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The Contradictions in Poynting Theorem and Classical Electromagnetic Field Theory

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Abstract When studying the energy principle of N current elements, the author found that the Poynting's theorem of N current elements is very similar to the principle of mutual energy, except that the subscript of the summation symbol is different. Because the mutual energy principle of N current elements and Poynting's law of N current elements are all energy conservation laws, the author considers that they should be the same. Due to this consideration, the author found that the self-energy flow should not transfer energy. Self-energy flow is the energy flow corresponding to the Poynting vector of a current element, but this energy flow is not zero for any antenna. The author believes that this is a loophole in classical electromagnetic theory. The author's previous solution was to add a time-reversal wave to Maxwell's theory. This paper studies this problem through the electromagnetic field of plane-sheet current. It is found that the magnetic field calculated by Maxwell's equations is still the magnetic field generated by the current itself, but the electric field calculated by Maxwell's equation is actually the advanced wave generated by the environment of the current.

Keywords: *Maxwell equation, reciprocity theorem, conservation of energy, Poynting theorem, energy flow, transformer, primary coil, secondary coil, transmitting antenna, receiving antenna, retarded wave, retarded potential, advanced wave, advanced potential, absorber, emitter, photons, quantum, electromagnetic wave, electromagnetic field, electromagnetics*

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1. Introduction

The author studied the mutual energy theory and puts forward the mutual energy theorem [1,2,3]. In fact, this theorem was put forward before and after the author, but it was put forward as the reciprocity theorem [4,5,6]. Later, the principle of mutual energy was put forward [7] by the author. There are two current elements in the mutual energy theorem, which is extended to N by the author. It is assumed that there are only these N charges in the universe, so all electromagnetic energy exchange can only be carried out in these N charges. At this time, the author found that this energy theorem is obviously the law of conservation of energy. It means that if the energy of one of the N charges increases, the energy of the other charges must decrease by the same amount, so the total energy remains unchanged. Therefore, the mutual energy theorem can be upgraded to the law of conservation of energy. The author found that the principle of mutual energy supports this law of energy conservation, so the principle of mutual energy is also a law of energy conservation in a broader sense. The author knows that Poynting's theorem in classical electromagnetic theory is the law of conservation of energy. Therefore, the author attempts to prove that the mutual energy theorem is the law of conservation of energy by Poynting theorem. It is proved from Poynting's theorem that the mutual energy theorem is an energy theorem, and there is no problem. However, to prove that the mutual energy theorem is the law of energy conservation, it requires that all self energy terms do not transfer energy. However, from any antenna, we know that the energy represented by Poynting vector is radiated outward. Therefore, this self energy term is to transfer energy. This leads to contradictions. In order to solve this contradiction, the author has put forward the concept of wave reverse collapse. Reverse collapse consists of time reversal waves. By adding the wave of reverse collapse, that is, the wave of time reversal, the energy flow represented by Poynting vector will no longer propagate energy [7,8,9,10].

However, the author is not very satisfied with his proposed time reversal wave scheme. He feels that this time reversal wave is like a loophole in programming, so he makes a patch. This patch solves one problem, but it may cause a series of other problems at the same time, so it is not a fundamental solution. The author discusses the loopholes of Poynting's theorem and hopes to put forward a better or more thorough solution. Therefore, this paper only discusses the loopholes of Poynting's theorem or the conflict in classical electromagnetic field theory detail. A example with Plane-sheet currents will be applied to show the problems. A new solution is given in which the electromagnetic fields of the solution of Maxwell's equations are reinterpreted.

By the way few years ago the author have a preprint about the bugs in Poynting theorem [11]. It should be noticed that this article has written earlier, however published later than some recent other articles of the author, which are [12,13,14,15,16]. There is a process in which the author's ideas mature. This paper is one of the author's fierce ideological struggles. Some views are not yet fully mature. However, the author still wants readers to understand the whole process of the development of the author's ideas.

2. The Author's Mutual Energy Theory

This loophole was discovered in 2016. Due to the discovery of this loophole, the author patched the classical electromagnetic theory and put forward the so-called self energy principle. There is a time reversal wave, which can offset the energy represented by the Poynting vector. In this way, this part of energy will not overflow the universe. The author thinks that our universe should be translucent. Because our universe does not have a shell, which can absorb all the energy of electromagnetic waves. There is dust in the universe, but light can always pass around it. There are stars in the universe, but in any surface surrounding the light source, stars always account for a very small part, and most of the light still passes through this surface. So no matter how big the universe is, there will always be some energy escaping from the universe. But this is impossible because it violates the principle of energy conservation. This requires a self energy principle to establish a time reversal wave to offset the energy of all waves. The propagation of energy is completed by mutual energy flow.

2.1. Axiom (1) the Law of Conservation of Energy

In 1987, the author proposed the mutual energy theorem [1,2,3], and after 2017, extended this theorem to the law of conservation of energy [7,8,9,10], and proposed the following law of conservation of energy for N current elements $\boldsymbol{J}_i$, $i=1,\cdots N$ in electromagnetic theory,

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\int_{t=-\infty}^{\infty}dt\iiint_V(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dV=0 \qquad (1)$$

The above law of conservation of energy is self-evident. This is because the above formula shows that if the charge $\boldsymbol{J}_i$ gives to $\boldsymbol{J}_j$ an action $\iiint_V(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dV$, there must be a reaction $\iiint_V(\boldsymbol{E}_j\cdot\boldsymbol{J}_i)dV$ makes the current element $\boldsymbol{J}_j$ increased energy and current element $\boldsymbol{J}_i$ the reduced energy, and the amount of increase and reduction are equal. So keep the total energy constant. The left side of the above formula (1) is the change of total energy. This energy change is equal to zero. This is correct and self-evident. There is no need to prove it. The author puts forward the formula (1) as the first axiom of electromagnetic field theory. It can be used to verify the classical electromagnetic theory. In other words, if the classical electromagnetic theory conflicts with this law of energy conservation, we need to revise the classical electromagnetic theory.

