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The Intrinsic Derivation of the Gravitational Constant G

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Abstract A mathematical investigation to derive the gravitational constant reveals new data. The concept is based on symmetry and *CTP*-operation together with a treatment related to the two light-speeds in the vacuum. With respect to a validity in space, an antispace incorporating matter, anti-matter and anything between this the study presents an intrinsic derivation based on natural constants alone. It does not disregard former achievements as the results strongly agree with others presented elsewhere. The advantage is the curious appearing numer for *G* can be demonstrated embedded in the theories touching gravity treatment in good conformance. An extension for application to already established space theories is discussed, which can then provide a basis for the Grand Unified Theories.

Keywords: *special relativity, classical field theory, gravitation*

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1. Introduction

It is a well-known problem to generate the gravitational constant *G* to any existing theory in gravito-mechanics as embedded in the theory of general relativity. *G* measurements have been carried out for a long time. It all started with Cavendish [1-5].

Even attempts on unifying the theory, the Grand Unified Theory, GUT [6,7,8] with the statements from quantum electrodynamics, QED introduced from [9] failed at these post-Newtonian theories [10], This is in particular due to the ambiguous solution of the anticipated quadrupole moment in the investigations in the field of geodesic space-time with respect to gravitational damping [11]. Since *G* is a more complicated typos than electrostatic interaction of two point charges [12,13,14] this constant can not be achieved by adapting to a simple form, such as the constant Alpha [15] in the electrostatic when brought to the existing conditions in gravitation. However, it was possible to derive theoretically the constant Alfa (α) as intrinsic on the basis of symmetry considerations [16-21] as it is crucial in the theories of general relativity (GR) introduced from Hilbert [22] and QED to open the challenge for a new theory. A particular example here is the vivid representation of an anti-Schwarzschild metric from the Schwarzschild metric obtained from those formalisms [17], as well as the simultaneous finding of an intermediate space with a quasi "mean" Schwarzschild radius $r \approx MG$ [18]. Consequently, this constant has long-ranging consequences. Since the present study does not refer to already existing masses [23], it is in contrary an intrinsic derivation on a mathematical path and completely dispenses with existing phenomena, which can only be explained by the constant *G*, then in reverse. The current theory is based on relativistic symmetry with an electrostatic concept. It relies exclusively on natural constants. The calculations are based on previous results and omit explicit citation in this paper.

Since repeated experiments from the 17-th century on have demonstrated *m* inertial and gravitational be identical this observation has been entailed *a priori* [21] in the equivalence principle [24,25] in GR observed more than 2500 years ago [26]. It will never, however be treated a scalar meaning vectorless or respective the same to all three directions in Euclidean space. More, gravitation will be shown a field effect valid in any space of any composition, for any value of mass being positive, negative, or even zero, and for any time interval running in positive or negative direction or oscillating. In the 2007 Fixler et al. [27] described a new measurement of the gravitational constant by atom interferometry, reporting a value of $G = 6.69\ 3(34) \cdot 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$. An improved cold atom measurement gives of $G = 6.67\ 191(99) \cdot 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$ [28]. In SI units the 2015 CODATA recommended value is G = 6.67 408 (31) · 10−11 $m^3\ kg^{-1}\ s^{-2}$ corresponding to a standard uncertainty $4.7 \cdot 10^{-5}$ [29].

In order to determine the basic requirements for a gravitational interaction, an intensive clarification of the electrostatic interaction between two bodies is required. This may initially appear a bit superfluous, but it is an essential point in the more far-reaching concept of the essence of gravitation. In the end, the theory will lead to the - here called Gravitoelectrodynamics – `The Theory of

the Nothing for Everything´ as based on previous results (citations see above).

At first glance, the following theory seems cumbersome and also too detailed. However, it is necessary to examine all the facts in detail in order to really ensure a later interpretation on the basis of all this. The following very detailed description may seem redundant, but is imperative for well understanding this particular fact.

2. Theory

In space-time nothing is at rest. It is important to follow the electromagnetic wave EM and the light beam both are treated in unison, in the same way and in the same meaning. A light beam passes through the shortest time period according to the Lagrangian extremal principle [30] when covering any path *s*. That statement is valid for any material, any mass *m*, and any property of space and applies to all materials and every track *s*. The distance *d* results from the Pythagoras. According to the above, this route can be closed according to time or time interval, respectively. It is valid for forward and backward running time, and for *m* of any value as (positive) matter, (negative) anti-matter, or 0 (zero), respectively.

The variables referring to a single electrical point-charge are indexed "e" to omit confusion with the momentum, whereas indexed "*g*" in case of gravitational considerations. As a consequence the distance between two points, either assigned to *m* or an electrical elementary charge e, are in the free vacuum under exclusion of any external conditions, *i. e.*, those outside the system, considered

$$m_{e,B} + m_{e,T} = 0 \tag{1a}$$

$$\Delta m_e = m_{e,B} - m_{e,T} \equiv +2 \cdot m_e \tag{1b}$$

$$\vec{d}_e = +\Delta\, \vec{r}_e = \vec{r}_B - \vec{r}_T \equiv +2r_e \tag{1c}$$

$$d_e = +\Delta\, |\vec{r}_e| = |\vec{r}_B| \cdot |\vec{r}_T| \equiv +\frac{\alpha^2}{2} \cdot 2r_e \tag{1d}$$

