B^2) - E^1] \\ -v_2 E + [(B^1 - B^3) - E^2] \\ -v_3 E + [(B^2 - B^1) - E^3] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} v.E + [E + (v \times B)] \\ -v_1 E^1 - v_1 E^2 - v_1 E^3 + [(B^3 - B^2) - E^1] \\ -v_2 E^1 - v_2 E^2 - v_1 E^3 + [(B^1 - B^3) - E^2] \\ -v_3 E^1 - v_3 E^2 - v_3 E^3 + [(B^2 - B^1) - E^3] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} v.E + [E + (v \times B)] \\ -v_1 E + [(B^3 - B^2) - E^1] \\ -v_2 E + [(B^1 - B^3) - E^2] \\ -v_3 E + [(B^2 - B^1) - E^3] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{i'\nu'} \end{bmatrix} = \begin{bmatrix} v.E + [E + (v \times B)] \\ -v_1 E + [(B^3 - B^2) - E^1] + [(B^1 - B^3) - E^2] + [(B^2 - B^1) - E^3] \end{bmatrix}$$

4-D Electric field

It is very important result that shows the origin of 4D Lorentz force as the consequence of transformation of 4D electric field in noninertial metric.

$$F^{0'\nu'} = v.E + [E + (v \times B)]$$

Here, Power and Lorentz force are unified

Magnetic Field

$$F^{i'\nu'} = -E - v.E$$

Electromagnetic Field

$$F^{\mu'\nu'} = (v \times B)$$

Dual of EMF

$$*F^{\mu'\nu'} = -(v \times E)$$

Electromagnetic field is transformed into magnetic force that has no counter example in contemporary literature.

3.2. Transformation of Maxwell's Equations

$$F^{\mu'\nu'}{}_{,\nu'} = g^{\mu'}_{\alpha} F^{\mu'\nu'}{}_{,\nu'}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \\ F^{2'\nu'}{}_{,\nu'} \\ F^{3'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} 1 & -v_1 & -v_2 & -v_3 \\ -v_1 & 1 & 0 & 0 \\ -v_2 & 0 & 1 & 0 \\ -v_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E^1{}_{,1} & E^2{}_{,2} & E^3{}_{,3} \\ -E^1{}_{,0} & 0 & B^3{}_{,2} & -B^2{}_{,3} \\ -E^2{}_{,0} & -B^3{}_{,1} & 0 & B^1{}_{,3} \\ -E^3{}_{,0} & B^2{}_{,1} & -B^1{}_{,1} & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \\ F^{2'\nu'}{}_{,\nu'} \\ F^{3'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} V.E_{,0} & [E^1{}_{,1} + (v_2 B^3{}_{,1} - v_3 B^2{}_{,1})] & [E^2{}_{,2} + (v_3 B^1{}_{,2} - v_1 B^3{}_{,2})] & [E^3{}_{,3} + (v_1 B^2{}_{,3} - v_2 B^1{}_{,3})] \\ -E^1{}_{,0} & -v_1 E^1{}_{,1} & -v_1 E^2{}_{,2} + B^3{}_{,2} & -v_1 E^3{}_{,3} - B^2{}_{,3} \\ -E^2{}_{,0} & -v_2 E^1{}_{,1} - B^3{}_{,1} & -v_2 E^2{}_{,2} & -v_2 E^3{}_{,3} + B^1{}_{,3} \\ -E^3{}_{,0} & -v_3 E^1{}_{,1} + B^2{}_{,1} & -v_3 E^2{}_{,2} - B^1{}_{,2} & -v_3 E^3{}_{,3} \end{bmatrix}$$

Four new symmetry terms also appear the diagonal having very strange physical significance.

$$F^{0'\nu'}{}_{,\nu'} = V.E_{,0}$$

$$F^{i'\nu'}{}_{,i'} = -\nabla[V.E]$$

$$F^{\mu'\mu'}{}_{,\mu'} = V.E_{,0} - \nabla[V.E]$$

Temporal singularity is transformed into time-varying electric power as a scalar quantity and spatial singularity into gradient of electric power or electrical Yank, a vector quantity.

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} v.E - v[(\nabla \times B) - E_{,0}] \\ -\nabla[v.E] + [(\nabla \times B) - E_{,0}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} v.E_{,0} - \nabla[(E + (v \times B) - E_{,0}] \\ -\nabla[v.E] + [(\nabla \times B) - E_{,0}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} v.E_{,0} + \nabla.E - v.[(\nabla \times B)] \\ -\nabla[v.E] + [(\nabla \times B) - E_{,0}] \end{bmatrix}$$

Gauss's Law

$$F^{0'\nu'}{}_{,\nu'} = v.E_{,0} - \nabla[(E + (v \times B)]$$

Gauss's law is transformed into time varying electric power and divergence of Lorentz force
We can write the above result as

$$F^{0'\nu'}{}_{,\nu'} = \nabla.E - v.[(v \times B) - E_{,0}]$$

Ampere's Law

$$F^{i'\nu'}{}_{,\nu'} = -\nabla[v.E] + [(\nabla \times B) - E_{,0}]$$

Ampere's law is transformed into electrical Yank and Ampere's law itself. These equations show that classical electrodynamics is the special case of this model.

3.3. Conservation Law by Matrix Method

$$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}^{\mu'}_{\alpha}\mathbf{F}^{\alpha\nu'}{}_{,\nu'\mu'}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \\ F^{2'\nu'}{}_{,\nu'} \\ F^{3'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} 1 & -v_1 & -v_2 & -v_3 \\ -v_1 & 1 & 0 & 0 \\ -v_2 & 0 & 1 & 0 \\ -v_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E^1{}_{,1} & E^2{}_{,2} & E^3{}_{,3} \\ -E^1{}_{,0} & 0 & B^3{}_{,2} & -B^2{}_{,3} \\ -E^2{}_{,0} & -B^3{}_{,1} & 0 & B^1{}_{,3} \\ -E^3{}_{,0} & B^2{}_{,1} & -B^1{}_{,1} & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \\ F^{2'\nu'}{}_{,\nu'} \\ F^{3'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} V.E_{,0} & [E^1{}_{,1} + (v_2 B^3{}_{,1} - v_3 B^2{}_{,1})] & [E^2{}_{,2} + (v_3 B^1{}_{,2} - v_1 B^3{}_{,2})] & [E^3{}_{,3} + (v_1 B^2{}_{,3} - v_2 B^1{}_{,3})] \\ -E^1{}_{,0} & -v_1 E^1{}_{,1} & -v_1 E^2{}_{,2} + B^3{}_{,2} & -v_1 E^3{}_{,3} - B^2{}_{,3} \\ -E^2{}_{,0} & -v_2 E^1{}_{,1} - B^3{}_{,1} & -v_2 E^2{}_{,2} & -v_2 E^3{}_{,3} + B^1{}_{,3} \\ -E^3{}_{,0} & -v_3 E^1{}_{,1} + B^2{}_{,1} & -v_3 E^2{}_{,2} - B^1{}_{,2} & -v_3 E^3{}_{,3} \end{bmatrix}$$

Hidden Symmetry or new symmetry terms along the diagonal of the above matrix represent EM waves.
By the definition of inhomogeneous wave equation, time varying current density is the source of 4D EM wave in terms of electric field i.e.

$$-J^i{}_{,0} = \Box^2(E)$$

Also, we know $v[\nabla.E_{,0}] = -J^i{}_{,0}$. Therefore $v[\nabla.E_{,0}] = -\Box^2(E)$
Temporal singularity is turned into 4D EM wave

$$F^{0'0'}{}_{,0'0'} = v[\nabla.E_{,0}] = -\Box^2(E)$$

Spatial singularity is also turned into 4D EM wave but with opposite sign

$$F^{i'i'}{}_{,i'i'} = -v[\nabla.E_{,0}] = \Box^2(E)$$

Action and reaction of 4D EM waves

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = -\Box^2(E) + \Box^2(E) = 0$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0} \\ F^{1'\nu'}{}_{,\nu'1} \\ F^{2'\nu'}{}_{,\nu'2} \\ F^{3'\nu'}{}_{,\nu'3} \end{bmatrix} = \begin{bmatrix} v[\nabla.E_{,0}] + \nabla.E_{,0} + v_1(B^2{}_{,31} - B^3{}_{,21}) + v_2(B^3{}_{,12} - B^1{}_{,32}) + v_3(B^1{}_{,23} - B^2{}_{,13}) \\ -v_1[\nabla.E_{,0}] + [(B^3{}_{,21} - B^2{}_{,31}) - E^1{}_{,01}] \\ -v_2[\nabla.E_{,0}] + [(B^1{}_{,32} - B^3{}_{,12}) - E^1{}_{,02}] \\ -v_3[\nabla.E_{,0}] + [(B^2{}_{,13} - B^1{}_{,23}) - E^3{}_{,03}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0} \\ F^{1'\nu'}{}_{,\nu'1} \\ F^{2'\nu'}{}_{,\nu'2} \\ F^{3'\nu'}{}_{,\nu'3} \end{bmatrix} = \begin{bmatrix} v[\nabla.E,_0] + \nabla.E,_0 - v_1(\nabla\times -B)^1,_1 - v_2(\nabla\times -B)^2,_2 - v_3(\nabla\times -B)^3_3 \\ -v_1[\nabla.E,_0] + [(\nabla\times -B)^1,_1 - E^1,_{01}] \\ -v_2[\nabla.E,_0] + [(\nabla\times -B)^2,_2 - E^2,_{02}] \\ -v_3[\nabla.E,_0] + [(\nabla\times -B)^3,_3 - E^3,_{03}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} \nabla.E,_0 - v\nabla.[(\nabla\times B) - E,_0] \\ -v[\nabla.E,_0] + \nabla.[(\nabla\times B) - E,_0] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} \nabla.E,_0 + v[\nabla.E,_0] \\ -v[\nabla.E,_0] + \nabla.[(\nabla\times B) - E,_0] \end{bmatrix}$$

$$-v[\nabla.E,_0] = {}^2(E), \; v[\nabla.E,_0] = -{}^2(E)$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} \nabla.E,_0 - {}^2(E) \\ {}^2(E) + \nabla . [(\nabla\times B) - E,_0] \end{bmatrix}$$

Conservation of Gauss's Law

$$F^{0'\nu'}{}_{,\nu'0'} = \nabla.E,_0 - \Box^2(E)$$

Very strange origin of 4D EM wave

$$F^{0'0'}{}_{,0'0'} = -\Box^2(E)$$

Conservation of Ampere's Law

$$F^{i'\nu'}{}_{,\nu'i'} = \Box^2(E) - \nabla.E,_0$$

Strange Origin of 4D EM wave

$$F^{i'\nu'}{}_{,\nu'i'} = \Box^2(E)$$

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = 0$$

In terms of sources, conservation law implies the same result

$$\begin{bmatrix} J^{0'}{}_{,0'} \\ \mathbf{J}^{i'}{}_{,i'} \end{bmatrix} = \begin{bmatrix} J^0{}_{,0} - \mathbf{v}[\nabla.\mathbf{J}] \\ -\mathbf{v}J^0{}_{,0} + \nabla.\mathbf{J} \end{bmatrix}$$

$$\mathbf{J}^{\mu'}{}_{,\mu'} = J^0{}_{,0} + \nabla.\mathbf{J}] - \mathbf{v}[J^0{}_{,0} + \nabla.\mathbf{J}]$$

$$\mathbf{J}^{\mu'}{}_{,\mu'} = \left[\mathbf{J}^{\mu}{}_{,\mu}\right] - \text{v}\left[\mathbf{J}^{\mu}{}_{,\mu}\right]$$

Conservation law is obtained in two parts. The first part is classical conservation law and the second part appears as an effect of velocity or consequence of inertia. Since $J^{\mu}{}_{,\mu} = 0$ so the conservation law holds as usual.

$$\mathbf{J}^{\mu'}{}_{,\mu'} = 0$$

3.4. Conservation Law in Noninertial Metric (Einstein summation Convention Method

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} V.E,_{00} & E^1,_{10} + (v\times B)^1,_{10} & E^2,_{20} + (v\times B)^2,_{20} & E^3,_{30} + (v\times B)^3,_{30} \\ -E^1,_{01} & -v_1E^1,_{11} & -v_1E^2,_{21} + B^3,_{21} & -v_1E^3,_{31} - B^2,_{31} \\ -E^2,_{02} & -v_2E^2,_{22} - B^3,_{12} & -v_2E^2,_{22} & -v_2E^3,_{32} + B^1,_{32} \\ -E^3,_{03} & -v_3E^1,_{13} + B^2,_{13} & -v_3E^2,_{23} - B^1,_{23} & -v_3E^3,_{33} \end{bmatrix}$$

Diagonal terms constitute 4D EM wave in terms of electric power

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = V.E,_{00} - \nabla^2(v.E) = \Box^2(v.E)$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0} \\ F^{1'\nu'}{}_{,\nu'1} \\ F^{2'\nu'}{}_{,\nu'2} \\ F^{3'\nu'}{}_{,\nu'3} \end{bmatrix} = \begin{bmatrix} v.E_{00} + \nabla.[E + (v\times B)],_0 \\ -v_1[\nabla.E,_1] + [(\nabla\times -B)^1,_1 - E^1,_{01}] \\ -v_2[\nabla.E,_2] + [(\nabla\times -B)^2,_2 - E^2,_{02}] \\ -v_3[\nabla.E,_3] + [(\nabla\times -B)^3,_3 - E^3,_{03}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0} \\ F^{1'\nu'}{}_{,\nu'1} \\ F^{2'\nu'}{}_{,\nu'2} \\ F^{3'\nu'}{}_{,\nu'3} \end{bmatrix} = \begin{bmatrix} v.E_{00} + \nabla.[E + (v\times B)],_0 \\ -v_1[\nabla.E,_1] + [(\nabla\times -B)^1,_1 - E^1,_{01}] \\ -v_2[\nabla.E,_2] + [(\nabla\times -B)^2,_2 - E^2,_{02}] \\ -v_3[\nabla.E,_3] + [(\nabla\times -B)^3,_3 - E^3,_{03}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0} \\ F^{1'\nu'}{}_{,\nu'1} \\ F^{2'\nu'}{}_{,\nu'2} \\ F^{3'\nu'}{}_{,\nu'3} \end{bmatrix} = \begin{bmatrix} v.E_{00} + \nabla.[E + (v\times B)],_0 \\ -v_1[\nabla.E,_1] + [(\nabla\times -B)^1,_1 - E^1,_{01}] \\ -v_2[\nabla.E,_2] + [(\nabla\times -B)^2,_2 - E^2,_{02}] \\ -v_3[\nabla.E,_3] + [(\nabla\times -B)^3,_3 - E^3,_{03}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0} \\ F^{i'\nu'}{}_{,\nu'i} \end{bmatrix} = \begin{bmatrix} v.E_{00} + \nabla.[E + (v\times B)],_0 \\ -\nabla^2(v.E) + \nabla.[(\nabla\times B) - E_0] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0} \\ F^{i'\nu'}{}_{,\nu'i} \end{bmatrix} = \begin{bmatrix} v.E_{00} + \nabla.[E + (v\times B)],_0 \\ -\nabla^2(v.E) + \nabla.[(\nabla\times B) + E_0] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0} \\ F^{i'\nu'}{}_{,\nu'i} \end{bmatrix} = \begin{bmatrix} v.E_{00} + \nabla.E - v.(\nabla \times B)_{,0} \\ -\nabla^2 (v.E) - \nabla.E_0 \end{bmatrix}$$

By Faraday's law

$$-v.(\nabla \times B)_{,0} = -\nabla^2 (v.E)$$

$$F^{0'\nu'}{}_{,\nu'0'} = \Box^2 (v.E) + \nabla.E_{,0}$$

$$F^{i'\nu'}{}_{,\nu'i} = -\nabla^2 (v.E) - \nabla.E_0$$

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = \Box^2 (v.E) - \nabla^2 (v.E)$$

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2]((v.E))$$

Conservation law consists of Laplacian wave and 4D wave of electric power. These terms seem to represent expansion of the universe

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = [\Box^2 ((v.E))$$

Results of Model-2

1. Temporal and Spatial singularities show the Action and Reaction of Electric Power balance

$$F^{\mu'\nu'} = v.E - v.E = 0$$

2. Origin of 4D Lorentz force $F^{0'\nu'} = v.E + [E + (v \times B)]$ The above relation can be written in compact form as

$$F^{0'\nu'} = g^{0'}_{\alpha} F^{\alpha\nu'}, \quad \left\{ g^0_0 = g^1_1 = g^2_2 = g^3_3 = 1, g^3_3 = g^3_3 = -v_i \right\}$$

3. New Form of EMF as a representative of magnetic force

$$F^{\mu'\nu'} = (v \times B)$$

4. Predictions in Electromagnetic Laws. Gauss's law is transformed into time varying electric power and divergence of Lorentz force. Ampere's law is transformed into negative of gradient of electric power and Ampere's law itself

5. New Forms of Conservation law

(a) Conservation law by matrix method holds as usual

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = 0$$

Strange origin of wave

$$F^{0'0'}{}_{,0'0'} = -\Box^2 (E)$$

Strange Origin of 4D EM wave

$$F^{i'i'}{}_{,i'i'} = \Box^2 (E)$$

Combination of waves is action and reaction implies balance

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = 0$$

(b) Conservation law by Einstein summation convention method Predicts new terms

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](v.E)$$

Conservation law consists of superposition of 4D wave of electric power and Laplacian wave of electric power.