Substituting $N=2$, the energy conservation law Eq.(1) becomes,

$$-\int_{t=-\infty}^{\infty}dt\iiint_{V_1}(\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dV=\int_{t=-\infty}^{\infty}dt\iiint_{V_2}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2)dV \qquad (2)$$

The above is Welch time-domain reciprocity theorem too [6]. The Fourier transform of the above formula is the following,

$$-\iiint_{V_1}(\boldsymbol{E}_2^*\cdot\boldsymbol{J}_1)dV=\iiint_{V_2}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2^*)dV \qquad (3)$$

The author call the above formula the mutual energy theorem [1]. Now it is energy conservation law for $N=2$.

2.2. Axiom (2) Radiant Energy should not Overflow the Universe

The author believes that if the electromagnetic field formula or viewpoint is self-evident, it should be put forward as an axiom of electromagnetic field. Axioms do not have to be limited to Maxwell's equations, axioms can be more. There are some axioms for verification. For example, the law of conservation of energy proposed by the author in the previous sub-section is one. Next, the author proposes another axiom, which should also be self-evident.

If there is a sphere with infinite radius, electromagnetic radiation should not overflow this sphere, because then electromagnetic energy will radiate outside the universe, and our universe will lose energy constantly, so energy will not be conserved. Therefore, this view of point should also be put forward as an axiom of electromagnetism. If the classical electromagnetic theory conflicts with this axiom, it is necessary to modify the classical electromagnetic theory. This axiom is self-evident. According to this axiom, in today's electromagnetic wave theory, an electromagnetic wave can be generated when there is a current change on an antenna. This electromagnetic wave can continue to propagate until it overflows the sphere with infinite radius, which must be problematic. Electromagnetic radiation should be related to electromagnetic absorption. Electromagnetic radiation will terminate on all absorbers. Electromagnetic radiation will not move to space without absorbers. This view is the view of Wheeler Feynman's absorber theory [17,18]. Our classical electromagnetic theory today does not have a response of this principle. Quantum mechanics has wave collapse. In quantum mechanics, all waves, including those describing photons in Maxwell's equations, collapse to a point. With the concept of collapse, at least energy will not overflow the universe.

With this principle, the classical electromagnetic theory is obviously wrong. The energy flow represented by Poynting vector will spread to the depths of the universe until it finally overflows the universe.

2.3. Axiom (3) the Mutual Energy Principle

The author found that the loophole of Poynting's theorem was around 2016. At that time, the author has established the mutual energy principle [7,8,9,10]. Suppose there are N current elements $\boldsymbol{J}_i$, $i=1,\cdots N$, their field $\xi_i=[\boldsymbol{E}_i,\boldsymbol{H}_i]^T$, superscript T indicates matrix transpose. ξ_i satisfying the following formula of mutual energy principle,

$$\begin{aligned}&-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma\\&=\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\iiint_V(\boldsymbol{E}_i\cdot\boldsymbol{J}_j+\boldsymbol{E}_i\cdot\frac{\partial}{\partial t}\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\frac{\partial}{\partial t}\boldsymbol{B}_j)dV\end{aligned}\tag{4}$$

Starting from this formula, the law of conservation of energy (1) can be proved [7]. Therefore, if (4) can be deduced (1), (4) should also be a broader law of energy conservation.

2.4. The Mutual Energy Flow Theorem

The mutual energy flow theorem can be proved by the mutual energy principle [7,8,9,10]

$$\begin{aligned}&-\int_{t=-\infty}^{\infty}dt\iiint_{V_1}(\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dV=(\xi_1,\xi_2)\\&=\int_{t=-\infty}^{\infty}dt\iiint_{V_2}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2)dV\end{aligned}\tag{5}$$

where

$$(\xi_1,\xi_2)=\int_{t=-\infty}^{\infty}dt\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma\tag{6}$$

Γ is a surface which can segment $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. For example, Γ is sphere surround $\boldsymbol{J}_1$ or $\boldsymbol{J}_2$. For example, Γ is plane separate $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. Of course, this mutual energy flow theorem is also the energy flow theorem. Its existence makes the law of conservation of energy called the law of localized conservation of energy. If in the Fourier frequency domain, the above two equations are,

$$-\iiint_{V_1}(\boldsymbol{E}_2^*\cdot\boldsymbol{J}_1)dV=(\xi_1,\xi_2)=\iiint_{V_2}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2^*)dV\tag{7}$$

where

$$(\xi_1,\xi_2)=\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma\tag{8}$$

The mutual energy flow theorem is proposed, which makes the author more convinced of the correctness of the law of energy conservation and the mutual energy principle Eq.(1, 4).

3. Bugs in Poynting's Theorem

Poynting theorem is the law of conservation of energy in classical electromagnetic theory. We often only use it to prove matters related to energy. Sometimes it is also used to calculate, for example, the energy per square meter of the sun to the earth. However, if we use the energy flow corresponding to the Poynting vector multiplied by the section area of the wire antenna as the energy received by the wire antenna, then this energy is much smaller than the energy actually received by the antenna. In order to still make Poynting's theorem available, we must use a so-called concept of effective scattering cross section. This effective scattering cross section is often hundreds of times that of the section area of the wire antenna. In fact, this has told us that Poynting's theorem is problematic. These problems belong to whether Poynting vector can represent energy flow. The author puts forward the concept of mutual energy flow and proves the theorem of mutual energy flow. Therefore, it is considered that mutual energy flow is the real electromagnetic energy flow. However, it seems that Poynting's theorem can still be used to calculate the radiant energy of the whole antenna. In this chapter, we will discuss the problems of Poynting's theorem in calculating the radiated energy of antenna.