$$\vec{d}_g = +\Delta\, \vec{r}_g = \vec{r}_1 - \vec{r}_2 \equiv +2r_g \tag{1e}$$

$$d_g = +\Delta\, |\vec{r}_g| \equiv |\vec{r}_1| - |\vec{r}_2| \equiv +\frac{2}{\alpha^2} \cdot 2r_g \tag{1f}$$

with respect to a radius *r* are verified in the text. The sub-indices strongly distinguish both linear 1-dimensional electrostatic interactions in those between two point charges INSIDE an interacting body **e**, whereas variables due to gravitation appearing between two spheres are explicitly marked ***g***. The distances and related variables immediately incorporated in gravitational aspects referring to generally large values are exposed upper case notation like *R* and *D*, respectively, to distinguish from those found independently on the inside of a body if not otherwise cited. Consequently, considerations within a body or respective "globe" are due to a sphere and have spherical properties, whereas those outside consider an anti-sphere with hyperbolic conditions. In other words: both systems are ideally mirrored to each another.

It is suggested a path to compare electrostatics with gravitational conditions. That supports later evaluation of the results and in-depth understanding. All parts highlighting special features are separated in sections.

2.1. Electrostatic Point-charge Interaction

It should not be left unmentioned that the mass is actually a vector of at least 3 dimensions, although the scalar formula $E = mc^2 \left(\leftrightarrow E \equiv mc^2\right)$ could cover it up. A variable *m* should therefore be handled with caution since the Newton's axiom $\vec{F} = m\vec{a}$ [31] points to the dependence on mechanical acceleration and can as well the same depend on a field demonstrating vector character, at least in applying to it.

The first two pairs in the above set distinguish between subluminal bradyon *B* and superluminal tachyon *T* and contrast to the second, where two real particles or "bodies" are distinguished 1 and 2, respectively like real masses *m* of any value.

In spatial separation $d = 2r_e$ the electrostatic energy between two point-charges of opposite elementary charges e points to

$$\begin{aligned} &\Delta V_e(r) = V_T(r) - V_B(r) \\ &\equiv \frac{\alpha\hbar c}{2\left(1-\alpha^2\right)^{1/2} r_e} - \frac{\alpha\hbar c}{2\left[2-\left(1-\alpha^2\right)^{1/2}\right] r_e} \\ &= \frac{\alpha\hbar c \cdot 2\left[1-\left(1-\alpha^2\right)^{1/2}\right]}{2r_e} \equiv \alpha\hbar c \frac{\alpha^2}{2 \cdot r_e} \equiv \alpha\hbar c \frac{\alpha^2}{d_e} \end{aligned} \tag{2}$$

with the dimensionless, non-oscillating factor

$$\alpha \gtrsim \left| \frac{-e \cdot e}{(4\pi\varepsilon_0)\hbar c} \right|, \tag{3}$$

is the Sommerfeld's constant. This constant considers at the same time the radius-circumference discrepancy (RS) for an ideal sphere [21] explaining the identity

$$k_{e,EM} =: -\alpha\hbar c \equiv \frac{-e \cdot e}{(4\pi\varepsilon_0)} \Leftrightarrow -\hbar c \equiv \frac{-e \cdot e}{\alpha \cdot (4\pi\varepsilon_0)} \tag{4}$$

It is the same in any space as derived theoretically from the two limit speeds in the vacuum for a particle of the elementary charge e,

$$v_B = \left(1-\alpha^2\right)^{+½} \cdot c \equiv [2-\left(1-\alpha^2\right)^{½}]^{-1} \cdot c, \tag{5}$$

$$v_T = [2-\left(1-\alpha^2\right)^{+½}] \cdot c \equiv \left(1-\alpha^2\right)^{-½} \cdot c \tag{6}$$

[21] with average

$$\frac{|v_T| + |v_B|}{2} \equiv \frac{v_T + v_B}{2} = +c \tag{7}$$

That establishes the enormous constancy in the propagation (!) speed *c* of an EM as the same for the always constant *c* of the static field. An EM oscillates at

$$v_{EM} = \left(1-\alpha^2\right)^{\pm ½} \cdot c \tag{8}$$

per one cycle within the speed gap

$$\Delta v = v_T - v_B \equiv 2[1-\left(1-\alpha^2\right)^{+½}] \cdot c \equiv \alpha^2 \tag{9}$$

between its half-waves for every single system covering $c = c_0$ in the vacuum covered by a subluminal bradyon (B) and a superluminal tachyon (T). Therefore, every point-charge of any mass m including zero (m = 0) traverses the light barrier at c from one speed limit to the other demonstrating entirely *CPT*-operation.

The addition of the total values of the respective distance from the center of one system to a particular virtual B and T particle results in twice the average distance to the this center, and the same is true for the associated momentae. All other variables are related in accordance as extensively discussed elsewhere (see citations). That points to the set of relations

$$\frac{v_B}{c} = \frac{c}{v_T}, \frac{r_B}{r} = \frac{r}{r_T}, \frac{p_B}{p} = \frac{p}{p_T}, \tag{10}$$

$$|2r| = |r_B| + |r_T|, \; |2p| = |p_B| + |p_T|, \tag{11}$$

$$\frac{r_T}{r_B} = \frac{p_B}{p_T} \equiv \frac{\left(1-\alpha^2\right)^{½}}{[2-\left(1-\alpha^2\right)^{½}]} \equiv \left(1-\alpha^2\right), \tag{12}$$