6. New Origin of 4D wave of Electric Power

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2 (v.E)$$

4. Electrodynamics in Noninertial Metric (Similarity Transformation Method

Electrodynamics in noninertial metric based on similarity transformation method is done in order to compare the results with the contemporary models that resemble with our results.

The non- zero terms of the noninertial metric are

$$g_0^0 = g_1^1 = g_2^2 = g_3^3 = 1, g_0^1 = g_1^0 = -v_i$$

4.1. Transformation of EMF

$$F^{\mu' \nu'} = g_\alpha^{\mu'} g_\beta^{\nu'} F^{\alpha\beta}$$

Expanding above by Einstein's summation convention method

$$F^{\mu' \nu'} = g_0^{\mu'} g_\beta^{\nu'} F^{0\beta} + g_1^{\mu'} g_\beta^{\nu'} F^{1\beta} + g_2^{\mu'} g_\beta^{\nu'} F^{2\beta} + g_3^{\mu'} g_\beta^{\nu'} F^{3\beta}$$

For simplicity writing all the values of components of electric and magnetic field in matrix form, we have

$$\begin{bmatrix} F^{0' \nu'} \\ F^{1' \nu'} \\ F^{2' \nu'} \\ F^{3' \nu'} \end{bmatrix} = \begin{bmatrix} 0 & v_1[v.E]+[E^1(v\times B)^1] & v_2[v.E]+[E^2(v\times B)^2] & v_3[v.E]+[E^3(v\times B)^3] \\ v_1[v.E]+[E^1(v\times B)^1] & 0 & [B^3(v\times B)^3] & -[B^2(v\times B)^2] \\ v_2[v.E]+[E^2(v\times B)^2] & [B^3(v\times B)^3] & 0 & [B^1(v\times B)^1] \\ v_3[v.E]+[E^2(v\times B)^2] & [B^2(v\times B)^2] & -[B^1(v\times B)^1] & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0' \nu'} \\ F^{1' \nu'} \\ F^{2' \nu'} \\ F^{3' \nu'} \end{bmatrix} = \begin{bmatrix} -v[v.E]+[E+(v\times B)] \\ v_1[v.E]-[E^1+(v\times B)^1]+[B^3-(v\times B)^3]-[B^2-(v\times B)^2] \\ v_2[v.E]-[E^2+(v\times B)^2]-[B^3-(v\times B)^3]+[B^1-(v\times B)^1] \\ v_3[v.E]-[E^2+(v\times B)^2]+[B^2-(v\times B)^2]-[B^1(v\times B)^1] \quad 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0' \nu'} \\ F^{i' \nu'} \end{bmatrix} = \begin{bmatrix} -v[v.E]+[E+(v\times B)] \\ v[v.E]-[E+(v\times B)]+[B-(v\times B)]-[B-(v\times B)] \end{bmatrix}$$

Electric Field

$$F^{0' \nu'} = -v[v.E]+[E+(v\times B)]$$

Electric field is transformed in to Lorentz force and directed power

$$F = q[E+(v\times B)]$$

Directed Power= $v[v.qE]$

Magnetic Field

$$F^{i' \nu'} = v[v.E]-[E+(v\times B)]+[B+(v\times E)]-[B-(v\times E)]$$

$$F^{\mu' \nu'} = -v[v.E]+[E+(v\times B)]+v[v.E]-[B+(v\times E)]+[B-(v\times E)]-[B-(v\times E)]$$

$$F^{\mu' \nu'} = 0$$

All the terms cancel out due to opposite sign so anti-symmetric electromagnetic field remains anti-symmetric

4.2. Transformation of Maxwell's Equations

$$F^{\mu' \nu'}{}_{,\nu'} = g_\alpha^{\mu'} F^{\mu' \nu'}{}_{,\nu'}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \\ F^{2'\nu'}{}_{,\nu'} \\ F^{3'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} -v^2[\nabla.E]+[E+(v\times B)] \\ v_1[v.E]_{,0} - [E^1+(v\times B)^1]_{,0} + [B^3-(v\times B)^3]_{,2} - [B^2-(v\times B)^2]_{,3} \\ v_2[v.E]_{,0} - [E^2+(v\times B)^2]_{,0} - [B^3-(v\times B)^3]_{,1} + [B^1-(v\times B)^1]_{,3} \\ v_3[v.E]_{,0} - [E^2+(v\times B)^2]_{,0} + [B^2-(v\times B)^2]_{,2} - [B^1(v\times B)^1]_{,2} \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{i'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} -v^2[\nabla.E]+[E+(v\times B)] \\ v_1[v.E]_{,0} - [E+(v\times B)^1]_{,0} + \nabla\times[B-(v\times E)]_{,2} \end{bmatrix}$$

Gauss's Law

$$F^{0'i'}{}_{,i'} = -v^2[\nabla.E]+\nabla.[E+(v\times B)]$$

Gauss's law is transformed in to energy density and divergence of Lorentz force

Ampere's Law

$$F^{i'\nu'}{}_{,\nu'} = v_1[v.E]_{,0} - [E+(v\times B)^1]_{,0} + \nabla\times[B-(v\times E)]_{,2}$$

Ampere's law is transformed in to time varying directed power, time varying Lorentz force and curl of dual Lorentz force

4.3. Transformation of Conservation Law

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = g^{\mu'}_{\alpha} g^{\nu'}_{\beta} F^{\alpha\beta}{}_{,\nu'\mu'}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} -v^2[\nabla.E]_{,0} + \nabla.[E+(v\times B)]_{,0} \\ v^2[\nabla.E]_{,0} - \nabla.[E+(v\times B)^1]_{,0} + \nabla.\nabla\times B - \nabla.\nabla\times(v\times E) \end{bmatrix}$$

Conservation of Gauss's Law

$$F^{0'\nu'}{}_{,\nu'0'} = -v^2[\nabla.E]_{,0} + \nabla.[E+(v\times B)]_{,0}$$

Conservation of Ampere's Law

$$F^{i'\nu'}{}_{,\nu'i'} = v^2[\nabla.E]_{,0} - \nabla.[E+(v\times B)^1]_{,0} + \nabla.\nabla\times B - \nabla.\nabla\times(v\times E)$$

$$F^{i'\nu'}{}_{,\nu'i'} = v^2[\nabla.E]_{,0} + \nabla.[E+(v\times B)]_{,0}$$

Conservation law contains time varying energy density, time varying divergence of Lorentz force with minus sign and terms with plus sign. All the terms cancel out due to opposite sign and vector identity property.

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = -v^2[\nabla.E]_{,0} + \nabla.[E+(v\times B)^1]_{,0} + v^2[\nabla.E]_{,0} - \nabla.[E+(v\times B)^1]_{,0}$$

Results of Model-3

1. New Forms of Electric Field, Magnetic Field and EMF

Electric field is transformed into Lorentz force and the term representing directed power both with minus sign as mentioned in equation.

Magnetic field is transformed into Lorentz force and directed power with plus sign and dual Lorentz forces with opposite sign via equation.

Electromagnetic field remains antisymmetric viz. Equation.

EMF is totally turned into home of Lorentz force, Dual Lorentz force supplemented with directed power.

2. New Form of Gauss's law and Ampere's Law.

Gauss's law is transformed in to energy density and divergence of Lorentz force via equation.

Ampere's law is transformed in to time varying directed power, time varying Lorentz force and curl of dual Lorentz force by equation.

3. Conservation law holds as usual.

5. Electrodynamics in Accelerating Metric

Spacetime physics in noninertial frame of reference is very vast field. The above three models are based on numerical vacuum metric and velocity metric. Accelerating, rotating and the combination of accelerating and rotating frames of reference are also very advance and challenging issues in the contemporary world.

Acceleration Metric

$$g_0^0 = g_1^1 = g_2^2 = g_3^3 = 1, g_i^0 = g_0^i = -v_i$$

$$\left[\mathbf{g}_\nu^\mu\right] == \begin{bmatrix} 1 & -a_1 & -a_2 & -a_3 \\ -a_1 & 1 & 0 & 0 \\ -a_2 & 0 & 1 & 0 \\ -a_3 & 0 & 0 & 1 \end{bmatrix}$$

Since the transformation of EMF, ME and EM conservation law in accelerating metric is same as above therefore, results are presented directly. This model incorporates 4D Lorentz force as an integral part of electrodynamics.

5.1. Transformation of EMF

$$F^{\mu'\nu'} = g_\alpha^{\mu'} F^\alpha{}_{\nu'}$$

$$\begin{bmatrix} F^{0'0'} & F^{0'1'} & F^{0'2'} & F^{0'3'} \\ F^{1'0'} & F^{1'1'} & F^{1'2'} & F^{1'3'} \\ F^{2'0'} & F^{2'1'} & F^{2'2'} & F^{2'3'} \\ F^{3'0'} & F^{3'1'} & F^{3'2'} & F^{3'3'} \end{bmatrix} = \begin{bmatrix} 1 & -a_1 & -a_2 & -a_3 \\ -a_1 & 1 & 0 & 0 \\ -a_2 & 0 & 1 & 0 \\ -a_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & B^3 & -B^2 \\ -E^2 & -B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} a.E & [E^1 + (a_2B^3 - a_3B^2)] & [E^2 + (a_3B^1 - a_1B^3)] & [E^3 + (a_1E^1 - a_2B^1)] \\ -E^1 & -a_1E^1 & -a_1E^2 + B^3 & -a_1E^3 - B^2 \\ -E^2 & -a_2E^1 - B^3 & -a_2E^2 & -a_2E^3 + B^1 \\ -E^3 & -a_3E^1 + B^2 & -a_3E^2 - B^1 & -a_3E^3 \end{bmatrix}$$

Electromagnetic field is transformed in to 16 components. 4 new symmetry terms are obtained along the diagonal. Origin of time varying electric power

$$F^{0'0'} = a.E$$

$$F^{i'i'} = -a.E$$

Action and reaction of time-varying electric power

$$F^{\mu'\mu'} = a.E - a.E = 0$$

These new terms have physical significance as time varying electrical power or hidden power. Temporal term and spatial terms have opposite sign so they balance the field.

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} a.E + [E^1 + (v\times B)^1] + [E^2 + (v\times B)^2] + [E^2 + (v\times B)^2] \\ -a_1E + [(B^3 - B^2) - E^1] \\ -a_2E + [(B^1 - B^3) - E^2] \\ -a_3E + [(B^2 - B^1) - E^3] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} a.E + [E + (a \times B)] \\ -a_1 E + [(B^3 - B^2) - E^1] \\ -a_2 E + [(B^1 - B^3) - E^2] \\ -a_3 E + [(B^2 - B^1) - E^3] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{i'\nu'} \end{bmatrix} = \begin{bmatrix} a.E + [E + (a \times B)] \\ -a.E + [(B^3 - B^2) - E^1] + [(B^1 - B^3) - E^2] + [(B^2 - B^1) - E^3] \end{bmatrix}$$

4-D Electric field

4D electric field is transformed into time varying electric power, electric field and time varying Lorentz force

$$F^{0'\nu'} = a.E + [E + (a \times B)]$$

Here, time-varying Power and time-varying magnetic force Lorentz force are unified

Magnetic Field

$$F^{i'\nu'} = -a.E + [(B^3 - B^2) - E^1] + [(B^1 - B^3) - E^2] + [(B^2 - B^1) - E^3]$$

Electromagnetic Field

$$F^{\mu'\nu'} = a \times B$$

Electromagnetic field is transformed into time varying magnetic force that has no counter example in contemporary literature.

5.2. Transformation of Maxwell's Equations

$$F^{\mu'\nu'},_{\nu'} = g^{\mu'}_{\alpha} F^{\mu'\nu'},_{\nu'}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} 1 & -a_1 & -a_2 & -a_3 \\ -a_1 & 1 & 0 & 0 \\ -a_2 & 0 & 1 & 0 \\ -a_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E^1,_1 & E^2,_2 & E^3,_3 \\ -E^1,_0 & 0 & B^3,_2 & -B^2,_3 \\ -E^2,_0 & -B^3,_1 & 0 & B^1,_3 \\ -E^3,_0 & B^2,_1 & -B^1,_2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} a.E,_0 & [E^1,_1 + (a_2 B^3,_1 - a_3 B^2,_1)] & [E^2,_2 + (a_3 B^1,_2 - a_1 B^3,_2)] & [E^3,_3 + (a_1 B^2,_3 - a_2 B^1,_3)] \\ -E^1,_0 & -a_1 E^1,_1 & -a_1 E^2,_2 + B^3,_2 & -a_1 E^3,_3 - B^2,_3 \\ -E^2,_0 & -a_2 E^1,_1 - B^3,_1 & -a_2 E^2,_2 & -a_2 E^3,_3 + B^1,_3 \\ -E^3,_0 & -a_3 E^1,_1 + B^2,_1 & -a_3 E^2,_2 - B^1,_2 & -a_3 E^3,_3 \end{bmatrix}$$

Four new symmetry terms also appear the diagonal having very strange physical significance.

$$F^{0'0'},_{0'} = a.E,_0$$

$$F^{i'i'},_{i'} = -a[\nabla.E]$$

$$F^{\mu'\mu'},_{\mu'} = a.E,_0 - a[\nabla.E]$$

Temporal singularity is transformed into time-varying electric power as a scalar quantity and spatial singularity into current density a vector quantity.

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \\ F^{2'\nu'}{}_{,\nu'} \\ F^{3'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} v.E_{,0} + \nabla.E + \nabla.(a \times B) \\ -v_1[\nabla.E] + [(B^3{}_{,2}) + (B^2{}_{,3}) - E^1{}_{,0}] \\ -v_2[\nabla.E] + [(B^1{}_{,3}) + (B^3{}_{,1}) - E^2{}_{,0}] \\ -v_3[\nabla.E] + [(B^2{}_{,1}) + (B^1{}_{,2}) - E^3{}_{,0}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} \nabla.E - a.[(\nabla \times B) - E_{,0}] \\ -\nabla[a.E] - a.[(\nabla \times B) - E_{,0}] \end{bmatrix}$$

As

$$-\mathbf{a}\left[\nabla.\mathbf{E}\right] = -\mathbf{v}\left[\nabla.\mathbf{E}_{,0}\right] - \left[\left(\nabla \times \mathbf{B}\right) - \mathbf{E}_{,0}\right]_{,0} = \Box^2(E)$$

So

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'} \\ F^{1'\nu'}{}_{,\nu'} \end{bmatrix} = \begin{bmatrix} a.E_{,0} + \nabla.E - a.[(\nabla \times B)] \\ \Box^2(E) - [(\nabla \times B) - E_{,0}] \end{bmatrix}$$

Gauss's Law

$$F^{0'\nu'}{}_{,\nu'} = a.E_{,0} + \nabla.[E + (a \times B)]$$

Gauss's law is transformed into time varying electric power and divergence of electric field and time-varying magnetic force

We can write the above result as

$$F^{0'\nu'}{}_{,\nu'} = \nabla.E - a.\ [(\nabla \times B) - E_{,0}]$$

Ampere's Law

$$F^{i'\nu'}{}_{,\nu'} =^2 (E) + [(\nabla \times B) - E_{,0}]$$

Where spatial singularity contributes as a wave

$$F^{i'i'}{}_{,i'} =^2 (E)$$

Ampere's law is transformed into 4D EM wave and Ampere's law itself. Classical laws of electrodynamics are contained in the model, a basic requirement of any generalized physical theory.

