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Doubly-Excited $^{1,3}S^e$, $^{1,3}P^0$, $^{1,3}D^e$, $^{1,3}F^0$ and $^{1,3}G^e$ Resonances States of Two-Electron Atoms below the N = 3 – 8 Hydrogenic Thresholds

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Abstract In this paper we present accurately calculated data on the resonance parameters (resonance energy and excitation energies) of the doubly excited singlet and triplet states $^{1,3}S^e$, $^{1,3}P^0$, $^{1,3}D^e$, $^{1,3}F^0$ and $^{1,3}G^e$ of helium and helium-like ions (Z = 2 – 10) located below the hydrogenic thresholds N = 3 – 8, using the variational method of the Screening Constant per Unit Nuclear Charge (SCUNC) formalism. These energies are calculated using special form wave functions of the Hylleraas type and a real Hamiltonian. The results obtained are in very good agreement with the experimental and theoretical values available in the literature. The results for the energies of the doubly excited $^{1,3}S^e$, $^{1,3}P^0$, $^{1,3}D^e$, $^{1,3}F^0$ and $^{1,3}G^e$ states associated with hydrogenic thresholds up to N = 8 listed for the first time in this paper may provide a useful guideline for future experimental and theoretical studies in the autoionization states of two-electron systems.

Keywords: *helium-like systems, energy resonances, excitation energy, screening constant by unit nuclear charge (SCUNC), doubly excited states, hylleraas type wave function*

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1. Introduction

The present paper is the third in a series of works devoted to the study of doubly excited states (DES) of two-electron atoms [1,2]. Very recently, we presented precise values on the resonance parameters (resonance energies and excitation energies) of the symmetric doubly excited (nl^2) $^1L^\pi$ states of helium-like ions (Z = 2 – 10) [1] and the doubly excited singlet and triplet states $^{1,3}S^e$, $^{1,3}P^0$, $^{1,3}D^e$, $^{1,3}F^0$, and $^{1,3}G^e$ of the helium isoelectronic sequence located below the N = 2 hydrogenic threshold [2].

The study of doubly excited states of two-electron atoms has become an important subject of theoretical study as it provides a fundamental testing ground for the accuracy of various theoretical calculations. Moreover, the precise determination of the resonance parameters of doubly excited states of different two-electron atoms is of great importance for the analysis of astrophysical data [3], for the diagnosis of lines observed in the solar corona [4], of high-temperature discharges [5] and for the diagnosis of plasmas [6]. It should also be recalled that electronic correlation effects have played a primary role in the dynamics of doubly excited states of helium-like ions, and that studies of these states have contributed considerably to the development of various computational methods in the theory of two-electron atoms [7,8,9]. Since the first observations of these states by Madden and Codling [10,11] in photoabsorption experiments on helium, the study of excited states in two-electron atomic systems remains an active area of research, both experimentally and theoretically. This is due to the multitude of theoretical methods that have been developed over the last two decades for a thorough understanding of these highly correlated states. We note, however, that most of the studies on doubly excited states of atomic systems in the literature concern the determination of the DES resonance parameters of helium-like ions associated with the N = 2, 3 and 4 hydrogenic thresholds.

There are very few results in the literature for higher values of N ($N \geq 5$) and recent interests are now focused on resonances associated with higher hydrogen thresholds (N = 5, 6, 7, 8...). It is within this framework that we decided to present in this paper precise values on the resonance parameters (resonance energies and excitation energies) of the doubly excited states $^{1,3}S^e$, $^{1,3}P^0$, $^{1,3}D^e$, $^{1,3}F^0$, and $^{1,3}G^e$ associated with hydrogenic thresholds N = 3 – 8.

The results are obtained using the variational method of the Screening Constant per Unit Nuclear Charge (SCUNC) formalism. This approach has been successfully applied in the determination of the resonance parameters of the symmetric doubly excited nl^2 (l = s, p, d, f, h and g) states and the singlet and triplet states of the different valence electrons of type $Nlnl'$ associated with the N = 2 hydrogenic threshold of helium and helium-like ions [1,2].

The goal of the present work is to extend our previous work [1,2] to the study of doubly excited states of two-electron atomic systems associated with hydrogenic thresholds $N = 3 – 8$ in order to provide accurate values on the resonance parameters that can be used as a reference for future theoretical and experimental work on doubly excited states for high values of N.

Section 2 presents the theoretical procedure used in this work. In Section 3, a presentation and discussion of our calculations with other theoretical calculations and experimental data are also made.

2. Theory and Calculation

The description of the properties of matter at the atomic scale can be accurately described, taking into account correlations, by solving the time-independent Schrödinger equation

$$\hat{H}\psi(r_1,r_2) = E\psi(r_1,r_2) \tag{1}$$

where $\hat{H}$ represents the Hamiltonian operator of the considered system (atom, molecule, solid), $\psi(r_1,r_2)$ the trial wave function and E the associated energy.

The Hamiltonian H of the helium isoelectronic series in given by (in atomic units)

$$H = -\frac{1}{2}\Delta_1 - \frac{1}{2}\Delta_2 - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r_{12}} \tag{2}$$

Where r_1 and r_2 are the radial coordinates of the two electrons and r_{12} is their relative distance.

However, it is not possible to solve such an equation rigorously, except for single-electron systems, because of the electronic correlation term $r_{12} = |r_1 - r_2|$. One is therefore obliged to resort to approximate solutions. In order to be able to solve this equation in an approximate way several approximations and several methods of theoretical calculations using correlated wave functions had to be proposed.

The trial wave function we used in this work is obtained from the simplest helium wavefunction of Hylleraas [12,13,14] which we modified to fit the doubly excited states $Nln'l'$ of different valence electrons of two-electron atoms (Z = 2 – 10). It is of the form [1,2]:

$$\psi(r_1,r_2) = \frac{1}{4\pi}\sum_{\upsilon=0}^{\upsilon=N-\ell-1}\left(N^2 r_0^2\right)^{\upsilon} \sum_{\upsilon'=0}^{\upsilon'=n-\ell'-1}\left(n^2 r_0^2\right)^{\upsilon'}\left(1+(-1)^S C_0 Z r_{12}\right) \times \exp\left[-\alpha(r_1+r_2)\right] \tag{3}$$

In this expression, N and n are the principal quantum numbers, l and l' are orbital quantum numbers, r_0 is Bohr radius, S is the total spin of atomic system, α and C_0 are the variational parameters to be determined by minimizing the energy, Z is the nuclear charge number, r_1 and r_2 are the coordinates of electrons with respect to the nucleus.

The interest of the Hylleraas type wave functions $\psi(r_1,r_2)$, is explained by the fact that they contain an electronic correlation term $r_{12} = |r_1 - r_2|$, which represents the angular part of the wave functions instead of the spherical harmonics as it is the case of the other wave functions used in the description of excited states. This electronic correlation term plays an important role in the test wave functions for the description of doubly excited states.

From the theoretical viewpoint, the Hylleraas variational method is based on the Hylleraas and Undheim theorem [15] according to which, a good approximation of the energy eigenvalue $E(\alpha, C_0)$ is obtained when the minima of the function $\left(\partial^2 E(\alpha,C_0)/\partial\alpha\,\partial C_0 = 0\right)$ converge with increasing values of the dimension D of the basis states and when the function exhibit a plateau.

Using this theorem, the values of the variational parameters α and C_0 can be determined by the following conditions:

$$\frac{\partial E(\alpha,C_0)}{\partial\alpha} = 0 \tag{4}$$

and

$$\frac{\partial E(\alpha,C_0)}{\partial C_0} = 0 \tag{5}$$

2.1. General Formalism of the SCUNC Method

In the framework of the Ritz' variation principle, the energy $E(\alpha) = \langle H\rangle(\alpha)$ is calculated from the relation:

$$E(\alpha) = \langle H\rangle(\alpha) = \frac{\langle \Psi(\alpha)|H|\Psi(\alpha)\rangle}{\langle \Psi(\alpha)|\Psi(\alpha)\rangle} \tag{6}$$

In this equation, the correlated wave functions are given by (3) and the Hamiltonian H of the helium isoelectronic series in given by (2) in atomic units.

Furthermore, the closure relation represents the fact that $|\vec{r}_1,\vec{r}_2\rangle$ are continuous bases in the space of the two – electron space, written as follow:

$$\iint dr_1^3 dr_2^3 |\vec{r}_1,\vec{r}_2\rangle\langle\vec{r}_1,\vec{r}_2| = 1 \tag{7}$$

Using this relation, according to (6), we obtain:

$$E(\alpha)\iint dr_1^3 dr_2^3 \langle\Psi(\alpha)||\vec{r}_1,\vec{r}_2\rangle\times\langle\vec{r}_1,\vec{r}_2||\Psi(\alpha)\rangle = \iint dr_1^3 dr_2^3 \langle\Psi(\alpha)||\vec{r}_1,\vec{r}_2\rangle\hat{H}\langle\vec{r}_1,\vec{r}_2||\Psi(\alpha)\rangle \tag{8}$$

By developping this expression (8), we find:

$$E(\alpha)\iint dr_1^3 dr_2^3 \Psi(\alpha)\times\Psi^*(\alpha) = \iint dr_1^3 dr_2^3 \Psi(\alpha)\hat{H}\,\Psi^*(\alpha) \tag{9}$$

This means:

$$N\times E(\alpha) = \iint dr_1^3 dr_2^3 \Psi(\alpha)\hat{H}\,\Psi^*(\alpha) \tag{10}$$

With the normalization constant

$$N = \iint dr_1^3 dr_2^3 \left|\Psi(\alpha)\right|^2 \tag{11}$$

To make it easier to integrate equation (9), we operate the variable changes in elliptic coordonates by:

$$s = r_1 + r_2; \quad t = r_1 - r_2; \quad u = r_{12} \tag{12}$$

On the basis of these variable changes, the elementary volume element

$$d\tau = d^3 r_1 d^3 r_2 = 2\pi^2 \left(s^2 - t^2\right) u\, ds\, du\, dt \tag{13}$$

Using these elliptical coordinates, Eq. (10) is rewritten as follows

$$\begin{aligned} & N \times E(\alpha) \\ & = \int_0^{\infty} ds \int_0^{s} du \int_0^{u} dt \left\{ u\left(s^2 - t^2\right) \left[\left(\frac{\partial \Psi}{\partial s}\right)^2 + \left(\frac{\partial \Psi}{\partial t}\right)^2 + \left(\frac{\partial \Psi}{\partial u}\right)^2 \right] \right. \\ & \left. + 2\left(\frac{\partial \Psi}{\partial u}\right) \times \begin{bmatrix} s\left(u^2 - t^2\right) \times \dfrac{\partial \Psi}{\partial s} + t\left(s^2 - u^2\right) \times \dfrac{\partial \Psi}{\partial t} \\ -\Psi^2 \times \left(4Zsu - s^2 + t^2\right) \end{bmatrix} \right\} \end{aligned} \tag{14}$$

With respect to the correlated wave functions given by expression (3), it is expressed as follows :

$$\begin{aligned} \psi(\alpha) = \frac{1}{4\pi} & \sum_{\upsilon=0}^{\upsilon = N-\ell-1} \left(N^2 r_0^2\right)^{\upsilon} \sum_{\upsilon'=0}^{\upsilon' = n-\ell'-1} \left(n^2 r_0^2\right)^{\upsilon'} \\ & \times \left(1 + (-1)^S C_0 Z u\right) \exp\left[-\alpha s\right] \end{aligned} \tag{15}$$

Furthermore, according to (12), the normamization constant is written in elliptic coordinates as:

$$N = \int_0^{\infty} ds \int_0^{s} du \int_0^{u} dt\, u\left(s^2 - t^2\right) \times \Psi^2 \tag{16}$$

In order to roughly resolve equation (1), we reduced the three-body interaction problem in a two-body interaction problem by introducing the concepts of internal and external interaction channels. These interaction channels are of three sorts: an internal interaction channel and two external interaction channels.

– The internal interaction channel reflects the attractive Coulomb nucleus–internal electron interaction, arbitrarily denoted “electron (1)”.

– The first external interaction channel takes account of the attractive Coulomb nucleus–external electron interaction, arbitrarily denoted “electron (2)”.

– The second external interaction channel takes account of the repulsive Coulomb internal electron (1)–external electron (2) interaction.

Using the two-body interaction model, we can then roughly resolve the Schrödinger equation (1) by combining the theory of stationary disturbances and the Ritz variation principle. This enables the screening constant by unit nuclear charge to be introduced.

Indeed, if ψ_0 designates the eigenwave function of the undisturbed operator, $\hat{H}_0$, then the following is obtained according to the eigenvalue equation:

$$\hat{H}_0 \psi_0 = E_0 \psi_0$$

where E_0 is the eigenvalue of $\hat{H}_0$, given by the relation:

$$E_0 = E_0^{(1)} + E_0^{(2)} = -Z^2 Ryd - Z^2 Ryd \tag{17}$$

To determine the eigenvalue, W_0, of the Hamiltonian $\hat{W}$ by the stationary disturbance theory, we write it in the form of a sum of eigenvalues obtained at various approximation orders, i.e.:

$$W_0 = W_0^{(1)} + W_0^{(2)} + W_0^{(3)} + \ldots\ldots\ldots\ldots W_0^{(q)} \tag{18}$$

where $W_0^{(1)}$ is the first-order approximation of the disturbance energy, $W_0^{(2)}$ is the second-order approximation and so on, up to the approximation of any order, q. The eigenvalue, E ($1s^2$; $^1S^e$), of the Hamiltonian $\hat{H}$ is given by the equation:

$$\hat{H}\psi_0 = E_0\left(1S^2;^1 S^e\right)\psi_0 = \left(\hat{H} + \hat{W}\right)\psi_0$$

This relation can be written using (17) and (18) in the form:

$$\begin{aligned} E\left(1s^2;^1 S^e\right) = & -Z^2 Ryd - Z^2 Ryd + W_0^{(1)} \\ & + W_0^{(2)} + W_0^{(3)} + \ldots\ldots + W_0^{(q)} \end{aligned} \tag{19}$$

In the first-order approximation, the stationary disturbance theory gives, for the energy, $W_0^{(1)}$, the value [16]:

$$W_0^{(1)} = \frac{5}{4} Z\, Ryd \tag{20}$$

Note that the first two terms on the right-hand side of equation (19) are proportional to Z^2; therefore, intuitively, result (20) can be transformed as follows:

$$W_0^{(1)} = \left(\frac{5}{4Z}\right) \times Z^2\, Ryd \tag{21}$$

We can thereupon introduce a parameter denoted $\varepsilon_p(Z)$, which we call the p*thorder approximation of the disturbance coefficient.* In the first-order approximation, this parameter has the following value: $\varepsilon_1(Z) = (5/4Z)$, which then enables result (21) to be written in the form:

$$W_0^{(1)} = \varepsilon_1 \times Z^2\, Ryd.$$

Equation (19) is then transformed as follows:

$$\begin{aligned} E\left(1s^2;^1 S^e\right) = & -Z^2 Ryd - Z^2 Ryd + \varepsilon_1 \times Z^2\, Ryd \\ & + \varepsilon_2 \times Z^2\, Ryd + \ldots\ldots\ldots\ldots + \varepsilon_q \times Z^2\, Ryd \end{aligned}$$

That is, in condensed form:

$$E\left(1s^2;^1 S^e\right) = -Z^2 Ryd - Z^2 Ryd + \sum_{p=1}^{q} \varepsilon_q(Z).Z^2\, Ryd \tag{22}$$

Using the concepts of interaction channels defined above, we consider expression (22) as the sum of the total energies of the hydrogen-like system {nucleus–electron (1)} and the {hydrogen-like–electron (2)} system. We can then write (22) in the form:

$$E\left(1s^2;^1S^e\right)=E^{(1)}\left(1s;^1S^e\right)+E^{(2)}\left(1s;^1S^e\right) \qquad (23)$$

whereby:

– $E^{(1)}\left(1s;^1S^e\right)$ is the energy of the hydrogen-like system (*in the internal interaction channel*), which is given by the well-known relation:

$$E^{(1)}\left(1s;^1S^e\right)=-Z^2Ryd \qquad (24)$$

– $E^{(2)}\left(1s;^1S^e\right)$ is the total energy in the external interaction channel, written in the ground state, by comparing (22), (23) and (24):

$$E^{(2)}\left(1s;^1S^e\right)=-Z^2Ryd+\sum_{p=1}^{q}\varepsilon_q(Z).Z^2\ Ryd$$

That is:

$$E^{(2)}\left(1s;^1S^e\right)=-Z^2\left(1-\sum_{p=1}^{q}\varepsilon_q(Z)\right)Ryd \qquad (25)$$

Introducing the effective charge, Z^*, within the framework of the present formalism, we write:

$$Z^{*2}=Z^2\left(1-\sum_{p=1}^{q}\varepsilon_q(Z)\right) \qquad (26)$$

Noting that the quantity:

$$\left(1-\sum_{p=1}^{q}\varepsilon_q(Z)\right)>0$$

(25) can be written as:

$$E^{(2)}\left(1s;^1S^e\right)=-Z^{*2}\ Ryd \qquad (27)$$

Written in this form, expression (27) reflects the energy of a hydrogen-like system of effective charge, Z^*. Let us thus write:

$$\left[1-\beta\left(1s^2;^1S^e;Z\right)\right]^2=\left(1-\sum_{p=1}^{q}\varepsilon_q(Z)\right) \qquad (28)$$

In this expression, $\beta\left(1s^2;^1S^e;Z\right)$ is a parameter that takes account of all of the electronic correlation effects. Its physical meaning is specified below. Considering relations (26) and (28), the following is obtained:

$$Z^{*2}=Z^2\left[1-\beta\left(1s^2;^1S^e;Z\right)\right]^2$$

That is:

$$Z^*=Z\left[1-\beta\left(1s^2;^1S^e;Z\right)\right] \qquad (29)$$

Result (29) gives the expression of the effective nuclear charge, Z^*, at the ground state. This result can become generalized in the case of the doubly excited states designated by the label ($Nlnl'$; $^{2S+1}L^{\pi}$) as follows:

$$Z^*=Z\left[1-\beta\left(Nl,nl';^{2S+1}L^{\pi};Z\right)\right] \qquad (30)$$

By substituting result (30) into expression (27) of the total energy in the external interaction channel, we obtain:

$$E^{(2)}\left(Nl,;nl'\ ^{2S+1}L^{\pi}\right)=-\frac{Z^2\left[1-\beta\left(Nl,nl';^{2S+1}L^{\pi}\right)\right]^2}{n^2}\ Ryd \qquad (31)$$

This relation makes it possible to specify the physical meaning of the parameter $\beta\left(Nl,nl';^{2S+1}L^{\pi};Z\right)$ by comparing it to the total energy of the {hydrogen-like ion-electron} system provided by Slater's atomic orbital theory [17,18]:

$$E=-\frac{(Z-\sigma_i)^2}{n^{*2}}\ \ Ryd \qquad (32)$$

The comparison of expressions (31) and (32) gives:

$$Z^2\left[1-\beta\left(Nl,nl';^{2S+1}L^{\pi}\right)\right]^2\equiv(Z-\sigma_i)^2=Z^2\left(1-\frac{\sigma_i}{Z}\right)^2$$

That is, thus:

$$\beta\left(Nl,nl';^{2S+1}L^{\pi}\right)\equiv\frac{\sigma_i}{Z} \qquad (33)$$

Considering that σ_i designates the screening constant and Z denotes the nuclear charge (*in elementary charge unit, e*), the physical meaning of the parameter, $\beta\left(Nl,nl';^{2S+1}L^{\pi};Z\right)$, according to relation (33), is then clear: it is the screening constant by unit nuclear charge. From this definition stems the name of the new method to roughly calculate energies of multi-electron atomic systems: the Screening Constant by Unit Nuclear Charge (SCUNC) method [19]

By substituting (24) and (27) into (23) and replacing the effective charge, Z*, by its expression (29), we establish the expression of the energy of the ground state of helium-like systems:

$$E\left(1s^2;^1S\right)=-Z^2Ryd-Z^2\left[1-\beta\left(1S_0,Z\right)\right]^2\ Ryd \qquad (34)$$

In this equation, the second term on the right-hand side corresponds to the firstionization energy. This makes it possible to generalize (34) to cases of autoionizing states of the type (Nl,nl') $^{2S+1}L^{\pi}$ with $n = N, N + 1, N + 2,...$, i.e.:

$$E\left(Nl,nl';\,^{2S+1}L^{\pi}\right) = -\frac{Z^2}{N^2}Ryd - \frac{Z^2}{n^2}\left[1-\beta\left(Nl,nl';\,^{2S+1}L^{\pi};Z\right)\right]^2 Ryd \quad (35)$$

In condensed form, we obtain, in Rydberg:

$$E\left(Nl,nl';\,^{2S+1}L^{\pi}\right) = -Z^2\left\{\frac{1}{N^2}+\frac{1}{n^2}\left[1-\beta\left(Nl,nl';\,^{2S+1}L^{\pi};Z\right)\right]^2\right\}Ryd \quad (36)$$

In this equation, N and n designate the principal quantum numbers of the inner and outer electrons, respectively, of helium and helium-like ions: on the basis of relation (33), the screening constant by unit nuclear charge is generally expressed in the form of a development in power of $1/Z$ [19], i.e.:

$$\beta\left(Nl,nl';\,^{2S+1}L^{\pi};Z\right) = \sum_{k=1}^{q} f_k\left(Nl,nl';\,^{2S+1}L^{\pi}\right)\times\left(\frac{1}{Z}\right)^k \quad (37)$$

In this expression, the f_k parameters are screening constants determined either theoretically or empirically. The order of development, q, is linked to the accuracy of the calculations and the number of experimental values to be used to determine the f_k parameters empirically. In the general case, the value of q is set at 2 if the semi-empirical procedure is adopted.

2.2. Energy Resonances of the *Nsnl* $^{2S+1}L^{\pi}$ and *Npnl* $^{2S+1}L^{\pi}$ Doubly Excited States of Heliumlike Ions

In the framework of the SCUNC formalism, the resonance energies of the doubly excited (*Nlnl'*) $^{2S+1}L^{\pi}$ states of two-electron atomic systems are expressed by the expression (36) above, which we recall.

$$E\left(N\ell n\ell',\,^{2s+1}L^{\pi}\right) = -Z^2\left(\frac{1}{N^2}+\frac{1}{n^2}\left[1-\beta\left(N\ell n\ell',\,^{2s+1}L^{\pi},Z\right)\right]^2\right)\mathrm{Ry}$$

where $\beta\left(N\ell n\ell',\,^{2s+1}L^{\pi},Z\right)$ is the screening constant by unit nuclear charge expressed as a function of the variational parameter α evaluated variationaly using a wave function.

In this work, the screening constants per unit nuclear charge $\beta\left(N\ell n\ell',\,^{2s+1}L^{\pi},Z\right)$ of the doubly excited *Nsnl* $^{2S+1}L^{\pi}$ (l = 0, 1, 2) and *Npnl* $^{2S+1}L^{\pi}$ (l = 1, 2, 3) states of helium-like ions (Z = 2 – 10) below the hydrogenic thresholds N = 3 – 8 are expressed as a function of the variational α-parameter as follows

- For *Nsns* $^{1,3}S^e$, *Nsnp* $^{1,3}P^0$ and *Nsnd* $^{1,3}D^e$ doubly excited states

$$\beta\left(Nsn\ell,\,^{2S+1}L^{\pi},Z,\alpha\right) = \frac{\alpha}{Z^2}\left(1+\frac{L+1}{N+L+S(S+1)+6}+\frac{L+1}{n+L+S(S+1)+6}\right) \quad (38)$$

- For *Npnp* $^{1,3}D^e$, *Npnd* $^{1,3}F^0$ and *Ndnd* $^{1,3}G^e$ doubly excited states

$$\beta\left(Npn\ell,\,^{2S+1}L^{\pi},Z,\alpha\right) = \frac{\alpha}{Z^2}\left(1+\frac{L-S}{N+n+S(S+1)+1}\right) \quad (39)$$

In these expressions, N and n denote the main quantum numbers of the inner and outer electrons respectively, L denotes the quantum state under consideration (*S, P, D, F* etc.), S is the total spin of the atomic system and α is the variational parameter.

Substituting expressions (38) and (39) into equation (36), the resonance energies of the doubly excited *Nsns* $^{1,3}S^e$, *Nsnp* $^{1,3}P^0$, *Nsnd* $^{1,3}D^e$, *Npnp* $^{1,3}D^e$, *Npnd* $^{1,3}F^0$ and *Ndnd* $^{1,3}G^e$ (N = 3 – 8 and n = N, N + 1, N + 2...) states in the helium-like ions are then expressed as follows (in *Ryd.*)

- For *Nsns* $^{1,3}S^e$, *Nsnp* $^{1,3}P^0$ and *Nsnd* $^{1,3}D^e$ doubly excited states

$$E\left(Nsn\ell,\,^{2S+1}L^{\pi},Z\right) = -Z^2\left(\frac{1}{N^2}+\frac{1}{n^2}\left[1-\frac{\alpha}{Z^2}\left(1+\frac{L+1}{N+L+S(S+1)+6}+\frac{L+1}{n+L+S(S+1)+6}\right)\right]^2\right) \quad (40)$$

- For *Npnp* $^{1,3}D^e$, *Npnd* $^{1,3}F^0$ and *Ndnd* $^{1,3}G^e$ doubly excited states

$$E\left(Npn\ell,\,^{2S+1}L^{\pi},Z\right) = -Z^2\left(\frac{1}{N^2}+\frac{1}{n^2}\left[1-\frac{\alpha}{Z^2}\left(1+\frac{L-S}{N+n+S(S+1)+1}\right)\right]^2\right) \quad (41)$$

In these equations, only the parameter α is unknown. Considering the 3s4s $^1S^e$ level of heliumlike ions (Z = 2 – 10), we calculated the values of the variational parameters α and C_0, the results are presented in Table 1 below. The details of the calculation of these variational parameters are well explained in our previous work [1,2].

Table 1. Values of variational parameters α and C_0 of helium-like ions (Z = 2 – 10).

Z	2	3	4	5	6	7	8	9	10
α	0.969	1.579	2.188	2.797	3.405	4.014	4.623	5.231	5.839
C_0	0.249	0.278	0.292	0.301	0.307	0.310	0.313	0.316	0.318

The equations (40) and (41) are used to calculate the resonance energies of the *Nsns* $^{1,3}S^e$, *Nsnp* $^{1,3}P^0$, *Nsnd* $^{1,3}D^e$, *Npnp* $^{1,3}D^e$, *Npnd* $^{1,3}F^0$ and *Ndnd* $^{1,3}G^e$ (N = 3 – 8, n =N, N + 1, N + 2...) doubly excited states of heliumlike (Z = 2 – 10) ions without a complex calculation program.

3. Results and Discussions

The results of the currently calculations for the energy resonances (in a.u.) of $^{1,3}S^e$, $^{1,3}P^0$, $^{1,3}D^e$, $^{1,3}D^e$, $^{1,3}F^0$ and $^{1,3}G^e$ states below the N = 3 – 8 hydrogenic thresholds are listed in Table 2 – Table 13.

Table 2 shows the comparison between current SCUNC calculations of the energy positions (– E, a.u) of doubly excited $^1S^e$ resonance states of helium-like ions (Z = 2 – 10) under N = 3 – 8 thresholds and other theoretical results. No experimental results are available for these states for direct comparison. Our present results are compared with the density function results of Roy *et al*. [20], the complex coordinate results of Ho [21], and the theoretical data from the time variation perturbation method of Ray and Mukherjee [22]. The comparison shows that the current SCUNC results are generally in very good agreement with those obtained by the above mentioned work. For 3*s*4*s* states up to Z = 5, the present results are in good agreement with the result of Roy *et al.* [20] and Ray and Mukherjee [22]. Moreover, the energy gaps almost never exceeded 0.02 a.u which proves the good accuracy of our present values.For the 3s5s levels, the comparison with the values of Ray and Mukherjee [22], the only ones available in the literature, shows a very good agreement up to Z = 5. Moreover the energy differences between our values and those of Ray and Mukherjee [22] never exceeded 0.01 a.u which allows us to consider our values as very accurate. For the 4*s*5*s* and 5*s*5*s* levels we compared our values with the results of Roy *et al* [20] and Ray and Mukherjee [22], we also note a very satisfactory agreement with a very good accuracy. The very good agreement noted for the levels 3*s*4*s*, 3*s*5*s*, 4*s*5*s* and 5*s*5*s*, allows us to consider our results as accurate for higher values of $N \geq 5$. Several new states are reported here, for example 5*sns*, 6*sns*, 7*sns* and 8*sns* with $5 \leq n \leq 11$ for helium and its isoelectronic series.

Results for *Nsns* $^3S^e$ states ($4 \leq n \leq 12$) below the N = 3 – 8 thresholds are shown in Table 3. For the 3*s*4*s* $^3S^e$ levels our present results compared with the density-functional theory (DFT) results of Roy *et al.* [20] and with the results of Bachau *et al.* [23] who applied the Feshbach projection operator (FPO) method. The comparison shows a very good agreement. It should be mentioned that our results are closer to the accurate results of the Feshbach projection operator method of Bachau *et al.* [23] and the largest difference between our values and those of these authors is 0.009 a.u which proves the good accuracy of our calculations. For the 3*s*5*s* $^3S^e$ states, we compared our present results (SCUNC) with the multi-configuration calculations of Lipsky *et al.* [24] and with the data of Pekeris [25]. Here again, we note a quite satisfactory agreement. Our values are closer to those of Lipsky *et al.* [24] with a maximum energy difference of 0.005 a.u which proves the very good precision of our present values and allows us to consider the new values presented for the first time in the literature for the 3*sns* $^3S^e$ levels with $n \geq 5$ as very precise. Our results on the calculation of the resonance energies (– E, a.u) of the doubly excited 4*s*5*s* $^3S^e$ states are with the data of Roy *et al.* [20] and Pekeris [25]. For these states too, we note a very good agreement and our values are of very high accuracy. Except Z = 2, where the energy difference between our present value and that of Pekeris [25] is 0.01 a.u, the large energy difference with respect to the values of Roy *et al.* [20] and Pekeris [25] is 0.007 a.u. This satisfactory agreement for the 4*s*5*s* $^3S^e$ states allows us to consider our values for the 4*sns* $^3S^e$ levels with $n \geq 5$ as very accurate. Generally, we note a very good agreement between our present SCUNC results and other theoretical reference values in the literature. To the best of our knowledge, some of the states calculated here are new for example 5*sns* $^3S^e$, 6*sns* $^3S^e$, 7*sns* $^3S^e$ and 8*sns* $^3S^e$ with $5 \leq n \leq 12$.

Table 4 reports the results for the *Nsnp* $^1P^0$ states (n = 4 – 13) for the isoelectronic series below the N =3 – 8 threshold. No experimental results are available for these states for direct comparison. For the 3*s*4*p* $^1P^0$ states, we compared our SCUNC results with the results of the semi - empirical procedure of the SCUNC method of Sakho *et al.* [26] and with the theoretical results of the Feshbach projection operator method of Bachau *et al.* [23]. We note an excellent agreement up to Z = 10. It should be mentioned that the values are closer to the precise results of Bachau *et al.* [22]. For the 3*s*5*p* $^1P^0$ levels, our results are also compared with the theoretical results of the semi-empirical procedure of the SCUNC method of Sakho *et al.* [26], the only ones available to our knowledge. Here also we note a very satisfactory agreement. Although in some cases our energy values are overestimated and in some cases underestimated, the overall agreement with the theoretical reference values is good. To our knowledge, some of the states calculated here are presented for the first time in the literature for example *Nsnp* $^1P^0$ associated with the thresholds $N \geq 4$. Thus we believe that these precise values listed in this table should be a good reference for future studies.