3.1. Poynting Theorem

Poynting's theorem can be deduced from Maxwell's equation [19],

$$\begin{aligned}&-\oiint_{\Gamma}(\boldsymbol{E}\times\boldsymbol{H})\cdot\hat{n}d\Gamma\\&=\iiint_V(\boldsymbol{E}\cdot\boldsymbol{J}+\boldsymbol{E}\cdot\frac{\partial}{\partial t}\boldsymbol{D}+\boldsymbol{H}\cdot\frac{\partial}{\partial t}\boldsymbol{B})dV\end{aligned}\tag{9}$$

3.2. Complex Poynting Theorem

The complex Poynting theorem can be written as,

$$\begin{aligned}&-\oiint_{\Gamma}(\boldsymbol{E}\times\boldsymbol{H}^*)\cdot\hat{n}d\Gamma\\&=\iiint_V(\boldsymbol{E}\cdot\boldsymbol{J}^*+j\omega(\mu_0\boldsymbol{H}\cdot\boldsymbol{H}^*-\epsilon_0\boldsymbol{E}\cdot\boldsymbol{E}^*))dV\end{aligned}\tag{10}$$

The energy of $j\omega(\mu_0\boldsymbol{H}\cdot\boldsymbol{H}^*-\epsilon_0\boldsymbol{E}\cdot\boldsymbol{E}^*)$ is a pure imaginary number, where $j=\sqrt{-1}$. Its real part is zero, so it has no contribution to energy transmission. If we only care about the active power part, the energy storage of inductance and capacitance can be ignored. So we get,

$$\oiint_{\Gamma}(\boldsymbol{E}\times\boldsymbol{H}^*)\cdot\hat{n}d\Gamma=-\iiint_V(\boldsymbol{E}\cdot\boldsymbol{J}^*)dV\tag{11}$$

If take the real part of the above formula, it is exact correct formula. In most situation we only care the real part of a complex formula, hence we can omit the image part in Eq.(10).

3.3. Poynting Theorem of N Current Elements

Considering that there are N current elements in V, considering the superposition principle, the corresponding electromagnetic fields are,

$$\boldsymbol{J}=\sum_{i=1}^{N}\boldsymbol{J}_i,\ \boldsymbol{E}=\sum_{i=1}^{N}\boldsymbol{E}_i,\ \boldsymbol{H}=\sum_{i=1}^{N}\boldsymbol{H}_i\tag{12}$$

The above formulas are substituted into Poynting's theorem to obtain the Poynting's theorem of N current elements,

$$-\sum_{i=1}^{N}\sum_{j=1}^{N}\oint\!\!\!\oint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma = \sum_{i=1}^{N}\sum_{j=1}^{N}\iiint_V(\boldsymbol{E}_i\cdot\boldsymbol{J}_j+\boldsymbol{E}_i\cdot\frac{\partial}{\partial t}\boldsymbol{D}_j+\boldsymbol{H}_i\cdot\frac{\partial}{\partial t}\boldsymbol{B}_j)dV \quad (13)$$

We find that the Poynting theorem (13) is very close to the formula of the mutual energy principle (4). The only difference is that the summation symbol is different,

$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \; vs \sum_{i=1}^{N}\sum_{j=1}^{N} \quad (14)$$

The difference between the two formula (4,13) is,

$$-\sum_{i=1}^{N}\oint\!\!\!\oint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma = \sum_{i=1}^{N}\iiint_V(\boldsymbol{E}_i\cdot\boldsymbol{J}_i+\boldsymbol{E}_i\cdot\frac{\partial}{\partial t}\boldsymbol{D}_i+\boldsymbol{H}_i\cdot\frac{\partial}{\partial t}\boldsymbol{B}_i)dV \quad (15)$$

The formula (15) is the sum of Poynting's theorem of N current elements. Therefore, it is a correct energy formula. From the above, we can conclude that Poynting's theorem can prove that the principle of mutual energy is an energy theorem. Because both formula (13) and formula (15) are Poynting's theorem. The principle of mutual energy (4) can be obtained by subtracting the two formulas. Therefore, the principle of mutual energy can be proved by Poynting's theorem. Poynting theorem is an energy theorem, so the principle of mutual energy (4) is also an energy theorem.

However, the author is not satisfied with proving that the principle of mutual energy is an energy theorem from Poynting's theorem. The author hopes to assume that Poynting's theorem is the law of conservation of energy and prove that the principle of mutual energy is also the law of conservation of energy. Let's compare the two laws of energy conservation.

We know that Poynting's theorem is the recognized law of energy conservation in electromagnetic field theory, and the principle of mutual energy is the law of energy conservation discovered by the author. They both should be correct. Therefore, their difference formula (15) should not transfer energy. But (15) is the sum of Poynting's theorem of N current elements. From the antenna radiation theory, we know that the energy flow corresponding to these Poynting vectors is not zero. It does deliver energy. This constitutes a conflict if Poynting's theorem Eq.(15) transfers energy. The formula (13) transmits more energy than (4). We have two different laws of conservation of energy. Our electromagnetic theory is in trouble!

The author believes that the law of conservation of energy (1) is reliable. (4) and (1) do not conflict, so they can be trusted. Then the problem must lie in Poynting's theorem (13) and hence (9).

3.4. The Author has Tried to Modify the Classical Electromagnetic Theory

The author's first attempt is to add a time reversal wave [7]. In this way, considering the time reversal wave, the self energy item corresponding to Poynting energy (15) does not transfer energy. With this time reversal wave, or the reverse collapse of wave, the wave collapse in quantum mechanics can be completed by the process of wave reverse collapse + mutual energy flow. This can be a good interpretation of quantum mechanics. However, the author is not very satisfied with this amendment. Because this correction is more like a patch to the classical electromagnetic theory. It corrects some problems and will cause other problems. The following is an example to illustrate the problem and give a new solution.

