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A Behavioral Portfolio Decision Model with Triangular Fuzzy Number Return and Investor's Sentiment

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Abstract This paper deals with a behavioral portfolio decision problem with triangular fuzzy number return. A fuzzy sentimental mean model for behavioral portfolio decision is proposed by taking into account investor's sentiment and multiple mental accounts. The presented behavioral portfolio decision model maximizes the fuzzy sentimental mean value of portfolio return and ensures the portfolio return of each mental account exceeding the given minimum triangular fuzzy number level with a given possibility degree. Then, multiple programming models are designed to solve the optimal behavioral portfolio strategy. Finally, a numerical example is given to illustrate the validity of the proposed approach.

Keywords: *behavioral portfolio model, fuzzy number, investor's sentiment, possibility degree, mental account*

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1. Introduction

In 1952 Markowitz [1] proposed the mean-variance portfolio decision model and provided a fundamental basis for modern portfolio selection theory by maximizing the expected return for a given level of risk. In addition, Shefrin and Statman [2] proposed behavioral portfolio framework for asset choice under uncertainty based on prospect theory. In the behavioral portfolio process, each portfolio layer is associated with a particular aspiration level and resembles a separate mental account [3]. After that, Ma [4] proposed a practical decision making method for behavioral portfolio choice, Yaz [5] studied a behavioral approach to efficient portfolio formation, Mukesh [6] and Amelia [7] developed multi-criteria behavioral portfolio decision models. Jin [8] developed multi-period and multi-objective behavioral portfolio approach. Also, Xie [9] studied the behavioral assets portfolio method based on sentiment recognition.

Recently, fuzzy set have been generally used in handling and describing imprecise and complex phenomena that often rise in business, financial and managerial systems. In uncertain portfolio decision scenario, the return of financial asset is conveniently evaluated by fuzzy number. Inspired by the idea of Markowitz's M-V model, a lot of fuzzy portfolio model extensions have been proposed to deal with portfolio decision with fuzzy return and risk under fuzzy uncertain environment. For example, Wang [10], Fang [11] studied the fuzzy portfolio selection problems. Bilbao Terol [12] and Gupta [13] studied the portfolio models based on fuzzy decision theory and fuzzy programming technique. Tsaur [14] and Zhou [15] investigated fuzzy portfolio model with different investor's attitudes. Zhang [16,17] proposed some portfolio models based on possibilistic mean and variance. Rupak[18] presented the portfolio selection model based on fuzzy entropy and skewness. Mukesh [19], Yue [20] and Zhang [21] proposed fuzzy higher order moment portfolio models. Liagkouras [22], Liu [23,24], Muresh [25] and Zhang [26] also discussed the fuzzy multi-period portfolio models.

However, in uncertain portfolio decision environment the portfolio return of each mental account are usually fuzzy, and the above-mentioned fuzzy portfolio decision models have not considered the investor's sentiment and investor's behavioral interacting factors. Therefore, we will provide a new methodology to build portfolios for behavioral investors that follow ethical, environmental and social considerations in their investment process. To do so, we construct fuzzy behavioral portfolio theory with mental accounts, multiple programming models and sentiment factors. In this work, we propose a fuzzy behavioral portfolio model to determine the asset allocation between the different mental accounts according to the investor's sentiment and the historical fuzzy return data in financial market.

2. Preliminaries

Let us first review some basic concepts of triangular fuzzy numbers, which will be utilized in the following sections about fuzzy behavioral portfolio decision model with fuzzy number return.

Definition 1 [27]. A fuzzy set of real line with a normal, fuzzy convex and continuous membership function of bounded support is called a triangular fuzzy number $\tilde{A}=(a,b,c)$, if its membership function satisfies the following form:

$$\mu_{\tilde{A}}(x)=\begin{cases}1-(b-x)/(b-a), & if \quad a\le x\le b,\\ 1, & if \quad x=b,\\ 1-(x-b)/(c-b), & if \quad b\le x\le c,\\ 0, & otherwise\end{cases}$$

and $\alpha=(b-a)$ and $\beta=(c-b)$ are the left width and right width of fuzzy number $\tilde{A}$, respectively.