Table 5 shows the calculated resonance energies for the *Nsnp* $^3P^o$ triplet states (n = 4 - 13) associated with the N = 3 – 8 hydrogenic thresholds. For the 3*s*4*p* $^3P^0$ levels our present results are compared with the values of Sakho *et al.* [26], Bachau *et al*. [23] and Roy *et al.* [20]. The comparison shows an excellent agreement since the energy differences between our values and those of the other authors cited never exceeded 0.01 a.u. sometimes even less. For the 4*s*5*p* $^3P^0$ states, we compared our results with the values of Roy *et al.* [20], the only theoretical values available to our knowledge. For these states too we find a very satisfactory agreement overall and up to Z = 5, the maximum energy difference in absolute value is 0.009 a.u which sufficiently proves the accuracy of our calculations. For the *Nsnp* $^3P^0$ states associated with the $N \geq 5$ thresholds no theoretical or experimental results have been found in the literature to our knowledge. Hence the precise results listed in this table should serve as a good reference for future studies.

Results for *Nsnd* $^1D^e$ states (n = 4 – 13) below the N = 3 – 8 threshold are cited in Table 6. A comparison is made with the theoretical results of Sakho *et al.* [26] and Bachau *et al*. [23]. For these states also we note a very

satisfactory agreement. A multitude of new values for $N \geq 4$ are presented for the first time in this table.

Table 2. Energy positions ($-E$) for doubly excited $Nsns\ ^1S^e$ (n = 4 – 11) states of Helium-like ions (Z = 2 – 10) below the N = 3 – 8 hydrogenic thresholds. Comparison is made with other theoretical results.The results are expressed in atomic units

3*sns*	3s4s $^1S^e$				3s5s $^1S^e$		3s6s $^1S^e$	3s7s $^1S^e$	3s8s $^1S^e$
Z	$-E^p$	$-E^a$	$-E^b$	$-E^c$	$-E^p$	$-E^c$	$-E^p$	$-E^p$	$-E^p$
2	0.285	0.267		0.289	0.262	0.270	0.250	0.243	0.238
3	0.674	0.654	0.685	0.680	0.612	0.622	0.578	0.558	0.544
4	1.237	1.216	1.332	1.246	1.112	1.113	1.044	1.003	0.977
5	1.973	1.950	2.201	1.968	1.764	1.762	1.650	1.581	1.536
6	2.882				2.566		2.393	2.289	2.222
7	3.965				3.519		3.276	3.130	3.035
8	5.221				4.623		4.298	4.101	3.974
9	6.651				5.878		5.458	5.205	5.040
10	8.254				7.285		6.758	6.439	6.233
4*sns*	4s5s $^1S^e$				4s6s $^1S^e$		4s7s $^1S^e$	4s8s $^1S^e$	4s9s $^1S^e$
Z	$-E^p$	$-E^a$	$-E^c$		$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.173	0.155			0.158		0.144	0.140	0.137
3	0.407	0.382	0.406		0.369		0.330	0.320	0.312
4	0.744	0.712	0.736		0.669		0.595	0.574	0.560
5	1.183	1.145	1.168		1.060		0.937	0.904	0.881
6	1.724				1.541		1.358	1.309	1.273
7	2.368				2.112		1.857	1.788	1.739
8	3.114				2.773		2.434	2.342	2.277
9	3.963				3.525		3.089	2.971	2.887
10	4.914				4.339		3.822	3.675	3.570
5*sns*	5s5s $^1S^e$				5s6s $^1S^e$		5s7s $^1S^e$	5s8s $^1S^e$	5s9s $^1S^e$
Z	$-E^p$	$-E^a$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.121				0.108		0.101	0.096	0.093
3	0.293	0.295			0.259		0.238	0.225	0.215
4	0.545	0.553			0.477		0.435	0.408	0.390
5	0.877	0.890			0.762		0.693	0.648	0.617
6	1.288				1.115		1.011	0.943	0.896
7	1.779				1.536		1.389	1.293	1.228
8	2.351				2.025		1.828	1.700	1.612
9	3.002				2.581		2.327	2.161	2.048
10	3.734				3.205		2.886	2.679	2.537
6*sns*	6s6s $^1S^e$				6s7s $^1S^e$		6s8s $^1S^e$	6s9s $^1S^e$	6s10s $^1S^e$
Z	$-E^p$				$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.084				0.077		0.072	0.068	0.066
3	0.204				0.183		0.170	0.160	0.154
4	0.379				0.338		0.311	0.292	0.279
5	0.610				0.540		0.495	0.464	0.442
6	0.896				0.791		0.723	0.677	0.643
7	1.237				1.090		0.994	0.929	0.882
8	1.634				1.437		1.309	1.221	1.158
9	2.087				1.832		1.667	1.554	1.472
10	2.595				2.276		2.068	1.926	1.824
7*sns*	7s7s $^1S^e$				7s8s $^1S^e$		7s9s $^1S^e$	7s10s $^1S^e$	7s11s $^1S^e$
Z	$-E^p$				$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.062				0.057		0.054	0.051	0.050
3	0.150				0.137		0.127	0.121	0.116
4	0.279				0.252		0.234	0.220	0.210
5	0.449				0.403		0.372	0.350	0.334
6	0.659				0.591		0.544	0.511	0.486
7	0.910				0.814		0.748	0.701	0.667
8	1.202				1.074		0.986	0.923	0.876
9	1.535				1.369		1.255	1.174	1.114
10	1.908				1.700		1.558	1.456	1.381
8*sns*	8s8s $^1S^e$				8s9s $^1S^e$		8s10s $^1S^e$	8s11s $^1S^e$	8s11s $^1S^e$
Z	$-E^p$				$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.048				0.044		0.042	0.040	0.039
3	0.115				0.106		0.099	0.094	0.090
4	0.214				0.195		0.182	0.172	0.165
5	0.344				0.313		0.291	0.274	0.262
6	0.505				0.458		0.425	0.400	0.381
7	0.697				0.632		0.584	0.550	0.523
8	0.921				0.833		0.770	0.723	0.688
9	1.176				1.062		0.981	0.920	0.875
10	1.462				1.319		1.217	1.142	1.084

[p] Present results
[a] Roy *et al.* [20]
[b] Ho [21]
[c] Ray and Mukherjee [22]

Table 3. Energy positions *(−E)* for doubly excited *Nsns* $^3S^e$ *(n* = 4 – 12*)* states of Helium-like ions (Z = 2 – 10) below the *N* = 3 – 8 hydrogenic thresholds. Comparison is made with other theoretical results.The results are expressed in atomic units

3*sns*	3s4s $^3S^e$			3s5s $^3S^e$			3s6s $^3S^e$	3s7s $^3S^e$	3s8s $^3S^e$
Z	$-E^p$	$-E^a$	$-E^b$	$-E^p$	$-E^c$	$-E^d$	$-E^p$	$-E^p$	$-E^p$
2	0.286	0.277		0.263	0.258	0.251	0.251	0.243	0.238
3	0.677	0.672	0.689	0.614	0.611	0.598	0.579	0.558	0.545
4	1.241	1.241	1.265	1.115	1.114	1.096	1.046	1.005	0.978
5	1.978	1.983	2.014	1.767	1.767	1.747	1.652	1.582	1.537
6	2.889			2.570			2.396	2.291	2.223
7	3.973			3.524			3.279	3.132	3.036
8	5.231			4.629			4.302	4.104	3.976
9	6.662			5.885			5.463	5.208	5.042
10	8.267			7.292			6.762	6.443	6.235
4*sns*	4s5s $^3S^e$			4s6s $^3S^e$			4s7s $^3S^e$	4s8s $^3S^e$	4s9s $^3S^e$
Z	$-E^p$	$-E^a$	$-E^d$	$-E^p$			$-E^p$	$-E^p$	$-E^p$
2	0.166	0.160	0.156	0.154			0.141	0.138	0.135
3	0.395	0.393	0.388	0.361			0.326	0.317	0.310
4	0.727	0.728	0.720	0.658			0.589	0.570	0.557
5	1.160	1.165	1.155	1.045			0.930	0.899	0.876
6	1.696			1.522			1.349	1.302	1.268
7	2.334			2.089			1.846	1.780	1.733
8	3.075			2.747			2.421	2.333	2.270
9	3.918			3.495			3.074	2.960	2.879
10	4.863			4.333			3.805	3.663	3.561
5*sns*	5s5s $^3S^e$			5s6s $^3S^e$			5s7s $^3S^e$	5s8s $^3S^e$	5s9s $^3S^e$
Z	$-E^p$			$-E^p$			$-E^p$	$-E^p$	$-E^p$
2	0.122			0.109			0.101	0.096	0.093
3	0.295			0.260			0.239	0.225	0.216
4	0.547			0.478			0.436	0.409	0.390
5	0.879			0.764			0.694	0.649	0.618
6	1.291			1.117			1.012	0.944	0.897
7	1.784			1.539			1.391	1.295	1.229
8	2.356			2.028			1.830	1.701	1.613
9	3.008			2.584			2.329	2.163	2.049
10	3.740			3.209			2.889	2.681	2.538
6*sns*	6s6s $^3S^e$			6s7s $^3S^e$			6s8s $^3S^e$	6s9s $^3S^e$	6s10s $^3S^e$
Z	$-E^p$			$-E^p$			$-E^p$	$-E^p$	$-E^p$
2	0.085			0.077			0.072	0.069	0.066
3	0.205			0.184			0.170	0.161	0.154
4	0.380			0.339			0.311	0.293	0.279
5	0.611			0.542			0.496	0.465	0.443
6	0.898			0.793			0.724	0.677	0.644
7	1.240			1.092			0.996	0.930	0.882
8	1.637			1.439			1.310	1.222	1.159
9	2.090			1.835			1.669	1.555	1.473
10	2.599			2.278			2.070	1.927	1.825
7*sns*	7s7s $^3S^e$			7s8s $^3S^e$			7s9s $^3S^e$	7s10s $^3S^e$	7s11s $^3S^e$
Z	$-E^p$			$-E^p$			$-E^p$	$-E^p$	$-E^p$
2	0.062			0.057			0.054	0.051	0.050
3	0.151			0.137			0.128	0.121	0.116
4	0.280			0.253			0.234	0.221	0.211
5	0.450			0.404			0.373	0.351	0.334
6	0.660			0.592			0.545	0.511	0.486
7	0.911			0.815			0.749	0.702	0.667
8	1204			1.075			0.987	0.923	0.877
9	1.537			1.370			1.257	1.175	1.115
10	1.910			1.702			1.559	1.457	1.381
8*sns*	8s8s $^3S^e$			8s9s $^3S^e$			8s10s $^3S^e$	8s11s $^3S^e$	8s12s $^3S^e$
Z	$-E^p$			$-E^p$			$-E^p$	$-E^p$	$-E^p$
2	0.048			0.044			0.042	0.040	0.039
3	0.116			0.106			0.099	0.094	0.091
4	0.215			0.196			0.182	0.172	0.165
5	0.345			0.313			0.291	0.274	0.262
6	0.506			0.459			0.425	0.400	0.381
7	0.698			0.632			0.585	0.550	0.523
8	0.922			0.834			0.770	0.724	0.688
9	1.177			1.063			0.981	0.921	0.875
10	1.463			1.320			1.218	1.142	1.085

[p] Present results
[a] Roy *et al.* [20]
[b] Bachau *et al.* [23]
[c] Lipsky *et al.* [24]
[d] Pekeris [25]

Table 4. Energy positions *(−E)* for doubly excited *Nsnp* $^1P^0$ *(n* = 4 – 13*)* states of Helium-like ions (Z = 2 – 10) below the *N* = 3 – 8 hydrogenic thresholds. Comparison is made with other theoretical results. The results are expressed in atomic units

3*snp*	3s4p $^1P^0$			3s5p $^1P^0$		3s6p $^1P^0$	3s7p $^1P^0$	3s8p $^1P^0$
Z	$-E^p$	$-E^a$	$-E^b$	$-E^p$	$-E^a$	$-E^p$	$-E^p$	$-E^p$
2	0.278	0.266	0.271	0.258	0.237	0.247	0.241	0.237
3	0.661	0.657	0.660	0.604	0.579	0.573	0.554	0.541
4	1.218	1.221	1.222	1.100	1.073	1.036	0.998	0.972
5	1.947	1.958	1.957	1.748	1.717	1.639	1.573	1.530
6	2.850	2.869	2.865	2.546	2.513	2.380	2.280	2.215
7	3.927	3.954	3.948	3.495	3.460	3.260	3.118	3.026
8	5.176	5.212	5.204	4.595	4.558	4.279	4.088	3.964
9	6.600	6.644	6.633	5.847	5.807	5.437	5.190	5.029
10	8.200	8.249	8.236	7.249	7.208	6.734	6.422	6.220
4*snp*	4s5p $^1P^0$			4s6p $^1P^0$		4s7p $^1P^0$	4s8p $^1P^0$	4s9p $^1P^0$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.161			0.150		0.144	0.140	0.137
3	0.386			0.355		0.335	0.323	0.314
4	0.713			0.648		0.609	0.584	0.567
5	1.142			1.032		0.966	0.923	0.894
6	1.673			1.507		1.406	1.341	1.296
7	2.307			2.071		1.929	1.836	1.772
8	3.043			2.726		2.534	2.410	2.324
9	3.881			3.471		3.222	3.061	2.950
10	4.823			4.306		3.994	3.791	3.652
5*snp*	5s6p $^1P^0$			5s7p $^1P^0$		5s8p $^1P^0$	5s9p $^1P^0$	5s10p $^1P^0$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.106			0.099		0.095	0.092	0.089
3	0.254			0.234		0.222	0.213	0.207
4	0.469			0.430		0.405	0.387	0.374
5	0.752			0.686		0.643	0.613	0.592
6	1.103			1.002		0.936	0.891	0.859
7	1.521			1.379		1.286	1.222	1.176
8	2.007			1.815		1.691	1.605	1.543
9	2.561			2.313		2.151	2.040	1.961
10	3.183			2.870		2.667	2.528	2.428
6*snp*	6s7p $^1P^0$			6s8p $^1P^0$		6s9p $^1P^0$	6s10p $^1P^0$	6s11p $^1P^0$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.075			0.070		0.067	0.065	0.064
3	0.180			0.167		0.158	0.152	0.148
4	0.333			0.307		0.289	0.277	0.267
5	0.534			0.490		0.461	0.439	0.423
6	0.783			0.717		0.672	0.639	0.615
7	1.080			0.987		0.923	0.877	0.843
8	1.425			1.300		1.214	1.153	1.107
9	1.819			1.657		1.546	1.466	1.407
10	2.261			2.057		1.917	1.817	1.743
7*snp*	7s8p $^1P^0$			7s9p $^1P^0$		7s10p $^1P^0$	7s11p $^1P^0$	7s12p $^1P^0$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.056			0.053		0.050	0.049	0.048
3	0.134			0.125		0.119	0.114	0.111
4	0.248			0.231		0.218	0.209	0.201
5	0.399			0.369		0.347	0.331	0.319
6	0.585			0.539		0.507	0.483	0.465
7	0.807			0.743		0.697	0.663	0.637
8	1.065			0.979		0.917	0.872	0.837
9	1.359			1.248		1.168	1.109	1.064
10	1.690			1.550		1.450	1.375	1.319
8*snp*	8s9p $^1P^0$			8s10p $^1P^0$		8s11p $^1P^0$	8s12p $^1P^0$	8s13p $^1P^0$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.043			0.041		0.039	0.038	0.037
3	0.104			0.098		0.093	0.089	0.087
4	0.193			0.180		0.171	0.163	0.158
5	0.309			0.288		0.272	0.260	0.250
6	0.454			0.421		0.397	0.379	0.364
7	0.626			0.580		0.546	0.520	0.500
8	0.827			0.765		0.719	0.684	0.657
9	1.055			0.975		0.916	0.871	0.836
10	1.311			1.211		1.137	1.080	1.036

[p] Present results
[a] Sakho *et al.* [26]
[b] Bachau *et al.* [23]

Table 5. Energy positions *(−E)* for doubly excited *Nsnp* $^3P^0$ *(n* = 4 – 13*)* states of Helium-like ions (Z = 2 – 10) below the *N* = 3 – 8 hydrogenic thresholds. Comparison is made with other theoretical results. The results are expressed in atomic units