4. Example Using Pane-sheet Currents

Suppose there is an infinite plane-sheet current, and the current direction is along the z axis. Current density $\boldsymbol{J}=\boldsymbol{J}_0\exp(j\omega t)$. The electromagnetic field of plane-sheet current is calculated according to the classical electromagnetic field theory. The classical electromagnetic field theory can first quantitatively obtain the magnetic field according to the integral Maxwell-Ampere circuital law, and then quantitatively obtain the electric field from the differential Maxwell-Ampere circuital law. We first obtain the electromagnetic field indicated by the current. Then consider that the electromagnetic field of plane-sheet current can only be plane wave, and calculate the electromagnetic field of other points in space. On the surface of the current,

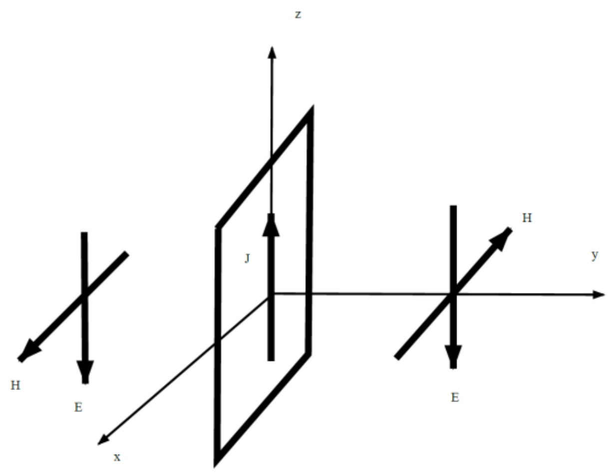

Figure 1. The retarded wave of the plane-sheet current

4.1. Electromagnetic Field of Plane-Sheet Current

$$\oint_C(\boldsymbol{H}_0\cdot d\boldsymbol{l})=\int_L J_0 dl \quad (16)$$

or $H_0=\frac{J_0}{2}$. Thus, the magnetic field at any point in space is,

$$\boldsymbol{H}=H_0\exp(j(\omega t-ky))(-\hat{x}) \quad (17)$$

Electric field can be obtained through $\nabla\times\boldsymbol{H}=\epsilon_0\frac{\partial\boldsymbol{E}}{\partial t}$,

$$(-j\boldsymbol{k})\times\boldsymbol{H}=\epsilon_0 j\omega\boldsymbol{E} \tag{18}$$

$$-\boldsymbol{k}\times\boldsymbol{H}=\epsilon_0\omega\boldsymbol{E} \tag{19}$$

$$\boldsymbol{E}=-\frac{1}{\epsilon_0\omega}\boldsymbol{k}\times\boldsymbol{H}=-\frac{k}{\epsilon_0\omega}\hat{y}\times\boldsymbol{H} \tag{20}$$

$$=\eta_0 H_0\exp(j(\omega t-ky))(-\hat{z}) \tag{21}$$

Among them the following is considered,

$$\boldsymbol{k}=k\hat{y}, k=\frac{\omega}{c}=\omega\sqrt{\epsilon_0\mu_0}, \eta_0=\sqrt{\frac{\mu_0}{\epsilon_0}} \tag{22}$$

The Poynting vector to the right of the current is,

$$\begin{aligned}\boldsymbol{S}_{right}&=\boldsymbol{E}\times\boldsymbol{H}^*\\&=(\eta_0 H_0\exp(j(\omega t-ky))(-\hat{z}))\\&\quad\times(H_0\exp(j(\omega t-ky))(-\hat{x}))^*\end{aligned} \tag{23}$$

$$=(\eta_0\times H_0(-\hat{z}))\times(H_0(-\hat{x}))=\eta_0 H_0^2\hat{y}=\frac{J_0^2}{4}\eta_0\hat{y} \tag{24}$$

The Poynting vector of the retarded wave points out of the current. This is correct. The above formula shows that there is on the right side of the current,

$$\boldsymbol{S}_{right}\cdot\hat{y}=\frac{J_0^2}{4}\eta_0 \tag{25}$$

Similarly, on the left side of the current,

$$\boldsymbol{S}_{left}\cdot(-\hat{y})=\frac{J_0^2}{4}\eta_0 \tag{26}$$

Calculate the power provided by the current to the system,

$$\begin{aligned}-\boldsymbol{E}\cdot\boldsymbol{J}^*&=-(\frac{J_0\eta_0}{2}\exp(j(\omega t))^*(-\hat{z}))\cdot J_0\exp(j\omega t)\hat{z}\\&=\frac{J_0^2\eta_0}{2}\end{aligned} \tag{27}$$

So we verified that,

$$\boldsymbol{S}_{right}\cdot\hat{y}+\boldsymbol{S}_{left}\cdot(-\hat{y})=-\boldsymbol{E}\cdot\boldsymbol{J}^* \tag{28}$$

For plane-sheet current, this means that complex Poynting's theorem Eq.(11) is satisfied.

4.2. Consideration of Advanced Wave

For magnetic field advanced wave the calculation of H_0 has not changed, $H_0=\frac{J_0}{2}$

$$\boldsymbol{H}=H_0\exp(j(\omega t+ky))(-\hat{x}) \tag{29}$$

In space, the current density is zero. According to Maxwell ampere theorem, there is $\nabla\times\boldsymbol{H}=\epsilon_0\frac{\partial\boldsymbol{E}}{\partial t}$

$$(j\boldsymbol{k})\times\boldsymbol{H}=\epsilon_0 j\omega\boldsymbol{E} \tag{30}$$

$$\begin{aligned}\boldsymbol{E}&=\frac{1}{\epsilon_0\omega}\boldsymbol{k}\times\boldsymbol{H}=\frac{k}{\epsilon_0\omega}\hat{y}\times\boldsymbol{H}\\&=\eta_0 H_0\exp(j(\omega t+ky))\hat{z}\end{aligned} \tag{31}$$