$$r_B \cdot r_T = r^2, \; p_B \cdot p_T = p^2, \tag{13}$$

with $\vec{p} \perp \vec{r}$ the average momentum. As a consequence, the absolute values point to

$$r_B = [2 \cdot \left(1-\alpha^2\right)^{+½}] \cdot r \equiv \left(1-\alpha^2\right)^{-½} \cdot r, \tag{14}$$

$$r_T = \left(1-\alpha^2\right)^{+½} \cdot r \equiv [2-\left(1-\alpha^2\right)^{-½}] \cdot r; \tag{15}$$

$$p_B = \left(1-\alpha^2\right)^{+½} \cdot p \equiv [2-\left(1-\alpha^2\right)^{-½}] \cdot p, \tag{16}$$

$$p_T = [2-\left(1-\alpha^2\right)^{+½}] \cdot p \equiv \left(1-\alpha^2\right)^{-½} \cdot p; \tag{17}$$

The two different light speeds are related to two different electrostatic energies related to the center of one single particle and refer to the respective half-waves in circumferencing it. Their difference within one electrostatic system is

$$\begin{aligned} &\Delta V_e(r) = V_T(r) - V_B(r) \\ &\equiv \frac{\alpha\hbar c}{2\left(1-\alpha^2\right)^{1/2} r_e} - \frac{\alpha\hbar c}{2\left[2-\left(1-\alpha^2\right)^{1/2}\right] r_e} \\ &= \frac{\alpha\hbar c \cdot 2\left[1-\left(1-\alpha^2\right)^{1/2}\right]}{2r_e} \equiv \alpha\hbar c \frac{\alpha^2}{2 \cdot r_e} \equiv \alpha\hbar c \frac{\alpha^2}{d_e} \end{aligned} \tag{18}$$

That value exceeds the average V and entails on the outside of a body an effective field. In other words, on the outside of the observed body it shows a kind of "rest" charged as "remnant" to leave virtually a naked charge producing in an "active" maximum value in the electrostatic potential. Though, the energy itself is always positive as revealing from the electrostatic fields between the two oppositing charges the resulting electrostatic field to an outside observer will oscillate $(+/-)$ around their average 0 (zero) in an amplitude entailed from the (small) difference between them, and m properly would still deserve vector character. Consequently, the study of gravitational interaction on a free object spinning arbitrarily around $\vec{r}$ describing an active $\vec{m}_e$ averaging in its electrostatic field can not be ignored rather the statistical appearance must be considered. In illustrating an object an ideal sphere or respective "globe" the probability in the appearance of one of the two extremae is 1 / π, which reduces to 1 / 2π in case it appears in either maximum (positive) or minimum, respectively. That is important due to the fact any interacting must obey the demand for validity independent on any distance, including long-range effects. It is not certain whether or not the objects will not "communicate" before or after engaged in the process. Though, the two light speeds determine propagation of an EM in oscillating the static field will not. It is constant and can not be described due to the wave particle formalism in contrast to any real m assigned to any value, including 0 (zero).

2.2. Forwarding Electrostatic Point-charge Interaction

It should not be left unmentioned that the mass is actually a vector of at least 3 dimensions, although the scalar formula $E = mc^2 \left(\leftrightarrow E \equiv mc^2\right)$ could cover it up. A variable m should therefore be handled with caution since the Newton's axiom $\vec{F} = m\vec{a}$ [31] points to the dependence on mechanical acceleration and can as well the same depend on a field demonstrating vector character, at least in applying to it.

The first two pairs in the above set distinguish between subluminal bradyon B and superluminal tachyon T and contrast to the second, where two real particles or "bodies" are distinguished 1 and 2, respectively like real masses m of any value.

In spatial separation $d = 2r_e$ the electrostatic energy between two point-charges of opposite elementary charges e points to

$$\begin{aligned} V_e(r) &= \frac{-e}{\sqrt{4\pi\varepsilon_0}} \frac{1}{2r_e} \frac{+e}{\sqrt{4\pi\varepsilon_0}} \equiv -\sqrt{\alpha\hbar c} \frac{1}{2r_e} \sqrt{\alpha\hbar c} \\ &= -\frac{\alpha\hbar c}{2r_e}, \end{aligned} \tag{19}$$

with $\alpha\hbar c \approx 0.230707751 \cdot 10^{-27} kg \cdot m^3 \cdot s^{-2}$.

$$V_e = -m_e \cdot c^2 \equiv -\frac{\alpha\hbar c}{d_e} \rightarrow m_e = \frac{\alpha\hbar}{c d_e}, \tag{20}$$

with the always positive dimensionless, non-oscillating factor

$$\alpha \gtrsim \frac{e \cdot e}{(4\pi\varepsilon_0)c\hbar} \tag{3}$$

the Sommerfeld's constant, which considers at the same time the radius-circumference, theoretically established before [32].

2.3. Introduction to Basic Gravitational Effects

In the following, a very attentive and detailed description must be included, although at first glance it may seem a bit too superfluous.

The gravity interaction is not to confuse with a pure and (generally expected) electrically neutral *m*, respectively. Consequently, the bodies can be assigned an "electrostatic *m*" exposed to a direction rather than a total or even true value generally valid as its intrinsic property, and it will be generally (!) never

$$m_{\text{total}} = \|\vec{m}_e\| = \sqrt{\frac{1}{i}\sum_{i=1}^{3} m_i^2} \quad \text{or } m_e = m_i,\ i = 1, 2, 3, \tag{21}$$

either.