5.3. Conservation Law by Matrix Method

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = g^{\mu'}_{\alpha} F^{\alpha}\nu'{}_{,\nu'\mu'}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} 1 & -a_1 & -a_2 & -a_3 \\ -a_1 & 1 & 0 & 0 \\ -a_2 & 0 & 1 & 0 \\ -a_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E^1{}_{,10} & E^2{}_{,20} & E^3{}_{,30} \\ -E^1{}_{,01} & 0 & B^3{}_{,21} & -B^2{}_{,31} \\ -E^2{}_{,02} & -B^3{}_{,12} & 0 & B^1{}_{,32} \\ -E^3{}_{,03} & B^2{}_{,13} & -B^1{}_{,23} & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} a[\nabla.E_{,0}] & [E^1{}_{,10} + (a_2B^3{}_{,12} - a_3B^2{}_{,13})] & [E^2{}_{,20} + (a_3B^1{}_{,23} - a_1B^3{}_{,21})] & [E^3{}_{,30} + (a_1B^2{}_{,31} - a_2B^1{}_{,32})] \\ -E^1{}_{,01} & -a_1E^1{}_{,10} & -a_1E^{20}{}_{,2} + B^3{}_{,21} & -a_1E^3{}_{,30} - B^2{}_{,31} \\ -E^2{}_{,02} & -a_2E^{10}{}_{,1} - B^3{}_{,12} & -a_2E^2{}_{,20} & -a_2E^3{}_{,30} + B^1{}_{,32} \\ -E^3{}_{,03} & -a_3E^1{}_{,10} + B^2{}_{,13} & -a_3E^{20}{}_{,2} - B^1{}_{,23} & -a_3E^3{}_{,30} \end{bmatrix}$$

Hidden Symmetry or new symmetry terms.

Temporal singularity of conservation law is turned in to 4D EM wave of time varying electric field

$$F^{0'0'}{}_{,0'0'} = \nabla[a.E_{,0}] =^2 (E_{,0})$$

Similarly, spatial singularity also turned in to wave as above but with opposite sign.

$$F^{i'i'}{}_{,i'i'} = -^2(E_{,0})$$

$$F^{\mu'\mu'}{}_{,\mu'\mu'} =^2 (E_{,0}) -^2 (E_{,0}) = 0$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} \Box^2(E_{,0}) + \nabla.(E_{,0}) + a_1(B^2{}_{,31} - B^3{}_{,21}) + a_2(B^3{}_{,12} - B^1{}_{,23}) + a_3(B^1{}_{,23} - B^2{}_{,13}) \\ -v_1[\nabla.E_{,0}] + [(B^3{}_{,21}) + (B^2{}_{,31}) - E^1{}_{,01}] \\ -v_2[\nabla.E_{,0}] + [(B^1{}_{,32}) + (B^3{}_{,12}) - E^2{}_{,02}] \\ -v_3[\nabla.E_{,0}] + [(B^2{}_{,13}) + (B^1{}_{,23}) - E^3{}_{,03}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} \Box^2(E_{,0}) + \nabla.(E_{,0}) - a_1(\nabla\times B)^1{}_{,1} - a_2(\nabla\times B)^2{}_{,2} - a_3(\nabla\times B)^3{}_{,3} \\ -a_1[\nabla.E_{,0}] + [(\nabla\times B)^1{}_{,1} - E^1{}_{,01}] \\ -a_2[\nabla.E_{,0}] + [(\nabla\times B)^2{}_{,2} - E^2{}_{,02}] \\ -a_3[\nabla.E_{,0}] + [(\nabla\times B)^3{}_{,3} - E^1{}_{,03}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} \nabla.\mathbf{E}_{,0} - \mathbf{a}\{\nabla.\left[\left(\nabla\times\mathbf{B}\right) - \mathbf{E}_{,0}\right] \\ -^2\left(\mathbf{E}_{,0}\right) + \nabla.\left[\left(\nabla\times\mathbf{B}\right) - \mathbf{E}_{,0}\right] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} \nabla.E_{,0} +^2 (E_{,0}) \\ -^2(E_{,0}) + \nabla\ .\ [(\nabla\times B) - E_{,0}] \end{bmatrix}$$

$$F_{,}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$$

Conservation law in terms of sources is given as

$$\begin{bmatrix} J^{0'}{}_{,0'} \\ J^{i'}{}_{,i'} \end{bmatrix} = \begin{bmatrix} J^0{}_{,0} - a[\nabla.J] \\ -aJ^0{}_{,0} + \nabla.J \end{bmatrix}$$

$$J^{\mu'}{}_{,\mu'} = [J^0{}_{,0} + \nabla.J] - a[J^0{}_{,0} + \nabla.J]$$

$$J^{\mu'}{}_{,\mu'} = [J^{\mu}{}_{,\mu}\}] - a[J^{\mu}{}_{,\mu}\}]$$

Conservation law is obtained in two parts. The first part is classical conservation law and the second part appears as an effect of acceleration or consequence of inertia. Since $J^{\mu},\mu = 0$ so the conservation law holds as usual.

$J^{\mu'}{}_{,\mu'} = 0$

5.4. Conservation Law in Noninertial Metric (Einstein Summation Convention Method

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} a.E,_{00} & E^1,_{10} + (a\times B)^1,_{10} & E^2,_{20} + (a\times B)^2,_{20} & E^3,_{30} + (a\times B)^3,_{30} \\ -E^1,_{01} & -a_1E^1,_{11} & -a_1E^2,_{21} + B^3,_{21} & -a_1E^3,_{31} - B^2,_{31} \\ -E^2,_{02} & -a_2E^2,_{22} - B^3,_{12} & -a_2E^2,_{22} & -a_2E^3,_{32} + B^1,_{32} \\ -E^3,_{03} & -a_3E^1,_{13} + B^2,_{13} & -a_3E^2,_{23} - B^1,_{23} & -a_3E^3,_{3} \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{1'\nu'}{}_{,\nu'1'} \\ F^{2'\nu'}{}_{,\nu'2'} \\ F^{3'\nu'}{}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} a.E,_{00} + \nabla.[E + (a\times B)],_0 \\ -a_1[\nabla.E,_1] + [(a\times B)^1,_1 - E^1,_{01}] \\ -a_2[\nabla.E,_2] + [(a\times B)^2,_2 - E^2,_{02}] \\ -a_3[\nabla.E,_3] + [(a\times B)^3,_3 - E^3,_{03}] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} a.E,_{00} + \nabla.[E + (a\times B)],_0 \\ -\nabla^2(a.E) + \nabla.[(a\times B) - \nabla.E,_0] \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}{}_{,\nu'0'} \\ F^{i'\nu'}{}_{,\nu'i'} \end{bmatrix} = \begin{bmatrix} a.E,_{00} + \nabla.[E + (a\times B)],_0 \\ -\nabla^2(a.E) - \nabla.E,_0 \end{bmatrix}$$

$F^{0'\nu'}{}_{,\nu'0'} = a.E,_{00} - \nabla.E,_0 - a(\nabla\times B),_0 = a.E,_0 + \Box^2(a.E)$ From Faraday's law

$$-\nabla.E,_0 = -\nabla^2(a.E)$$

$$F^{i'\nu'}{}_{,\nu'i'} = -\Box^2(a.E) - a.E,_0$$

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = -\Box^2(a.E) - \nabla^2(a.E)$$

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](a.E)$$

Conservation law consists of 4D wave minus Laplacian wave of time varying electric power. These terms seem to represent accelerating expansion of the universe.

4D EM wave of time-varying electric power

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = \Box\Box^2(a.E)$$

5.5. Discovery of ULTM for Noninertial

5.5.1. Bridging the Gap Between Special and General Relativity

By choosing $x^0 = c\theta t = c\omega t^2$ and $x^i = (x, y, z)$ have the dimensions of distance so 4-position coordinate x^μ = $(c\omega t^2, x^i)$ is considered such that 4-velocity $V^\mu = (c\omega t, V^i)$ and 4-acceleration becomes $a^\mu = (c\omega, a^i)$. Here $c\omega$ is time component of 4-acceleration. In this way, special relativity and general relativity are unified. Constancy of speed of light is not affected by this choice.

In order to do spacetime physics in noninertial frame, ULTM in 2D and 4D is developed where constant translational acceleration and angular velocity are incorporated as a dimensionless parameter. This approach is a natural way to develop general relativity in the language of special relativity

5.5.2. ULTM for Noninertial Frame in 2D

$$\left[\mathbf{L}^{\mu}_{\nu}\right] = \begin{bmatrix} \dfrac{1}{(1-\frac{a^2}{c^2\omega^2})} & -\dfrac{a^2}{(1-\frac{a^2}{c^2\omega^2})} & 0 & 0 \\ -\dfrac{a^2}{(1-\frac{a^2}{c^2\omega^2})} & \dfrac{1}{(1-\frac{a^2}{c^2\omega^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Det L} = \frac{(1+\frac{a^2}{c^2\omega^2})}{(1-\frac{a^2}{c^2\omega^2})}$$

Its inverse is

$$\left(\mathbf{L}^{\mu}_{\nu}\right)^{-1} = \begin{bmatrix} \dfrac{1}{(1+\frac{a^2}{c^2\omega^2})} & \dfrac{\frac{a^2}{c^{2\omega^2}}}{(1-\frac{a^2}{c^2\omega^2})} & 0 & 0 \\ -\dfrac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & \dfrac{1}{(1+\frac{a^2}{c^2\omega^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Det\left(\mathbf{L}^{\mu}_{\nu}\right)^{-1} = \frac{(1-\frac{a^2}{c^2\omega^2})}{(1+\frac{a^2}{c^2\omega^2})}$$

$$\begin{bmatrix} \frac{1}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & 0 & 0 \\ -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & \frac{1}{(1-\frac{a^2}{c^2\omega^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{(1+\frac{a^2}{c^2\omega^2})} & \frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & 0 & 0 \\ -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & \frac{1}{(1+\frac{a^2}{c^2\omega^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5.5.3. ULTM for Noninertial Frame in 4D

This matrix is nothing but 4 by 4 physical identity matrix for noninertial frame

$$\left[\mathbf{L}^{\mu}_{\nu}\right] = \begin{bmatrix} \frac{1}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} \\ -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & \frac{1}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} \\ -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & \frac{1}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} \\ -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & \frac{1}{(1-\frac{a^2}{c^2\omega^2})} \end{bmatrix}$$

Its inverse again is a 4 by 4 physical identity matrix

$$\left(\mathbf{L}^{\mu}_{\nu}\right)^{-1} = \begin{bmatrix} \frac{1}{(1+\frac{a^2}{c^2\omega^2})} & \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} & \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} & \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} \\ \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} & \frac{1}{(1+\frac{a^2}{c^2\omega^2})} & \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} & \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} \\ \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} & \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} & \frac{1}{(1+\frac{a^2}{c^2\omega^2})} & \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} \\ \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} & \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} & \frac{\frac{a^2}{c^2\omega^2}}{(1+\frac{a^2}{c^2\omega^2})} & \frac{1}{(1+\frac{a^2}{c^2\omega^2})} \end{bmatrix}$$

In this paper, we present these matrices just for the interest of the readers and a separate model is in process for publication due to long calculations. Spacetime laws of physics remain same in their original form after transformation under ULTM based on UTL for 4-vectors and tensors. It is the basic requirement of principle of relativity and symmetry principle.

5.6. Results of Model 4

5.6.1. Electric and Magnetic field in Accelerating metric

4-D Electric field

$$F^{0'\nu'} = E + [a.E(a \times B)]$$

$$F^{0'0'} = a.E$$

Magnetic Field

$$F^{i',v'} = -a.E + [(B^3 - B^2) - E^1] + [(B^1 - B^3) - E^2] + [(B^2 - B^1) - E^3]$$

$$F^{i',i'} = -a.E$$

5.6.2. Electromagnetic Field in Accelerating Metric

$$F^{\mu'\nu'} = (a \times B)$$

5.6.3. Maxwell's Equations in Accelerating Metric

Gauss's Law

$$\mathbf{F}^{0'\nu'}{}_{,\nu'} = \nabla.\mathbf{E} \square a.[(\nabla \times \mathbf{B}) - \mathbf{E}_{,0}\, b$$

Ampere's Law

$$F^{i',v'}{}_{,v'} = ^2 (E) + [(\nabla \times B) - E_{,0}]$$

Gauss's Law for Magnetism

$$*F^{0'\nu'}{}_{,\nu'} = \nabla.B + a.[(\nabla \times E) + B_{,0}]$$

Faraday's Law

$$*F^{i',v'}{}_{,v'} = ^2 (B) - [(\nabla \times E) - B_{,0}]$$

5.6.4. Conservation Law by Matrix Method

4D EM wave of time-varying electric field

$$F^{0'0'}{}_{,0'0'} = \square^2 (E_{,0})$$

4D EM wave of time-varying electric field

$$F^{i'i'}{}_{,i'i'} = -^2 (E_{,0})$$

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = 0$$

5.6.5. Conservation Law by Einstein Convention Method

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = [\square^2 - \nabla^2](a.E)$$

Conservation law consists of 4D wave minus Laplacian wave of time varying electric power. These terms seem to represent a new gauge that will explain accelerating expansion of the universe.

$$F^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(a.E)$$

6. A Comparison of Theory of Electrodynamics, Inertia and Gravitation in Noninertial Metrices

1. Theory of inertial and gravitation possess same mathematical structure as that of electrodynamics but physically different. Such type of efforts being done in the contemporary world viz. Gravitoelectromagnetism (GEM) at a very large scale [21,22,23,24]. Here we study them by establishing mathematical correspondence but physically different. Since the pattern of all these theories is same so only results are presented to avoid repletion of calculations. The detailed version of theory of inertia and gravitation will be presented in a separate paper. The purpose of presenting here to use some results from the theory of inertia and gravitation required in the comparison of 4D Lorentz force of electrodynamics with that of inertia and gravitation. Inertia field tensor $\Omega^{\mu\nu}$ and gravitational field tensor $G^{\mu\nu}$ are taken as antisymmetric tensor just like EMF tensor $F^{\mu\nu}$.

2. Electrodynamics in noninertial metric uncovers very important aspects like the origin of 4D Lorentz force as a consequence of transformation of EMF in this formulation. Similarly, 4D Lorentz force for inertia and gravitation are emerged. Furthermore, wave equations and equation of continuity appear as the part of the theory.

6.1. Generalization of Lorentz Transformation Matrix from 2D to 4D

It is one of the basic requirements of any general physical theory must contain classical theory as special case. Usual Lorentz matrix (LT) is limited up to two dimensions viz. time axis and one space axis. Due to this limitation, we do not get complete form of 4D Lorentz force as for example. One can ask a natural question about the relation between noninertial velocity metric given in Table 2 and LT in 4D in Table 3. It is very interesting thing to note that generalized LT in 4D reduces to noninertial velocity metric by putting $\gamma = 1$. In Table 3, all the results of Table 2 are same Lorentz factor γ. For simplicity, we consider $c = 1$ and $B = v$

Lorentz Transformation in 2D

$$L^{\mu}_{\alpha} = \begin{bmatrix} \gamma & -\gamma B & 0 & 0 \\ -\gamma B & \gamma & 0 & 0 \\ 0 & 0 & 10 & \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Generalization of Lorentz transformation from 2D to 4D

$$L^0{}_{,0} = L^1{}_{,1} = L^2{}_{,2} = L^3{}_{,3} = \gamma,\ L^0{}_{,i} = L^i{}_{,0} = -\gamma v, c = 1$$

$$L^{\mu}_{\alpha} = \begin{bmatrix} \gamma & -\gamma B_1 & -\gamma B_2 & -\gamma B_3 \\ -\gamma B_1 & \gamma & 0 & 0 \\ -\gamma B_2 & 0 & \gamma & 0 \\ -\gamma B_3 & 0 & 0 & \gamma \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma v_1 & -\gamma v_2 & -\gamma v_3 \\ -\gamma v_1 & \gamma & 0 & 0 \\ -\gamma v_2 & 0 & \gamma & \\ -\gamma v_3 & 0 & 0 & \gamma \end{bmatrix}$$

6.2. Universal Transformation Matrices

In the whole literature on physics and mathematics, an identity matrix is that which contain 1 along the principal diagonal and zeroes above and below the diagonal. It is very surprising that there exist identity matrices unlike usual identity matrix in the sense that spacetime laws of physics remain same in their original form after transformation. In spacetime physics, we do need physical identity matrices to keep the spacetime laws as it is. In Table 5, physical identity matrices in 2D and 4d are presented. Another interesting observation after calculations, it is noted that the inverses also behave as a physical identity matrix. These matrices are independent of the nature of frame of reference. In other words, they are valid for inertial as well as for noninertial frame of reference.