Definition 2 [27]. $\tilde{A}=(a,b,c)$ is a triangular fuzzy number, the λ -level cut set of $\tilde{A}$ can be computed as

$$\tilde{A}_\lambda=\{x/\mu_{\tilde{A}}(x)\ge\lambda\}=[a_-(\lambda),a^-(\lambda)]$$
$$=[b-\alpha(1-\lambda),b+\beta(1-\lambda)],\ \forall\lambda\in[0,1]$$

Definition 3 [14]. Let $\tilde{A}_1=(a_1,b_1,c_1)$, $\tilde{A}_2=(a_2,b_2,c_2)$ be any two triangular fuzzy numbers, some basic operators are defined as

(1) $\tilde{A}_1+\tilde{A}_2=(a_1+a_2,b_1+b_2,c_1+c_2)$,

(2) $x\tilde{A}_1=(xa_1,xb_1,xc_1),\ \forall x>0.$

Theorem 1 [17]. Let $\tilde{A},\tilde{B}$ be any two triangular fuzzy numbers, then we can prove that

(1)

$$(\tilde{A}+\tilde{B})_\lambda=\tilde{A}_\lambda+\tilde{B}_\lambda$$
$$=[a_-(\lambda)+b_-(\lambda),a^-(\lambda)+b^-(\lambda)],\ \forall\lambda\in[0,1],$$

(2) $(x\tilde{A})_\lambda=x\tilde{A}_\lambda=[xa_-(\lambda),xa^-(\lambda)],\ \forall x>0,\ \lambda\in[0,1].$

The proof is easy and omitted.

Definition 4 [28]. Let $\tilde{A}=(a,b,c)$ be a triangular fuzzy numbers with λ -level set

$$\tilde{A}_\lambda=[a_-(\lambda),a^-(\lambda)]$$
$$=[b-\alpha(1-\lambda),b+\beta(1-\lambda)],\ \lambda\in[0,1],$$

then the possibilistic mean value of fuzzy number $\tilde{A}$ is defined as

$$M(\tilde{A})=\int_0^1\lambda[a_-(\lambda)+a^-(\lambda)]d\lambda$$
$$=\int_0^1\lambda[2b+(\beta-\alpha)(1-\lambda)]d\lambda=b+(\beta-\alpha)/6$$
$$=[4b+(a+c)]/6.$$

Definition 5 [29]. Let $\tilde{A}_1=(a_1,b_1,c_1)$, $\tilde{A}_2=(a_2,b_2,c_2)$ be any two triangular fuzzy numbers, the degree of possibility of $\tilde{A}_1\ge\tilde{A}_2$ is defined as

$$P(\tilde{A}_1\ge\tilde{A}_2)$$
$$=0.5\max\{1-\max\{\frac{b_2-a_1}{(b_2-a_2)+(b_1-a_1)},0\},0\}$$
$$+0.5\max\{1-\max\{\frac{c_2-b_1}{(c_2-b_2)+(c_1-b_1)},0\},0\}.$$

Theorem 2 [29]. Let $\tilde{A}_1=(a_1,b_1,c_1)$, $\tilde{A}_2=(a_2,b_2,c_2)$, $\tilde{A}_3=(a_3,b_3,c_3)$ be three triangular fuzzy numbers, then

(1) $0\le P(\tilde{A}_1\ge\tilde{A}_2)\le 1$;

(2) If $c_2\le a_1, P(\tilde{A}_1\ge\tilde{A}_2)=1$; If $c_1\le a_2, P(\tilde{A}_1\ge\tilde{A}_2)=0$;

(3) $P(\tilde{A}_1\ge\tilde{A}_2)+P(\tilde{A}_2\ge\tilde{A}_1)=1$; $P(\tilde{A}_1\ge\tilde{A}_1)=1/2$;

(4) If $\tilde{A}_1\ge\tilde{A}_2$, then $P(\tilde{A}_1\ge\tilde{A}_3)\ge P(\tilde{A}_2\ge\tilde{A}_3)$.

It can be proved easily by Definition 5 of possibility degree of triangular fuzzy numbers.

Definition 6 [9] Let s_j be the sentiment of investor on asset j, $\tilde{R}_j$ be the fuzzy number return of asset j, the sentiment influential function $f(s_j)$ and the sentiment-adjusted fuzzy number return $\hat{R}_j$ of asset j, respectively, are defined as

$$f(s_j)=e^{\gamma s_j},(\gamma>0),\ \hat{R}_j=f(s_j)\tilde{R}_j.$$

Remark 1. The sentiment influential function $f(s_j)$ is a increasing function. The higher is the investor's sentiment on asset j, the higher is the estimated fuzzy return of asset j. And one can easily get the following results.