The further illustration should support in which every body has a fit radius or diameter that does not have a real measurable size, but rather a size representative of its issue. This value serves to determine an electrostatic potential or the energy of a body, from which a mass is derived. The whole mass is focused on the middle as a focus, as if then the entire body would consist only of one point, which either merges to zero or, in the other extreme case, extends infinitely far. These two extremae would correspond principally to an infinitely high mass or vanishingly small. The charge on the "outer skin" of the bodies is generated from the outside by either another charged body or a pure e alone, respectively located in the near or distant environment. If the outside component is missing there will be no field. The electrical charges on the outer skin of a body should be symmetrical to each other, meaning that a positive and a negative charge oppositely oppose each other and then cause an electrostatic attraction to each another inside and to the outside as well. The actual field is, therefore not originated from the inside and is not the autonomous cause of the body rather an external effect. Nevertheless, an interaction is always an attribute of the body itself. Then, process in gravity can be illustrated in the three chains:

a) electrostatic pure point-point for any *D*

$$\mathrm{e}_0(\text{start}) \to \phi_0(\mathrm{e}_0) \to D_x \to \phi_x(e_x) \to e(\text{end}), \tag{22}$$

b) electrostatics point-body for shorter *D*

$$\mathrm{e}_0(\text{start}) \to \phi_0(\mathrm{e}_0) \to D_1 \to \phi_1(\mathrm{e}_1) \to \mathrm{e}_1(\text{body}) \to r_{e,1}$$
$$r_{e,1} \to |m_{e,1}|, \tag{23}$$

c) electrostatics point-body for larger *D*

$$\mathrm{e}_0(\text{start}) \to \phi_0(\mathrm{e}_0) \to D_2 \to \phi_2(\mathrm{e}_2) \to \mathrm{e}_2(\text{body} \to r_{e,2}$$
$$r_{e,2} \to |m_{e,2}|. \tag{24}$$

In those succeeding the field decreases with *D* or a respective *R* producing a different charge value on the "skin" or shield of the body distinct from the origin. Then, the body strives to maintain the value of its proper field in direction to its center achieved in a decrease of r_e to compensate the lower shield charge around. The charges should comply the above situation as opposite charges of the same absolute values facing each another on opposite sides referred to the body´s center. That decrease, imagined a body's squeezing, entails a change in the original value of m_e depending on *D*; it is an indirect effect, thus. The properly inside *E* must perpetuate in accordance to the Lavoisier's discovery [33]). The "active" mass, not those with a stamped value in the print, is therefore inextricably linked to the distance. However, that is not an *m* any more but rather a product, the reason explicitly marking this feature to resume in short. The distinct results referring to $D_1 < D_2$ are collocated in the list

$$\begin{aligned}
&|e(D_1)| \to |e_1| > |e_2| \leftarrow |e(D_2)| \\
&|\phi(D_1)| \to |\phi_1| > |\phi_2| \leftarrow |\phi(D_2)| \\
&|r_{e,0}(D_1)| \to |r_{e,1}| > |r_{e,2}| \leftarrow |r_{e,0}(D_2)| \\
&|m_{e,0}(r_{e1})| \to |m_{g,1}| < |m_{g,2}| \leftarrow |m_{e,0}(r_{e,2})|.
\end{aligned} \tag{25}$$

Those relations are still undetermined and must be referred to a basis, which requires a calibration of the terms. It can be recognized the effect of gravitation seems to do exactly the opposite, as evidently run to a deformation of the field due to the distance from the center would have been properly expected. A squeezing of the body is therefore much more crucial. The further, therefore, the two bodies of the effect of the potential are the more like those, with increasing *D* the two bodies become more and more similar. With increasing *R* from the center the paths of both objects, then move more and more parallel to finally end up at two parallel rays or respective beams, not following a crush, the energy expenditure for the "jump" over the light barrier. It is getting smaller until finally vanished tiny or respective even disappear. Consequently, two effects coincide as far from the center an interacting body becomes smaller and tends to a perfectly shaped sphere aware of no external field meaning no more possible "squeezing". There are two effects of squeezing to distinguish: a) the pure light-effect from, in the model, circumferencing a center as an external phenomenon; b) the internal change in the individual from gravity.

2.4. The Feature in Gravitation

In resuming this means the relation between two, at a first internal r_e depends on the ratio of potentials ϕ_e acting on them from outside,

$$\frac{D^{\text{large}}}{D^{\text{short}}} \sim \frac{\overset{\text{weak}}{e}}{\underset{e}{\text{strong}}} \sim \frac{r_e^{\text{short}}}{r_e^{\text{large}}} \qquad \frac{D^{\text{large}}}{D^{\text{short}}} \propto \frac{r_e^{\text{large}}}{r_e^{\text{short}}} \tag{26}$$

as the lengths inside and out then behave inversely proportional to the fraction of the outside potentials. This compensates the external influence in maintain the

constitution of the interacting body inside as stated already. In a field, the above object would really and literally squeezed to almost nothing, *i. e.*, to an extent r_e of 1 due to the cancel out if not the Heisenberg's statistics spoke against it avoiding this singularity. As a result, it can never ever be traded a common, real mass but only exclusively a representative "virtual" mass based on electrostatics, instead as to introduce below.