3s*np*	3s4p $^3P^0$				3s5p $^3P^0$		3s6p $^3P^0$	3s7p $^3P^0$	3s8p $^3P^0$
Z	$-E^p$	$-E^a$	$-E^b$	$-E^c$	$-E^p$	$-E^a$	$-E^p$	$-E^p$	$-E^p$
2	0.280	0.270	0.279	0.272	0.260	0.240	0.248	0.242	0.237
3	0.666	0.660	0.676	0.663	0.607	0.583	0.575	0.555	0.542
4	1.225	1.224	1.245	1.227	1.105	1.076	1.039	1.000	0.974
5	1.956	1.962	1.989	1.966	1.753	1.721	1.642	1.576	1.532
6	2.862	2.873	2.906		2.553	2.517	2.385	2.283	2.217
7	3.940	3.958	3.997		3.503	3.464	3.266	3.122	3.029
8	5.192	5.216	5.261		4.605	4.562	4.285	4.093	3.967
9	6.618	6.648	6.699		5.858	5.811	5.444	5.195	5.032
10	8.217	8.254	8.311		7.262	7.211	6.742	6.428	6.224
4s*np*	4s5p $^3P^0$				4s6p $^3P^0$		4s7p $^3P^0$	4s8p $^3P^0$	4s9p $^3P^0$
Z	$-E^p$	$-E^c$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.163	0.158			0.151		0.144	0.140	0.137
3	0.389	0.388			0.356		0.337	0.324	0.315
4	0.717	0.721			0.651		0.611	0.585	0.568
5	1.147	1.156			1.036		0.969	0.925	0.895
6	1.679				1.511		1.409	1.343	1.297
7	2.314				2.076		1.932	1.839	1.774
8	3.052				2.731		2.538	2.412	2.326
9	3.891				3.477		3.227	3.064	2.953
10	4.834				4.313		3.999	3.795	3.655
5s*np*	5s6p $^3P^0$				5s7p $^3P^0$		5s8p $^3P^0$	5s9p $^3P^0$	5s10p $^3P^0$
Z	$-E^p$				$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.107				0.100		0.095	0.092	0.090
3	0.255				0.236		0.223	0.214	0.207
4	0.471				0.432		0.406	0.388	0.375
5	0.755				0.688		0.644	0.614	0.593
6	1.107				1.005		0.938	0.893	0.860
7	1.526				1.382		1.288	1.224	1.177
8	2.013				1.819		1.693	1.607	1.545
9	2.567				2.317		2.154	2.042	1.962
10	3.190				2.875		2.671	2.530	2.430
6s*np*	6s7p $^3P^0$				6s8p $^3P^0$		6s9p $^3P^0$	6s10p $^3P^0$	6s11p $^3P^0$
Z	$-E^p$				$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.075				0.071		0.068	0.065	0.064
3	0.181				0.168		0.159	0.153	0.148
4	0.334				0.308		0.290	0.277	0.268
5	0.536				0.492		0.462	0.440	0.424
6	0.785				0.719		0.673	0.640	0.616
7	1.083				0.989		0.925	0.878	0.844
8	1.429				1.303		1.216	1.154	1.108
9	1.823				1.660		1.548	1.468	1.409
10	2.265				2.060		1.920	1.819	1.745
7s*np*	7s8p $^3P^0$				7s9p $^3P^0$		7s10p $^3P^0$	7s11p $^3P^0$	7s12p $^3P^0$
Z	$-E^p$				$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.056				0.053		0.051	0.049	0.048
3	0.135				0.126		0.120	0.115	0.111
4	0.249				0.231		0.219	0.209	0.202
5	0.400				0.370		0.348	0.332	0.320
6	0.586				0.541		0.508	0.484	0.465
7	0.809				0.744		0.698	0.664	0.638
8	1.067				0.981		0.919	0.873	0.838
9	1.362				1.250		1.170	1.110	1.065
10	1.692				1.552		1.451	1.377	1.320
8s*np*	8s9p $^3P^0$				8s10p $^3P^0$		8s11p $^3P^0$	8s12p $^3P^0$	8s13p $^3P^0$
Z	$-E^p$				$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.043				0.041		0.039	0.038	0.037
3	0.105				0.098		0.093	0.090	0.087
4	0.193				0.180		0.171	0.164	0.158
5	0.310				0.288		0.272	0.260	0.251
6	0.455				0.422		0.398	0.379	0.365
7	0.627				0.581		0.547	0.521	0.501
8	0.828				0.766		0.720	0.685	0.658
9	1.057				0.976		0.917	0.872	0.837
10	1.313				1.212		1.138	1.081	1.037

[p] Present results
[a] Sakho *et al.* [26]
[b] Bachau *et al.* [23]
[c] Roy *et al.* [20]

Table 6. Energy positions *(−E)* for doubly excited *Nsnd* $^1D^e$ *(n* = 4 – 13*)* states of Helium-like ions (Z = 2 – 10) below the *N* = 3 – 8 hydrogenic thresholds. Comparison is made with other theoretical results. The results are expressed in atomic units

3*snd*	3s4d $^1D^e$			3s5d $^1D^e$		3s6d $^1D^e$	3s7d $^1D^e$	3s8d $^1D^e$
Z	$-E^p$	$-E^a$	$-E^b$	$-E^p$	$-E^a$	$-E^p$	$-E^p$	$-E^p$
2	0.272	0.259	0.276	0.261	0.232	0.245	0.239	0.235
3	0.651	0.649	0.669	0.609	0.574	0.568	0.551	0.539
4	1.202	1.213	1.236	1.109	1.068	1.030	0.993	0.969
5	1.927	1.949	1.976	1.759	1.712	1.630	1.567	1.526
6	2.824	2.861	2.889	2.559	2.508	2.369	2.272	2.209
7	3.895	3.946	3.975	3.512	3.455	3.247	3.109	3.019
8	5.140	5.204	5.236	4.615	4.553	4.264	4.077	3.956
9	6.558	6.636	6.670	5.869	5.802	5.419	5.177	5.019
10	8.150	8.241	8.277	7.274	7.202	6.714	6.408	6.209
4*snd*	4s5d $^1D^e$			4s6d $^1D^e$		4s7d $^1D^e$	4s8d $^1D^e$	4s9d $^1D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.158			0.148		0.142	0.138	0.136
3	0.380			0.350		0.332	0.321	0.312
4	0.704			0.642		0.605	0.581	0.564
5	1.129			1.024		0.960	0.919	0.890
6	1.657			1.496		1.399	1.335	1.291
7	2.288			2.058		1.920	1.829	1.767
8	3.021			2.711		2.523	2.402	2.318
9	3.856			3.454		3.210	3.052	2.943
10	4.794			4.287		3.980	3.781	3.644
5*snd*	5s6d $^1D^e$			5s7d $^1D^e$		5s8d $^1D^e$	5s9d $^1D^e$	5s10d $^1D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.103			0.097		0.093	0.091	0.089
3	0.250			0.232		0.220	0.212	0.206
4	0.463			0.426		0.401	0.384	0.372
5	0.744			0.680		0.638	0.610	0.589
6	1.093			0.995		0.931	0.887	0.856
7	1.509			1.370		1.279	1.217	1.172
8	1.993			1.805		1.683	1.599	1.539
9	2.545			2.301		2.142	2.033	1.955
10	3.164			2.857		2.657	2.520	2.422
6*snd*	6s7d $^1D^e$			6s8d $^1D^e$		6s9d $^1D^e$	6s10d $^1D^e$	6s11d $^1D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.073			0.069		0.066	0.064	0.063
3	0.177			0.165		0.157	0.151	0.146
4	0.328			0.304		0.287	0.275	0.266
5	0.528			0.486		0.457	0.437	0.421
6	0.776			0.712		0.668	0.636	0.613
7	1.071			0.981		0.918	0.873	0.840
8	1.415			1.293		1.209	1.148	1.104
9	1.807			1.648		1.539	1.461	1.403
10	2.248			2.048		1.910	1.812	1.739
7*snd*	7s8d $^1D^e$			7s9d $^1D^e$		7s10d $^1D^e$	7s11d $^1D^e$	7s12d $^1D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.055			0.052		0.050	0.048	0.047
3	0.132			0.124		0.118	0.113	0.110
4	0.245			0.228		0.216	0.207	0.200
5	0.394			0.366		0.345	0.329	0.318
6	0.580			0.535		0.504	0.480	0.462
7	0.801			0.738		0.693	0.660	0.635
8	1.058			0.973		0.913	0.868	0.834
9	1.351			1.242		1.163	1.105	1.061
10	1.680			1.542		1.444	1.371	1.315
8*snd*	8s9d $^1D^e$			8s10d $^1D^e$		8s11d $^1D^e$	8s12d $^1D^e$	8s13d $^1D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.042			0.040		0.039	0.038	0.037
3	0.102			0.096		0.092	0.089	0.086
4	0.190			0.178		0.169	0.162	0.157
5	0.306			0.285		0.270	0.258	0.249
6	0.450			0.418		0.395	0.377	0.363
7	0.621			0.576		0.543	0.518	0.498
8	0.821			0.760		0.716	0.681	0.655
9	1.049			0.970		0.912	0.868	0.833
10	1.304			1.205		1.132	1.076	1.033

[p] Present results
[a] Sakho *et al.* [26]
[b] Bachau *et al.* [23]

Table 7. Energy positions *(−E)* for doubly excited *Nsnd* $^3D^e$ *(n* = 4 – 12*)* states of Helium-like ions (Z = 2 – 10) below the *N* = 3 – 8 hydrogenic thresholds. Comparison is made with other theoretical results. The results are expressed in atomic units

3*snd*	3s4d $^3D^e$			3s5d $^3D^e$		3s6d $^3D^e$	3s7d $^3D^e$	3s8d $^3D^e$
Z	$-E^p$	$-E^a$	$-E^b$	$-E^p$	$-E^c$	$-E^p$	$-E^p$	$-E^p$
2	0.275	0.268	0.266	0.256	0.235	0.246	0.240	0.236
3	0.657	0.653	0.654	0.601	0.577	0.571	0.552	0.540
4	1.211	1.212	1.216	1.096	1.071	1.033	0.995	0.971
5	1.938	1.944	1.951	1.742	1.716	1.635	1.570	1.528
6	2.839	2.850		2.538	2.511	2.375	2.276	2.212
7	3.912	3.929		3.486	3.458	3.254	3.114	3.022
8	5.160	5.181		4.585	4.556	4.272	4.083	3.960
9	6.581	6.606		5.835	5.805	5.428	5.183	5.024
10	8.176	8.205		7.235	7.206	6.724	6.415	6.215
4*snd*	4s5d $^3D^e$			4s6d $^3D^e$		4s7d $^3D^e$	4s8d $^3D^e$	4s9d $^3D^e$
Z	$-E^p$	$-E^b$		$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.160	0.156		0.149		0.143	0.139	0.136
3	0.383	0.384		0.352		0.334	0.322	0.313
4	0.708	0.716		0.645		0.607	0.582	0.565
5	1.136	1.149		1.028		0.963	0.921	0.892
6	1.665			1.501		1.402	1.338	1.293
7	2.297			2.065		1.924	1.832	1.770
8	3.032			2.718		2.529	2.405	2.321
9	3.869			3.462		3.216	3.056	2.947
10	4.808			4.296		3.986	3.785	3.647
5*snd*	5s5d $^3D^e$			5s6d $^3D^e$		5s7d $^3D^e$	5s8d $^3D^e$	5s9d $^3D^e$
Z	$-E^p$	$-E^b$		$-E^p$	$-E^b$	$-E^p$	$-E^p$	$-E^p$
2	0.115	0.120		0.104	0.102	0.098	0.094	0.091
3	0.282	0.299		0.252		0.233	0.221	0.212
4	0.529	0.558		0.466		0.428	0.403	0.386
5	0.856	0.896		0.748		0.683	0.640	0.611
6	1.262			1.097		0.998	0.933	0.889
7	1.748			1.515		1.374	1.282	1.219
8	2.314			2.000		1.810	1.686	1.601
9	2.960			2.552		2.306	2.146	2.036
10	3.686			3.173		2.863	2.662	2.523
6*snd*	6s6d $^3D^e$			6s7d $^3D^e$		6s8d $^3D^e$	6s9d $^3D^e$	6s10d $^3D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.080			0.074		0.070	0.067	0.065
3	0.197			0.178		0.166	0.157	0.151
4	0.369			0.330		0.305	0.288	0.276
5	0.596			0.530		0.488	0.459	0.438
6	0.878			0.779		0.714	0.669	0.637
7	1.216			1.075		0.983	0.920	0.875
8	1.610			1.420		1.296	1.211	1.150
9	2.059			1.812		1.652	1.542	1.463
10	2.564			2.253		2.052	1.913	1.814
7*snd*	7s7d $^3D^e$			7s8d $^3D^e$		7s9d $^3D^e$	7s10d $^3D^e$	7s11d $^3D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.059			0.055		0.052	0.050	0.048
3	0.145			0.133		0.124	0.118	0.114
4	0.272			0.247		0.229	0.217	0.208
5	0.439			0.396		0.367	0.346	0.330
6	0.647			0.582		0.537	0.505	0.481
7	0.895			0.803		0.740	0.695	0.661
8	1.185			1.061		0.976	0.915	0.870
9	1.515			1.354		1.244	1.165	1.107
10	1.886			1.684		1.545	1.446	1.372
8*snd*	8s8d $^3D^e$			8s9d $^3D^e$		8s10d $^3D^e$	8s11d $^3D^e$	8s12d $^3D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.046			0.043		0.041	0.039	0.038
3	0.112			0.103		0.097	0.092	0.089
4	0.209			0.191		0.179	0.170	0.163
5	0.337			0.307		0.286	0.271	0.259
6	0.496			0.451		0.419	0.395	0.377
7	0.687			0.623		0.578	0.544	0.519
8	0.908			0.823		0.762	0.717	0.682
9	1.161			1.051		0.972	0.913	0.869
10	1.446			1.307		1.207	1.134	1.078

[p] Present results
[a] Bachau *et al.* [23]
[b] Roy *et al.* [20]
[c] Sakho *et al.* [26].

Table 8. Energy positions *(−E)* for doubly excited *Npnp* $^1D^e$ *(n = 3 – 12)* states of Helium-like ions (Z = 2 – 10) below the *N* = 3 – 8 hydrogenic thresholds. Comparison is made with other theoretical results. The results are expressed in atomic units