The electric field and current of the advanced wave are in the same direction. The electric field of the retarded wave is opposite to the direction of the current. Poynting vector,

$$\boldsymbol{S}_{right}=\boldsymbol{E}\times\boldsymbol{H}^*=-\eta_0 H_0^2\hat{y}=-\frac{J_0^2}{4}\eta_0\hat{y} \tag{32}$$

$$\boldsymbol{S}_{right}\cdot\hat{y}=-\frac{J_0^2}{4}\eta_0 \tag{33}$$

It is also correct that the Poynting vector of the advanced wave points to the interior of the current. Similarly,

$$\boldsymbol{S}_{left}\cdot(-\hat{y})=-\frac{J_0^2}{4}\eta_0 \tag{34}$$

calculate

$$-\boldsymbol{E}\cdot\boldsymbol{J}^*=-\frac{J_0^2\eta_0}{2} \tag{35}$$

In this way, we verify that the following formula is true,

$$\boldsymbol{S}_{right}\cdot\hat{y}+\boldsymbol{S}_{left}\cdot(-\hat{y})=-\boldsymbol{E}\cdot\boldsymbol{J}^* \tag{36}$$

The above formula shows that complex Poynting's theorem Eq.(11) is satisfied for advanced wave. It can be seen that for the electromagnetic field of plane-sheet current, the Poynting theorem of retarded wave and advanced wave is satisfied.

4.3. Leftward and Rightward Waves

Current can send out retarded wave or advanced wave. According to Wheeler and Feynman's absorber theory, the current can send out half retarded and half advanced waves. So when does the current send out retarded wave and advanced wave? When will half retarded and half advanced waves be sent out? In fact, the wave generated by the current itself belongs to self energy flow. Self energy flow does not transfer energy. This self energy either has a time reversal wave to offset it, or it itself is a reactive power wave, so the energy generated will return automatically. For this reason, the wave emitted by the current can be selected arbitrarily. If it is not selected, it is considered as an invalid wave. This wave can be emitted, but it does not work. It is an invalid wave.

In the author's mutual energy theory the advanced wave is accept as physical objective existence. Hence, the possible wave forms of plane-sheet current can be the retarded wave, see upper left of the Figure 2, advanced wave upper right of the Figure 2, half retarded wave and half advanced wave, see the middle of the Figure 2. Rightward wave, see lower left of the Figure 2. Leftward wave, see lower right of the Figure 2.

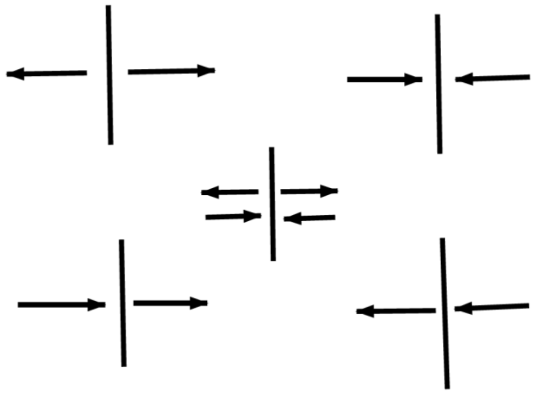

Figure 2. Possible waves of plane-sheet currents

For this reason, the current can also send out a rightward wave or a leftward wave. The rightward wave is a wave propagating to the right. This wave is a retarded wave on the right side of the current and a advanced wave on the left side of the current. The same is true for the leftward wave. The left side of the current is the retarded wave and the right side is the advanced wave. Next we will study the transformer. In this case, the electromagnetic field generated by the current is a leftward wave or a rightward wave.

4.4. Double Plane-sheet Current Transformer

It is assumed that the primary coil and secondary coil of the transformer are infinite plane-sheet currents. The secondary coil is at the left of the primary coil. Of course, the energy is transferred from the primary coil to the secondary coil. Therefore, we choose the primary coil of the transformer to generate the rightward wave. In this way, energy can be sent to the secondary coil. The secondary coil also is selected the as rightward wave, so that the wave of the secondary coil can be synchronized with the primary coil to absorb electromagnetic energy. See Figure 3.

For the primary coil, we assume that it is powered by a current source, so the current is a constant alternating current. For the secondary coil, we assume that its conductivity is σ. It is assumed that the two ends of the secondary coil are connected by superconducting wires. Therefore, the resistance R_2 of the secondary coil is determined by conductivity. Assuming this conductivity is small enough, $R_2 \gg j\omega L_2$, L_2 is the self inductance of the secondary coil. Therefore, the impedance of the secondary coil is resistive. Therefore, the current of the secondary coil is consistent with the direction of the induced electric field on the secondary coil.

From the above, we know that the electromagnetic field on the right side of the primary coil is

$$\boldsymbol{H}_1 = \frac{J_{10}}{2}\exp(j(\omega t - ky))(-\hat{x}) \quad (37)$$

$$\boldsymbol{E}_1 = \eta_0 \frac{J_{10}}{2}\exp(j(\omega t - ky))(-\hat{z}) \quad (38)$$