=In all those considerations, the background thought is still the principle of equivalence [24,25,26]. If that is the case, then the field acting on a body will have to demonstrate the exactly same effect as acceleration, because both are strictly Newtonian and of the same force, so it is an equivalent phenomenon. Since that must be true for a single body of any value *m* it immediately leads to the question for interacting. Since it is now clear which way the free, *i.*, *e.*, not gravitationally influenced *m* is originated from its own movement alone, a unification of the two influences must prove itself in a next step. Of course, the body caused from an EM will be deformed showing a light effect. At increasing distances from each other they will more and more claim perfect spherical character as in eqs. (21-23). In these extremae no orientation or respective direction is exposed, and an *m* takes over the definition in eq. (24). If, however the already mentioned change occurs from the outside indirectly through an artificially and still anticipated "gravitational-force effect" to then become squashed from the outside, this gravitational effect will contrast with the pure light effect to a certain extent. The task is to amalgamate both effects in order to finally be able making a true statement on the influence of the gravitational effect super-sheathing, *i. e.* super-posing the first (Figure 1). A final goal could be an altogether statement, then.

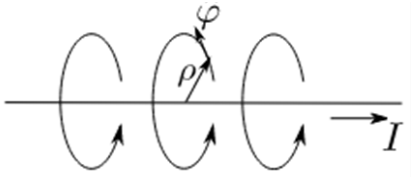

Figure 1. A gravitational flow viewed in a gravitational tube with its orthogonal effect

The further calculation could be carried out either with only one body oscillating between *B* and *T*, or concerning two separate systems (Figure 2), if the energies were then also included separately and explicitly treated. Due to its complexity this step, again requires in-depth explanations accompanying the understanding. The two interacting m_g are not real in measurable values and rather virtual objects. However, these can not simply disappear from shortening their proper *r* as determined from an external field, which does not disappear; it very same causes the special nature of this special kind of interaction (Figure 2). Since the charge on the skin, directly on the outer layer of those virtual *m* is caused, the field at this point will act in full force as generated from the outside. It remains entailed from that. In the absence of an external force, such a pseudo-mass will hover around as an undefined something ("some thing") in the huge quantum vacuum. However, the result of the given derivation is a scalar, not yet directed as a vector oriented body, since no appearing field has been applied, yet. But if an external charge is present, then weakening appears as with the coupled bodies being freely selectable, but in any case no α factor. Certainly, it must be obvious a field quoted above appears with an enormous amount. This is important to note in the radius-circumference (Figure 1), because although a particle never reaches the speed of light, comparable to an atomic model (recognized in the α), the field originated from a source at a thought center is higher by the same amount (see: eq. 25), is say demonstrating the reverse as "swallowing in consuming" the α factor as an electrostatic field effect. Moreover, those *m* are pseudo or virtual $m_0 = m'_g$, respectively, because the *D* determining their values cancels with the real large *D* of the entire system. The above scalar relations will apply to all values and valid for any object, regardless of nature, composition, and intrinsic character. On the other side, in the absence of an external force such a pseudo-mass will hover around as an undefined something in the huge quantum vacuum.

Now, items and information collected in this long text in a kind of construction kit must now be picked up and put together into a complete puzzle, compiled continuing every *m* and variable provided with a stroke or asterisk expose not a pure or freely closable, either. The so-called bodies always consist of that, each forming a closer to itself product and contain, in addition to its "real" property additionally an explaining factor, which can create a field or received it from another body, so can be charged. It must therefore be carefully sorted out the expressions and put together accordingly, in order to include them in the formulas and assign them. Then, there remain the distances, *R* or *D*, which have not been explicitly written in the last lines, because the coupling term itself must not contain any variables to consist only of exclusively natural constants, whereby an oscillation will of course be included. In other words said products have to be separated in such a way that those variables, *i. e.*, freely selectable. More markable said it has to be exonerate the interacting m and to exempt them from the central or respective coupling term.still without *D* resumes the above seeming 1 "mechanical" or "classical" *m* term in the nominator, but superposing electrostatics apply onto wave mechanics make them appear 2. Those formulae clearly illustrates the circumstances the components appear as later multiplications outside the (brace) coupling bracket, whereas the interacting ones carry with them the character of all variables involved in the coupling process into it; they are perpetually affiliated (Figure 2). Here, the gravitational interaction of two *m* with the brace bracket points to a coupling term to appear in appropriate dimension in obeying the relativistic energy expression displayed in principle. The from here ongoing theoretical development does not mean under no circumstances two *m* attract with just the same sign. Who says that ? In accordance to Newton it is *actio equi re-actio* ?! The minus sign in the present theory is not and never conventional, as usual; if it appears it must be always alone a strict mathematical consequence. In addition, every pre-sign (+ or −) should be of equal worth, none preferred, and the definition in a conventional sense is clearly interpreted man-made; a positive *m* is merely an

experience and must not disguise the real and real character. On a first view, those *m* assign a kind unity representing itself a basis does not seem probable, but is: as they reveal from the interaction process they are inseparable part of it.

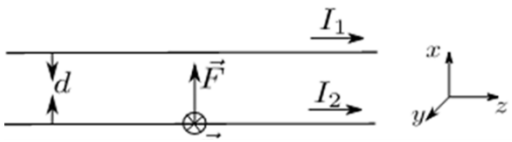

Figure 2. The interaction of two gravitational tubes flown through from a gravitational wave

2.5. Real Gravitational Interaction

A first simple estimation on gravitational interaction is

$$E_g(D) = -(m)\cdot\left\{\frac{c^2}{m_g}\right\}\cdot(+m),$$
$$E_g(D) = +\left(-m_e^*\right)\cdot\left\{\frac{c^2}{m_g^*}\right\}\cdot\left(+m_e^*\right), \tag{27}$$

and the development of this basic equation provides all the prerequisites for the discussion of all causes and connections in relation to the gravitational interaction (Figure 2). Furthermore, an interaction between two *m* of exactly the same composition is introduced and assumed in this study.