(i) If $s_j>0$, then $f(s_j)>1$, the fuzzy return of asset j will increase with the positive sentiment on asset j.

(ii) If $s_j=0$, then $f(s_j)=1$, the fuzzy return of asset j will be unchanged with the rational sentiment on asset j.

(iii) If $s_j<0$, then $f(s_j)<1$, the fuzzy return of asset j will decrease with the negative sentiment on asset j.

3. The Formulation of Fuzzy Behavioral Portfolio Decision Model

In this section, we discuss the behavioral portfolio selection problem with fuzzy returns and investor's sentiments. We first introduce the problem description and notations used in the following section. Then, we formulate the fuzzy behavioral portfolio model by maximizing the fuzzy sentimental mean of portfolio return.

3.1. Problem Description and Notations

Let us consider a behavioral portfolio selection problem with t mental accounts. Each mental account consists n_i risky assets. The return rates of risky assets are evaluated by triangular fuzzy numbers. Assume that the investor intends to allocate his/her wealth among the n_i risky assets for making accounting investment plan in t mental accounts. To make it easier to follow our exposition, we put together all the notations that will be used hereafter.

x_{ij}: the investment proportion of risky asset j in mental account i;

l_{ij}: the lower boundary of investment proportion of risky asset j in mental account i;

u_{ij} : the upper boundary of investment proportion of risky asset j in mental account i ;

w_i : the importance degree of the holding mental account i ;

d_{ij} : the unit transaction cost of risky asset j in mental account i ;

s_{ij} : the sentiment of investor on asset j in mental account i ;

$f(s_{ij})$: the sentiment influential function of investor on asset j in mental account i .

3.2. Sentiment-adjusted Mean of Fuzzy Return for Portfolio

Assume that the whole investment process is self-financing, that is, the investor does not invest the additional capital during the portfolio selection process. Let $\tilde{R}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ be the triangular fuzzy number return of asset j at mental account i . According to the previous section, the sentimental mean value of triangular fuzzy number return for portfolio $x_i = (x_{i1}, x_{i2}, \cdots, x_{in_i})$ at mental account i is determined by

$$M(\tilde{R}_{pi}) = \sum_{j=1}^{n_i} x_{ij} M(\tilde{R}_{ij}) f(s_{ij})$$
$$= \sum_{j=1}^{n_i} x_{ij} f(s_{ij})[b_{ij} + (\beta_{ij} - \alpha_{ij})/6]$$
$$= \sum_{j=1}^{n_i} f(s_{ij}) x_{ij} [4b_{ij} + (a_{ij} + c_{ij})]/6.$$

3.3. Construction of Fuzzy Behavioral Portfolio Decision Model with Mental Accounts and Investor's Sentiment

Assume that the objective of the investor wants to maximize the expected sentimental return of portfolio over the whole t mental accounts. At the same time, the fuzzy portfolio return at each mental account must achieve or exceed the given minimum fuzzy number return level with a certain possibility degree. Thus, the fuzzy behavioral portfolio decision selection problem with multi-accounts can be formulated as the following programming model denoted by (P1):

$$\max \sum_{i=1}^{t} w_i \sum_{j=1}^{n_i} x_{ij} (M(\tilde{R}_{ij}) f(s_{ij})) - \sum_{i=1}^{t} \sum_{j=1}^{n_i} d_{ij} x_{ij}$$

s.t. $P(\sum_{j=1}^{n_i} x_{ij} \tilde{R}_{ij} \ge \tilde{r}_i) \ge \alpha_i, \quad i = 1, 2, \cdots, t$

$$\sum_{i=1}^{t} \sum_{j=1}^{n_i} x_{ij} = 1,$$
$$0 \le l_{ij} \le x_{ij} \le u_{ij} \le 1,$$

where $W = (w_1, w_2, \cdots, w_t)$ is the weight vector of all the mental accounts, $\sum_{i=1}^{t} w_i = 1, w_i \in [0,1],$ w_i is the importance degree of mental acc oun t i. And $\tilde{r}_i = (a_{r_i}, b_{r_i}, c_{r_i})$ represents the given minimum aspiration fuzzy return level of the portfolio wealth regarding the i -th mental account; α_i is the given possibility degree level assuring that the fuzzy return of i -th mental account greater than the given minimum aspiration fuzzy return level $\tilde{r}_i$. In general, the lower level is the account mental, the greater is the parameter α_i .