The induced electric field on the secondary coil caused by the current of the primary coil is, $E_1\left(y = L\right)$. The current of the secondary coil is,

$$\boldsymbol{J}_2 = E_1(y = L)\sigma(-\hat{z}) \quad (39)$$

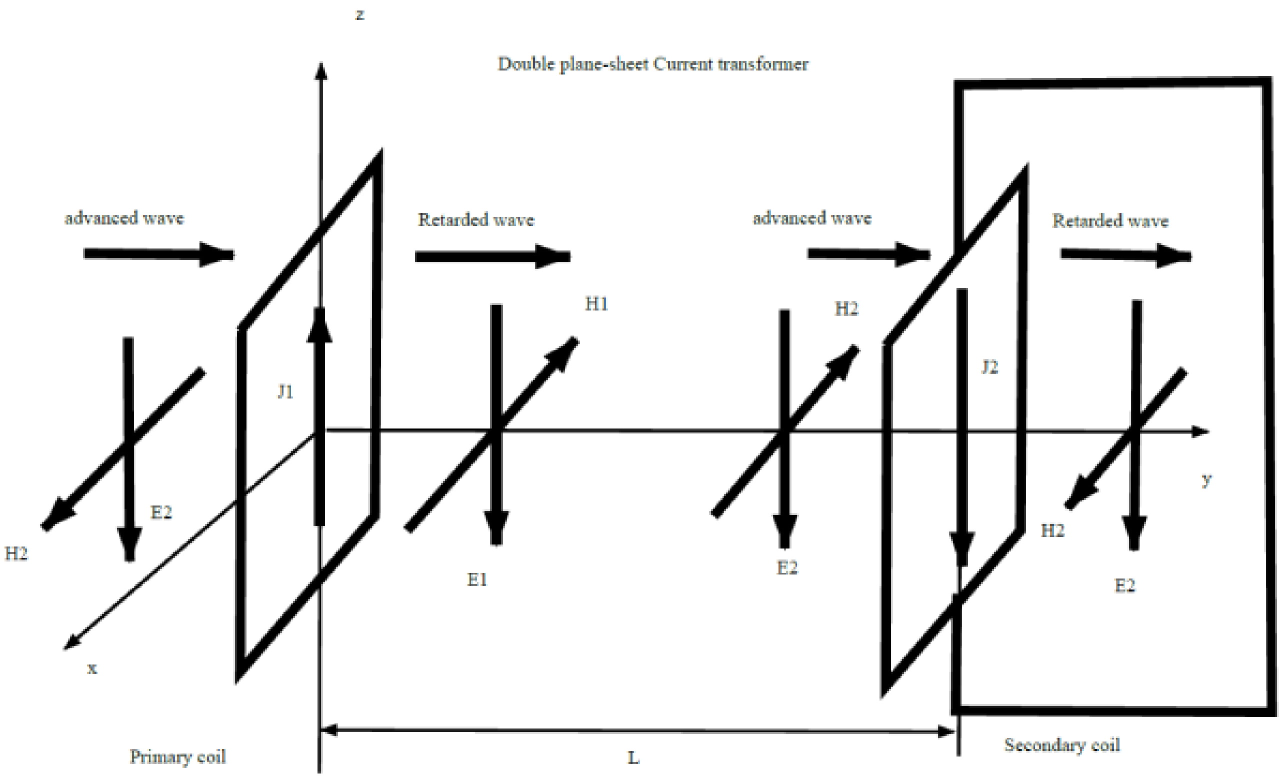

Figure 3. Double plane-sheet current transformer

For simplicity, we can choose the secondary coil and the primary coil close together, i.e. $L=0$. The magnetic field generated by the secondary coil to the left of its current is

$$H_2 = \frac{J_{20}}{2}\exp(j(\omega t - ky))(-\hat{x}) \tag{40}$$

$$\begin{aligned}\boldsymbol{E}_2 &= \frac{\eta_0 J_{20}}{2}\exp(j(\omega t - ky))(-\hat{y})\times(-\hat{x}) \\ &= \frac{\eta_0 J_{20}}{2}\exp(j(\omega t - ky))(-\hat{z})\end{aligned}$$

$J_{20} = E_{10}(y=L)\sigma$. When $L=0$,

$$J_{20} = \eta_0 \frac{J_{10}}{2}\sigma \tag{41}$$

Calculate the mutual energy flow,

$$\begin{aligned}\boldsymbol{S}_m &= \boldsymbol{S}_{12} + \boldsymbol{S}_{21} = \boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1 \\ &= \eta_0 \frac{J_{10}}{2}\exp(j(\omega t - ky))(-\hat{z}) \\ &\quad \times \frac{J_{20}}{2}\exp(j(\omega t - ky))^*(-\hat{x}) \\ &\quad + \frac{\eta_0 J_{20}}{2}\exp(j(\omega t - ky))^*(-\hat{z}) \\ &\quad \times \frac{J_{10}}{2}\exp(j(\omega t - ky))(-\hat{x}) \\ &= \eta_0 \frac{1}{2}J_{10}J_{20}\hat{y} = \eta_0 \frac{1}{2}J_{10}(\eta_0 \frac{J_{10}}{2}\sigma)\hat{y} = \frac{1}{4}\eta_0^2 J_{10}^2 \sigma \hat{y}\end{aligned} \tag{42}$$

It can be seen that the mutual energy flow is from the primary to the secondary coil. In the above calculation, we consider $L=0$, even if the $L \neq 0$ conclusion will not change. In addition, we also chose $R_2 \gg j\omega L_2$ this will not affect the calculation results. The mutual energy flow is always from the primary coil to the secondary coil, and it is active power.

Next, let's verify that the mutual energy flow theorem is satisfied. Calculate the power provided by the primary coil per unit area,

$$P_1 = -\boldsymbol{E}_2^* \cdot \boldsymbol{J}_1 \tag{43}$$

$$\begin{aligned}&= -(\frac{\eta_0 J_{20}}{2}\exp(j(\omega t))(-\hat{z}))^* \cdot \boldsymbol{J}_{10}\exp(j\omega t)\hat{z} \\ &= \frac{\eta_0 J_{20}\boldsymbol{J}_{10}}{2} = \frac{\eta_0 \boldsymbol{J}_{10}}{2}(\eta_0 \frac{J_{10}}{2}\sigma) = \frac{1}{4}\eta_0^2 J_{10}^2 \sigma\end{aligned} \tag{44}$$

Calculate the power received by the secondary coil per unit area,

$$\begin{aligned}P_2 &= \boldsymbol{E}_1 \cdot \boldsymbol{J}_2^* \\ &= (\eta_0 \frac{J_{10}}{2}\exp(j(\omega t))(-\hat{z})) \cdot (E_1(y=0)\sigma(-\hat{z}))^* \\ &= (\eta_0 \frac{J_{10}}{2}) \cdot (\eta_0 \frac{J_{10}}{2}\sigma) = \frac{\sigma \eta_0^2 J_{10}^2}{4}\end{aligned} \tag{45}$$

This means that,

$$P_1 = S_m = P_2 \tag{46}$$

Thus, the following mutual energy flow theorem Eq.(7) is verified. It can be seen that the mutual energy flow theorem is satisfied for the double plane-sheet current transformer.