With the above results regarding the definition and introduction of a quasi electrodynamics m_e – in both interactive and gravitation m_g – yields

$$E_g(D) = -m_e^*\frac{\alpha^2}{e}\cdot\left\{\frac{c^2}{m_g^*}\right\}\cdot\frac{\alpha^2}{e}m_e^*,$$
$$\rightarrow$$
$$E_g(D) = -\left(m_e^*\right)^2\frac{\alpha^4}{e^2}\cdot\left\{\frac{c^2}{m_g^*}\right\},$$

and expansion gets

$$E_g(D) = -\left(m_e^*\right)^2\frac{\alpha^4}{e^2}\cdot\left\{\frac{m_g^*\cdot c^2}{m_g^*\cdot m_g^*}\right\}, \tag{28}$$

it will be shown below how the *D* follows from the λ.

2.6. Complete Electrostatic Interaction

Every half-wave can be considered separately, every wave follows the de Broglie's ascertainment (*e. g.*, [34]). When compared to atom theory both of these systems together obey the Dirac's formalism [35,36] based on the differential-equation twin. They are orthogonal to each another (Figure 1, Figure 2) as well as their proper ("eigen-") values, however permit linear expressions in *m*, *p*, and *V* or respective *E* in contrast to their origin

$$E^2 = \underbrace{[p\cdot c]^2}_{\text{kinetic}} + \underbrace{[m_0 c\cdot c]^2}_{\text{static}} \tag{29}$$

– before arriving at $E = mc^2$ – consisting of quadratic terms via the Pythagoras leading to the Klein-Gordon-Fock equation (KGF). From the "gap" between the speeds [16] the above "artificial" photon-rest mass can be claimed depending on *R* or the diameter *D* = 2*R*, respectively. The last equation is very significant because the first summand can be interpreted in terms of the two interacting m_e^* whereas the second summand refers to the "rigid" m_g^* in the nominator sought. These two m are therefore of very different character in their nature, thus related to the multiplier in front of the active m_e^* with e and α^2.

It comes to the forth, the above gathers knowledge producing

$$\frac{\alpha\hbar}{cd_e} = -\left\{\frac{\alpha^6\hbar^2\cdot(4\pi\varepsilon_0)}{c^2e^2}\frac{1}{\left(m_g^*\right)^2}\right\}\frac{1}{d_e}$$
$$1 = -\left\{\frac{\alpha^5\hbar\cdot(4\pi\varepsilon_0)}{ce^2}\frac{1}{\left(m_g^*\right)^2}\right\}$$

with oscillation **i**

$$m_g^* = \sqrt{-\frac{\alpha^5\hbar\cdot(4\pi\varepsilon_0)}{ce^2}} \equiv \pm\mathbf{i}\cdot\frac{\alpha^{5/2}\hbar^{1/2}\cdot(4\pi\varepsilon_0)^{1/2}}{c^{1/2}e}. \tag{30}$$

In previous investigations [16] a relationship comes into field pointing to the relationship between static and dynamic behaviors and deduced from the formula given above in eq. (28) with [16]

$$\mathrm{V_{T/B}} = c \pm 8.06\,766\,28\,5\cdot10^{+3}\,m/s$$

in the numerical value

$$\frac{\Delta\,\mathrm{V_{T,B}}}{c} = 0.26\,910\,82\,658\,92\cdot10^{-3}. \tag{31}$$

This explains a circumference around the static point of *c* allowing a gravitational wave (GW) in condition of a "screw" forming a gravitational wave (Figure 1), GW – in analogy to an EM. Such a previously predicted statement for a gravitational wave is justified via the operator equation $\mathrm{d}/\mathrm{dt} = \mathbf{i}\omega_g$. It is, by the way

$$\alpha\hbar \triangleq 7.69\,558\,22\,357\,42\cdot10^{-37}\ \mathrm{Js}.$$

The last arrows, above or below, clearly mark the wave character of the interaction between the two interacting *m*. The circular frequency ω is actually a vector, but due to the clarity the vector symbols (for mass etc.) have been avoided beforehand. Owing to

$$\vec{X}_0 = i\vec{\omega}t \tag{32}$$

this variable is orthogonal or respective perpendicular to the other three vectors, which as variables $X_1 = x$, $X_2 = y$, $X_3 = z$ point to x, y, z directions [17, 18] and will not interfere with anything in x-direction describing D, the interaction. However, this omega is very important because it describes a torsion or respective rotation in the form of a screw in interaction propagation direction, *i.e.*, all around, as a beam of light requires (Figure 1). From eq. (23) can be discussed on the basis via generally

$$D = \frac{\lambda_g}{\pi} \equiv d_e \Leftrightarrow \lambda_g = \pi D \equiv \pi d_e \tag{33}$$