3*pnp*	3p3p ^{1}D^e			3p4p ^{1}D^e			3p5p ^{1}D^e	3p6p ^{1}D^e	3p7p ^{1}D^e
Z	$-E^p$	$-E^a$	$-E^b$	$-E^p$	$-E^c$	$-E^d$	$-E^p$	$-E^p$	$-E^p$
2	0.328			0.283	0.285	0.276	0.262	0.250	0.243
3	0.800	0.802	0.796	0.671	0.670	0.669	0.611	0.578	0.558
4	1.493	1.510	1.499	1.233	1.218	1.234	1.111	1.044	1.004
5	2.407	2.440	2.423	1.967	1.951	1.974	1.762	1.649	1.581
6	3.543			2.875			2.563	2.393	2.290
7	4.901			3.956			3.516	3.276	3.130
8	6.482			5.211			4.620	4.297	4.102
9	8.284			6.639			5.874	5.458	5.205
10	10.308			8.241			7.280	6.757	6.440
4*pnp*	4p4p $^1D^e$			4p5p $^1D^e$			4p6p $^1D^e$	4p7p $^1D^e$	4p8p $^1D^e$
Z	$-E^p$	$-E^a$	$-E^e$	$-E^p$	$-E^a$	$-E^e$	$-E^p$	$-E^p$	$-E^p$
2	0.187		0.199	0.165	0.153	0.177	0.153	0.146	0.141
3	0.455	0.455	0.467	0.393	0.378	0.400	0.360	0.339	0.326
4	0.847	0.855	0.867	0.724	0.706	0.727	0.656	0.615	0.589
5	1.363	1.380	1.393	1.156	1.137	1.150	1.043	0.974	0.929
6	2.005			1.691			1.519	1.416	1.348
7	2.771			2.328			2.086	1.940	1.845
8	3.663			3.068			2.744	2.548	2.420
9	4.679			3.910			3.491	3.238	3.073
10	5.820			4.855			4.329	4.011	3.805
5*pnp*	5p5p $^1D^e$			5p6p $^1D^e$			5p7p $^1D^e$	5p8p $^1D^e$	5p9p $^1D^e$
Z	$-E^p$	$-E^a$		$-E^p$			$-E^p$	$-E^p$	$-E^p$
2	0.121			0.109			0.101	0.096	0.093
3	0.293	0.292		0.259			0.238	0.225	0.216
4	0.545	0.549		0.477			0.436	0.409	0.391
5	0.877	0.885		0.763			0.694	0.649	0.618
6	1.288			1.116			1.012	0.944	0.897
7	1.779			1.537			1.390	1.295	1.229
8	2.351			2.025			1.829	1.701	1.613
9	3.002			2.582			2.328	2.163	2.050
10	3.734			3.206			2.888	2.681	2.538
6*pnp*	6p6p $^1D^e$			6p7p $^1D^e$			6p8p $^1D^e$	6p9p $^1D^e$	6p10p $^1D^e$
Z	$-E^p$			$-E^p$			$-E^p$	$-E^p$	$-E^p$
2	0.084			0.077			0.072	0.069	0.066
3	0.205			0.184			0.170	0.161	0.154
4	0.380			0.338			0.311	0.293	0.280
5	0.611			0.541			0.496	0.465	0.443
6	0.897			0.792			0.724	0.677	0.644
7	1.239			1.091			0.996	0.930	0.883
8	1.636			1.439			1.310	1.222	1.159
9	2.089			1.834			1.669	1.555	1.474
10	2.597			2.278			2.070	1.928	1.826
7*pnp*	7p7p $^1D^e$			7p8p $^1D^e$			7p9p $^1D^e$	7p10p $^1D^e$	7p11p $^1D^e$
Z	$-E^p$			$-E^p$			$-E^p$	$-E^p$	$-E^p$
2	0.062			0.057			0.054	0.052	0,050
3	0.151			0.137			0.128	0.121	0.116
4	0.280			0.253			0.234	0.221	0.211
5	0.450			0.404			0.373	0.351	0.334
6	0.660			0.592			0.545	0.512	0.487
7	0.911			0.816			0.750	0.702	0.667
8	1.204			1.075			0.987	0.924	0.877
9	1.537			1.371			1.257	1.176	1.115
10	1.910			1.702			1.560	1.458	1.382
8*pnp*	8p8p $^1D^e$			8p9p $^1D^e$			8p10p $^1D^e$	8p11p $^1D^e$	8p12p $^1D^e$
Z	$-E^p$			$-E^p$			$-E^p$	$-E^p$	$-E^p$
2	0.048			0.044			0.042	0.040	0.039
3	0.116			0.106			0.100	0.095	0.091
4	0.215			0.196			0.183	0.173	0.165
5	0.345			0.314			0.291	0.275	0.262
6	0.506			0.459			0.426	0.401	0.382
7	0.699			0.633			0.585	0.550	0.524
8	0.923			0.834			0.771	0.724	0.688
9	1.178			1.064			0.982	0.922	0.876
10	1.464			1.321			1.219	1.143	1.086

[p] Present results
[a] Roy *et al.* [20]
[b] Ho and Bhatia [28]
[c] Ray *et al.* [29]
[d] Lipsky *et al.* [24]
[e] Ray and Mukherjee [22].

Table 9. Energy positions *(−E)* for doubly excited *Nppn* $^3D^e$ *(n* = 4 – 12*)* states of Helium-like ions (Z = 2 – 10) below the *N* = *3 – 8* hydrogenic thresholds. Comparison is made with other theoretical results. The results are expressed in atomic units

3*pnp*	3p4p ^{3}D^e			3p5p ^{3}D^e		3p6p ^{3}D^e	3p7p ^{3}D^e	3p8p ^{3}D^e
Z	$-E^p$	$-E^a$	$-E^b$	$-E^p$	$-E^b$	$-E^p$	$-E^p$	$-E^p$
2	0.289	0.274	0.267	0.266	0.245	0.252	0.245	0.239
3	0.683	0.667	0.652	0.618	0.587	0.582	0.560	0.546
4	1.250	1.233	1.211	1.121	1.078	1.050	1.008	0.980
5	1.990	1.973	1.943	1.774	1.718	1.657	1.586	1.540
6	2.903			2.579		2.403	2.296	2.227
7	3.990			3.535		3.287	3.138	3.041
8	5.250			4.642		4.311	4.111	3.981
9	6.685			5.900		5.473	5.216	5.048
10	8.292			7.309		6.774	6.452	6.242
4*pnp*	4p5p $^3D^e$			4p6p $^3D^e$		4p7p $^3D^e$	4p8p $^3D^e$	4p9p $^3D^e$
Z	$-E^p$	$-E^a$	$-E^c$	$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.168	0.159	0.155	0.155		0.147	0.142	0.139
3	0.399	0.390	0.384	0.363		0.342	0.328	0.318
4	0.732	0.724	0.714	0.661		0.619	0.591	0.572
5	1.167	1.160	1.148	1.049		0.979	0.933	0.901
6	1.704			1.528		1.421	1.352	1.305
7	2.344			2.096		1.947	1.850	1.783
8	3.086			2.755		2.555	2.426	2.337
9	3.931			3.504		3.247	3.080	2.965
10	4.878			4.344		4.021	3.812	3.668
5*pnp*	5p5p $^3D^e$			5p6p $^3D^e$		5p7p $^3D^e$	5p8p $^3D^e$	5p9p $^3D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.124			0.110		0.102	0.097	0.094
3	0.298			0.262		0.241	0.227	0.217
4	0.553			0.482		0.439	0.411	0.392
5	0.887			0.769		0.698	0.652	0.620
6	1.301			1.124		1.017	0.948	0.900
7	1.795			1.546		1.396	1.299	1.232
8	2.369			2.037		1.836	1.706	1.617
9	3.023			2.595		2.337	2.169	2.054
10	3.756			3.221		2.897	2.687	2.543
6*pnp*	6p6p $^3D^e$			6p7p $^3D^e$		6p8p $^3D^e$	6p9p $^3D^e$	6p10p $^3D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.086			0.078		0.073	0.069	0.067
3	0.208			0.186		0.172	0.162	0.155
4	0.384			0.342		0.314	0.295	0.281
5	0.617			0.545		0.499	0.467	0.445
6	0.904			0.797		0.728	0.680	0.646
7	1.247			1.097		1.000	0.933	0.885
8	1.646			1.446		1.315	1.226	1.162
9	2.100			1.842		1.674	1.559	1.477
10	2.610			2.287		2.077	1.932	1.829
7*pnp*	7p7p $^3D^e$			7p8p $^3D^e$		7p9p $^3D^e$	7p10p $^3D^e$	7p11p $^3D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.063			0.058		0.055	0.052	0.050
3	0.153			0.139		0.129	0.122	0.117
4	0.283			0.255		0.236	0.222	0.212
5	0.453			0.407		0.375	0.352	0.336
6	0.665			0.595		0.548	0.513	0.488
7	0.917			0.819		0.753	0.705	0.669
8	1.210			1.080		0.990	0.926	0.879
9	1.544			1.376		1.261	1.178	1.118
10	1.919			1.708		1.564	1.461	1.385
8*pnp*	8p8p $^3D^e$			8p9p $^3D^e$		8p10p $^3D^e$	8p11p $^3D^e$	8p12p $^3D^e$
Z	$-E^p$			$-E^p$		$-E^p$	$-E^p$	$-E^p$
2	0.049			0.045		0.042	0.040	0.039
3	0.117			0.107		0.100	0.095	0.091
4	0.217			0.197		0.184	0.174	0.166
5	0.347			0.316		0.293	0.276	0.263
6	0.509			0.462		0.427	0.402	0.383
7	0.702			0.635		0.588	0.552	0.525
8	0.927			0.837		0.773	0.726	0.690
9	1.183			1.067		0.985	0.924	0.877
10	1.469			1.325		1.222	1.146	1.087

[p] Present results
[a] Roy *et al.* [20]
[b] Lipsky *et al.* [24]
[c] Pekeris [25].

Table 10. Energy positions *(−E)* for doubly excited *Npnd* $^1F^0$ *(n* = 3 – 12*)* states of Helium-like ions (Z = 2 – 10) below the *N* = *3 – 8* hydrogenic thresholds. Comparison is made with other theoretical results.The results are expressed in atomic units

3*pnd*	3p3d $^1F^0$			3p4d $^1F^0$			3p5d $^1F^0$			3p6d $^1F^0$	3p7d $^1F^0$
Z	$-E^p$	$-E^a$	$-E^b$	$-E^p$	$-E^a$	$-E^c$	$-E^p$	$-E^a$	$-E^c$	$-E^p$	$-E^p$
2	0.317	0.278		0.278	0.260	0.278	0.259	0.248	0.228	0.248	0.242
3	0.781	0.744	0.771	0.662	0.641	0.673	0.606	0.591	0.570	0.575	0.555
4	1.464	1.433	1.458	1.219	1.197	1.242	1.103	1.086	1.064	1.039	1.000
5	2.369	2.343	2.366	1.948	1.926	1.984	1.751	1.732	1.708	1.642	1.577
6	3.496			2.851			2.550		2.504	2.385	2.284
7	4.845			3.928			3.500		3.451	3.266	3.123
8	6.415			5.178			4.601		4.549	4.285	4.094
9	8.208			6.602			5.853		5.798	5.444	5.196
10	10.223			8.199			7.256		7.198	6.742	6.430
4*pnd*	4p4d $^1F^0$			4p5d $^1F^0$			4p6d $^1F^0$			4p7d $^1F^0$	4p8d $^1F^0$
Z	$-E^p$	$-E^a$		$-E^p$	$-E^a$		$-E^p$	$-E^a$		$-E^p$	$-E^p$
2	0.182	0.159		0.163	0.152		0.152	0.144		0.145	0.140
3	0.446	0.423		0.389	0.378		0.357			0.337	0.324
4	0.834	0.812		0.716	0.701		0.652			0.612	0.586
5	1.347	1.326		1.146	1.137		1.037			0.970	0.927
6	1.984			1.679			1.512			1.411	1.345
7	2.746			2.314			2.077			1.934	1.841
8	3.633			3.051			2.733			2.541	2.415
9	4.645			3.891			3.479			3.230	3.068
10	5.782			4.833			4.315			4.002	3.798
5*pnd*	5p5d $^1F^0$			5p6d $^1F^0$			5p7d $^1F^0$			5p8d $^1F^0$	5p9d $^1F^0$
Z	$-E^p$			$-E^p$			$-E^p$			$-E^p$	$-E^p$
2	0.118			0.107			0.100			0.096	0.092
3	0.289			0.256			0.236			0.224	0.215
4	0.538			0.473			0.433			0.407	0.389
5	0.868			0.757			0.690			0.646	0.616
6	1.277			1.109			1.007			0.940	0.895
7	1.766			1.528			1.384			1.290	1.226
8	2.336			2.016			1.822			1.696	1.610
9	2.985			2.571			2.320			2.158	2.046
10	3.714			3.194			2.879			2.674	2.534
6*pnd*	6p6d $^1F^0$			6p7d $^1F^0$			6p8d $^1F^0$			6p9d $^1F^0$	6p10d $^1F^0$
Z	$-E^p$			$-E^p$			$-E^p$			$-E^p$	$-E^p$
2	0.083			0.076			0.071			0.068	0.066
3	0.202			0.182			0.169			0.160	0.153
4	0.376			0.336			0.310			0.292	0.279
5	0.605			0.538			0.494			0.463	0.441
6	0.890			0.788			0.721			0.675	0.642
7	1.231			1.086			0.992			0.927	0.881
8	1.627			1.432			1.306			1.219	1.157
9	2.078			1.827			1.664			1.551	1.471
10	2.585			2.270			2.065			1.924	1.823
7*pnd*	7p7d $^1F^0$			7p8d $^1F^0$			7p9d $^1F^0$			7p10d $^1F^0$	7p11d $^1F^0$
Z	$-E^p$			$-E^p$			$-E^p$			$-E^p$	$-E^p$
2	0.061			0.057			0.053			0.051	0.049
3	0.149			0.136			0.127			0.120	0.115
4	0.277			0.251			0.233			0.220	0.210
5	0.446			0.402			0.371			0.350	0.333
6	0.656			0.589			0.543			0.510	0.485
7	0.907			0.812			0.747			0.700	0.666
8	1.198			1.071			0.984			0.921	0.875
9	1.530			1.366			1.253			1.173	1.113
10	1.903			1.697			1.556			1.455	1.380
8*pnd*	8p8d $^1F^0$			8p9d $^1F^0$			8p10d $^1F^0$			8p11d $^1F^0$	8p12d $^1F^0$
Z	$-E^p$			$-E^p$			$-E^p$			$-E^p$	$-E^p$
2	0.047			0.044			0.042			0.040	0.039
3	0.115			0.105			0.099			0.094	0.090
4	0.213			0.195			0.182			0.172	0.165
5	0.343			0.312			0.290			0.274	0.261
6	0.503			0.457			0.424			0.399	0.381
7	0.695			0.630			0.584			0.549	0.523
8	0.919			0.831			0.769			0.722	0.687
9	1.173			1.060			0.980			0.920	0.874
10	1.459			1.317			1.216			1.141	1.084

[p] Present results
[a] Roy *et al.* [20]
[b] Bachau *et al.* [23]
[c] Sakho *et al.* [26].

Table 11. Energy positions *(−E)* for doubly excited *Npnd* $^3F^0$ *(n* = 3 – 11*)* states of Helium-like ions (Z = 2 – 10) below the *N* = *3 – 8* hydrogenic thresholds. Comparison is made with other theoretical results. The results are expressed in atomic units

3*pnd*	3p3d $^3F^0$				3p4d $^3F^0$			3p5d $^3F^0$			3p6d $^3F^0$
Z	$-E^p$	$-E^a$	$-E^b$	$-E^c$	$-E^p$	$-E^a$	$-E^b$	$-E^p$	$-E^a$	$-E^d$	$-E^p$
2	0.332	0.334	0.326	0.325	0.285	0.265	0.271	0.263	0.249	0.238	0251
3	0.809	0.828	0.823	0.815	0.675	0.652	0.660	0.613	0.596	0.581	0.579
4	1.506	1.545	1.536	1.528	1.238	1.212	1.222	1.114	1.093	1.074	1.046
5	2.424	2.483	2.471	2.463	1.975	1.947	1.958	1.766	1.741	1.719	1.651
6	3.564		3.629	3.620	2.884		2.868	2.568		2.515	2.396
7	4.927		5.008	4.999	3.967		3.951	3.522		3.462	3.279
8	6.511		6.610	6.601	5.224		5.208	4.627		4.559	4.301
9	8.318		8.434	8.425	6.654		6.639	5.882		5.810	5.462
10	10.347		10.480	10.471	8.258		8.243	7.289		7.209	6.762
4*pnd*	4p4d $^3F^0$				4p5d $^3F^0$			4p6d $^3F^0$			4p7d $^3F^0$
Z	$-E^p$	$-E^a$			$-E^p$	$-E^a$		$-E^p$	$-E^a$		$-E^p$
2	0.189	0.194			0.166	0.155		0.154	0.145		0,146
3	0.458	0.476			0.395	0.383		0.361			0,340
4	0.851	0.883			0.726	0.714		0.658			0,616
5	1.370	1.415			1.159	1.147		1.045			0,975
6	2.013				1.695			1.522			1,417
7	2.780				2.333			2.089			1,942
8	3.673				3.073			2.747			2,550
9	4.691				3.916			3.495			3,240
10	5.834				4.862			4.333			4,014
5*pnd*	5p5d $^3F^0$				5p6d $^3F^0$			5p7d $^3F^0$			5p8d $^3F^0$
Z	$-E^p$				$-E^p$			$-E^p$			$-E^p$
2	0.122				0.109			0.101			0.097
3	0.295				0.260			0.239			0.225
4	0.547				0.478			0.437			0.410
5	0.879				0.764			0.695			0.649
6	1.291				1.118			1.013			0.945
7	1.784				1.539			1.391			1.296
8	2.356				2.028			1.831			1.702
9	3.008				2.585			2.330			2.164
10	3.740				3.210			2.890			2.682
6*pnd*	6p6d $^3F^0$				6p7d $^3F^0$			6p8d $^3F^0$			6p9d $^3F^0$
Z	$-E^p$				$-E^p$			$-E^p$			$-E^p$
2	0.085				0.077			0.072			0.069
3	0.205				0.184			0.170			0.161
4	0.381				0.339			0.312			0.293
5	0.612				0.542			0.497			0.466
6	0.899				0.793			0.725			0.678
7	1.241				1.093			0.997			0.930
8	1.638				1.440			1.311			1.223
9	2.091				1.836			1.670			1.556
10	2.600				2.280			2.072			1.929
7*pnd*	7p7d $^3F^0$				7p8d $^3F^0$			7p9d $^3F^0$			7p10d $^3F^0$
Z	$-E^p$				$-E^p$			$-E^p$			$-E^p$
2	0.063				0.058			0.054			0.052
3	0.151				0.137			0.128			0.121
4	0.280				0.253			0.234			0.221
5	0.450				0.405			0.374			0.351
6	0.661				0.593			0.546			0.512
7	0.913				0.816			0.750			0.703
8	1.205				1.076			0.988			0.924
9	1.538				1.372			1.258			1.176
10	1.912				1.704			1.561			1.458
8*pnd*	8p8d $^3F^0$				8p9d $^3F^0$			8p10d $^3F^0$			8p11d $^3F^0$
Z	$-E^p$				$-E^p$			$-E^p$			$-E^p$
2	0.048				0.045			0.042			0.040
3	0.116				0.106			0.100			0.095
4	0.215				0.196			0.183			0.173
5	0.345				0.314			0.292			0.275
6	0.507				0.460			0.426			0.401
7	0.699				0.633			0.586			0.551
8	0.923				0.835			0.771			0.724
9	1.179				1.064			0.983			0.922
10	1.465				1.322			1.219			1.144