Table 1

$$(\mathbf{g}_0^0 = 1,\ \mathbf{g}_1^1 = \mathbf{g}_2^2 = \mathbf{g}_3^3 = -1,\ \ \mathbf{g}_i^0 = \ \mathbf{g}_0^i \ = -1)$$

$$[\mathbf{g}_\nu^\mu] = \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 \end{bmatrix}$$

Electrodynamics	**Theory of Inertia**	**Theory of Gravitation**
$[\mathbf{F}^{\mu\nu}] = \begin{bmatrix} 0 & \mathbf{E}^1 & \mathbf{E}^2 & \mathbf{E}^3 \\ -\mathbf{E}^1 & 0 & \mathbf{B}^3 & -\mathbf{B}^2 \\ -\mathbf{E}^2 & -\mathbf{B}^3 & 0 & \mathbf{B}^1 \\ -\mathbf{E}^3 & \mathbf{B}^2 & -\mathbf{B}^1 & 0 \end{bmatrix}$	$\Omega^{\mu\nu} = \begin{bmatrix} 0 & \mathbf{a}^1 & \mathbf{a}^2 & \mathbf{a}^3 \\ -\mathbf{a}^1 & 0 & \boldsymbol{\omega}^3 & -\boldsymbol{\omega}^2 \\ -\mathbf{a}^2 & -\boldsymbol{\omega}^3 & 0 & \boldsymbol{\omega}^1 \\ -\mathbf{a}^3 & \boldsymbol{\omega}^2 & -\boldsymbol{\omega}^1 & 0 \end{bmatrix}$	$\mathbf{G}^{\mu\nu} = \begin{bmatrix} 0 & \mathbf{g}^1 & \mathbf{g}^2 & \mathbf{g}^3 \\ -\mathbf{g}^1 & 0 & \boldsymbol{\omega}^3 & -\boldsymbol{\omega}^2 \\ -\mathbf{g}^2 & -\boldsymbol{\omega}^3 & 0 & \boldsymbol{\omega}^1 \\ -\mathbf{g}^3 & \boldsymbol{\omega}^2 & -\boldsymbol{\omega}^1 & 0 \end{bmatrix}$
Electromagnetic Field	**Inertia Field**	**Gravitational Field**
$\mathbf{F}^{\mu'\nu'} = \mathbf{g}_\alpha^{\mu'}\mathbf{F}^{\alpha\nu'}$	$\Omega^{\mu'\nu'} = \mathbf{g}_\alpha^{\mu'}\Omega^{\alpha\nu'}$	$G^{\mu'\nu'} = \mathbf{g}_\alpha^{\mu'}G^{\alpha\nu'}$
Origin of Electric Field	Origin of Acceleration Field	Origin of Gravitational Field
$\mathbf{F}^{0'0'} = \mathbf{E}$	$\Omega^{0'0'} = \mathbf{a}$	$\mathbf{G}^{0'0'} = \mathbf{g}$
Origin of Opposite EF	Origin of Opposite Inertial Field	Origin of Opposite GF
$\mathbf{F}^{i'i'} = -\mathbf{E}$	$\Omega^{i'i'} = -\mathbf{a}$	$\mathbf{G}^{i'i'} = -\mathbf{g}$
Origin of Action and reaction	Origin of Action and reaction	Origin of Action and reaction
$\mathbf{F}^{\mu'\mu'} = \mathbf{E}-\mathbf{E}$	$\Omega^{\mu'\mu'} = \mathbf{a}-\mathbf{a}$	$\mathbf{G}^{\mu'\mu'} = \mathbf{g}-\mathbf{g}$
$\mathbf{F}^{\mu'\nu'} = 2\mathbf{E} - 2\mathbf{E} = 0$	$\Omega^{\mu'\nu'} = 2\mathbf{a} - 2\mathbf{a} = 0$	$\mathbf{G}^{\mu'\nu'} = 2\mathbf{g} - 2\mathbf{g} = 0$
Maxwell's Equations	**Maxwell's Equations Inertia**	**Maxwell's Eq. Gravitation**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'} = \mathbf{g}_\alpha^{\mu'}\mathbf{F}^{\alpha\nu'}{}_{,\nu'}$	$\Omega^{\mu'\nu'}{}_{,\nu'} = \mathbf{g}_\alpha^{\mu'}\Omega^{\alpha\nu'}{}_{,\nu'}$	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'} = \mathbf{g}_\alpha^{\mu'}\mathbf{G}^{\alpha\nu'}{}_{,\nu'}$
Origin of Electric Jerk	Origin of Inertial Jerk	Origin of Gravitational Jerk
$\mathbf{F}^{0'0'}{}_{,0'} = \mathbf{E}_{,0}$	$\Omega^{0'0'}{}_{,0'} = \mathbf{a}_{,0}$	$\mathbf{G}^{0'0'}{}_{,0'} = \mathbf{g}_{,0}$
Origin of Gauss's Law	Origin of Gauss's Law	Origin of Gauss's Law
$\mathbf{F}^{i'i'}{}_{,i'} = -\boldsymbol{\nabla}.\mathbf{E}$	$\Omega^{i'i'}{}_{,i'} = -[\boldsymbol{\nabla}.\mathbf{a}]$	$\mathbf{G}^{i'i'}{}_{,i'} = -[\boldsymbol{\nabla}.\mathbf{g}]$
$\mathbf{F}^{\mu'\mu'}{}_{,\mu'} = \mathbf{E}_{,0} - [\boldsymbol{\nabla}.\mathbf{E}]$	$\Omega^{\mu'\mu'}{}_{,\mu'} = \mathbf{a}_{,0} - [\boldsymbol{\nabla}.\mathbf{a}]$	$\mathbf{G}^{\mu'\mu'}{}_{,\mu'} = \mathbf{g}_{,0} - [\boldsymbol{\nabla}.\mathbf{g}]$
Gauss's Law	Gauss's Law for Inertia	Gauss's Law for Gravitation
$\mathbf{F}^{0'\nu'}{}_{,\nu'} = \boldsymbol{\nabla}.\mathbf{E} - [(\boldsymbol{\nabla}\times\mathbf{B}) - \mathbf{E}_{,0}]$	$\Omega^{0'\nu'}{}_{,\nu'} = -[\boldsymbol{\nabla}.\mathbf{a} + (\boldsymbol{\nabla}\times\boldsymbol{\omega}) - \mathbf{a}_{,0}]$	$\mathbf{G}^{0'\nu'}{}_{,\nu'} = -[\boldsymbol{\nabla}.\mathbf{g} + (\boldsymbol{\nabla}\times\boldsymbol{\omega}) - \mathbf{g}_{,0}]$
Ampere's law	Ampere's Law for Inertia	Ampere's Law for Gravitation
$\mathbf{F}^{i'\nu'}{}_{,\nu'} = -3\boldsymbol{\nabla}.\mathbf{E} - [(\boldsymbol{\nabla}\times\mathbf{B}) - \mathbf{E}_{,0}]$	$\Omega^{i'\nu'}{}_{,\nu'} = -3\boldsymbol{\nabla}.\mathbf{a} - [(\boldsymbol{\nabla}\times\boldsymbol{\omega}) - \mathbf{a}_{,0}]$	$\mathbf{G}^{i'\nu'}{}_{,\nu'} = -3\boldsymbol{\nabla}.\mathbf{g} - [(\boldsymbol{\nabla}\times\boldsymbol{\omega}) - \mathbf{g}_{,0}]$
Sum of Gauss and Ampere's Law	Sum of Gauss and Ampere's Law	Sum of Gauss and Ampere's law
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'} = -2[\boldsymbol{\nabla}.\mathbf{E} + [(\boldsymbol{\nabla}\times) - \mathbf{E}_{,0}]$	$\Omega^{\mu'\nu'}{}_{,\nu'} = -2\,\Omega^{\mu\nu}{}_{,\nu}$	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'} = -2\,\mathbf{G}^{\mu\nu}{}_{,\nu}$
Conservation Law	**Conservation Law**	**Conservation Law**
By Matrix Method	**By Matrix Method**	**By Matrix Method**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = 0$	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$
By Einstein Convention Method	**By Einstein Convention Method**	**By Einstein Convention Method**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}_\alpha^{\mu'}\mathbf{F}^{\alpha\nu'}{}_{,\nu'\mu'}$	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}_\alpha^{\mu'}\Omega^{\alpha\nu'}{}_{,\nu'\mu'}$	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}_\alpha^{\mu'}\mathbf{G}^{\alpha\nu'}{}_{,\nu'\mu'}$
4D Wave of EM Field	4D Wave of Inertial Field	4D Wave of Gravitational Field
$\mathbf{F}^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(\mathbf{E})$	$\Omega^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(\mathbf{a})$	$\mathbf{G}^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(\mathbf{g})$
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](\mathbf{E})$	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](\mathbf{a})$	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](\mathbf{g})$

Table 2. Electrodynamics, Inertia and Gravitation in Noninertial Velocity Metric

$$(\mathbf{g}^0_0 = \mathbf{g}^1_1 = \mathbf{g}^2_2 = \mathbf{g}^3_3 = 1,\ \ \mathbf{g}^0_i = \mathbf{g}^i_0 = -\mathbf{v}_i)$$

$$[\mathbf{g}^{\mu}_{\nu}] = \begin{bmatrix} 1 & -\mathbf{v}_1 & -\mathbf{v}_2 & -\mathbf{v}_3 \\ -\mathbf{v}_1 & 1 & 0 & 0 \\ -\mathbf{v}_2 & 0 & 1 & 0 \\ -\mathbf{v}_3 & 0 & 0 & 1 \end{bmatrix}$$

Theory of Electrodynamics	**Theory of Inertia**	**Theory of Gravitation**
Electromagnetic Field	**Inertia Field**	**Gravitational Field**
$\mathbf{F}^{\mu'\nu'} = \mathbf{g}^{\mu'}_{\alpha}\mathbf{F}^{\alpha\nu'}$	$\Omega^{\mu'\nu'} = \mathbf{g}^{\mu'}_{\alpha}\Omega^{\alpha\nu'}$	$\mathbf{G}^{\mu'\nu'} = \mathbf{g}^{\mu'}_{\alpha}\mathbf{G}^{\alpha\nu'}$
Origin of Electric Power	Origin of Inertial Power	Origin of Gravitational Power
$\mathbf{F}^{0'0'} = (\mathbf{v}.\mathbf{E})$ (1)	$\Omega^{0'0'} = (\mathbf{v}.\mathbf{a})$ (1)	$\mathbf{G}^{0'0'} = (\mathbf{v}.\mathbf{g})$ (1)
$\mathbf{F}^{i'i'} = -(\mathbf{v}.\mathbf{E})$	$\Omega^{i'i'} = -(\mathbf{v}.\mathbf{a})$	$\mathbf{G}^{i'i'} = -(\mathbf{v}.\mathbf{g})$
Action and reaction of power	Action and reaction of power	Action and reaction of power
$\mathbf{F}^{\mu'\mu'} = (\mathbf{v}.\mathbf{E}) - (\mathbf{v}.\mathbf{E})$	$\Omega^{\mu'\mu'} = [(\mathbf{v}.\mathbf{a}) - (\mathbf{v}.\mathbf{a})]$	$\mathbf{G}^{\mu'\mu'} = (\mathbf{v}.\mathbf{g}) - (\mathbf{v}.\mathbf{g})$
Electromagnetic Field	Inertia Field Tensor	Gravitational Field tensor
$\mathbf{F}^{\mu'\nu'} = (\mathbf{v}\times\mathbf{B})$ (2)	$\Omega^{\mu'\nu'} = (\mathbf{v}\times\boldsymbol{\omega})$ (2)	$\mathbf{G}^{\mu'\nu'} = (\mathbf{v}\times\boldsymbol{\omega})$ (2)
4D Lorentz Force	4D Lorentz Force for Inertia	4D Lorentz Force for Gravitation
$\mathbf{F}^{0'\nu'} = \mathbf{v}.\mathbf{E} + [\mathbf{E} + (\mathbf{v}\times\mathbf{B})]$ (3)	$\Omega^{0'\nu'} = \mathbf{v}.\mathbf{a} + [\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]$ (3)	$\mathbf{G}^{0'\nu'} = \mathbf{v}.\mathbf{g} + [\mathbf{g} + (\mathbf{v}\times\boldsymbol{\omega})]$ (3)
Dual Lorentz force	Dual Lorentz Force	Dual Lorentz Force
$*\,\mathbf{F}^{0'\nu'} = \mathbf{v}.\mathbf{B} + [\mathbf{B} - (\mathbf{v}\times\mathbf{E})]$ (4)	$*\,\Omega^{0'\nu'} = \mathbf{v}.\boldsymbol{\omega} + [\boldsymbol{\omega} - (\mathbf{v}\times\mathbf{a})]$(4)	$*\,\Omega^{0'\nu'} = \mathbf{v}.\boldsymbol{\omega} + [\boldsymbol{\omega} - (\mathbf{v}\times\mathbf{g})]$(4)
$\mathbf{F}^{i'\nu'} = -\mathbf{E} - (\mathbf{v}.\mathbf{E})$	$\Omega^{i'\nu'} = -\mathbf{a} - (\mathbf{v}.\mathbf{a})$	$\mathbf{G}^{i'\nu'} =- \mathbf{g} - (\mathbf{v}.\mathbf{g})$
Maxwell's Equations	**Maxwell's Eq. for Inertia**	**Maxwell's Eq. Gravitation**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'} = \mathbf{g}^{\mu'}_{\alpha}\mathbf{F}^{\alpha\nu'}{}_{,\nu'}$	$\Omega^{\mu'\nu'}{}_{,\nu'} = \mathbf{g}^{\mu'}_{\alpha}\Omega^{\alpha\nu'}{}_{,\nu'}$	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'} = \mathbf{g}^{\mu'}_{\alpha}\mathbf{G}^{\alpha\nu'}{}_{,\nu'}$
Origin of Time varying power	Origin of Time varying power	Origin of Time varying Power
$\mathbf{F}^{0'0'}{}_{,0'} = \mathbf{v}.\mathbf{E}_{,0}$ (5)	$\Omega^{0'0'}{}_{,0'} = \mathbf{v}.\mathbf{a}_{,0}$ (5)	$\mathbf{G}^{0'0'}{}_{,0'} = \mathbf{v}.\mathbf{g}_{,0}$ (5)
Origin of Electric Yank	Origin of Inertial Yank	Origin of Gravitational Yank
$\mathbf{F}^{i'i'}{}_{,i'} = -\nabla(\mathbf{v}.\mathbf{E})$ (6)	$\Omega^{i'i'}{}_{,i'} = -\nabla(\mathbf{v}.\mathbf{a})$ (6)	$\mathbf{G}^{i'i'}{}_{,i'} = -\nabla(\mathbf{v}.\mathbf{g})$ (6)
Origin of Time varying Power-Yank	Origin of Time Varying Power-Yank	Origin of Time Varying Power-Yank
$\mathbf{F}^{\mu'\mu'}{}_{,\mu'} = \mathbf{v}.\mathbf{E}_{,0} - \nabla(\mathbf{v}.\mathbf{E})$ (7)	$\Omega^{\mu'\mu'}{}_{,\mu'} = \mathbf{v}.\mathbf{a}_{,0} - \nabla(\mathbf{v}.\mathbf{a})]$ (7)	$\mathbf{G}^{\mu'\mu'}{}_{,\mu'} = \mathbf{v}.\mathbf{g}_{,0} - \nabla(\mathbf{v}.\mathbf{g})]$ (7)
Gauss's Law	Gauss's Law	Gauss's Law
$\mathbf{F}^{0'\nu'}{}_{,\nu'} = [\mathbf{v}.\mathbf{E}_{,0} + \nabla.[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]$ (8)	$\Omega^{0'\nu'}{}_{,\nu'} = \mathbf{v}.\mathbf{a}_{,0} + \nabla.[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]$ (8)	$\mathbf{G}^{0'\nu'}{}_{,\nu'} = \mathbf{v}.\mathbf{g}_{,0} + \nabla.[\mathbf{g} + (\mathbf{v}\times\boldsymbol{\omega})]$ (8)
Ampere's Law	Ampere's Law for Inertia	Ampere's Law Gravitation
$\mathbf{F}^{i'\nu'}{}_{,\nu'} = [-\nabla(\mathbf{v}.\mathbf{E}) + [(\nabla\times\mathbf{B}) - \mathbf{E}_{,0}]$ (9)	$\Omega^{i'\nu'}{}_{,\nu'} = [-\nabla(\mathbf{v}.\mathbf{a}) + [(\nabla\times\boldsymbol{\omega}) - \mathbf{a}_{,0}]$(9)	$\mathbf{G}^{i'\nu'}{}_{,\nu'} = [-\nabla(\mathbf{v}.\mathbf{g}) + [(\nabla\times\boldsymbol{\omega}) - \mathbf{g}_{,0}]$ (9)
Conservation Law	**Conservation Law**	**Conservation Law**
By Matrix Method	**By Matrix Method**	**By Matrix Method**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$ (10)	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = 0$ (10)	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$ (10)
By Einstein Convention Method	**By Einstein Convention Method**	**By Einstein Convention Method**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}^{\mu'}_{\alpha}\mathbf{F}^{\alpha\nu'}{}_{,\nu'\mu'}$ (11)	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}^{\mu'}_{\alpha}\Omega^{\alpha\nu'}{}_{,\nu'\mu'}$ (11)	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}^{\mu'}_{\alpha}\mathbf{G}^{\alpha\nu'}{}_{,\nu'\mu'}$ (11)
4D Wave of Electrical Power	4D Wave of Inertial Power	4D Wave of Gravitational Power
$\mathbf{F}^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(\mathbf{v}.\mathbf{E})$ (12)	$\Omega^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(\mathbf{v}.\mathbf{a})$ (12)	$\mathbf{G}^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(\mathbf{v}.\mathbf{g})$ (12)
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](\mathbf{v}.\mathbf{E})$ (13)	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](\mathbf{v}.\mathbf{a})$ (13)	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](\mathbf{v}.\mathbf{g})$ (13)