[16], if "calibration" for the shortest contact between the two interacting m is taken into account. The eq. (32) leaves, with regard to the two mechanic interacting particles

$$\sqrt{\frac{\Delta V_{T,B}}{c}} \triangleq 0.01\,640\,45\,196\,78 \equiv 1.64\,945\,19\,678 \cdot 10^{-2}$$

and regarding the circumpherence of the two interacting particles m_e^* as the two bodies´ surfaces interacting reveals

$$4\pi \cdot \sqrt{\frac{\Delta V_{T,B}}{c}} \triangleq 1.29\,524\,89\,316\,08\,568, \tag{34}$$

and this is the static variable enveloped in eq. (30). Consequently, subtraction leads directly to

$$C = \left(\frac{m_e^* \alpha^2}{e}\right)^2 \cdot \frac{ce^2}{\alpha^5 \hbar \cdot (4\pi\varepsilon_0)} \triangleq 79.05\,691\,49\,302\,32 \cdot 10^{-12}\,\frac{\mathrm{m}^3}{\mathrm{kg} \times \mathrm{s}^2} \tag{35}$$

and with regard to the circumpherence touching the surfaces of the two interacting "globes" in accordance to eq. (30), due to the unit radius r=1

$$4\pi \triangleq 12.56\,637\,06\,143\,59\,172 \tag{36}$$

brings into their difference

$$C - 4\pi \triangleq 66.49\,054\,43\,158\,72\,83 \cdot 10^{-12}\,\frac{\mathrm{m}^3}{\mathrm{kg} \times \mathrm{s}^2} \tag{37}$$

in accordance to eq. (29). This refers to the exact experimental value and reveals in this theory

$$G \triangleq 66.74\,083\,1 \cdot 10^{-12}\,\frac{\mathrm{m}^3}{\mathrm{kg} \times \mathrm{s}^2} \tag{38}$$

generally called the gravitation constant.

Quod erat demonstrandum

3. Results

As G reveals originates from quadratic terms the associated relations in gravitation are linear leading to quadratic effects. It should be emphasized the result for G is a positive constant, although it is perpendicular (orthogonal) to the direction, say x, of propagation with an oscillation of ω eqs. (32, 34), *i. e.*, the interaction between the two interacting m through a "gravitational wave" - not some pure EM appears.

The calculation presented here means the following. It is a summary of atomic physics, spin, and gravitational interaction, which finally leads to gravitoelektrodynamics (GED) as a representation of a grand unified theory , GUT, in a simple style This can be clearly illustrated in

$$\mathfrak{q} = (\mathfrak{H} \times \mathfrak{r}_s) \times \mathfrak{\gamma}.$$

Here, the symbols reflect

$\mathfrak{H}$ - the Hilbert space, spanned in n countable infinite dimensions by Schroedinger's H-wave functions, *i. e.*, n-dimensional

$\mathfrak{r}_s$ -the spin space, in quantum physics essentially related to the electron spin as proposed from Kudar (1926) and mathematically proven in the Dirac´s theory [35,36] being 2-dimensional due to the two magnetic spin-quantum numbers (positions)

$\mathfrak{\gamma}$ - a gravitational space, which refers to a character 2-dimensional

or respective with their independent variables. It shows, then for a

$$\mathfrak{q} = (\mathfrak{H} \times \mathfrak{r}_s) \times \mathfrak{\gamma}$$

to get into

$$\mathfrak{q} = (\mathfrak{H} \times \mathfrak{r}_s) \times \mathfrak{\gamma} \triangleq n \times 2 \times 2 \ (\rightarrow 4n) \text{ dimensions}$$

$\mathfrak{q}(\mathfrak{r}_e, \mathfrak{r}_s, g)$ - Gravitoelectrodynmics depending on

$\mathfrak{H}(\mathfrak{r}_e)$ - Hilbert space from the H-wave functions of Schroedinger´s half-relativistic theory

$\mathfrak{r}_s$ - spin space from Dirac´s exact relativistic representation via the Liouville-von Neumann equation

$\mathfrak{\gamma}(g)$ - pure gravity space in the view of the nominator $\left(m_g^*\right)^2$ of the above (original) expressions for the two waves going forth and back between the two interacting m objects in spin 2 wave

EM- electromagnetic wave

GW- gravitation wave

GED- gravito electrodynamics

QED- quantum electrodynamics

GUT- grand unified theory

4. Discussion

In comparison to electrodynamics or respective pure atomic physics, there are significant differences to gravitational interaction. A recent work was based on the twin pair of Dirac's equations a light beam. In atomic physics this is sufficient and formed the two EM, each with the spin 1/2, formed the EM with spin 1 – so be described by the Fermi-Dirac statistics [36,38]. It is clear when factoring out the Dirac equation information is lost, which, of course, in the KGF [39] retains. One effect is the trembling motion (*e. g.*, [40-43] cannot be represented by this, though it is omnipresent – also in gravitation. In this case, the GW would also not show any "helical movement" or respective a vertically (orthogonally) oriented helix during propagation as it certainly appears in

the EM eq. (46) above, $i\vec{\omega}_g(D)\cdot t$. The propagation speed c, which - seen as a single beam – represents the superposition of both half-waves, has an integer spin 1 and obeys the Bose-Einstein statistics [44]. In contrast to this, under the gravitational interaction, one wave of spin 1 can be assigned the respective m object. Therefore, again through a superposition, and here of the two "beams" in the GW, a spin 2 results (compare [45]).

The rigid, non-deformable objects, such as a point charge, are expected not be assigned a “sub-atomic” gravitational constant, then it is another effect rather than explicable from gravitation characteristics. For better understanding it needs to be clearly stated once again: when the two m bodies come closer to their minimal possible distance they will never exactly “look” like two flat coins, rather a certain finite minimal radius in form of spheres remains. In contrast to this, the two bodies, with increasing distance from each other, will resemble more and more perfect spheres that are becoming smaller and smaller.