[p] Present results
[a] Roy *et al.* [20]
[b] Bachau *et al.* [23]
[c] Ivanov and Safronova [27]
[d] Sakho *et al.* [26]

Table 12. Energy positions *(−E)* for doubly excited *Ndnd* $^1G^e$ *(n* = 3 – 12*)* states of Helium-like ions (Z = 2 – 10) below the *N = 3 – 8* hydrogenic thresholds. Comparison is made with other theoretical results. The results are expressed in atomic units

3*dnd*	3d3d ^{1}G^e				3d4d ^{1}G^e		3d5d ^{1}G^e		3d6d ^{1}G^e	3d7d ^{1}G^e
Z	$-E^p$	$-E^a$	$-E^b$	$-E^c$	$-E^p$	$-E^a$	$-E^p$	$-E^c$	$-E^p$	$-E^p$
2	0.307	0.293		0.310	0.273	0.280	0.256	0.267	0.246	0.241
3	0.762	0.762	0.769	0.777	0.653	0.650	0.600	0.609	0.571	0.553
4	1.437	1.454	1.466	1.464	1.205	1.202	1.095	1.098	1.034	0.997
5	2.332	2.368	2.377	2.373	1.930	1.922	1.740	1.741	1.636	1.572
6	3.450				2.828		2.537		2.376	2.279
7	4.789				3.900		3.484		3.256	3.117
8	6.350				5.146		4.583		4.274	4.086
9	8.133				6.565		5.832		5.431	5.187
10	10.139				8.157		7.233		6.727	6.420
4*dnd*	4d4d $^1G^e$				4d5d $^1G^e$		4d6d $^1G^e$		4d7d $^1G^e$	4d8d $^1G^e$
Z	$-E^p$	$-E^a$	$-E^c$		$-E^p$	$-E^c$	$-E^p$		$-E^p$	$-E^p$
2	0.178	0.173	0.195		0.160	0.175	0.150		0.144	0.140
3	0.438	0.443	0.460		0.384	0.397	0.354		0.335	0.323
4	0.822	0.839	0.857		0.709	0.722	0.647		0.609	0.584
5	1.330	1.359	1.380		1.137	1.143	1.031		0.966	0.924
6	1.964				1.667		1.504		1.406	1.341
7	2.722				2.299		2.068		1.928	1.836
8	3.605				3.034		2.722		2.533	2.410
9	4.612				3.872		3.467		3.222	3.062
10	5.745				4.811		4.302		3.993	3.792
5*dnd*	5d5d $^1G^e$				5d6d $^1G^e$		5d7d $^1G^e$		5d8d $^1G^e$	5d9d $^1G^e$
Z	$-E^p$	$-E^a$			$-E^p$		$-E^p$		$-E^p$	$-E^p$
2	0.116	0.113			0.105		0.099		0.095	0.092
3	0.284	0.287			0.253		0.235		0.222	0.214
4	0.532	0.543			0.469		0.430		0.405	0.388
5	0.859	0.876			0.751		0.686		0.643	0.614
6	1.266				1.102		1.002		0.937	0.892
7	1.753				1.520		1.379		1.286	1.223
8	2.320				2.006		1.816		1.691	1.606
9	2.967				2.560		2.313		2.152	2.042
10	3.694				3.181		2.871		2.668	2.529
6*dnd*	6d6d $^1G^e$				6d7d $^1G^e$		6d8d $^1G^e$		6d9d $^1G^e$	6d10d $^1G^e$
Z	$-E^p$	$-E^a$			$-E^p$		$-E^p$		$-E^p$	$-E^p$
2	0.081	0.079			0.075		0.071		0.068	0.065
3	0.199	0.201			0.180		0.168		0.159	0.153
4	0.372	0.378			0.333		0.308		0.290	0.277
5	0.600	0.611			0.534		0.491		0.461	0.440
6	0.884				0.783		0.718		0.673	0.640
7	1.223				1.081		0.988		0.924	0.878
8	1.618				1.426		1.302		1.216	1.154
9	2.068				1.820		1.659		1.548	1.468
10	2.574				2.262		2.059		1.919	1.819
7*dnd*	7d7d $^1G^e$				7d8d $^1G^e$		7d9d $^1G^e$		7d10d $^1G^e$	7d11d $^1G^e$
Z	$-E^p$	$-E^a$			$-E^p$		$-E^p$		$-E^p$	$-E^p$
2	0.060	0.058			0.056		0.053		0.051	0.049
3	0.147	0.148			0.135		0.126		0.120	0.115
4	0.275	0.278			0.249		0.231		0.219	0.209
5	0.443	0.450			0.400		0.370		0.348	0.332
6	0.652				0.586		0.541		0.508	0.484
7	0.902				0.808		0.744		0.698	0.664
8	1.192				1.067		0.981		0.919	0.873
9	1.523				1.361		1.250		1.170	1.111
10	1.895				1.692		1.552		1.452	1.377
8*dnd*	8d8d $^1G^e$				8d9d $^1G^e$		8d10d $^1G^e$		8d11d $^1G^e$	8d12d $^1G^e$
Z	$-E^p$				$-E^p$		$-E^p$		$-E^p$	$-E^p$
2	0.047				0.043		0.041		0.040	0.038
3	0.113				0.105		0.098		0.093	0.090
4	0.211				0.194		0.181		0.171	0.164
5	0.340				0.310		0.289		0.273	0.261
6	0.501				0.455		0.422		0.398	0.380
7	0.692				0.628		0.582		0.547	0.521
8	0.915				0.828		0.767		0.721	0.686
9	1.169				1.057		0.977		0.918	0.872
10	1.454				1.314		1.213		1.139	1.082

[p] Present results
[a] Roy *et al.* [20]
[b] Bachau *et al.* [23]
[c] Ray and Mukherjee [22].

Table 13. Energy positions *(−E)* for doubly excited *Ndnd* $^3G^e$ *(n* = 3 – 13*)* states of Helium-like ions (Z = 2 – 10) below the *N = 3 – 8* hydrogenic thresholds. Comparison is made with other theoretical results. The results are expressed in atomic units

3*dnd*	3d4d $^3G^e$		3d5d $^3G^e$		3d6d $^3G^e$	3d7d $^3G^e$	3d8d $^3G^e$	3d9d $^3G^e$
Z	$-E^p$	$-E^a$	$-E^p$	$-E^a$	$-E^p$	$-E^p$	$-E^p$	$-E^p$
2	0.281	0.263	0.261	0.240	0.249	0.242	0.238	0,235
3	0.668	0.647	0.609	0.590	0.576	0.556	0.544	0,535
4	1.227	1.205	1.107	1.080	1.042	1.002	0.976	0,958
5	1.959	1.937	1.757	1.730	1.646	1.579	1.535	1,505
6	2.865		2.557		2.389	2.287	2.220	2,175
7	3.945		3.509		3.271	3.127	3.033	2,968
8	5.198		4.611		4.291	4.098	3.972	3,885
9	6.624		5.865		5.451	5.200	5.038	4,926
10	8.224		7.269		6.749	6.435	6.230	6,089
4*dnd*	4d4d $^3G^e$		4d5d $^3G^e$		4d6d $^3G^e$	4d7d $^3G^e$	4d8d $^3G^e$	4d9d $^3G^e$
Z	$-E^p$		$-E^p$	$-E^a$	$-E^p$	$-E^p$	$-E^p$	$-E^p$
2	0.185		0.164	0.153	0.152	0.145	0.141	0.138
3	0.451		0.391	0.390	0.358	0.338	0.325	0.316
4	0.841		0.720	0.706	0.654	0.614	0.587	0.569
5	1.356		1.151	1.135	1.039	0.972	0.928	0.897
6	1.995		1.685		1.515	1.413	1.346	1.300
7	2.760		2.321		2.082	1.937	1.843	1.778
8	3.649		3.059		2.738	2.544	2.417	2.330
9	4.664		3.900		3.485	3.233	3.070	2.958
10	5.803		4.844		4.322	4.006	3.801	3.660
5*dnd*	5d5d $^3G^e$		5d6d $^3G^e$		5d7d $^3G^e$	5d8d $^3G^e$	5d9d $^3G^e$	5d10d $^3G^e$
Z	$-E^p$		$-E^p$		$-E^p$	$-E^p$	$-E^p$	$-E^p$
2	0.119		0.108		0.101	0.096	0.093	0.090
3	0.291		0.257		0.237	0.224	0.215	0.208
4	0.541		0.475		0.434	0.408	0.390	0.377
5	0.872		0.759		0.691	0.647	0.616	0.595
6	1.282		1.112		1.009	0.942	0.896	0.862
7	1.772		1.532		1.387	1.292	1.227	1.180
8	2.343		2.020		1.825	1.698	1.611	1.548
9	2.993		2.576		2.323	2.160	2.047	1.966
10	3.723		3.199		2.882	2.677	2.535	2.434
6*dnd*	6d6d $^3G^e$		6d7d $^3G^e$		6d8d $^3G^e$	6d9d $^3G^e$	6d10d $^3G^e$	6d11d $^3G^e$
Z	$-E^p$		$-E^p$		$-E^p$	$-E^p$	$-E^p$	$-E^p$
2	0.084		0.076		0.072	0.068	0.066	0.064
3	0.203		0.183		0.169	0.160	0.154	0.149
4	0.378		0.337		0.310	0.292	0.279	0.269
5	0.607		0.539		0.495	0.464	0.442	0.426
6	0.893		0.789		0.722	0.676	0.643	0.618
7	1.234		1.088		0.993	0.928	0.881	0.847
8	1.630		1.435		1.308	1.220	1.158	1.111
9	2.082		1.830		1.665	1.553	1.472	1.412
10	2.590		2.273		2.067	1.925	1.824	1.749
7*dnd*	7d7d $^3G^e$		7d8d $^3G^e$		7d9d $^3G^e$	7d10d $^3G^e$	7d11d $^3G^e$	7d12d $^3G^e$
Z	$-E^p$		$-E^p$		$-E^p$	$-E^p$	$-E^p$	$-E^p$
2	0.062		0.057		0.054	0.051	0.049	0.048
3	0.150		0.136		0.127	0.121	0.116	0.112
4	0.278		0.252		0.233	0.220	0.210	0.203
5	0.447		0.403		0.372	0.350	0.334	0.321
6	0.657		0.590		0.544	0.510	0.486	0.467
7	0.908		0.813		0.748	0.701	0.666	0.640
8	1.200		1.072		0.985	0.922	0.876	0.840
9	1.532		1.368		1.255	1.174	1.114	1.068
10	1.905		1.699		1.557	1.456	1.380	1.323
8*dnd*	8d8d $^3G^e$		8d9d $^3G^e$		8d10d $^3G^e$	8d11d $^3G^e$	8d12d $^3G^e$	8d13d $^3G^e$
Z	$-E^p$		$-E^p$		$-E^p$	$-E^p$	$-E^p$	$-E^p$
2	0.047		0.044		0.042	0.040	0.039	0.038
3	0.115		0.106		0.099	0.094	0.090	0.087
4	0.214		0.195		0.182	0.172	0.165	0.159
5	0.343		0.312		0.290	0.274	0.262	0.252
6	0.504		0.458		0.424	0.400	0.381	0.366
7	0.696		0.631		0.584	0.549	0.523	0.502
8	0.920		0.832		0.769	0.723	0.687	0.660
9	1.175		1.061		0.980	0.920	0.875	0.839
10	1.460		1.318		1.217	1.141	1.084	1.040

[p] Present results
[a] Pekeris [25].

The results for the triplet states *Nsnd* $^3D^e$ (n = 4 – 12) associated with thresholds N = 3 -8 are shown in Table 7. For the 3s4d $^3D^e$ states, a comparison is made with the results of Bachau *et al*. [23] and Roy *et al.* [20]. The agreement is globally satisfactory and the difference between the results never exceeded 0.02 a.u. Sometimes we even note a difference of less than 0.01 a.u. which justifies the high precision of our calculations. For the 3s5d $^3D^e$ levels our SCUNC results are compared with the semi-empirical procedure values of Sakho *et al.* [26]. Here also the agreement is considered good. For the 4s5d $^3D^e$, 5s5d $^3D^e$ and 5s6d $^3D^e$ states, we compared our values with the only available values from Roy *et al.* [20]. For all these states also we note an excellent agreement with the reference values [19]. Moreover, the differences between our values and those of the reference [20] never reached 0.02 a.u or less. For this reason we consider our results to be very accurate. The values presented for the first time for $N \geq 5$ will be, in my opinion, very useful for future studies on these states on both experimental and theoretical levels.

The doubly excited *Npnp* $^1D^e$ (n = 3 – 12) states below the N = 3 – 8 threshold are reported in Table 8. For the 3p3p $^1D^e$ states the comparison made with the values of the Density Functional formalism of Roy *et al.* [20] and the values of Ho and Bhatia [28] who used the complex rotation method with wave functions containing up to 1230 Hylleraas functions shows a very good agreement and moreover, the energy deviations with the quoted reference values never went beyond 0.03 a.u which testifies to the very good accuracy of our calculations. For the 3p4p $^1D^e$ levels, the comparison of our results with those of Ray *et al.* [30] and Lipsky *et al.* [23] indicates a satisfactory agreement up to Z = 5. However it should be mentioned that our values are closer to those of Lipsky et al. [23] with a maximum energy difference of 0.007 a.u. Our results for the 4p4p $^1D^e$ states are compared with the only available values of Roy *et al.* [20] and Ray and Mukherjee [22] here also, we note a very good agreement. For the 4p5p $^1D^e$ states, a comparison is made with the values of Roy *et al.* [20] and Ray and Mukherjee [22] we also note for these states a very good agreement and the energy difference between our present calculations and that of the authors mentioned never exceeded 0.02 a.u which allows us to consider our values as very precise.For the 5p5p $^1D^e$ states, the comparison made with the only ones available from Roy *et al.* [20] shows a very good agreement and the maximum deviation between our values and those of the reference which is 0.007 a.u proves sufficiently the precision of our calculations. Although we notice both an overestimation and an underestimation of the calculated positions, the overall agreement is again good, considering the simplicity of the present formalism. Moreover, the good agreement of our results with the cited reference values allows us to consider our presented values for the *Npnp* $^1D^e$ levels associated with the higher N thresholds (N = 5, 6, 7 and 8) as accurate.

Results for $^3D^e$ states arising from *Npnp* configurations associated with N = 3 – 8 thresholds are shown in Table 9. A comparison is made with the results of Roy *et al*. [20], Lipsky *et al.* [23] and of Pekeris [25]. The overall agreement is quite satisfactory. For the *Npnp* $^3D^e$ states with $N \geq 5$, there are no values in the literature. These latter states are reported here for the first time.