If we let $f(s_{ij}) = e^{0.5 s_{ij}}$ be the sentiment function of investor on asset j at mental account i , then the above programming model (P1) can be transformed to the following optimization models (P11)-(P12) according to Definition 5 of possibility degree of fuzzy number returns and Theorem 1, 2.

(P11)

$$\max \sum_{i=1}^{t} w_i \sum_{j=1}^{n_i} \frac{x_{ij}(4b_{ij} + a_{ij} + c_{ij})}{6} f(s_{ij}) - \sum_{i=1}^{t} \sum_{j=1}^{n_i} d_{ij} x_{ij}$$

s.t.

$$\begin{cases} P\{(\sum_{j=1}^{n_i} x_{ij} a_{ij}, \sum_{j=1}^{n_i} x_{ij} b_{ij}, \sum_{j=1}^{n_i} x_{ij} c_{ij}) \ge \tilde{r}_i\} \ge \alpha_i, \\ i = 1, 2, \cdots, t; \\ \sum_{i=1}^{t} \sum_{j=1}^{n_i} x_{ij} = 1 \\ 0 \le l_{ij} \le x_{ij} \le u_{ij} \le 1, \ i = 1, 2 \cdots, t; \ j = 1, 2 \cdots n_i. \end{cases}$$

The above model is equivalent to the following programming model.

(P12)

$$\max \sum_{i=1}^{t} w_i \sum_{j=1}^{n_i} \frac{x_{ij}(4b_{ij} + a_{ij} + c_{ij})}{6} e^{0.5 s_{ij}} - \sum_{i=1}^{t} \sum_{j=1}^{n_i} d_{ij} x_{ij}$$

s.t.

$$\begin{cases} 0.5 \max\{1 - \max\{\dfrac{b_{r_i} - \sum_{j=1}^{n_i} x_{ij} a_{ij}}{\begin{pmatrix} b_{r_i} - a_{r_i} + \sum_{j=1}^{n_i} x_{ij} b_{ij} \\ -\sum_{j=1}^{n_i} x_{ij} a_{ij} \end{pmatrix}}, 0\}, 0\} \\ +0.5 \max\{1 - \max\{\dfrac{c_{r_i} - \sum_{j=1}^{n_i} x_{ij} b_{ij}}{\begin{pmatrix} c_{r_i} - b_{r_i} + \sum_{j=1}^{n_i} x_{ij} c_{ij} \\ -\sum_{j=1}^{n_i} x_{ij} b_{ij} \end{pmatrix}}, 0\}, 0\} \ge \alpha_i, \\ \forall i = 1, 2 \cdots, t; \\ \sum_{i=1}^{t} \sum_{j=1}^{n_i} x_{ij} = 1 \quad 0 \le l_{ij} \le x_{ij} \le u_{ij} \le 1 \end{cases}$$

4. Illustrative Example

Example 1. In order to express the idea of our model and the effectiveness of the proposed fuzzy behavioral portfolio method, we give an example for simulating the real transaction. For simplicity, in the example we consider two-mental accounting behavioral portfolio decision problem with fuzzy number returns. Assume that the financial market has two mental accounts MA_1, MA_2 . The lower-level mental account MA_1 has three alternative financial assets A_{11}, A_{12}, A_{13} . The high-level mental account MA_2 has three alternative financial assets A_{21}, A_{22}, A_{23} . All the financial assets in the above two mental accounts are selected from Shanghai Stock Exchange in China. To simulate the transaction, we collect the weekly closing pricing of assets from Jun 2018 to Jun 2019, with 1 yearly observations. By analyzing the stock historical data, the corresponding corporations' financial reports and the future information, we can utilize the simple statistical frequency method [17] to assess the fuzzy return $\tilde{R}_{ij}=(a_{ij},b_{ij},c_{ij})$ of assets A_{ij} in the above two mental accounts, $i=1,2;\quad j=1,2,3$. The evaluated fuzzy number return are listed in the following Table 1.