4.5. Outside of Double Plane-sheet Current Transformer

Now consider changing the direction of the magnetic field of the secondary coil on the right side of the secondary coil and on the left side of the primary coil of the transformer, the direction of the $\boldsymbol{H}_2$ is reversed. The sign of $\boldsymbol{E}_1 \times \boldsymbol{H}_2{}^*$ changes. Similarly, on the left side of the primary coil, due to the reversal of the magnetic field of the primary coil, the sign of $\boldsymbol{E}_2^* \times \boldsymbol{H}_1$ is changed. Therefore, the mutual energy flow outside the transformer is zero. So, the mutual energy flow is,

$$S_m = \begin{cases} 0 & -\infty < y < 0 \\ \frac{1}{4}\eta_0^2 J_{10}^2 \sigma \hat{y} & 0 < y < L \\ 0 & L < y < \infty \end{cases} \tag{47}$$

Above, it is assumed that the primary coil is at $y=0$ and the secondary coil is at $y=L$. However, L can be large, small or even close to zero. When $L=0$, the system is a transformer including primary coil and secondary coil. When the distance of L is very large, the system has actually become an antenna system, including a transmitting antenna at $y=0$ and a receiving antenna at $y=L$. Therefore, in fact, antenna system is the promotion of transformer system.

The above formula shows that the mutual energy flow is generated on the primary coil and annihilated on the secondary coil. Therefore, the mutual energy has the natures of photons, which are also generated by the primary coil (light source) and then annihilated on the secondary coil (light sink). The description of photons by mutual energy flow is very close to that of John Cramer in the transactional interpretation of quantum mechanical [20,21].

4.6. Self-energy Flow is Reactive Power

The author calls the energy flow described by Poynting theorem as self energy flow. From the above, we can clearly see that self energy flow is energy flow density, and Poynting vector describes the external radiation of current. We can also consider that the mutual energy flow transfers energy. In this way, the mutual energy flow describes that the energy flow is generated on the primary coil and annihilated on the secondary coil. Self energy flow and mutual energy flow are all good. However, if we think that both self energy flow and mutual energy flow transfer energy flow, it constitutes a contradiction. Because the self energy flow alone just describes the radiation of current. Mutual energy flow just describes the energy flow from primary to secondary. If they are added up, the sum of mutual energy flow and self energy flow is greater than the energy generated by the primary coil,

$$\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_1^*)\cdot nd\Gamma+\iint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma > -\iiint_V(\boldsymbol{E}_1\cdot\boldsymbol{J}_1^*)dV \tag{48}$$

The self energy flow and the mutual energy flow together will lager than the mutual energy created on the plane sheet current,

$$\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_1^*)\cdot nd\Gamma+\iint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma > -\iiint_V(\boldsymbol{E}_2\cdot\boldsymbol{J}_1^*)dV \tag{49}$$

Therefore, only one of self energy or mutual energy flow can really transfer energy, and they cannot both transfer energy.

For the traditional electromagnetic field theory, it is considered to be self energy flow, that is, Poynting vector transmits energy. The author believes that this is incorrect. The author supports mutual energy flow to transfer energy.

Since mutual energy flow transfers energy, how can we make self energy flow not transfer energy? The previous author introduced a method to add a time reversal wave to Maxwell's theory. Let the energy flow of time reversal wave offset the self energy flow. However, the author is not satisfied with this method, because the self energy flow is offset, and the time reversed wave may also produce the time reversed mutual energy flow, which can also offset the mutual energy flow. In this way, the whole system has no energy flow at all. It is difficult to find a way to only offset the self energy flow without offsetting the mutual energy flow.

Another way is to reinterpret Maxwell's theory. It is believed that Maxwell's theory is still correct, but the Poynting vector in Poynting theorem is not a self energy flow, but some kind of the mutual energy flow. Poynting vector

$$\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}\neq\boldsymbol{E}_1\times\boldsymbol{H}_1 \tag{50}$$

$$\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}=\boldsymbol{E}_2\times\boldsymbol{H}_1 \tag{51}$$

Here, it is assumed that there are two secondary coil on both sides of the plane-sheet current, and the two secondary coils have a good absorption of the energy flow emitted by the primary coil. $\boldsymbol{H}_1$ is the magnetic field of primary coil. Hence, $\boldsymbol{H}=\boldsymbol{H}_1$ it shows that the magnetic field in Poynting vector is the magnetic field of the plane-sheet current. $\boldsymbol{E}=\boldsymbol{E}_2$ it shows that the electric field in the Poynting vector is actually the electric field generated by such a secondary coil, not the electric field of the primary coil itself!

