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On the Role of Schwarzschild Interaction in Understanding Strong Interaction and Nuclear Binding Energy

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Abstract In this paper the authors reviewed the basics of final unification with respect to Schwarzschild interaction and strong interaction. In the earlier published papers the authors suggested that, strength of any interaction can be defined as the ratio of the operating force magnitude and the magnitude of $\left(c^4/G\right)$. If strength of the Schwarzschild interaction is assumed to be unity, then weak interaction strength seems to be 'squared Avogadro number $\left(N_A^2\right)$' times less than the Schwarzschild interaction. 'Inverse' of the strong coupling constant can be considered as the "natural logarithm of square root of ratio of gravitational and electromagnetic force ratio of down quark mass where the operating gravitational constant is squared Avogadro number times the gravitational constant. With the earlier proposed two new grand unified back ground numbers $\left(x \cong 38.72479081 \text{ and } y \cong 47.41543166\right)$ and the unified force $\left(c^4/N_A^2 G\right)$, attempt is made to fit and understand the mystery of Up and Down quarks, strong coupling constant, nuclear stability, nuclear binding energy. It is very strange and very interesting to say that, at the stable mass number, nuclear binding energy is approximately equal to the sum of rest energy of $2Z$ up quarks and $Z\left(1+\alpha_s\right)$ down quarks where α_s is the strong coupling constant.

Keywords: *gravitational constant, astrophysical force limit, avogadro number, schwarzschild's interaction, strong interaction, SEMF, atomic radii, final unification*

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1. Introduction

From final unification point of view, it is very much essential to couple the universal gravitational constant with the elementary physical constants. Then only the essence of unification can be understood. So far scientists proposed several interesting models [1-8]. In this context, readers may go through the authors published papers [9-18]. By introducing two new back ground unified numbers (x, y), in the published paper authors expressed their views [9] on final unification and proposed three characteristic relations for connecting, fitting and verifying the Newtonian gravitational constant in a unified approach via the Avogadro number N_A. In this paper, the topics covered and reviewed are: Schwarzschild interaction strength, meaning of strength of interaction in atomic physics, significance of Avogadro number, fitting of the gravitational constant, muon an tau rest masses, strong coupling constant, fine structure ratio, reduced Planck's constant, rest masses of Up & Down quarks, Nucleon rest masses, rms radius of proton, nuclear charge radius, nuclear stability, nuclear binding energy, atomic radii and atomic mass unit. Important points can be expressed as follows.

1) Note that, as per the basic concepts of final unification, there exists a fundamental unified force from which all the observed forces emerged. If so, magnitude of the unified force can be assumed to be equal to the astrophysical force limit $\left(c^4/G\right)$. Note that, magnitude of the radial inward force acting on any black hole surface [11] is of the order of $\left(c^4/G\right)$.
2) By considering the squared Avogadro number [9] as a characteristic proportionality ratio, the characteristic magnitude of the unified atomic force can be assumed to be $\left(c^4/N_A^2 G\right)$.

2. The Classical Limits of Force and Power

Without considering the current notion of black hole physics, Schwarzschild radius of black hole can be estimated with the characteristic astrophysical limiting force of magnitude $\left(c^4/G\right)$. The outstanding problem in particle physics today is the inclusion of gravity in a single, unified quantum theory of all the fundamental interactions. Particle physicists have long suggested that the four observed fundamental forces of nature (the gravitational, electromagnetic, weak nuclear and strong nuclear forces) are separate, low energy manifestations of what was once a single force at times close to the Big Bang. It is postulated that as the universe expanded and cooled, this single force gradually broke down into the four separate interactions as observed today. However, unification theories that seek to unify the force of gravity with all the other forces (Theories of Everything) remain elusive, as the gravitational interaction lacks a quantum formulation.