Table 1. The assessed fuzzy return of the selected stock assets from two mental accounts

Mental account 1	Fuzzy number return	Mental account 2	Fuzzy number return
Asset 11	(0.12, 0.28, 0.43)	Asset 21	(0.34, 0.53, 0.61)
Asset 12	(0.23, 0.35, 0.51)	Asset 22	(0.27, 0.48, 0.58)
Asset 13	(0.31, 0.46, 0.60)	Asset 23	(0.42, 0.58, 0.62)

Suppose that the investor's initial sentiment vector on the selected six financial assets is

$$S=(s_{11},s_{12},s_{13},s_{21},s_{22},s_{23})=(1,0,1,0,1,2)..$$

If we choose $f(s)=e^{0.5s}$ as the sentiment influential function, we can compute the sentiment influential function value vector as

$$\begin{aligned}&f(S)=(f(s_{11}),f(s_{12}),f(s_{13}),f(s_{21}),f(s_{22}),f(s_{23}))\\&=(e^{0.5s_{11}},e^{0.5s_{12}},e^{0.5s_{13}},e^{0.5s_{21}},e^{0.5s_{22}},e^{0.5s_{23}})\\&=(1.6487,1,1.6487,1,1.6487,2.7183).\end{aligned}$$

Also we can compute the possibilistic mean vector of the selected stock assets as

$$\begin{aligned}M(\tilde{R}_{ij})&=\begin{pmatrix}M(\tilde{R}_{11}),M(\tilde{R}_{12}),M(\tilde{R}_{13}),\\M(\tilde{R}_{21}),M(\tilde{R}_{22}),M(\tilde{R}_{23})\end{pmatrix}\\&=(0.2783,0.3567,0.4583,0.5117,0.4617,0.56)\end{aligned}$$

In this example we assume the lower boundary l_{ij} and upper boundary u_{ij} of investment proportion of risky asset j at mental account i are 0.1 and 0.3, respectively. Suppose $\tilde{r}_1=(0,0.1,0.25)$, $\tilde{r}_2=(0.15,0.3,0.45)$ are the given minimum expected fuzzy return of the portfolio for mental account 1 and 2, respectively. And we let $\alpha_1=0.85$ be the possibility degree that the fuzzy return of the first mental account MA_1 exceeds the minimum aspiration $\tilde{r}_1=(0,0.1,0.25)$, and $\alpha_2=0.05$ is the given possibility degree that the fuzzy return of the second mental account MA_2 exceeds the minimum aspiration $\tilde{r}_2=(0.15,0.3,0.45)$. Suppose $d_{1j}=0.003$ is the transaction costs of assets A_{1j} in MA_1 and $d_{2j}=0.005$ is the transaction costs of assets A_{2j} in MA_2, $(j=1,2,3)$.

In order to obtain the corresponding portfolio strategy $x=(x_{11},x_{12},x_{13},x_{21},x_{22},x_{23})$, we construct the following sentiment-adjusted fuzzy behavioral portfolio model.

(P2)

$$\begin{aligned}\max\sum_{i=1}^{2}w_i\sum_{j=1}^{3}\{\frac{x_{ij}[4b_{ij}+(a_{ij}+c_{ij})]}{6}e^{0.5s_{ij}}\}\\-0.003\sum_{j=1}^{3}x_{1j}-0.005\sum_{j=1}^{3}x_{2j}\end{aligned}$$

s.t.

$$\begin{cases}P\left\{\begin{pmatrix}\sum_{j=1}^{3}x_{1j}a_{1j},\\\sum_{j=1}^{3}x_{1j}b_{1j},\\\sum_{j=1}^{3}x_{1j}c_{1j}\end{pmatrix}\geq(0,0.1,0.25)\right\}\geq0.85,\\P\left\{\begin{pmatrix}\sum_{j=1}^{3}x_{2j}a_{2j},\\\sum_{j=1}^{3}x_{2j}b_{2j},\\\sum_{j=1}^{3}x_{2j}c_{2j}\end{pmatrix}\geq(0.15,0.3,0.45)\right\}\geq0.05,\\\sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij}=1\\0.1=l_{ij}\leq x_{ij}\leq u_{ij}=0.3,\ i=1,2;\ j=1,2,3.\end{cases}$$

Substituting data (a_{ij},b_{ij},c_{ij}) of fuzzy number return $\tilde{R}_{ij}(i=1,2;j=1,2,3)$ of assets A_{ij} in Table 1 into the model (P2) and according to Definition 4, 5 and Theorem 2 we can easily transform the above fuzzy behavioral portfolio model to the following four linear programming models **(P21)-(P24)**.