Then the electric field $\boldsymbol{E}_1$ of the original primary coil, what should it be? $\boldsymbol{E}_1$ is a plane wave. We just need to find the electric field on the current surface $\boldsymbol{E}_{10}$. Hence,

$$\boldsymbol{E}_1=\boldsymbol{E}_{10}\exp(j(\omega t-ky)) \tag{52}$$

Under the method of magnetic static field, according to Faraday's law

$$\boldsymbol{E}_1=-j\omega\boldsymbol{A}=-j\omega\frac{\mu_0}{4\pi}\iint_{\Gamma}\frac{(J\hat{z})d\Gamma}{r} \tag{53}$$

Please note that we use the magnetic quasi-static potential to determine the phase of the electric field and not the retarded potential.

$$\boldsymbol{E}_1\sim -j\boldsymbol{J}_1 \tag{54}$$

The symbol ~ is only phase sensitive and not value sensitive. So there is,

$$\boldsymbol{E}_1=j\eta_0\frac{J_{10}}{2}\exp(j(\omega t-ky))(-\hat{z}) \tag{55}$$

In this way, the electric and magnetic fields,

$$\boldsymbol{H}_1=\frac{J_{10}}{2}\exp(j(\omega t-ky))(-\hat{x}) \tag{56}$$

There is a phase difference (j). Therefore

$$\Re(\boldsymbol{E}_1\times\boldsymbol{H}_1^*)=0 \tag{57}$$

$\Re$ is to take the real part. In this way, the self energy flow is reactive power. Similarly, there is $\Re(\boldsymbol{E}_2\times\boldsymbol{H}_2^*)=0$.

4.7. Recalculate Mutual Energy Flow

Because we adjust the electric field and add a phase factor of *j*, the mutual energy flow must also be recalculated.

$$\boldsymbol{H}_1=\frac{J_{10}}{2}\exp(j(\omega t-ky))(-\hat{x}) \tag{58}$$

$$\boldsymbol{E}_1=j\eta_0\frac{J_{10}}{2}\exp(j(\omega t-ky))(-\hat{z}) \tag{59}$$

$\boldsymbol{J}_2=\boldsymbol{E}_1(y=L)$. For simplicity, $L=0$. Indicates that the secondary coil is next to the primary coil. The magnetic field is obtained according to the original method.

$$\boldsymbol{H}_2=j\frac{J_{20}}{2}\exp(j(\omega t-ky))(-\hat{x}) \tag{60}$$

Considering a phase difference of (-*j*) in the electric field,

$$\begin{aligned}\boldsymbol{E}_2&=(-j)j\frac{\eta_0 J_{20}}{2}\exp(j(\omega t-ky))(-\hat{y})\times(-\hat{x})\\&=\frac{\eta_0 J_{20}}{2}\exp(j(\omega t-ky))(-\hat{z})\end{aligned} \tag{61}$$

Calculation of mutual energy flow,

$$\begin{aligned}\boldsymbol{S}_m&=\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1\\&=(j\eta_0\frac{J_{10}}{2}\exp(j(\omega t-ky))(-\hat{z}))\\&\quad\times(j\frac{J_{20}}{2}\exp(j(\omega t-ky))(-\hat{x}))^*\\&\quad+(\frac{\eta_0 J_{20}}{2}\exp(j(\omega t-ky))(-\hat{z}))^*\\&\quad\times\frac{J_{10}}{2}\exp(j(\omega t-ky))(-\hat{x})\\&=\frac{1}{4}\eta_0^2 J_{10}^2\sigma\hat{y}\end{aligned} \tag{62}$$

It can be seen that the mutual energy flow does not change because we redefine the electric field. On the outside of the transformer, the mutual energy flow still as 0. Hence there are still,

$$S_m = \begin{cases} 0 & -\infty < y < 0 \\ \frac{1}{4}\eta_0^2 J_{10}^2 \sigma \hat{y} & 0 < y < L \\ 0 & L < y < \infty \end{cases} \quad (63)$$

This is because the magnetic field reverses on both sides of the current. Therefore, the mutual energy flow is zero outside the transformer.

When we consider adding a phase factor j to the electric field, so that the electric field and magnetic field maintain a 90 degree phase difference, so the Poynting vector is reactive power. In this way, the self energy flow does not transfer energy. Self energy flow does not transfer energy flow does not affect mutual energy flow can transfer energy. This reinterpretation of electromagnetic field avoids the loophole of Poynting's theorem, and does not need to introduce time reversal waves. The only thing to do is to reinterpret the electric field calculated by Maxwell's equation. This electric field is not the electric field of the current itself, but the electric field of the advanced wave generated by the environment of the current.

5. Conclusion

This paper expounds that the Poynting theorem of classical electric field theory is actually flawed, there is loophole. This paper describes the reasons for the discovery of this loophole. Because the author introduced the principle of mutual energy and the theorem of energy conservation. Comparing the Poynting theorem of N current elements and the mutual energy principle of N current elements, it is found that the self energy flow should not transfer energy, because the mutual energy already contains all the energy terms that should appear. However, for the transmitting antenna, the self energy flow term corresponding to Poynting's theorem is obviously not zero. This leads a contradiction. In order to solve the contradiction, the author has introduced time reversal wave. However, the time reversal wave may also produce the mutual energy flow of the time reversal wave, so as to offset the mutual energy flow. The result become a zero solution. Of course, the zero solution is wrong. This paper reinterpret the electromagnetic field solved by Maxwell's equations.

The author re-interpreted the electromagnetic fields obtained by the Maxwell's equation. The magnetic field obtained by the Maxwell's equation still the magnetic field of the plane-sheet current. But the electric field obtained by Maxwell's equations actually is not the electric field of the plane-sheet current, but the electric field of the environment. Here assume the environment can produce advanced wave and can absorb all electromagnetic field radiate out by the plane-sheet current. The author only studied the example of plane-sheet current, but believe this result can be extended to the more general situation.

A example plane-sheet current is applied to show the author's view of point. the example shows the mutual energy flow can explain the energy flow produced from the primary coil and annihilated at the secondary coil. The energy flow corresponding to the Poynting vector cannot achieve this. It is clear that the mutual energy flow has the properties of the photon.

This properties of the mutual energy flow is very similar to the properties of photon in the transactional interpretation of quantum mechanics introduced by John Cramer [20,21]. Hence, the theory of the mutual energy can be seen as a concrete implementation of the transactional interpretation of the quantum mechanics.

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