Table 2a. Dual of Electrodynamics, Inertia and Gravitation in Noninertial Velocity Metric

$$(\mathbf{g}_0^0 = \mathbf{g}_1^1 = \mathbf{g}_2^2 = \mathbf{g}_3^3 = 1,\ \ \mathbf{g}_i^0 = \mathbf{g}_0^i = -\mathbf{v}_i)$$

$$[\mathbf{g}_\nu^\mu] = \begin{bmatrix} 1 & -\mathbf{v}_1 & -\mathbf{v}_2 & -\mathbf{v}_3 \\ -\mathbf{v}_1 & 1 & 0 & 0 \\ -\mathbf{v}_2 & 0 & 1 & 0 \\ -\mathbf{v}_3 & 0 & 0 & 1 \end{bmatrix}$$

Dual of Electrodynamics	**Dual Field Theory of Inertia**	**Dual Theory of Gravitation**
$[*\,\mathbf{F}^{\mu\nu}] = \begin{bmatrix} 0 & \mathbf{B}^1 & \mathbf{B}^2 & \mathbf{B}^3 \\ -\mathbf{B}^1 & 0 & -\mathbf{E}^3 & \mathbf{E}^2 \\ -\mathbf{B}^2 & \mathbf{E}^3 & 0 & -\mathbf{E}^1 \\ -\mathbf{B}^3 & -\mathbf{E}^2 & \mathbf{E}^1 & 0 \end{bmatrix}$	$*\,\Omega^{\mu\nu} = \begin{bmatrix} 0 & \boldsymbol{\omega}^1 & \boldsymbol{\omega}^2 & \boldsymbol{\omega}^3 \\ -\boldsymbol{\omega}^1 & 0 & -\mathbf{a}^3 & \mathbf{a}^2 \\ -\boldsymbol{\omega}^2 & \mathbf{a}^3 & 0 & -\mathbf{a}^1 \\ -\boldsymbol{\omega}^3 & -\boldsymbol{\omega}^2 & \mathbf{a}^1 & 0 \end{bmatrix}$	$*\,\mathbf{G}^{\mu\nu} = \begin{bmatrix} 0 & \boldsymbol{\omega}^1 & \boldsymbol{\omega}^2 & \boldsymbol{\omega}^3 \\ -\boldsymbol{\omega}^1 & 0 & -\mathbf{g}^3 & \mathbf{g}^2 \\ -\boldsymbol{\omega}^2 & \mathbf{g}^3 & 0 & -\mathbf{g}^1 \\ -\boldsymbol{\omega}^3 & -\boldsymbol{\omega}^2 & \mathbf{g}^1 & 0 \end{bmatrix}$
Dual Electromagnetic Field	**Dual Inertia Field**	**Dual Gravitational Field**
$*\,\mathbf{F}^{\mu'\nu'} = \mathbf{g}_\alpha^{\mu'} *\,\mathbf{F}^{\alpha\nu'}$	$*\,\Omega^{\mu'\nu'} = \mathbf{g}_\alpha^{\mu'} *\,\Omega^{\alpha\nu'}$	$*\,\mathbf{G}^{\mu'\nu'} = \mathbf{g}_\alpha^{\mu'} *\,\mathbf{G}^{\alpha\nu'}$
Origin of Electric Power	Origin of Centripetal Force	Origin of Centripetal Force
$*\,\mathbf{F}^{0'0'} = (\mathbf{v}.\mathbf{B})$ (1)	$(\Omega^{0'0'} = (\mathbf{v}.\boldsymbol{\omega}) = \mathrm{r}\boldsymbol{\omega}^2$ (1)	$*\ \mathbf{G}^{0'0'} = (\mathrm{r}\boldsymbol{\omega}^2) = \mathrm{r}\boldsymbol{\omega}^2$ (1)
$*\,\mathbf{F}^{i'i'} = -(\mathbf{v}.\mathbf{B})$	$(\Omega^{i'i'} = -(\mathbf{v}.\boldsymbol{\omega}) = -\mathrm{r}\boldsymbol{\omega}^2$	$*\ \mathbf{G}^{i'i'} = -\mathrm{r}\boldsymbol{\omega}^2$
Action and reaction of Force	Action and reaction of Force	Action and reaction of power
$*\,\mathbf{F}^{\mu'\mu'} = (\mathbf{v}.\mathbf{B}) - (\mathbf{v}.\mathbf{B})$	$*\,\Omega^{\mu'\mu'} = [(\mathrm{r}\boldsymbol{\omega}^2) - (\mathrm{r}\boldsymbol{\omega}^2)]$	$*\,\mathbf{G}^{\mu'\mu'} = \mathrm{r}\boldsymbol{\omega}^2 - \mathrm{r}\boldsymbol{\omega}^2$
Electromagnetic Thomas Precession	Inertial Thomas Precession	Gravitational Thomas Precession
$*\,\mathbf{F}^{\mu'\nu'} = \boldsymbol{\omega}_T = (\mathbf{E}\times\mathbf{v})$ (2)	$*\,\Omega^{\mu'\nu'} = \boldsymbol{\omega}_T{}' = (\mathbf{a}\times\mathbf{v})$ (2)	$*\,\mathbf{G}^{\mu'\nu'} = \boldsymbol{\omega}_T{}' = (\mathbf{g}\times\mathbf{v})$ (2)
4D Dual Lorentz force	4D Dual Lorentz force Inertia	4D Dual Lorentz Force Gravity
$*\,\mathbf{F}^{0'\nu'} = \mathbf{v}.\mathbf{B} + [\mathbf{B} + (\mathbf{E}\times\mathbf{v})]$ (3)	$*\,\Omega^{0'\nu'} = \mathrm{r}\boldsymbol{\omega}^2 + [\boldsymbol{\omega} + (\mathbf{a}\times\mathbf{v})]$(3)	$*\,\Omega^{0'\nu'} = \mathrm{r}\boldsymbol{\omega}^2 + [\boldsymbol{\omega} + (\mathbf{g}\times\mathbf{v})]$(3)
$*\,\mathbf{F}^{i'\nu'} = -\mathbf{B} - (\mathbf{v}.\mathbf{B})$ (4)	$\Omega^{i'\nu'} = -\boldsymbol{\omega} - \mathrm{r}\boldsymbol{\omega}^2$ (4)	$*\,\mathbf{G}^{i'\nu'} = -\boldsymbol{\omega} - (\mathbf{v}.\boldsymbol{\omega})$ (4)
Dual Maxwell's Equations	**Dual Maxwell's Eq. for Inertia**	**Dual Maxwell's Eq. Gravitation**
$*\,\mathbf{F}^{\mu'\nu'}{}_{,\nu'} = \mathbf{g}_\alpha^{\mu'} *\,\mathbf{F}^{\alpha\nu'}{}_{,\nu'}$	$*\,\Omega^{\mu'\nu'}{}_{,\nu'} = \mathbf{g}_\alpha^{\mu'} *\,\Omega^{\alpha\nu'}{}_{,\nu'}$	$*\,\mathbf{G}^{\mu'\nu'}{}_{,\nu'} = \mathbf{g}_\alpha^{\mu'} *\,\mathbf{G}^{\alpha\nu'}{}_{,\nu'}$
$*\,\mathbf{F}^{0'0'}{}_{,0'} = \mathbf{v}.\mathbf{B}_{,0}$ (5)	$*\,\Omega^{0'0'}{}_{,0'} = \mathbf{v}.\boldsymbol{\omega}_{,0}$ (5)	$*\,\mathbf{G}^{0'0'}{}_{,0'} = \mathbf{v}.\boldsymbol{\omega}_{,0}$ (5)
$*\,\mathbf{F}^{i'i'}{}_{,i'} = -\nabla(\mathbf{v}.\mathbf{B})$ (6)	$*\,\Omega^{i'i'}{}_{,i'} = -\nabla(\mathrm{r}\boldsymbol{\omega}^2)$ (6)	$*\,\mathbf{G}^{i'i'}{}_{,i'} = -\nabla(\mathrm{r}\boldsymbol{\omega}^2)$ (6)
$*\,\mathbf{F}^{\mu'\mu'}{}_{,\mu'} = \mathbf{v}.\mathbf{B}_{,0} - \nabla(\mathbf{v}.\mathbf{B})$ (7)	$*\,\Omega^{\mu'\mu'}{}_{,\mu'} = \mathbf{v}.\boldsymbol{\omega}_{,0} - \nabla(\mathrm{r}\boldsymbol{\omega}^2)]$ (7)	$*\,\mathbf{G}^{\mu'\mu'}{}_{,\mu'} = \mathbf{v}.\boldsymbol{\omega}_{,0} - \nabla(\mathrm{r}\boldsymbol{\omega}^2)]$ (7)
Gauss's Law for Magnetism	Dual Gauss's Law for Inertia	Dual Gauss's Law
$*\,\mathbf{F}^{0'\nu'}{}_{,\nu'} = \mathbf{v}.\mathbf{B}_{,0} + \nabla.[\mathbf{B} + (\mathbf{E}\times\mathbf{v})]$8)	$*\,\Omega^{0'\nu'}{}_{,\nu'} = \mathbf{v}.\boldsymbol{\omega}_{,0} + \nabla.[\boldsymbol{\omega} + (\mathbf{a}\times\mathbf{v})]$(8)	$*\,\mathbf{G}^{0'\nu'}{}_{,\nu'} = \mathbf{v}.\boldsymbol{\omega}_{,0} + \nabla.[\boldsymbol{\omega} + (\mathbf{g}\times\mathbf{v})]$ 8)
Faraday's Law	Faraday's Law for Inertia	Faraday's Law of Gravitation
$*\,\mathbf{F}^{i'\nu'}{}_{,\nu'} = -\nabla(\mathbf{v}.\mathbf{B}) - [(\nabla\times\mathbf{E}) + \mathbf{B}_{,0}]$ (9)	$*\,\Omega^{i'\nu'}{}_{,\nu'} = -\nabla(\mathrm{r}\boldsymbol{\omega}^2) - [(\nabla\times\mathbf{a})+]$(9)	$*\,\mathbf{G}^{i'\nu'}{}_{,\nu'} = -\nabla(\mathrm{r}\boldsymbol{\omega}^2) - [(\nabla\times\boldsymbol{\omega}) + \boldsymbol{\omega}_{,0}]$9)
Dual Conservation Law **By Matrix Method**	**Dual Conservation Law** **By Matrix Method**	**Conservation Law** **By Matrix Method**
$*\,\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$ (10)	$*\,\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = 0$ (10)	$*\,\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$ (10)
By Einstein Convention Method	**By Einstein Convention Method**	**By Einstein Convention Method**
$*\,\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}_\alpha^{\mu'} *\,\mathbf{F}^{\alpha\nu'}{}_{,\nu'\mu'}$ (11)	$*\,\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}_\alpha^{\mu'} *\,\Omega^{\alpha\nu'}{}_{,\nu'\mu'}$ (11)	$*\,\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}_\alpha^{\mu'}\,\mathbf{G}^{\alpha\nu'}{}_{,\nu'\mu'}$ (11)
4D Wave of Centripetal Force	4D Wave of Centripetal Force	4D Wave of Centripetal Force
$*\ \mathbf{F}^{\mu'\mu'}{}_{,\mu'\mu'} = \square^2(\mathbf{v}.\mathbf{B})$ (12)	$*\,\Omega^{\mu'\mu'}{}_{,\mu'\mu'} = \square^2(\mathrm{r}\boldsymbol{\omega}^2)$ (12)	$*\,\mathbf{G}^{\mu'\mu'}{}_{,\mu'\mu'} = \square^2(\mathrm{r}\boldsymbol{\omega}^2)$ (12)
$*\,\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = [\square^2 - \nabla^2](\mathbf{v}.\mathbf{B})$ (13)	$*\,\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = [\square^2 - \nabla^2](\mathrm{r}\boldsymbol{\omega}^2)$ (13)	$*\,\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = [\square^2 - \nabla^2](\mathrm{r}\boldsymbol{\omega}^2)$ (13)

Table 3. Electrodynamics, Inertia and Gravitation in Generalized Lorentz Matrix (UTL Method)

$$(\mathbf{L}^0_0 = \mathbf{L}^1_1 = \mathbf{L}^2_2 = \mathbf{L}^3_3 = \gamma,\ \ \mathbf{L}^0_i = \mathbf{L}^i_0 = -\gamma\mathbf{v}_i)$$

$$[\mathbf{L}^{\mu}_{\nu}] = \begin{bmatrix} \gamma & -\gamma\mathbf{v}_1 & -\gamma\mathbf{v}_2 & -\gamma\mathbf{v}_3 \\ -\gamma\mathbf{v}_1 & \gamma & 0 & 0 \\ -\gamma\mathbf{v}_2 & 0 & \gamma & 0 \\ -\gamma\mathbf{v}_3 & 0 & 0 & \gamma \end{bmatrix}$$