In the event there are very close contacts between the two m, quantum effects appear and could lead to a certain confusion and rejection; such phenomena cannot be dealt with here, and that is the crux. In any case, a massive spin-2 field results from the interaction of the two gravitational forces of the opposing and therefore mutually emitting GWs where Bose-Einstein statistics (e.g., [46]) takes place. In contrast, a half wave of the EM (*e. g*., [17]) strictly obeys a Fermi-Dirac statistic [38], though their total and added two half-spins of the oscillating light beam (EM) together fall back into a spin 1.

It is true, when compared to an EM with superposed spin 1 an anticipated gravitational wave (GW) can be concluded a twice spin, *i. e.*, 2 instead of 1. This statement is more directly justified from the quadrupole emission discussed a first order result in electrodynamics or respective QED illustrated in twice a circumference frequency of a particle in comparison to that of the dipolar radiation, *e. g*., in the Zeeman effect. Since the rotation of an object entails acceleration, also true for a circumferencing EM, it should radiate GW, whereas a uniformly accelerated electrical charge, here incorporated in an EM, does not [25]. Consequently, quantum gravity extremely high concentrations of mass or energy in a very small space are not experimentally accessible, for the description of which quantum effects must be taken into account in addition to gravity. Attempts at a quantum field theory of gravity are rudimentary. However, there is a lack of predictions that are both predictable and observable. The basic problem is that at such concentrations black holes quickly form, inside of which quantum effects take place that cannot be observed.

The members of the group “Theory of Gravitation: Massive Spin 2 Fields” deal with extended theories of gravitation. The focus is on particles with a spin number of 2, which, in contrast to Einstein's theory, have mass. A massive particle with spin 2 would, so to speak, fill a gap (*e. g*., [47]).

This is due the fact in the standard model there are theories with lower spin numbers both in massless and in massive form. The corresponding particles could be proven experimentally. Examples for the spin number 1 are the massless photon or the massive W- and Z-bosons.

The scientists are working on laying the mathematical foundation for a theory of gravity with massive spin-2 particles. They are also investigating how such a particle could affect existing models of particle physics and cosmology.

An oscillation of the coupling fraction points to the introduced model of a wave circling around a center. This can be assigned to a mass oscillating between B and T interacting with itself. That is, this one oscillating mass simultaneously represents the two sought after in the nominator, which are coupled by the energy apparent in the denominator justifying the square root eq. (22) at the same time. Consequently, this question can be answered by a square of m, which is formed from the two remaining "borrowed" R´s. The radius RE of this MG is tiny due to gravitational crushing compared to the huge R and D of the whole system. The field, created externally on the skin of the body and acting within it, is enormously large. Therefore, a tiny body is to be expected with an extremely large own energy. The derivation bases only on the two natural constants e, c, $\hbar$ and it involves the matching constant ε_0 in the electric patch factor $(4\pi\varepsilon_0)$.

From theory GW are interpreted a nonlinear plane waive in the vacuum representing space-time to satisfy Riccy´s flat metrics as the algebraic fundaments in the GR nonlinear regime and describe a 5-dimensional group of isometrics [45]. This statement is entailed from the plane of an EM, which *e.*, *g*, appears in the quantum mechanical solutions of the Klein-Gordon-Fock equation (KGF) as the same isomorphic in accordance to the Maxwell’s symmetry group in electrodynamics. It is manifested to carry energy analogue to an EM demonstrated in the approach on perturbation calculations [48,49] and spinor treatment [50,52].

Where the Maxwell’s theory permits the existence of propagating waves in the EM field the GR predicts fluctuations itself can propagate as GW in space-time metric. In case of existence the latter are capable of probing the universe back to the Planck’ scale. The field equations are derived from Hilbert [22] on the principle of least action states [52].

In brief, space-time is considered a Lorentzian manifold to look for asymptotically flat solutions of the Einstein equation with the added property allowing a given Riemannian manifold to be embedded as a space-like hyper surface such that each causal curve intersects it only once pointing to the “Cauchy problem”. This embears a critical aspect in itself as directly leading to a crux in gravitational theory and can not be simply answered in the Penrose-Hawking singularity theorems [53-57].

If the interaction always involves two half waves in the interaction (compare Figure 2), it can be assumed that this will also apply analogously to a type of a gravitational wave. A half-wave alone in this case is not enough for a safe description of the process, which should lead to a double spin in comparison to the simple one, *i.*, *e*,, spin 1 resulting from superposition of the two ½ of the half waves each in an EM.

5. Conclusion

The theory presented consists of the intrinsic derivation of the gravitational constant G. Starting from the both

Dirac's differential equations – resulting from the (linear) out-factorization of the KGF – a gravitational wave can be described. There is an obvious analogy with classical electrodynamics, which is why the action of G occurs exactly perpendicular to the propagation speed c of the EMW. Although, the influence of G is much stronger than the interaction between two electrically charged particles, *i. e*., ***e***, the results are comparable to QED. The calculation allows evidence of a spin 2 for the gravitational wave, which is in contrast to a spin 1 for an EMW, respectively light. Finally, the constant G is determined taking into account the surface area of the two interacting particles and gives an exact agreement with the numerical value recommended by CODATA.

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