(P21)

$$\begin{aligned}\max\ &w_1(0.4588x_{11}+0.3567x_{12}+0.7556x_{13})\\&+w_2(0.5117x_{21}+0.7612x_{22}+1.5222x_{23})\\&-0.003x_{11}-0.003x_{12}-0.003x_{13}\\&-0.005x_{21}-0.005x_{22}-0.005x_{23}\end{aligned}$$

s.t.,

$$\begin{cases}0.5\{1-\dfrac{b_{r_1}-\sum_{j=1}^{3}x_{1j}a_{1j}}{b_{r_1}-a_{r_1}+\sum_{j=1}^{3}x_{1j}b_{1j}-\sum_{j=1}^{3}x_{1j}a_{1j}}\}\\+0.5\{1-\dfrac{c_{r_1}-\sum_{j=1}^{3}x_{1j}b_{1j}}{c_{r_1}-b_{r_1}+\sum_{j=1}^{3}x_{1j}c_{1j}-\sum_{j=1}^{3}x_{1j}b_{1j}}\}\geq0.85\\0.5\{1-\dfrac{b_{r_2}-\sum_{j=1}^{3}x_{2j}a_{2j}}{b_{r_2}-a_{r_2}+\sum_{j=1}^{3}x_{2j}b_{2j}-\sum_{j=1}^{3}x_{2j}a_{2j}}\}\\+0.5\{1-\dfrac{c_{r_2}-\sum_{j=1}^{3}x_{2j}b_{2j}}{c_{r_2}-b_{r_2}+\sum_{j=1}^{3}x_{2j}c_{2j}-\sum_{j=1}^{3}x_{2j}b_{2j}}\}\geq0.05\\\sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij}=1\\0.1\leq x_{ij}\leq0.3\end{cases}$$

which is equivalent the following model **(** P_{21}' **)**:

$$\max w_1(0.4588x_{11}+0.3567x_{12}+0.7556x_{13}) \\ +w_2(0.5117x_{21}+0.7612x_{22}+1.5222x_{23}) \\ -0.003x_{11}-0.003x_{12}-0.003x_{13} \\ -0.005x_{21}-0.005x_{22}-0.005x_{23}$$

s.t.

$$\begin{cases} \dfrac{0.1-\sum_{j=1}^{3}x_{1j}a_{1j}}{(0.1-0)+(\sum_{j=1}^{3}x_{1j}b_{1j}-\sum_{j=1}^{3}x_{1j}a_{1j})} \\ +\dfrac{0.25-\sum_{j=1}^{3}x_{1j}b_{1j}}{(0.25-0.1)+\sum_{j=1}^{3}x_{1j}c_{1j}-\sum_{j=1}^{3}x_{1j}b_{1j}} \le 0.3 \\ \dfrac{0.3-\sum_{j=1}^{3}x_{2j}a_{2j}}{(0.3-0.15)+\sum_{j=1}^{3}x_{2j}b_{2j}-\sum_{j=1}^{3}x_{2j}a_{2j}} \\ +\dfrac{0.45-\sum_{j=1}^{3}x_{2j}b_{2j}}{(0.45-0.3)+\sum_{j=1}^{3}x_{2j}c_{2j}-\sum_{j=1}^{3}x_{2j}b_{2j}} \le 1.9 \\ \sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij}=1 \\ 0.1\le x_{ij}\le 0.3 \end{cases}$$

(P22)

$$\max w_1(0.4588x_{11}+0.3567x_{12}+0.7556x_{13}) \\ +w_2(0.5117x_{21}+0.7612x_{22}+1.5222x_{23}) \\ -0.003x_{11}-0.003x_{12}-0.003x_{13} \\ -0.005x_{21}-0.005x_{22}-0.005x_{23}$$

$$\text{s.t.,}\begin{cases} \sum_{j=1}^{3}x_{1j}a_{1j}\ge 0.25 \\ \sum_{j=1}^{3}x_{2j}a_{2j}\ge 0.45 \\ \sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij}=1 \\ 0.1\le x_{ij}\le 0.3. \end{cases}.$$