Electrodynamics	**Theory of Inertia**	**Theory of Gravitation**
Electromagnetic Field	**Inertia Field**	**Gravitational Field**
$\mathbf{F}^{\mu'\nu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{F}^{\alpha\nu'}$	$\Omega^{\mu'\nu'} = \mathbf{L}^{\mu'}_{\alpha}\Omega^{\alpha\nu'}$	$\mathbf{G}^{\mu'\nu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{G}^{\alpha\nu'}$
Origin of Electric Power	Origin of Inertial Power	Origin of Gravitational Power
$\mathbf{F}^{0'0'} = (\gamma\mathbf{v}.\mathbf{E})$ (1)	$\Omega^{0'0'} = \gamma(\mathbf{v}.\mathbf{a})$ (1)	$\mathbf{G}^{0'0'} = \gamma(\mathbf{v}.\mathbf{g})$ (1)
$\mathbf{F}^{i'i'} = -(\gamma\mathbf{v}.\mathbf{E})$	$\Omega^{i'i'} = -\gamma(\mathbf{v}.\mathbf{a})$	$\mathbf{G}^{i'i'} = -\gamma(\mathbf{v}.\mathbf{g})$
Action and reaction of power	Action and reaction of power	Action and reaction of power
$\mathbf{F}^{\mu'\mu'} = (\gamma\mathbf{v}.\mathbf{E}) - (\gamma\mathbf{v}.\mathbf{E})$	$\Omega^{\mu'\mu'} = [\gamma(\mathbf{v}.\mathbf{a}) - \gamma(\mathbf{v}.\mathbf{a})]$	$\mathbf{G}^{\mu'\mu'} = \gamma(\mathbf{v}.\mathbf{g}) - \gamma(\mathbf{v}.\mathbf{g})$
Electromagnetic Field	Inertia Field Tensor	Gravitational Field tensor
$\mathbf{F}^{\mu'\nu'} = \gamma(\mathbf{v}\times\mathbf{B})$ (2)	$\Omega^{\mu'\nu'} = \gamma\,(\mathbf{v}\times\boldsymbol{\omega})$ (2)	$\mathbf{G}^{\mu'\nu'} = \gamma(\mathbf{v}\times\boldsymbol{\omega})$ (2)
Dual of Electromagnetic Field	Dual of Inertia Field Tensor	Dual of Gravitation Field Tensor
$*\,\mathbf{F}^{\mu'\nu'} = \gamma(\mathbf{E}\times\mathbf{v})$ (3)	$*\,\Omega^{\mu'\nu'} = \gamma\,(\mathbf{a}\times\mathbf{v})$ (3)	$*\,\mathbf{G}^{\mu'\nu'} = \gamma(\mathbf{g}\times\mathbf{v})$ (3)
4D Lorentz Force	4D Lorentz Force for Inertia	4D Lorentz Force for Gravitation
$\mathbf{F}^{0'\nu'} = \gamma\mathbf{v}.\mathbf{E} + \gamma[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]$ (4)	$\Omega^{0'\nu'} = \gamma\mathbf{v}.\mathbf{a} + \gamma[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]$ (4)	$\mathbf{G}^{0'\nu'} = \gamma\mathbf{v}.\mathbf{g} + \gamma[\mathbf{g} + (\mathbf{v}\times\boldsymbol{\omega})]$ (4)
Dual of 4D Lorentz Force	4D Dual Lorentz Force of Inertia	Dual of Grav. Lorentz Force
$*\,\mathbf{F}^{0'\nu'} = \gamma\mathbf{v}.\mathbf{B} + \gamma[\mathbf{B} + (\mathbf{E}\times\mathbf{v})]$ (5)	$*\,\Omega^{0'\nu'} = \gamma\mathbf{v}.\boldsymbol{\omega} + \gamma[\boldsymbol{\omega} + (\mathbf{a}\times\mathbf{v})]$ (5)	$*\,\mathbf{G}^{0'\nu'} = \gamma\mathbf{v}.\boldsymbol{\omega} + \gamma[\boldsymbol{\omega} + (\mathbf{g}\times\mathbf{v})]$ (5)
$\mathbf{F}^{i'\nu'} = -\gamma\mathbf{E} - (\gamma\mathbf{v}.\mathbf{E})$ (6)	$\Omega^{i'\nu'} = -\gamma\mathbf{a} - (\gamma\mathbf{v}.\mathbf{a})$ (6)	$\mathbf{G}^{i'\nu'} =- \gamma\mathbf{g} - \gamma(\mathbf{v}.\mathbf{g})$ (6)
Maxwell's Equations	**Maxwell's Eq. for Inertia**	**Maxwell's Eq. Gravitation**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{F}^{\alpha\nu'}{}_{,\nu'}$	$\Omega^{\mu'\nu'}{}_{,\nu'} = \mathbf{L}^{\mu'}_{\alpha}\Omega^{\alpha\nu'}{}_{,\nu'}$	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{G}^{\alpha\nu'}{}_{,\nu'}$
Origin of Time varying power	Origin of Time varying power	Origin of Time varying Power
$\mathbf{F}^{0'0'}{}_{,0'} = \gamma\mathbf{v}.\mathbf{E}_{,0}$ (7)	$\Omega^{0'0'}{}_{,0'} = \gamma\mathbf{v}.\mathbf{a}_{,0}$ (7)	$\mathbf{G}^{0'0'}{}_{,0'} = \gamma\mathbf{v}.\mathbf{g}_{,0}$ (7)
Origin of Electric Yank	Origin of Inertial Yank	Origin of Gravit. Yank
$\mathbf{F}^{i'i'}{}_{,i'} = -\gamma\nabla(\mathbf{v}.\mathbf{E})$ (8)	$\Omega^{i'i'}{}_{,i'} = -\gamma\nabla(\mathbf{v}.\mathbf{a})$ (8)	$\mathbf{G}^{i'i'}{}_{,i'} = -\gamma\nabla(\mathbf{v}.\mathbf{g})$ (8)
Origin of Time varying Power-Yank	Origin of Time Varying Power-Yank	Origin of Time Varying Power-Yank
$\mathbf{F}^{\mu'\mu'}{}_{,\mu'} = \gamma\mathbf{v}.\mathbf{E}_{,0} - \gamma\nabla(\mathbf{v}.\mathbf{E})$ (9)	$\Omega^{\mu'\mu'}{}_{,\mu'} = \gamma\mathbf{v}.\mathbf{a}_{,0} - \gamma\nabla(\mathbf{v}.\mathbf{a})]$ (9)	$\mathbf{G}^{\mu'\mu'}{}_{,\mu'} = \gamma\mathbf{v}.\mathbf{g}_{,0} - \gamma\nabla(\mathbf{v}.\mathbf{g})]$ (9)
Gauss's Law	Gauss's Law	Gauss's Law
$\mathbf{F}^{0'\nu'}{}_{,\nu'} = \gamma\mathbf{v}.\mathbf{E}_{,0} + \gamma\nabla.[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]$ 10)	$\Omega^{0'\nu'}{}_{,\nu'} = \gamma\mathbf{v}.\mathbf{a}_{,0} + \gamma\nabla.[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]$ (10)	$\mathbf{G}^{0'\nu'}{}_{,\nu'} = \gamma\mathbf{v}.\mathbf{g}_{,0} + \gamma\nabla.[\mathbf{g} + (\mathbf{v}\times\boldsymbol{\omega})]$ (10)
Ampere's Law	Ampere's Law for Inertia	Ampere's Law Gravitation
$\mathbf{F}^{i'\nu'}{}_{,\nu'} = -\gamma\nabla(\mathbf{v}.\mathbf{E}) + \gamma[(\nabla\times\mathbf{B}) - \mathbf{E}_{,0}]$ (11)	$\Omega^{i'\nu'}{}_{,\nu'} = -\gamma\nabla(\mathbf{v}.\mathbf{a}) + \gamma[(\nabla\times\boldsymbol{\omega}) - \mathbf{a}_{,0}]$ 11)	$\mathbf{G}^{i'\nu'}{}_{,\nu'} = -\gamma\nabla(\mathbf{v}.\mathbf{g}) + \gamma[(\nabla\times\boldsymbol{\omega}) - \mathbf{g}_{,0}]$ 11)
Conservation Law	**Conservation Law**	**Conservation Law**
By Matrix Method	**By Matrix Method**	**By Matrix Method**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$ (12)	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = 0$ (12)	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$ (12)
By Einstein Convention Method	**By Einstein Convention Method**	**By Einstein Convention Method**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{F}^{\alpha\nu'}{}_{,\nu'\mu'}$ (13)	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{L}^{\mu'}_{\alpha}\Omega^{\alpha\nu'}{}_{,\nu'\mu'}$ (13)	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{G}^{\alpha\nu'}{}_{,\nu'\mu'}$ (13)
4D Wave of Electrical Power	4D Wave of Inertial Power	4D Wave of Gravitational Power
$\mathbf{F}^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(\gamma\mathbf{v}.\mathbf{E})$ (14)	$\Omega^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(\gamma\mathbf{v}.\mathbf{a})$ (14)	$\mathbf{G}^{\mu'\mu'}{}_{,\mu'\mu'} = \Box^2(\gamma\mathbf{v}.\mathbf{g})$ (14)
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](\mathbf{v}.\mathbf{E})$ (15)	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](\gamma\mathbf{v}.\mathbf{a})$ (15)	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](\gamma\mathbf{v}.\mathbf{g})$ (15)

Table 4. Electrodynamics, inertia and gravitation by Similarity transformation in generalized LT

$$(\mathbf{L}^0_0 = \mathbf{L}^1_1 = \mathbf{L}^2_2 = \mathbf{L}^3_3 = \gamma,\ \mathbf{L}^0_i = \mathbf{L}^i_0 = -\gamma\mathbf{v}_i)$$

$$[\mathbf{L}^{\mu}_{\nu}] = \begin{bmatrix} \gamma & -\gamma\mathbf{v}_1 & -\gamma\mathbf{v}_2 & -\gamma\mathbf{v}_3 \\ -\gamma\mathbf{v}_1 & \gamma & 0 & 0 \\ -\gamma\mathbf{v}_2 & 0 & \gamma & 0 \\ -\gamma\mathbf{v}_3 & 0 & 0 & \gamma \end{bmatrix}$$

Theory of Electrodynamics	Theory of Inertia	Theory of Gravitation
Electromagnetic Field	**Inertia Field**	**Gravitational Field**
$\mathbf{F}^{\mu'\nu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{L}^{\nu'}_{\beta}\mathbf{F}^{\alpha\beta}$	$\Omega^{\mu'\nu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{L}^{\nu'}_{\beta}\Omega^{\alpha\nu'}$	$\mathbf{G}^{\mu'\nu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{L}^{\nu'}_{\beta}\mathbf{G}^{\alpha\nu'}$
Electric Field	Acceleration Field	Gravitational Acc. Field
$\mathbf{F}^{0'i'} = \gamma^2\{[\mathbf{E} + (\mathbf{v}\times\mathbf{B})] - \mathbf{v}[\mathbf{v}.\mathbf{E}]\}$	$\Omega^{0'\nu'} = \gamma^2\{[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})] - [\mathbf{v}.\mathbf{a}]\}$**(1)**	$\mathbf{G}^{0'\nu'} = \gamma^2\{[\mathbf{g} + (\mathbf{v}\times\boldsymbol{\omega})] - \mathbf{v}[\mathbf{v}.\mathbf{g}]\}$
Dual Electric Field	**Dual Acceleration field**	**Dual of Gravitation Field**
$*\,\mathbf{F}^{0'i'} = \gamma^2\{[\mathbf{B} + (\mathbf{E}\times\mathbf{v})] - \mathbf{v}[\mathbf{v}.\mathbf{B}]\}$	$*\,\Omega^{0'\nu'} = \gamma^2\{[\boldsymbol{\omega} + (\mathbf{a}\times\mathbf{v})] - \mathbf{v}[\mathbf{v}.\boldsymbol{\omega}]\}$ **(2)**	$*\,\mathbf{G}^{0'\nu'} = \gamma^2\{[\boldsymbol{\omega} + (\mathbf{g}\times\mathbf{v})] - \mathbf{v}[\mathbf{v}.\boldsymbol{\omega}]\}$
Magnetic field	Angular Velocity Field	Angular Velocity Field
$\mathbf{F}^{i\nu'} = \gamma^2\,\mathbf{v}[\mathbf{v}.\mathbf{E}] - \gamma^2[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]$	$\Omega^{i\nu'} = \gamma^2\mathbf{v}[\mathbf{v}.\mathbf{a}] - \gamma^2[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]$	$\mathbf{G}^{i\nu'} = \gamma^2\mathbf{v}[\mathbf{v}.\mathbf{g}] - \gamma^2[\mathbf{g} + (\mathbf{v}\times\boldsymbol{\omega})]$
Maxwell's Equations	**Maxwell's Eq. for Inertia**	**Maxwell's Eq. Gravitation**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{L}^{\nu'}_{\beta}\mathbf{F}^{\alpha\nu'}{}_{,\nu'}$	$\Omega^{\mu'\nu'}{}_{,\nu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{L}^{\nu'}_{\beta}\Omega^{\alpha\nu'}{}_{,\nu'}$	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{L}^{\nu'}_{\beta}\mathbf{G}^{\alpha\nu'}{}_{,\nu'}$
Gauss's Law	Gauss's Law	Gauss's Law
$\mathbf{F}^{0'i'}{}_{,i'} = -\gamma^2\mathbf{v}^2[\boldsymbol{\nabla}.\mathbf{E}] + \gamma^2\boldsymbol{\nabla}.[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]$	$\Omega^{0'\nu'}{}_{,\nu'} = -\gamma^2\mathbf{v}^2[\boldsymbol{\nabla}.\mathbf{a}] + \gamma^2\boldsymbol{\nabla}.[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]$	$\mathbf{G}^{0'\nu'}{}_{,\nu'} = -\gamma^2\mathbf{v}^2[\boldsymbol{\nabla}.\mathbf{g}] + \gamma^2\boldsymbol{\nabla}.[\mathbf{g} + (\mathbf{v}\times\boldsymbol{\omega})]$
Ampere's Law	Ampere's Law for Inertia	Ampere's Law Gravitation
$\mathbf{F}^{i'\nu'}{}_{,\nu'} = \gamma^2\mathbf{v}[\mathbf{v}.\mathbf{E}]_{,0} - \gamma^2[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]_{,0} + \gamma^2\boldsymbol{\nabla}\times[\mathbf{B} - (\mathbf{v}\times\mathbf{E})]$	$\Omega^{i'\nu'}{}_{,\nu'} = \gamma^2\mathbf{v}[\mathbf{v}.\mathbf{a}]_{,0} - \gamma^2[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]_{,0} + \gamma^2\boldsymbol{\nabla}\times[\boldsymbol{\omega} - (\mathbf{v}\times\mathbf{a})])$	$\mathbf{G}^{i'\nu'}{}_{,\nu'} = \gamma^2\mathbf{v}[\mathbf{v}.\mathbf{a}]_{,0} - \gamma^2[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]_{,0} + \gamma^2\boldsymbol{\nabla}\times[\boldsymbol{\omega} - (\mathbf{v}\times\mathbf{a})])$
Conservation Law	**Conservation Law**	**Conservation Law**
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{L}^{\nu'}_{\beta}\mathbf{F}^{\alpha\nu'}{}_{,\nu'\mu'}$	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{L}^{\nu'}_{\beta}\Omega^{\alpha\nu'}{}_{,\nu'\mu'}$	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{L}^{\mu'}_{\alpha}\mathbf{L}^{\nu'}_{\beta}\mathbf{G}^{\alpha\nu'}{}_{,\nu'\mu'}$
Conservation of Gauss's Law	Conservation of Gauss's Law	Conservation of Gauss's Law
$\mathbf{F}^{0'\nu'}{}_{,\nu'0'} = -\gamma^2\mathbf{v}^2[\boldsymbol{\nabla}.\mathbf{E}]_{,0} + \gamma^2\boldsymbol{\nabla}.[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]_{,0}$	$\Omega^{0'\nu'}{}_{,\nu'0'} = -\gamma^2\mathbf{v}^2[\boldsymbol{\nabla}.\mathbf{a}]_{,0} + \gamma^2\boldsymbol{\nabla}.[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]_{,0}$	$\mathbf{G}^{0'\nu'}{}_{,\nu'0'} = -\gamma^2\mathbf{v}^2[\boldsymbol{\nabla}.\mathbf{a}]_{,0} + \gamma^2\boldsymbol{\nabla}.[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]_{,0}$
Conservation of Ampere's Law	Conservation of Ampere's Law	Conservation of Ampere's Law
$\mathbf{F}^{i'\nu'}{}_{,\nu'i'} = \gamma^2\{\mathbf{v}^2[\boldsymbol{\nabla}.\mathbf{E}]_{,0} - \boldsymbol{\nabla}.[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]_{,0}\}$	$\Omega^{i'\nu'}{}_{,\nu'i'} = \gamma^2\{\mathbf{v}^2[\boldsymbol{\nabla}.\mathbf{a}]_{,0} - \boldsymbol{\nabla}.[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]_{,0}$	$\mathbf{G}^{i'\nu'}{}_{,\nu'i'} = \gamma^2\{\mathbf{v}^2[\boldsymbol{\nabla}.\mathbf{a}]_{,0} - \boldsymbol{\nabla}.[\mathbf{a} + (\mathbf{v}\times\boldsymbol{\omega})]_{,0}\}$
$\mathbf{F}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$	$\Omega^{\mu'\nu'}{}_{,\nu'\mu'} = 0$	$\mathbf{G}^{\mu'\nu'}{}_{,\nu'\mu'} = 0$

7. Discussion and Comparison

The results of this models are entirely new and there doesn't exist a single model that could be compared with it. This model shows the origin of electric field, and Newton's view of action and reaction of electric field in terms of temporal and spatial singularities of EMF.

Time derivative of temporal singularity is the origin of time varying electric field and space derivative of spatial singularities of EMF is equal to Gauss's law.

Maxwell's equations are completely symmetrized. Electromagnetic laws are doubled due to the contribution of terms of singularities.

The emergence of 4D EM wave as the representative of Singularities of EM conservation law as an integral part of the theory is very strange result.

$$F,^{\mu'\mu'}{}_{,\mu'\mu'} = \square\square^2(E)$$

EM conservation law contain two types of waves. 4D EM wave and Laplacian wave that is also very interesting result.