Table 10 shows the results for the *Npnd* $^1F^0$ (n = 3 – 12) states of helium and its isoelectronic series below the N = 3 – 8 threshold. No experimental results are available for these states for direct comparison. Our present results are compared with the values of Roy *et al.* [20], Bachau *et al*. [23] and Sakho *et al*. [26]. For all the levels listed in this table our agreements are very satisfactory and our calculations are very accurate up to Z = 10. Note that for the *Npnd* $^1F^0$ states with $N \geq 5$ (N = 5, 6, 7, 8...) we did not find any results in the literature hence the results presented for the first time in this table will be of great use for future studies.

In Table 11, a comparison is made with the theoretical results obtained by Roy *et al*. [20], Bachau *et al*. [23], Ivanov and Safronova [27] and Sakho *et al.* [26]. For the 3p3d $^3F^0$ state our results are in good agreement with those of Roy *et al*. [20], Bachau *et al.* [23] and Ivanov and Safronova [27]. For the 3p4d $^3F^0$ state our results are in perfect agreement with the results of Roy *et al.* [20] and Bachau *et al.* [23]. For the 3p5d $^3F^0$ state, the comparison with the values of Roy *et al.* [20] and Sakho *et al.* [26]. For the 4p4d $^3F^0$, 4p5d $^3F^0$ and 4p6d $^3F^0$ states, the comparison made with the only available values of Roy *et al.* [20] showed a very good agreement. Although we note slight discrepancies, the overall agreement with the theoretical reference values is good. To our knowledge, in this table also some of the calculated states are presented for the first time in the literature.

In Table 12 and Table 13, we present a comparison of the present SCUNC calculations of the energy positions ($-E$, a.u) of the doubly excited $^{1,3}G^e$ resonance states of helium-like ions (Z = 2 – 10) under the N = 3 – 8 thresholds with the results of Roy *et al.* [20] who applied density functional theory (DFT), with data from the Feschbach projection formalism of Bachau *et al.* [23], with values from the time-independent variational perturbation theory of Ray and Mukherjee [22], and with results from Pekeris [25]. Here again, the agreements between our calculations and those of these authors are generally good and with very good accuracy. It should be mentioned that for these states a multitude of new values never published in the literature are reported for the first time in this article.

In Table 14 – Table 19 are quoted our results for the excitation energies of the doubly excited *Nsns* $^{1,3}S^e$, *Nsnp* $^{1,3}P^0$, *Nsnd* $^{1,3}D^e$, *Npnp* $^{1,3}D^e$, *Npnd* $^{1,3}F^0$, and *Ndnd* $^{1,3}G^e$ states of helium and heliumlike ions with nuclear charge $Z \leq 10$ below the hydrogenic thresholds N = 3 – 8. Our excitation energies are calculated with respect to the precise ground state energies of Frankowski and Pekeris [32]. Note that our excitation energies ΔE are deduced from the values of the resonance energies E (*Nlnl'*) from which the precise values of the Frankowski and Pekeris ground state energy E ($1s^2$) of the atomic system under consideration must be subtracted. Let us say by using the following relation:

$$\Delta E = E\left(N\ell\, n\ell'\right) - E\left(1s^2\right) \tag{42}$$

The Frankowski and Pekeris [32] ground state energies are expressed in atomic units (a.u): – 2.90372 (*He*), – 7.27991 (Li^+), – 13.65556 (Be^{2+}) and – 22.03097 (B^{3+}). The comparison of our present results with those of the authors quoted in these different tables shows a very good

agreement and the energy differences between our present predictions and the results of other methods reported in these tables are very small as shown by the values given in the last three columns of Table 14 to Table 19. Analysis of these values clearly shows the very good accuracy of our calculations.

Table 14. Comparison of excitation energy of $Nsns\ ^{1,3}S^e$ (N = 3 – 5 and n = 4 – 6) resonances of helium-like ions (Z = 2 – 5) with some experimental and theoretical results. Our excitation energies are obtained with respect to the accurate ground-state energies of Frankowski and Pekeris [32]. The figures in the last three columns show the absolute deviation of the excitation energy from the reference values shown in columns 5, 6 and 7

Z	$Nlnl'\ ^{2S+1}L^{\pi}$	Total energy ($-E$, *a.u*)	Excitation energy (ΔE, a.u)				Deviation		
			Present work	Literature values					
2	3s4s $^1S^e$	0.285	2.619	2.615[a]	2.615[b]	2.622[c]	0.004	0.004	0.003
	3s5s $^1S^e$	0.262	2.642	2.649[e]	2.633[b]		0.007	0.009	
	3s4s $^3S^e$	0.286	2.618	2.627[d]	2.616[c]		0.009	0.002	
	3s5s $^3S^e$	0.263	2.641	2.646[e]			0.005		
	4s5s $^1S^e$	0.173	2.731	2.738[a]	2.739[f]	2.749[d]	0.007	0.008	0.018
	4s5s $^3S^e$	0.166	2.738	2.743[d]	2.747[g]		0.005	0.009	
	5s5s $^1S^e$	0.121	2.783						
	5s6s $^1S^e$	0.108	2.796						
	5s5s $^3S^e$	0.122	2.782						
	5s6s $^3S^e$	0.109	2.795						
3	3s4s $^1S^e$	0.674	6.606	6.600[b]	6.626[d]	6.595[c]	0.006	0.020	0.011
	3s5s $^1S^e$	0.612	6.668	6.658[b]			0.010		
	3s4s $^3S^e$	0.677	6.603	6.608[d]	6.591[c]	6.611[g]	0.005	0.012	0.008
	3s5s $^3S^e$	0.614	6.666	6.669[e]			0.003		
	4s5s $^1S^e$	0.407	6.873	6.874[b]	6.898[d]		0.001	0.019	
	4s5s $^3S^e$	0.395	6.885	6.887[d]	6.892[g]		0.002	0.007	
	5s5s $^1S^e$	0.293	6.987	6.985[d]	6.962[b]		0.002	0.025	
	5s6s $^1S^e$	0.259	7.021						
	5s5s $^3S^e$	0.295	6.985						
	5s6s $^3S^e$	0.260	7.020						
4	3s4s $^1S^e$	1.237	12.418	12.409[b]	12.440[d]		0.009	0.022	
	3s5s $^1S^e$	1.112	12.544	12.543[b]			0.001		
	3s4s $^3S^e$	1.241	12.415	12.415[d]	12.424[g]	12.391[c]	0.001	0.009	0.024
	3s5s $^3S^e$	1.115	12.541	12.541[e]			0.000		
	4s5s $^1S^e$	0.744	12.912	12.919[b]	12.944[d]		0.007	0.032	
	4s5s $^3S^e$	0.727	12.929	12.928[d]	12.936[f]		0.001	0.007	
	5s5s $^1S^e$	0.545	13.111	13.103[d]	13.081[b]		0.008	0.030	
	5s6s $^1S^e$	0.477	13.179						
	5s5s $^3S^e$	0.547	13.109						
	5s6s $^3S^e$	0.478	13.178						
5	3s4s $^1S^e$	1.973	20.058	20.063[b]	20.081[d]		0.005	0.023	
	3s5s $^1S^e$	1.764	20.267	20.269[b]			0.002		
	3s4s $^3S^e$	1.978	20.052	20.048[d]	20.017[c]	20.077[g]	0.004	0.035	0.025
	3s5s $^3S^e$	1.767	20.264	20.264[e]			0.000		
	4s5s $^1S^e$	1.183	20.848	20.886[d]	20.863[b]		0.038	0.015	
	4s5s $^3S^e$	1.160	20.871	20.876[g]	20.866[d]		0.005	0.005	
	5s5s $^1S^e$	0.877	21.154	21.141[d]	21.123[b]		0.013	0.031	
	5s6s $^1S^e$	0.762	21.269						
	5s5s $^3S^e$	0.879	21.152						
	5s6s $^3S^e$	0.764	21.267						

[a] Koyama *et al*. [33]
[b] Ray et Mukherjee [22]
[c] Bachau *et al*. [23]
[d] Roy *et al*. [20]
[e] Lipsky *et al*. [24]
[f] Herrick and Sinanoglu [31]
[g] Das *et al*. [34].

Table 15. Comparison of excitation energy of $Nsnp\ ^{1,3}P^0$ (N = 3 – 5 and n = 4 – 7) resonances of helium-like ions (Z = 2 - 5) with some experimental and theoretical results. Our excitation energies are obtained with respect to the accurate ground-state energies of Frankowski and Pekeris [32]. The figures in the last three columns show the absolute deviation of the excitation energy from the reference values shown in columns 5, 6 and 7

Z	$Nlnl'\ ^{2S+1}L^{\pi}$	Total energy (–E, a.u)	Excitation energy (ΔE, a.u)				Deviation		
			Present work	Literature values					
2	3s4p $^1P^0$	0.278	2.626						
	3s5p $^1P^0$	0.258	2.646						
	3s4p $^3P^0$	0.280	2.624	2.632[a]	2.624[b]		0.008	0.000	
	3s5p $^3P^0$	0.260	2.644						
	4s5p $^1P^0$	0.161	2.743						
	4s5p $^3P^0$	0.163	2.741	2.746 [a]			0.005		
	5s6p $^1P^0$	0.106	2.798						
	5s7p $^1P^0$	0.099	2.805						
	5s6p $^3P^0$	0.107	2.797						
	5s7p $^3P^0$	0.100	2.804						
3	3s4p $^1P^0$	0.661	6.619						
	3s5p $^1P^0$	0.604	6.676						
	3s4p $^3P^0$	0.666	6.614	6.618[a]	6.604[b]	6.605[c]	0.004	0.010	0.013
	3s5p $^3P^0$	0.607	6.673						
	4s5p $^1P^0$	0.386	6.894						
	4s5p $^3P^0$	0.389	6.891	6.892[a]			0.001		
	5s6p $^1P^0$	0.254	7.026						
	5s7p $^1P^0$	0.234	7.046						
	5s6p $^3P^0$	0.255	7.025						
	5s7p $^3P^0$	0.236	7.044						
4	3s4p $^1P^0$	1.218	12.438						
	3s5p $^1P^0$	1.100	12.556						
	3s4p $^3P^0$	1.225	12.431	12.428[a]	12.410[b]	12.413[c]	0.003	0.021	0.018
	3s5p $^3P^0$	1.105	12.551						
	4s5p $^1P^0$	0.713	12.943						
	4s5p $^3P^0$	0.717	12.939	12.935[a]			0.004		
	5s6p $^1P^0$	0.469	13.187						
	5s7p $^1P^0$	0.430	13.226						
	5s6p $^3P^0$	0.471	13.185						
	5s7p $^3P^0$	0.432	13.224						
5	3s4p $^1P^0$	1.947	20.084						
	3s5p $^1P^0$	1.748	20.283						
	3s4p $^3P^0$	1.956	20.075	20.065[a]	20.041[b]	20.048[c]	0.010	0.034	0.027
	3s5p $^3P^0$	1.753	20.278						
	4s5p $^1P^0$	1.142	20.889						
	4s5p $^3P^0$	1.147	20.884	20.875[a]			0.009		
	5s6p $^1P^0$	0.752	21.279						
	5s7p $^1P^0$	0.686	21.345						
	5s6p $^3P^0$	0.755	21.276						
	5s7p $^3P^0$	0.688	21.343						

[a] Roy *et al*. [20]
[b] Bachau *et al*. [23]
[c] Lipsky *et al*. [24].

Table 16. Comparison of excitation energy of $Nsnd\ ^{1,3}D^e$ (N = 3 – 5 and n = 4 – 7) resonances of helium-like ions (Z = 2 – 5) with some experimental and theoretical results. Our excitation energies are obtained with respect to the accurate ground-state energies of Frankowski and Pekeris [32]. The figures in the last three columns show the absolute deviation of the excitation energy from the reference values shown in columns 5, 6 and 7

Z	$Nlnl'\ ^{2S+1}L^{\pi}$	Total energy ($-E$, *a.u*)	Excitation energy (ΔE, a.u)				Deviation		
			Present work	Literature values					
2	3s4d $^1D^e$	0.272	2.632						
	3s5d $^1D^e$	0.255	2.649						
	3s4d $^3D^e$	0.275	2.629	2.621[a]	2.637[b]	2.636[c]	0.008	0.008	0.007
	3s5d $^3D^e$	0.256	2.648	2.648[a]	2.654[b]	2.648[d]	0.000	0.006	0.000
	3s6d $^3D^e$	0.246	2.658	2.662[b]			0.004		
	4s5d $^1D^e$	0.158	2.746						
	4s5d $^3D^e$	0.160	2.744	2.744[a]	2.748[b]		0.000	0.004	
	4s6d $^3D^e$	0.149	2.755	2.758[b]			0.003		
	5s6d $^1D^e$	0.103	2.801						
	5s7d $^1D^e$	0.097	2.807						
	5s6d $^3D^e$	0.115	2.789	2.783[b]			0.006		
	5s7d $^3D^e$	0.104	2.800	2.801[b]			0.001		
3	3s4d $^1D^e$	0.651	6.629						
	3s5d $^1D^e$	0.598	6.682						
	3s4d $^3D^e$	0.657	6.623	6.626[b]	6.627[c]	6.628[d]	0.003	0.004	0.005
	3s5d $^3D^e$	0.601	6.679	6.673[d]	6.684[b]		0.006	0.005	
	4s5d $^1D^e$	0.380	6.899						
	4s5d $^3D^e$	0.383	6.897						
	5s6d $^1D^e$	0.250	7.030						
	5s7d $^1D^e$	0.232	7.048						
	5s5d $^3D^e$	0.282	6.998	6.981[b]			0.017		
	5s6d $^3D^e$	0.252	7.028						
4	3s4d $^1D^e$	1.202	12.454						
	3s5d $^1D^e$	1.091	12.565						
	3s4d $^3D^e$	1.211	12.445	12.444[c]	12.446[d]	12.440[b]	0.001	0.001	0.005
	3s5d $^3D^e$	1.096	12.559	12.562[b]			0.003		
	4s5d $^1D^e$	0.704	12.952						
	4s5d $^3D^e$	0.708	12.948	12.940[b]			0.008		
	5s6d $^1D^e$	0.463	13.193						
	5s7d $^1D^e$	0.426	13.229						
	5s5d $^3D^e$	0.529	13.127	13.098[b]			0.029		
	5s6d $^3D^e$	0.466	13.189						
5	3s4d $^1D^e$	1.927	20.104						
	3s5d $^1D^e$	1.735	20.296						
	3s4d $^3D^e$	1.938	20.093	20.080[b]	20.087[c]		0.013	0.006	
	3s5d $^3D^e$	1.742	20.289	20.288[b]	20.299[d]		0.001	0.010	
	4s5d $^1D^e$	1.129	20.902						
	4s5d $^3D^e$	1.136	20.895	20.882[b]			0.013		
	5s6d $^1D^e$	0.744	21.287						
	5s7d $^1D^e$	0.680	21.351						
	5s5d $^3D^e$	0.856	21.175	21.134[b]			0.041		
	5s6d $^3D^e$	0.748	21.283						

[a] Shearer-Izumi [35]
[b] Roy *et al.* [20]
[c] Bachau *et al.* [23]
[d] Lispky *et al.* [24].