(P23)

$$\max w_1(0.4588x_{11}+0.3567x_{12}+0.7556x_{13}) \\ +w_2(0.5117x_{21}+0.7612x_{22}+1.5222x_{23}) \\ -0.003x_{11}-0.003x_{12}-0.003x_{13} \\ -0.005x_{21}-0.005x_{22}-0.005x_{23}$$

s.t.,

$$\begin{cases} \sum_{j=1}^{3}x_{1j}a_{1j}\ge 0.25 \\ \dfrac{0.3-\sum_{j=1}^{3}x_{2j}a_{2j}}{(0.3-0.15)+\sum_{j=1}^{3}x_{2j}b_{2j}-\sum_{j=1}^{3}x_{2j}a_{2j}} \\ +\dfrac{0.45-\sum_{j=1}^{3}x_{2j}b_{2j}}{(0.45-0.3)+\sum_{j=1}^{3}x_{2j}c_{2j}-\sum_{j=1}^{3}x_{2j}b_{2j}} \le 1.9 \\ \sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij}=1,\ 0.1\le x_{ij}\le 0.3 \end{cases}$$

(P24)

$$\max w_1(0.4588x_{11}+0.3567x_{12}+0.7556x_{13}) \\ +w_2(0.5117x_{21}+0.7612x_{22}+1.5222x_{23}) \\ -0.003x_{11}-0.003x_{12}-0.003x_{13} \\ -0.005x_{21}-0.005x_{22}-0.005x_{23}$$

s.t.,

$$\begin{cases} \dfrac{0.1-\sum_{j=1}^{3}x_{1j}a_{1j}}{(0.1-0)+(\sum_{j=1}^{3}x_{1j}b_{1j}-\sum_{j=1}^{3}x_{1j}a_{1j})} \\ +\dfrac{0.25-\sum_{j=1}^{3}x_{1j}b_{1j}}{(0.25-0.1)+\sum_{j=1}^{3}x_{1j}c_{1j}-\sum_{j=1}^{3}x_{1j}b_{1j}} \le 0.3 \\ \sum_{j=1}^{3}x_{2j}a_{2j}\ge 0.45 \\ \sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij}=1,\ \ 0.1\le x_{ij}\le 0.3. \end{cases}$$

Since the different importance agree of each mental account affects the behavioral portfolio solution, in this paper we shall consider four types of investment importance vectors of the two mental accounts as W1=(0.1,0.9) , W2=(0.5,0.5), W3=(0.9,0.1), W4=(0.6,0.4).

Then, we apply the nonlinear optimization tools in Matlab software package to solve the above-mentioned programming models. Finally, we obtain the optimal behavioral portfolio strategy, which is the solver corresponding to maximum objective function value of portfolio. The optimal behavioral investment portfolio solution $x^*=(x_{11}^*,x_{12}^*,x_{13}^*,x_{21}^*,x_{22}^*,x_{23}^*)$ corresponding to the maximum sentimental mean regarding the different weight vector of mental accounts are easily computed as listed in the following Table 2.

Table 2. Optimal behavioral portfolio strategy regarding different weight vectors of two mental accounts

Weight vector of mental accounts	Sentimental Mean value of portfolio	x_{11}^*	x_{12}^*	x_{13}^*	x_{21}^*	x_{22}^*	x_{23}^*
W=(0.1,0.9)	0.574	0.1	0.1	0.2643	0.1	0.1357	0.3
W=(0.5,0.5)	0.4421	0.1	0.1	0.2649	0.1	0.1351	0.3
W=(0.9,0.1)	0.3789	0.2794	0.1	0.3	0.1	0.1	0.1206
W=(0.6,0.4)	0.4145	0.1	0.1	0.3	0.1	0.1	0.3

5. Summary and Conclusion

In this paper, we consider the multi-account behavioral portfolio selection problem under fuzzy uncertain environment. We use the sentiment-adjusted mean value to measure triangular fuzzy number return of the behavioral portfolio. Furthermore, based on the possibility degree of triangular fuzzy number return of each mental account exceeding the given minimum fuzzy number level we present a sentiment-adjusted behavioral portfolio model with fuzzy return and investor's sentiment. In order to solve the proposed model, we transform it into the equivalent programming models. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

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