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = [\square^2 - \nabla^2]E$$

In the whole literature on relativistic electrodynamics, no one has ever worked out EM conservation law in tensor components form. The operator $[\square^2 - \nabla^2]$ resembles with Klein-Gorden operator. Here, it is representative of vacuum wave that might explain expansion of the universe.

7.1. Predictions of Model-2

The model developed in sec-II, shows that 4D Lorentz force is an integral part of electrodynamics in noninertial velocity metric as the consequence of transformation of 4D electric field whose time component is electric power and space component is Lorentz force. This could be possible due to the application of UTL on EMF in noninertial metric.

$$F^{0'\nu'} = v.E + [E + (v\times B)]$$

It can be written as under in terms of 4 force. We need to multiply by charge q

Table 5. Universal Transformation Matrices

Universal Physical Matrices

Universal Lorentz Transformation in2D

$$[L^{\mu}_{\nu}] = \begin{bmatrix} \frac{1}{(1-\beta^2)} & -\frac{\beta^2}{(1-\beta^2)} & 0 & 0 \\ -\frac{\beta^2}{(1-\beta^2)} & \frac{1}{(1-\beta^2)} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

$\det L = \gamma^2 (1 + \beta^2)$

Inverse of ULTM in 2D

$$\left[L^{\mu -1}_{\nu}\right] = \begin{bmatrix} \frac{1}{(1+\beta^2)} & \frac{\beta^2}{(1+\beta^2)} & 0 & 0 \\ \frac{\beta^2}{(1+\beta^2)} & \frac{1}{(1+\beta^2)} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

$\det L^{\mu -1}_{\nu} = \frac{(1-\beta^2)}{(1+\beta^2)}$

Universal Lorentz Transformation Matrix in 4D

$$[L^{\mu}_{\nu}] = \begin{bmatrix} \frac{1}{(1-\beta^2)} & -\frac{\beta_1^2}{(1-\beta^2)} & -\frac{\beta_2^2}{(1-\beta^2)} & -\frac{\beta_3^2}{(1-\beta^2)} \\ -\frac{\beta_1^2}{(1-\beta^2)} & \frac{1}{(1-\beta^2)} & -\frac{\beta_3^2}{(1-\beta^2)} & -\frac{\beta_2^2}{(1-\beta^2)} \\ -\frac{\beta_2^2}{(1-\beta^2)} & -\frac{\beta_3^2}{(1-\beta^2)} & \frac{1}{(1-\beta^2)} & -\frac{\beta_1^2}{(1-\beta^2)} \\ -\frac{\beta_3^2}{(1-\beta^2)} & -\frac{\beta_2^2}{(1-\beta^2)} & -\frac{\beta_1^2}{(1-\beta^2)} & \frac{1}{(1-\beta^2)} \end{bmatrix}$$

(3)

Putting c = 1 and $\beta = \mathbf{v}$

$$[L^{\mu}_{\nu}] = \begin{bmatrix} \frac{1}{(1-\mathbf{v}^2)} & -\frac{\mathbf{v}_1^2}{(1-\mathbf{v}^2)} & -\frac{\mathbf{v}_2^2}{(1-\mathbf{v}^2)} & -\frac{\mathbf{v}_3^2}{(1-\mathbf{v}^2)} \\ -\frac{\mathbf{v}_1^2}{(1-\mathbf{v}^2)} & \frac{1}{(1-\mathbf{v}^2)} & -\frac{\mathbf{v}_3^2}{(1-\mathbf{v}^2)} & -\frac{\mathbf{v}_2^2}{(1-\mathbf{v}^2)} \\ -\frac{\mathbf{v}_2^2}{(1-\mathbf{v}^2)} & -\frac{\mathbf{v}_3^2}{(1-\mathbf{v}^2)} & \frac{1}{(1-\mathbf{v}^2)} & -\frac{\mathbf{v}_1^2}{(1-\mathbf{v}^2)} \\ -\frac{\mathbf{v}_3^2}{(1-\mathbf{v}^2)} & -\frac{\mathbf{v}_2^2}{(1-\mathbf{v}^2)} & -\frac{\mathbf{v}_1^2}{(1-\mathbf{v}^2)} & \frac{1}{(1-\mathbf{v}^2)} \end{bmatrix}$$

(3)

Inverse of $[L^{\mu}_{\nu}]$

$$(L^{\mu}_{\nu})^{-1} = \begin{bmatrix} \frac{1}{(1+\beta^2)} & \frac{\beta_1^2}{(1+\beta^2)} & \frac{\beta_2^2}{(1+\beta^2)} & \frac{\beta_3^2}{(1+\beta^2)} \\ \frac{\beta_1^2}{(1+\beta^2)} & \frac{1}{(1+\beta^2)} & \frac{\beta_3^2}{(1+\beta^2)} & \frac{\beta_2^2}{(1+\beta^2)} \\ \frac{\beta_2^2}{(1+\beta^2)} & \frac{\beta_3^2}{(1+\beta^2)} & \frac{1}{(1+\beta^2)} & \frac{\beta_1^2}{(1+\beta^2)} \\ \frac{\beta_3^2}{(1+\beta^2)} & \frac{\beta_2^2}{(1+\beta^2)} & \frac{\beta_1^2}{(1+\beta^2)} & \frac{1}{(1+\beta^2)} \end{bmatrix}$$

(4)

Universal Numerical Matrices

Universal Transformation Matrix

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

Det L = 3

Inverse Universal Transformation Matrix

$$[L^{-1}] = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

$\det L^{-1} = \frac{1}{3}$

UTM in 4D

$$K^{\mu}_{\nu} = \begin{bmatrix} 2 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & 2 & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & 2 & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 2 \end{bmatrix} \qquad (7)$$

Det K = 9

Inverse of K in 4D

$$(K^{\mu}_{\nu})^{-1} = \begin{bmatrix} \frac{6}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{6}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{6}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{6}{9} \end{bmatrix} \qquad (8)$$

$$K^{\mu}_{\alpha} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ \frac{1}{3} & 0 & 1 & -1 \\ \frac{1}{3} & -1 & 0 & 1 \\ \frac{1}{3} & 1 & -1 & 0 \end{bmatrix} \qquad (9)$$

Det $(K^{\mu}_{\alpha}) = -3$

$$F^{\mu} = \mathbf{v}.q\mathbf{E} + \mathbf{q}\left[\mathbf{E} + (\mathbf{v} \times \mathbf{B})\right]$$

In relativity theory, we know that, energy and momentum are unified. Here the next step says that power and force should appear in unified manner.

Majority of the physicists write 4D Lorentz force, compatible with rate of change of 4-momentum with respect to proper time [4-8] as for example Ohanian

$$\frac{dp^{\mu}}{dt} = \frac{q}{mc} P_{\nu} F^{\mu\nu}$$

Spacetime theory of inertia contains 4D Lorentz force for inertia exactly compatible 4D Lorentz for electromagnetism

$$\Omega^{0'v'} = \mathbf{v}.\mathbf{a} + \left[\mathbf{a} + (\mathbf{v} \times \boldsymbol{\omega})\right]$$

$$F^{\mu} = \mathbf{v}.\mathbf{F} + \mathrm{m}\left[\mathbf{a} + (\mathbf{v} \times \boldsymbol{\omega})\right]$$

This equation represents unification of power and Lorentz force.

It contains Newton’s send law of motion as a special case.

$$F = ma$$

We can easily not only deduce Newton's first law of motion but also 4D Lorentzian momentum on multiplying by time on both sides

$$P^{\mu} = \mathbf{x}.\mathbf{F} + \mathrm{m}\left[\mathbf{v} + (\mathbf{x} \times \boldsymbol{\omega})\right]$$

The above equation is the expression for energy and momentum more general definition than usual definition of energy and momentum in special relativity P^{μ} = $[\frac{E}{c}, P]$ Newton's first law is actually definition of linear momentum

$P = mv$

We can write down 4D Lorentz force for electromagnetism and inertia in a compact form as a natural comparison

$$g_{\alpha}^{0} F^{\alpha\nu} = g_{\alpha}^{0} \Omega^{\alpha\nu}$$

Maxwell's equations in noninertial metric from table-2, are

Gauss's Law

$$F^{0'\nu'}{}_{,\nu'} = \mathbf{v}.\mathbf{E}_{,0} + \nabla.\mathbf{E} - \mathbf{v}.(\nabla \times \mathbf{B})$$

Ampere's Law

$$F^{0'\nu'}{}_{,\nu'} = -\nabla(\mathbf{v}.\mathbf{E}) - \mathbf{v}.\left[(\nabla \times \mathbf{B}) - \mathbf{E}_{,0}\right]$$

There is only one model of electrodynamics by Atwater who has worked out Maxwell's equations considering Galilean metric as noninertial metric, His equation contains only one component of Lorentz force.

$$(\nabla.E)' = \frac{1}{\varepsilon_0} \rho v.(\nabla \times B)_1$$

His Ampere's law terms do not represent a complete description due to its limitation in one dimension. Scorgie [3] employs Frenet equations to introduce intrinsic coordinates. But what is required is that the frame should be constructed. Our noninertial metric is generalization of Lorentz transformation and Galilean metric therefore, it represents complete description of Maxwell's equations in noninertial metric.

EM conservation law by matrix method hold as usual but by Einstein's convention method is d'Alembert Laplace gauge invariant expression of electric power.

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = [\Box^2 - \nabla^2](v.E)$$

4D EM wave along the diagonal of EM conservation law

$$F^{\mu'}{}_{,\nu'} = \Box^2 (v.E)$$

This relation has no counter example anywhere in physics.

7.2. Predictions of Model-3

This model is based on similarity transformation of electrodynamics in noninertial metric. Here electric field is transformed into complete Lorentz force and directed power

Electric Field

$$F^{0'i'} = [E + (v \times B)] - v[v.E]$$

Transformation of acceleration field by similarity transformation of inertia field tensor gives

$$\Omega^{0'\nu'} = \left\{\left[\mathbf{a} + (\mathbf{v} \times \boldsymbol{\omega})\right] - \mathbf{v}(\mathbf{v}.\mathbf{a})\right\}$$

comparing given equations, we get

$$\{\mathbf{a} + (\mathbf{v} \times \boldsymbol{\omega})] - \mathbf{v}(\mathbf{v}.\mathbf{a})\} = \{\left[\mathbf{E} + (\mathbf{v} \times \mathbf{B})\right] - -\mathbf{v}[\mathbf{v}.\mathbf{E}]\}$$

There are three models that contain terms like above equation Landau and Lifshitz [9] by employing Lagrange equation

$$\dot{\mathbf{v}} = \frac{e}{m} \sqrt{\left(1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}\right)} \left\{\mathbf{E} + \frac{1}{\mathbf{c}} \mathbf{v} \times \mathbf{H} - \frac{1}{\mathbf{c}^2} \mathbf{v}(\mathbf{v}.\mathbf{E})\right\}$$

Similar result is pointed out by Griffith D. J. [12]

$$\mathbf{a} = \frac{q}{m} = \sqrt{\left(1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}\right)} \left\{\mathbf{E} + \frac{1}{\mathbf{c}} \mathbf{u} \times \mathbf{B} - \frac{1}{\mathbf{c}^2} \mathbf{u}(\mathbf{u}.\mathbf{E})\right\}$$

and by Barnett, Stephen M. [27]

$$\mathbf{a} = \frac{Q}{m} = \sqrt{\left(1 - \frac{\mathbf{v}^2}{\mathbf{c}^2}\right)} \left\{\mathbf{E} + \frac{1}{\mathbf{c}} \mathbf{u} \times \mathbf{B} - \mathbf{u}(v.\mathbf{E})\right\}$$

The above three results contain inverse Lorentz factor whereas our all result including do not contain Lorentz factor. The left-hand side of above three do not have corresponding relation for acceleration like our

Magnetic Field

$$F^{i'\nu'} = v[v.E] - [E + (v \times B)] - [B - (v \times E)] - [B - (v \times E)]$$

The relation for magnetic field via equation (3.5) is entirely new and the contemporary world is devoid of it.

Mansurpuri [20] asserted to ban Lorentz force law as it is incompatible with special relativity due to its problem of not obeying conservation law. As a reaction to this statement, a lot of comments appeared in the literature [21,22,23]. The conservation law of our model -3, that satisfy EM conservation law including Lorentz force dispels this debate forever.

$$\begin{aligned} &F^{\mu'\nu'}{}_{,\nu'\mu'} \\ &= -v^2[\nabla.E]_{,0} - \nabla[E + (v \times B)]_{,0} \\ &-[B - (v \times E)] + v^2[\nabla.E]_{,0} - [B - (v \times E)] = 0 \end{aligned}$$

7.3. Predictions of Model-4

Possibility of Electromagnetic Engine

4-dimensional Poynting vector S^{μ} = (S^0, S^i) = $(E^2 + B^2, E \times B)$. Its temporal component is $[E^2 + B^2]$ that represents electromagnetic energy and spatial component is $E \times B$ called Poynting vector or energy flux.

The transformation of S^{μ} in accelerating metric gives us the following expression

$$S^{\mu'} = [E^2 + B^2] - a[E^2 + B^2] - 2(a.v)[E^2 + B^2]$$

The second term on right hand side represents accelerating electromagnetic energy that is required to propel an electromagnetic engine. The third term contains twice of electromagnetic power associated with electromagnetic energy. This relation strongly predicts the possibility of electromagnetic engines which are badly needed in the field of space exploration.

7.3.1. Electrodynamics in Accelerating Metric

$$\mathrm{F}^{\mu'\nu'} = (\mathrm{a} \times \mathrm{B})$$

Maxwell's Equations in Accelerating Metric

7.3.2. Electric and Magnetic field in Accelerating metric

4-D Electric field

$$F^{0'\nu'} = \mathbf{E} + \left[\mathbf{a}.\mathbf{E} + (\mathbf{a} \times \mathbf{B})\right]$$

$$F^{0'0'} = \mathbf{a}.\mathbf{E}$$

Magnetic Field

$$F^{\mathrm{i}',\nu'} = -\mathbf{a}.\mathbf{E} + \left[\left(\mathbf{B}^3 - \mathbf{B}^2\right) - \mathbf{E}^1\right] + \left[\left(\mathbf{B}^1 - \mathbf{B}^3\right) - \mathbf{E}^2\right] + \left[\left(\mathbf{B}^2 - \mathbf{B}^1\right) - \mathbf{E}^3\right]$$

$$F^{\mathrm{i}',\mathrm{i}'} = -\mathbf{a}.\mathbf{E}$$

Gauss's Law

$$F^{0'\nu'}{}_{,\nu'} = \nabla.\mathbf{E} \,\square\, \mathrm{a}.[(\nabla \times \mathbf{B}) - \mathbf{E}_{,0}]$$

Ampere's Law

$$F^{\mathrm{i}',\mathrm{v}'}{}_{,\mathrm{v}'} = {}^2\,(\mathbf{E}) + \left[(\nabla \times \mathbf{B}) - \mathbf{E}_{,0}\right]$$

Gauss's Law for Magnetism

$$*F^{0'\nu'}{}_{,\nu'} = \nabla.\mathbf{B} + a.[(\nabla \times \mathbf{E}) + \mathbf{B}_{,0}$$

Faraday's Law

$$*F^{\mathrm{i}',\mathrm{v}'}{}_{,\mathrm{v}'} = {}^2\,(\mathrm{B}) - \left[(\nabla \times \mathrm{E}) + \mathrm{B}_{,0}\right]$$

7.3.3. Conservation Law by Matrix Method

$$F^{0'0'}{}_{,0'0'} = {}^2\,(\mathbf{E}_{,0})$$

4D EM wave of time-varying electric field

$$F^{i'i'}{}_{,i'i'} = -{}^2\,(\mathbf{E}_{,0})$$

$$F^{\mu'\mu}{}_{,\mu'\mu'} = 0$$

7.3.4. Conservation Law by Einstein Convention Method

$$F^{\mu'\nu'}{}_{,\nu'\mu'} = [\square^2 - \nabla^2](\mathbf{a}.\mathbf{E})$$

Conservation law consists of 4D wave minus Laplacian wave of time varying electric power. These terms seem to represent a new gauge that will explain accelerating expansion of the universe.

$$F^{\mu'\mu}{}_{,\mu'\mu'} = \square^2(\mathbf{a}.\mathbf{E})$$

The results of electrodynamics in accelerating metric are the generalization of Maxwell's equation in noninertial frame by [24] case-7,

Maxwell's equations in noninertial frame are obtained by taking the covariant derivative of electromagnetic field tensor where semi-colon; represents partial derivative in curved spacetime and comma, denotes partial derivative in flat spacetime

$$F^{\mu\nu}{}_{,\nu} = F^{\mu\nu}{}_{,\nu} + \Gamma^{\nu}{}_{,\alpha\nu} F^{\mu\alpha}$$

The set of connection coefficients for noninertial frame for which Reimann curvature is zero.

$$\Gamma^0{}_{,10} = -a^1 \;,\; \Gamma^0{}_{,20} = -a^2 \;,\; \Gamma^0{}_{,30} = -a^3$$

Putting the values of connection coefficients in equation (A), we are at the following equations

$$\nabla.E - a.E = \rho$$

$$\nabla \times B - a \times B = J + \frac{\partial E}{\partial t}$$

$$\nabla.B - a.B = 0$$

$$\nabla \times E - a \times E = -\frac{\partial B}{\partial t}$$

Conservation Law

$$\frac{\partial(\nabla.E)}{\partial t} + \nabla.[(\nabla \times B) - E_{,0}] - a.[(\nabla \times B) - E_{,0}] = 0$$

The term (E\times a) *represents time-varying electromagnetic Thomass Precession.*

The terms a.E and a×B from equations (1) and (2) are contained in the formula of 4D electric field. The second term of conservation law $a.[(\nabla \times B) - E_{,0}]$ of equation is contained in Gauss's law via equation (4.6.1). physically, $a.[(\nabla \times B) - E_{,0}]$ represents power density. The terms $a.E$ and $a \times B$ time-varying electric power and time-varying magnetic force.