Table 17. Comparison of excitation energy of $Npnp\ ^{1,3}D^e$ (N = 3 – 5 and n = 3 – 6) resonances of helium-like ions (Z = 2 – 5) with some experimental and theoretical results. Our excitation energies are obtained with respect to the accurate ground-state energies of Frankowski and Pekeris [32]. The figures in the last three columns show the absolute deviation of the excitation energy from the reference values shown in columns 5, 6 and 7

Z	$Nlnl'\ ^{2S+1}L^{\pi}$	Total energy ($-E$, a.u)	Excitation energy (ΔE, a.u)				Deviation		
			Present work	Literature values					
2	3p3p $^1D^e$	0.329	2.575	2.575[a]	2.548[b],	2.559[c]	0.000	0.027	0.016
	3p4p $^1D^e$	0.284	2.619	2.619[d]	2.619[b]	2.629[c]	0.000	0.000	0.010
	3p5p $^1D^e$	0.262	2.642	2.634[d]	2.643[b]	2.651[c]	0.008	0.001	0.009
	3p4p $^3D^e$	0.289	2.615	2.611[e]	2.642[f]	2.630[a]	0.004	0.027	0.015
	3p5p $^3D^e$	0.266	2.638	2.633[e]	2.657[g]	2.658[f]	0.005	0.019	0.019
	4p5p $^1D^e$	0.165	2.739	2.727[d]	2.729[b]	2.740[c]	0.012	0.010	0.001
	4p5p $^3D^e$	0.168	2.736	2.749[g]	2.744[a]		0.013	0.008	
	5p5p $^1D^e$	0.121	2.783						
	5p6p $^1D^e$	0.109	2.795						
	5p5p $^3D^e$	0.124	2.780						
	5p6p $^3D^e$	0.110	2.794						
3	3p3p $^1D^e$	0.803	6.477	6.467[d]	6.478[a]	6.484[h]	0.010	0.001	0.007
	3p4p $^1D^e$	0.673	6.607	6.611[d]	6.634 [a]	6.637[h]	0.004	0.027	0.030
	3p4p $^3D^e$	0.683	6.597	6.596[e]	6.613[a]		0.001	0.016	
	3p5p $^3D^e$	0.618	6.662	6.658[e]			0.004		
	4p5p $^1D^e$	0.393	6.887	6.880 [e]	6.902[a]	6.879[h]	0.007	0.015	0.008
	4p5p $^3D^e$	0.399	6.881	6.868[e]	6.889[a]		0.013	0.008	
	5p5p $^1D^e$	0.293	6.987	6.988[a]	6.971[d]		0.001	0.016	
	5p6p $^1D^e$	0.259	7.021						
	5p5p $^3D^e$	0.298	6.982						
	5p6p $^3D^e$	0.262	7.018						
4	3p3p $^1D^e$	1.497	12.159	12.132[d]	12.145[a]	12.158[h]	0.027	0.005	0.001
	3p4p $^1D^e$	1.234	12.422	12.439[d]	12.453[a]	12.459[h]	0.017	0.031	0.037
	3p4p $^3D^e$	1.250	12.406	12.407[e]	12.422[a]		0.001	0.016	
	3p5p $^3D^e$	1.121	12.535	12.538[e]			0.005		
	4p5p $^1D^e$	0.723	12.933	12.928[d]	12.949[a]		0.005	0.016	
	4p5p $^3D^e$	0.732	12.924	12.942[g]	12.931[a]		0.000	0.007	
	5p5p $^1D^e$	0.545	13.111	13.107[a]	13.166[d]		0.004	0.055	
	5p6p $^1D^e$	0.477	13.179						
	5p5p $^3D^e$	0.553	13.103						
	5p6p $^3D^e$	0.482	13.174						
5	3p3p $^1D^e$	2.412	19.619	19.599[d]	19.591[a]	19.608[h]	0.020	0.028	0.011
	3p4p $^1D^e$	1.969	20.062	20.085[d]	20.098[a]	20.107[h]	0.023	0.036	0.045
	3p4p $^3D^e$	1.990	20.041	20.081[g]	20.058[a]		0.040	0.017	
	3p5p $^3D^e$	1.774	20.257	20.292[g]			0.035		
	4p5p $^1D^e$	1.155	20.876	20.881[d]	20.894[a]		0.005	0.018	
	4p5p $^3D^e$	1.167	20.864	20.883[g]	20.871[a]		0.019	0.007	
	5p5p $^1D^e$	0.877	21.154	21.146[a]	21.131[d]		0.008	0.023	
	5p6p $^1D^e$	0.763	21.268						
	5p5p $^3D^e$	0.887	21.144						
	5p6p $^3D^e$	0.769	21.262						

[a] Roy *et al.* [20].
[b] Fukuda *et al.* [30]
[c] Herrick and Sinanoglu [31]
[d] Ray and Mukherjee [22]
[e] Mukherji [36]
[f] Lispky *et al.* [24]
[g] Pekeris [25]
[h] Bachau *et al.* [23].

Table 18. Comparison of excitation energy of *Npnd* $^{1,3}F^0$ (*N* = 3 – 4 *and n* = 3 – 6) resonances of helium-like ions (Z = 2 - 5) with some experimental and theoretical results. Our excitation energies are obtained with respect to the accurate ground-state energies of Frankowski and Pekeris [32]. The figures in the last three columns show the absolute deviation of the excitation energy from the reference values shown in columns 5, 6 and 7

Z	$Nlnl'\ ^{2S+1}L^{\pi}$	Total energy (–*E*, a.u)	Excitation energy (Δ*E*, a.u)				Deviation		
			Present Work	Literature values					
	3p3d $^1F^0$	0.317	2.587	2.626[a]	2.599[b]	2.592[c]	0.039	0.012	0.005
	3p4d $^1F^0$	0.278	2.626	2.644[a]	2.626[b]	2.623[c]	0.018	0.000	0.003
	3p5d $^1F^0$	0.259	2.645	2.656[a]	2.653[d]	2.646[c]	0.011	0.008	0.001
	3p6d $^1F^0$	0.248	2.656	2.663[a]			0.007		
	3p3d $^3F^0$	0.332	2.572	2.570[a]	2.571[b]		0.002	0.001	
	3p4d $^3F^0$	0.285	2.619	2.638[a]	2.632[b]		0.019	0.013	
	3p5d $^3F^0$	0.263	2.641	2.654[a]	2.653[d]		0.013	0.012	
	3p6d $^3F^0$	0.251	2.653	2.662[a]			0.009		
2	4p4d $^1F^0$	0.181	2.723	2.744[a]	2.718[c]		0.021	0.005	
	4p5d $^1F^0$	0.162	2.742	2.752[a]	2.732[c]		0.010	0.010	
	4p6d $^1F^0$	0.151	2.753	2.759[a]			0.006		
	4p4d $^3F^0$	0.188	2.715	2.710[a]			0.005		
	4p5d $^3F^0$	0.166	2.738	2.749[a]			0.011		
	4p6d $^3F^0$	0.154	2.750	2.759[a]			0.009		
	3p3d $^1F^0$	0.781	6.499	6.535[a]	6.509[b]	6.501[c]	0.036	0.010	0.002
	3p4d $^1F^0$	0.662	6.618	6.639[a]	6.607[b]	6.621[c]	0.021	0.011	0.003
	3p5d $^1F^0$	0.606	6.674	6.689[a]	6.675[c]		0.015	0.001	
	3p3d $^3F^0$	0.809	6.471	6.452[a]	6.456[b]	6.457[d]	0.019	0.019	0.014
	3p4d $^3F^0$	0.675	6.605	6.628[a]	6.620[b]	6.621[d]	0.023	0.015	0.016
	3p5d $^3F^0$	0.613	6.667	6.684[a]	6.683[d]		0.017	0.016	
3	4p4d $^1F^0$	0.443	6.837	6.857[a]	6.832[c]		0.020	0.005	
	4p5d $^1F^0$	0.387	6.893	6.902[a]	6.889[c]		0.009	0.004	
	4p4d $^3F^0$	0.457	6.823	6.804[a]			0.019		
	4p5d $^3F^0$	0.394	6.886	6.897[a]			0.011		
	3p3d $^1F^0$	1.464	12.192	12.223[a]	12.198[b]	12.199[c]	0.031	0.006	0.007
	3p4d $^1F^0$	1.219	12.437	12.459[a]	12.414 [b]	12.459[c]	0.022	0.023	0.022
	3p5d $^1F^0$	1.103	12.553	12.569[a]	12.559 [c]		0.016	0.006	
	3p3d $^3F^0$	1.506	12.149	12.111[a]	12.118 [b]	12.121[d]	0.038	0.031	0.028
	3p4d $^3F^0$	1.238	12.418	12.443[a]	12.434 [b]	12.437[d]	0.025	0.016	0.019
	3p5d $^3F^0$	1.114	12.542	12.563[a]	12.562 [d]		0.021	0.020	
4	4p4d $^1F^0$	0.830	12.826	12.843[a]	12.815 [c]		0.017	0.011	
	4p5d $^1F^0$	0.714	12.942	12.949[a]	12.988 [c]		0.007	0.046	
	4p4d $^3F^0$	0.849	12.807	12.773[a]			0.034		
	4p5d $^3F^0$	0.725	12.931	12.942[a]			0.011		
	3p3d $^1F^0$	2.369	19.662	19.688[a]	19.665[b]	19.663[c]	0.026	0.003	0.001
	3p4d $^1F^0$	1.948	20.083	20.105[a]	20.047[b]	20.116[c]	0.022	0.036	0.033
	3p5d $^1F^0$	1.751	20.280	20.299[a]	20.293[c]		0.019	0.013	
	3p3d $^3F^0$	2.424	19.607	19.548[a]	19.558[b]	19.564[d]	0.059	0.049	0.040
	3p4d $^3F^0$	1.975	20.056	20.084[a]	20.073[b]	20.079[d]	0.028	0.017	0.023
	3p5d $^3F^0$	1.765	20.266	20.290[a]	20.291[d]		0.024	0.025	
5	4p4d $^1F^0$	1.341	20.689	20.705[a]	20.673 [c]		0.016	0.016	
	4p5d $^1F^0$	1.143	20.888	20.894[a]	20.910 [c]		0.006	0.022	
	4p4d $^3F^0$	1.367	20.664	20.616[a]			0.048		
	4p5d $^3F^0$	1.158	20.873	20.884[a]			0.011		

[a] Roy *et al*. [20]
[b] Bachau *et al.* [23]
[c] Ray *et al.* [29].
[d] Lispky *et al.* [24].

Table 19. Comparison of excitation energy *of Ndnd* $^{1,3}G^e$ (*N* = 3 – 7 and *n* = 3 – 7) resonances of helium-like ions (Z = 2 - 5) with some experimental and theoretical results. Our excitation energies are obtained with respect to the accurate ground-state energies of Frankowski and Pekeris [32]. The figures in the last three columns show the absolute deviation of the excitation energy from the reference values shown in columns 5, 6 and 7

Z	$Nlnl'\ ^{2S+1}L^{\pi}$	Total energy (–*E*, a.u)	Excitation energy (Δ*E*, a.u)				Deviation		
			Present work	Literature values					
2	3d3d $^1G^e$	0.307	2.597	2.595[a]	2.592[b]	2.611[c]	0.002	0.005	0.014
	3d4d $^1G^e$	0.273	2.631	2.624[b]			0.007		
	3d5d $^1G^e$	0.256	2.648	2.637[b]			0.011		
	3d4d $^3G^e$	0.281	2.623	2.641[d]	2.645[e]		0.018	0.022	
	3d5d $^3G^e$	0.261	2.643	2.660[d]			0.023		
	4d4d $^1G^e$	0.176	2.728	2.731[c]	2.709[b]		0.003	0.019	
	4d5d $^1G^e$	0.159	2.745	2.729[b]			0.016		
	4d4d $^3G^e$	0.184	2.720						
	4d5d $^3G^e$	0.163	2.741	2.751[d]	2.753[e]		0.010	0.012	
	5d5d $^1G^e$	0.116	2.788	2.791[c]	2.781[b]		0.003	0.007	
	6d6d $^1G^e$	0.081	2.823	2.825[c]			0.002		
	7d7d $^1G^e$	0.060	2.844	2.845[c]			0.001		
3	3d3d $^1G^e$	0.762	6.518	6.518[c]	6.503[a]	6.503[b]	0.000	0.015	0.015
	3d4d $^1G^e$	0.653	6.627	6.625[b]			0.002		
	3d5d $^1G^e$	0.600	6.679	6.671[b]			0.008		
	3d4d $^3G^e$	0.668	6.612	6.633[d]			0.021		
	3d5d $^3G^e$	0.609	6.671	6.690[d]			0.019		
	4d4d $^1G^e$	0.434	6.846	6.837[c],	6.820[b]		0.009	0.026	
	4d5d $^1G^e$	0.382	6.898	6.820[b]			0.078		
	4d4d $^3G^e$	0.184	7.096						
	4d5d $^3G^e$	0.163	7.117						
	5d5d $^1G^e$	0.284	6.996	6.993[c]	6.976[b]		0.003	0.020	
	6d6d $^1G^e$	0.199	7.081	7.079[c]			0.002		
	7d7d $^1G^e$	0.147	7.133	7.132[c]			0.001		
4	3d3d $^1G^e$	1.437	12.219	12.201[c],	12.190[a]	12.192[b]	0.018	0.029	0.027
	3d4d $^1G^e$	1.205	12.451	12.454[b]			0.003		
	3d5d $^1G^e$	1.095	12.561	12.558[b]			0.003		
	3d4d $^3G^e$	1.227	12.429	12.450[d]			0.021		
	3d5d $^3G^e$	1.107	12.549	12.571[d]			0.022		
	4d4d $^1G^e$	0.816	12.839	12.817[c]	12.798[b]		0.022	0.041	
	4d5d $^1G^e$	0.706	12.949	12.934[b]			0.015		
	4d4d $^3G^e$	0.838	12.818						
	4d5d $^3G^e$	0.718	12.938	12.950[d]			0.012		
	5d5d $^1G^e$	0.532	13.124	13.114[c]	13.170[b]		0.010	0.046	
	6d6d $^1G^e$	0.372	13.284	13.278[c]			0.006		
	7d7d $^1G^e$	0.275	13.381	13.377[c]			0.004		
5	3d3d $^1G^e$	2.332	19.699	19.663[c]	19.654[a]	19.658[b]	0.036	0.045	0.005
	3d4d $^1G^e$	1.930	20.101	20.109[b]			0.008		
	3d5d $^1G^e$	1.740	20.291	20.290[b]			0.001		
	3d4d $^3G^e$	1.959	20.072	20.094[d]			0.022		
	3d5d $^3G^e$	1.757	20.274	20.301[d]			0.027		
	4d4d $^1G^e$	1.322	20.709	20.672[c]	20.651[b]		0.037	0.058	
	4d5d $^1G^e$	1.133	20.898	20.888[b]			0.010		
	4d4d $^3G^e$	1.352	20.679						
	4d5d $^3G^e$	1.149	20.882	20.895[d]			0.013		
	5d5d $^1G^e$	0.859	21.172	21.155[c]	21.133[b]		0.017	0.039	
	6d6d $^1G^e$	0.600	21.431	21.420[c]			0.011		
	7d7d $^1G^e$	0.443	21.588	21.581[c]			0.007		

[a] Bachau *et al.* [23]
[b] Ray and Mukherjee [22]
[c] Roy *et al.* [20].
[d] Pekeris [25]
[e] Ho [21].

Overall, the very good agreements between the present SCUNC calculations of energy positions (– E, a.u) and excitation energies of the doubly excited $^{1,3}S^e$, $^{1,3}P^0$, $^{1,3}D^e$, $^{1,3}F^0$ and $^{1,3}G^e$ resonance states of helium-like ions (Z = 2 – 10) under the N = 3 – 8 thresholds and the various *ab-initio* results we have cited, sufficiently justify the validity of the present variational procedure of the screening constant per unit nuclear charge (SCUNC) formalism applied to the study of doubly excited states of two-electron atomic systems. It is worth mentioning that the calculations are directly obtained from simple analytical formulas, unlike all *ab-initio* methods cited in this paper.

4. Conclusion

In this paper, we have performed precise calculations on the resonance parameters (resonance energies and excitation energies) of the doubly excited states *Nsns* $^{1,3}S^e$, *Nsnp* $^{1,3}P^0$, *Nsnd* $^{1,3}D^e$, *Npnp* $^{1,3}D^e$, *Npnd* $^{1,3}F^0$ and *Ndnd* $^{1,3}G^e$ two-electron atoms (Z = 2 – 10) below the N = 3 – 8 hydrogenic thresholds in the framework of the variational method of the Screening Constant per Unit Nuclear Charge (SCUNC) formalism. The results presented in this work are in very good agreement with the results of other authors in the literature. Moreover, in this paper, we have reported for the first time in the literature a multitude of new values on the resonance parameters (resonance energies and excitation energies) of the doubly excited $^{1,3}S^e$, $^{1,3}P^0$, $^{1,3}D^e$, $^{1,3}F^0$ and $^{1,3}G^e$ states of the two-electron systems located below the hydrogenic thresholds N = 3 – 8. It is believed that the results reported in this work are quite accurate and should serve as useful references for future theoretical and experimental work but also fill the gap in results on the resonance parameters of doubly excited states of two-electron systems for high N values.

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