Among other models on electrodynamics in noninertial frames [25,26,27,28], Nikolie's model [25] and Costa's model [26] are found having similarity with ours but based on very advance mathematics viz. Monad Lie derivative Fermi walker derivatives. The set of Maxwell's equations of Nikoli's model page 78-79, and Costa's model page-30, equations (86-89), contains terms like ours with more terms. Nikolei points out the physical significance of extra terms of inertia as effective magnetic charges and currents but do not tell the physical meanings of the terms like (**ω**. B) represents what. Similar problem is with Costa's model. They employ Fermi-Walker derivative machinery and get the similar to that of Nikolie. In our model, each extra term has its corresponding origin whereas in Nikolie's model is not defined. Our model is based on metric in matrix and electrodynamical relations namely, EMF, ME and EM conservation law are obtained easily. The application of UTL for 4-vectors and tensors has brought these wonderful results. References [27,28] are based on differential forms. Their Maxwell's equations do not possess simplicity with the origin of new terms as in our model.

7.4. Predictions of Model-5: Theory of Inertia and Gravitation in Noninertial Metrices

Near about all the important results about the comparison of electrodynamics, inertia and gravitation are given in the tables 1 to 4. Table-2a, concerned with dual of these theories. In this part, we will discuss the issue of generalization of usual Lorentz transformation matrix from 2D to 4D. Furry's work on generalized LT and calculation of Thomas pression relation is very popular in the contemporary literature [29].

7.4.1. Thomas Precession Relation from 2D LT and Generalized LT in 4D

Furry's generalized boost matrix in 4D is given as

$$B(v)=\begin{bmatrix} \gamma & -\gamma\frac{v_1}{c} & -\gamma\frac{v_2}{c} & -\gamma\frac{v_3}{c} \\ -\gamma\frac{v_1}{c} & 1+(\gamma-1)\frac{(v_1^2)}{v^2} & (\gamma-1)\frac{v_1v_2}{v^2} & (\gamma-1)\frac{v_1v_3}{v^2} \\ -\gamma\frac{v_2}{c} & (\gamma-1)\frac{v_2v_1}{v^2} & (\gamma-1)\frac{v_2v_3}{v^2} & \\ -\gamma\frac{v_3}{c} & (\gamma-1)\frac{v_3v_1}{v^2} & (\gamma-1)\frac{v_3v_2}{v^2} & 1+(\gamma-1)\frac{(v_3^2)}{v^2} \end{bmatrix}$$

By using above matrix, he obtained the Thomas precession relation through very complex calculation

$$\omega_T=\frac{1}{c^2}\left(\frac{\gamma^2}{(\gamma+1)}\right)a\times v$$

In our formulation, this relation is present as a special case in the formula of dual Lorentz force of inertia in noninertial velocity metric and in our genderized LT. From table-2a, column-2, equations (2) and (3) in theory of inertia, Thomas precession formula is derived by transformation of dual of inertia field tensor in velocity metric. Remember that these calculations are done in natural unit system where all universal constants are taken equal to 1 for simplicity.

Thomas Precession resulted from dual field of inertia in velocity metric

$$*\Omega^{\mu'v'}{}_{,}=\boldsymbol{\omega}_T{}'=(\mathbf{a}\times\mathbf{v})$$

$$*\Omega^{0'v'}{}_{,}=\boldsymbol{\omega}_T{}'=\mathbf{v}.\boldsymbol{\omega}+\left[\boldsymbol{\omega}+(\mathbf{a}\times\mathbf{v})\right]$$

Thomas Precession resulted from dual field of gravitation in velocity metric

$$*G^{\mu'v'}=\omega_{T'}=(g\times v)$$

Thomas Precession resulted from dual field of electrodynamics in velocity metric

$$*F^{\mu'v'}=\omega_{T'}=(E\times v)$$

The same result is obtained in generalized LT based on UTL method table-3. Column-2 of theory of inertia. In this case, the above results gives the same results but with Lorenz factor

$$*\Omega^{\mu'v'}{}_{,}=\gamma(\mathbf{a}\times\mathbf{v})$$

$$*\Omega^{\mu'v'}=\gamma\mathbf{v}.\boldsymbol{\omega}+\gamma\left[\boldsymbol{\omega}+(\mathbf{a}\times\mathbf{v})\right]$$

In the same way, theory of dual inertia field in generalized LT based on similarity transformation, method given in table-4, column-2. The dual of Lorentz force for inertia in generalized LT contains Thomass relation as a special case.

$$*\Omega^{0'v'}=\gamma^2\{[\boldsymbol{\omega}+(\mathbf{a}\times\mathbf{v})]-\mathbf{v}[\mathbf{v}.\boldsymbol{\omega}]\}$$

The first term in square bracket contains Thomass precession as a special case where $c=1$ and $=v$

$$\omega_T=\gamma^2(a\times v)$$

One can easily compare Furry's matrix with our matrix given below equation (5.2)

7.4.2. Generalized Lorentz Transformation Matrix in 4D

$$L^0{}_{,0}=L^1{}_{,1}=L^2{}_{,2}=L^3{}_{,3}=\gamma=L^1{}_{,i}=L^i{}_{,1}=-\gamma v, c=1$$

$$L^\mu_\alpha=\begin{bmatrix} \gamma & -\gamma v_1 & -\gamma v_2 & -\gamma v_3 \\ -\gamma v_1 & \gamma & 0 & 0 \\ -\gamma v_2 & 0 & \gamma & 0 \\ -\gamma v_3 & 0 & 0 & \gamma \end{bmatrix}$$

$$L^\mu_\alpha==\begin{bmatrix} \frac{1}{1-v^2} & -\frac{v_1^2}{1-v^2} & -\frac{v_2^2}{1-v^2} & -\frac{v_3^2}{1-v^2} \\ -\frac{v_1^2}{1-v^2} & \frac{1}{1-v^2} & -\frac{v_3^2}{1-v^2} & -\frac{v_2^2}{1-v^2} \\ -\frac{v_2^2}{1-v^2}v_2 & -\frac{v_3^2}{1-v^2} & \frac{1}{1-v^2} & -\frac{v_1^2}{1-v^2} \\ -\frac{v_3^2}{1-v^2} & -\frac{v_2^2}{1-v^2} & -\frac{v_1^2}{1-v^2} & \frac{1}{1-v^2} \end{bmatrix}$$

Table-5, consists of universal transformation matrices physical and numerical. The matrix (5) is generalized to ULTM in 4D whose each column and row gives us 1 so that it acts as a 4 by 4 physical identity matrix. All the spacetime laws of physics remain same in their original form after transformation with the application of UTL.

In order to do spacetime physics, we do need a universal Lorentz transformation matrix for noninertial frame or in an accelerating and rotating matrix like above matrix.

7.4.3. Universal Lorentz Transformation Matrix for Noninertial Frame

This matrix is nothing but 4 by 4 physical identity matrix for noninertial frame

$$\left[[\mathbf{L}^{\mu}_{\nu}]\right]=\begin{bmatrix} \frac{1}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} \\ -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & \frac{1}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} \\ -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & \frac{1}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} \\ -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & -\frac{\frac{a^2}{c^2\omega^2}}{(1-\frac{a^2}{c^2\omega^2})} & \frac{1}{(1-\frac{a^2}{c^2\omega^2})} \end{bmatrix}$$

All the spacetime laws of physics remain same in their original form after transformation. This matrix has made possible to do general relativity in the language of special relativity.

7.4.4. Consequences of ULTM for Noninertial Frame

Special Relativity in an Accelerating and Rotating Frame

We do need Lorentz transformation matrix to do physics in the context of special relativity

Lorentz Transformation Matrix for Accelerating and Rotating Bodies

$$\left[[L^{\mu},_{\nu}]\right]=\begin{bmatrix} \cosh\psi & -\sinh\psi & 0 & 0 \\ -\sinh\psi & \cosh\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\psi=\frac{a}{c\omega}$$

$$\left[\mathbf{L}^{\mu}_{\nu}\right]=\begin{bmatrix} \frac{1}{\sqrt{1-\frac{\mathbf{a}^2}{c^2\omega^2}}} & -\frac{\frac{a}{c\omega}}{\sqrt{1-\frac{\mathbf{a}^2}{c^2\omega^2}}} & 0 & 0 \\ -\frac{\frac{a}{c\omega}}{\sqrt{1-\frac{\mathbf{a}^2}{c^2\omega^2}}} & \frac{1}{\sqrt{1-\frac{\mathbf{a}^2}{c^2\omega^2}}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here a is constant translational acceleration, ω is constant angular speed and c is the speed of light. ψ is a dimensionless parameter.

$$DetL=1$$

$$\left(\mathbf{L}^{\mu}_{\nu}\right)^{-1}=\begin{bmatrix} \frac{1}{\sqrt{\left(1-\frac{\mathbf{a}^2}{c^2\omega^2}\right)}} & \frac{\frac{a}{c\omega}}{\sqrt{\left(1-\frac{\mathbf{a}^2}{c^2\omega^2}\right)}} & 0 & 0 \\ \frac{\frac{a}{c\omega}}{\sqrt{\left(1-\frac{\mathbf{a}^2}{c^2\omega^2}\right)}} & \frac{1}{\sqrt{\left(1-\frac{\mathbf{a}^2}{c^2\omega^2}\right)}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{Det}\left(\mathbf{L}^{\mu}_{\nu}\right)^{-1}=1$$

Results of SR in Noninertial Frame

Length Contraction

$$L'=\mathrm{L}\sqrt{\left(1-\frac{\mathbf{a}^2}{c^2\omega^2}\right)}$$

Time Dilation in Noninertial Frame

$$t'=\frac{1}{\sqrt{\left(1-\frac{\mathbf{a}^2}{c^2\omega^2}\right)}}t$$

Transformation of Coordinates in Noninertial Frame

$$\begin{bmatrix} x^{0'} \\ x^{1'} \\ x^{2'} \\ x^{3'} \end{bmatrix}=\begin{bmatrix} \frac{1}{\sqrt{1-\frac{a^2}{c^2\omega^2}}} & -\frac{\frac{a}{c\omega}}{\sqrt{1-\frac{a^2}{c^2\omega^2}}} & 0 & 0 \\ -\frac{\frac{a}{c\omega}}{\sqrt{1-\frac{a^2}{c^2\omega^2}}} & \frac{1}{\sqrt{1-\frac{a^2}{c^2\omega^2}}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix}$$

$$x^{0'}=\frac{1}{\sqrt{1-\frac{a^2}{c^2\omega^2}}}x^0-\frac{\frac{a}{c\omega}}{\sqrt{1-\frac{a^2}{c^2\omega^2}}}x^1$$

$$x^{1'}=\frac{1}{\sqrt{1-\frac{a^2}{c^2\omega^2}}}x^1-\frac{\frac{a}{c\omega}}{\sqrt{1-\frac{a^2}{c^2\omega^2}}}x^0$$

$$x^{2'} = x^2$$

$$x^{3'} = x^3$$

Calculation of Relative Acceleration in Noninertial Frame

$$\tanh\psi_1 = \frac{a_1}{c\omega}, \quad \tanh\psi_2 = \frac{a_2}{c\omega}$$

$$\tanh(\psi_1+\psi_2) = \frac{\tanh(\psi_1)+\tanh(\psi_2)}{1+\frac{\tanh(\psi_1)\tanh(\psi_2)}{c^2\omega^2}}$$

$$\frac{a_R}{c\omega} = \frac{\frac{a_1}{c\omega}+\frac{a_2}{c\omega}}{1+\frac{a_1a_2}{c^2\omega^2}}$$

Multiplying by cω on both sides

$$a_R = \frac{a_1+a_2}{1+\frac{a_1a_2}{c^2\omega^2}}$$

For $a_1 = c\omega$ and $a_2 = c\omega$, plugging in above gives the expression for absolute acceleration.

$$a_R = \frac{c\omega + c\omega}{1+\frac{c^2\omega^2}{c^2\omega^2}} = c\omega$$

$$a_R = c\omega$$

Product of speed of light and angular speed is a universal constant independent of the motion of the source or the observer. c is the time component of 4 acceleration. $\mathbf{a}^\mu = (c\omega, \mathbf{a})$. 4-acceleration $\mathbf{a}^\mu$ doesn't remain same.

For $\omega > 1rev.s^{-1}, \omega$ is faster than light. It also shows that light moves on a helical way or in circular paths. Speed of recession of galaxies from each other is thought to be faster than light. Accelerating expansion of the universe is now an empirically established fact. Speed of light c and $c\omega$ are the time components of 4-velocity and 4-acceleration remain same but 4-velocity and 4-acceleration do not.

But in the context of ULTM based on UTL, 4-velocity and 4-acceleration remain same.

Contravariant 4-velocity $\mathbf{v}^\mu$ remains same

$$c' + v' = c + v$$

$$v^{\mu'} = v^\mu$$

Covariant 4-velocity remains covariant in its original form after transformation

$$c' - v' = c - v$$

$$v_{\mu'} = v_\mu$$

Similarly, Contravariant 4-acceleration $\mathbf{a}^\mu$ remains contravariant

$$(c\omega)' + a' = c\omega + a$$

$$a^{\mu'} = a^\mu$$

Covariant 4-acceleration a_μ remains covariant

$$(c\omega)' - a' = (c\omega) - a$$

$$a_{\mu'} = a_\mu$$

Nelson's model [30] is resembles with our approach in the of LT and the idea of Thomas precession. Sfarti's work [31] is to be improved according to the standard mathematics.

N.B. The first postulate of Einstein's relativity that is principle of relativity is completely satisfied if and only if the space time laws of physics are transformed under universal Lorentz transformation based on UTL for 4 vectors and tensors. One can see the detailed calculations given in [1].

8. Conclusion

Five models of electrodynamics in noninertial metrics and in generalized LT have shown that 4D EM wave and 4D Lorentz force emerge as an integral part of electrodynamics. These models have shown that the complete picture of 4D Lorentz force is the property of noninertial frame where noninertial metric contains all components of velocity so it is true generalization of usual LT and GT. All the results are independent of Lorentz factor. Theory of inertia and gravitation in noninertial metrices have solved the problem of comparison of electromagnetic quantities with their mechanical analogue like comparison of 4D Lorentz force. Matrix method and Einstein's summation method both are useful where former method obey conservation law while the later predicts new results that have never been noticed in the contemporary literature. The results of electrodynamics in accelerating metric predicts the possibility of electromagnetic energy driven engine. A basic framework of special relativity in accelerating and rotating frame is also worked out where time component of 4-acceleration has become a universal constant independent of the motion of the source or observer. It is an attribute for the possibility of faster than speed of light within the frame work of special relativity in noninertial frame of reference The future work is to present theory of inertia and gravitation in accelerating metric, Einstein's relativity in accelerating and rotating frame of reference and derivation of electromagnetic laws from each other based on spacetime exchanger matrices that is almost ready.

Dedicated to the ideal of my Artist and His all students

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