To unify cosmology, quantum mechanics and the four observed fundamental cosmological interactions – certainly a 'unified force' is required. In this connection $\left(c^4/G\right)$ can be considered as the classical force or astrophysical force limit. Similarly $\left(c^5/G\right)$ can be considered as the classical power limit. If it is true that c and G are fundamental physical constants in physics, then $\left(c^4/G\right)$ and $\left(c^5/G\right)$ can also be considered as fundamental compound physical constants. These classical limits are more powerful than the Uncertainty limit. Note that by considering the classical force limit $\left(c^4/G\right)$, the famous Planck mass can be obtained.

2.1. Simple Applications of $\left(c^4/G\right)$

a. Magnitude of force of attraction or repulsion between any two charged particles never exceeds $\left(c^4/G\right)$.

b. Magnitude of gravitational force of attraction between any two massive bodies never exceeds $\left(c^4/G\right)$.

c. Magnitude of mechanical force on a revolving/rotating body never exceeds $\left(c^4/G\right)$.

d. Magnitude of electromagnetic force on a revolving body never exceeds $\left(c^4/G\right)$.

2.2. Simple Applications of $\left(c^5/G\right)$

a) Mechanical power never exceeds $\left(c^5/G\right)$

b) Electromagnetic power never exceeds $\left(c^5/G\right)$

c) Thermal radiation power never exceeds $\left(c^5/G\right)$

d) Gravitational radiation power never exceeds $\left(c^5/G\right)$

3. Understanding the Role of $\left(c^4/G\right)$ in Black Hole Formation and Planck Mass Generation

3.1. Schwarzschild Radius of a Black Hole

The four basic physical properties of a rotating black hole [19,20] are its mass, size, angular velocity and temperature. Without going deep into the mathematics of black hole physics in this section an attempt is made to understand the Schwarzschild radius of a black hole. In all directions, if a force of magnitude $\left(c^4/G\right)$ acts on the mass-energy content of the assumed celestial body it approaches a minimum radius of $\left(GM/c^2\right)$ in the following way. Origin of the force $\left(c^4/G\right)$ may be due to self-weight or internal attraction or external compression or something else.

$$R_{\min} \cong \frac{Mc^2}{\left(c^4/G\right)} \cong \frac{GM}{c^2} \quad (1)$$

If no force (of zero magnitude) acts on the mass content M of the assumed massive body, its radius becomes infinity. With reference to the average magnitude of $\left(0, \frac{c^4}{G}\right) \cong \frac{c^4}{2G}$, the presently believed Schwarzschild radius can be obtained as

$$(R)_{ave} \cong \frac{Mc^2}{\left(c^4/2G\right)} \cong \frac{2GM}{c^2} \quad (2)$$

This proposal is very simple and seems to be different from the existing concepts and may be a unified form of the Newton's law of gravity, Special theory of relativity and General theory of relativity.

3.2. To Derive the Planck Mass

So far no theoretical model proposed a derivation for the Planck mass. To derive the Planck mass the following two conditions can be given a chance.

Assuming that gravitational force of attraction between two Planck particles of mass $\left(M_P\right)$ separated by a minimum distance ($r_{\min}$) be,

$$\left[\frac{GM_P M_P}{r_{\min}^2}\right] \equiv \left(\frac{c^4}{G}\right) \quad (3)$$

With reference to wave mechanics, let

$$2\pi . r_{\min} \cong \lambda_P = \left[\frac{h}{c.M_P}\right] \quad (4)$$

Here, λ_P represents the wavelength associated with the Planck mass. With these two assumed conditions Planck mass can be obtained as follows.

$$M_P = \sqrt{\frac{hc}{2\pi G}} \cong \sqrt{\frac{\hbar c}{G}} \quad (5)$$

3.3. Understanding the Strength of any Interaction

From the above relations it is reasonable to say that,

1) If it is true that c and G are fundamental physical constants, then $\left(c^4/G\right)$ can be considered as a fundamental compound constant related to a characteristic limiting force.
2) Black holes are the ultimate state of matter's geometric structure.
3) Magnitude of the operating force at the black hole surface is of the order of $\left(c^4/G\right)$.
4) Gravitational interaction taking place at black holes can be called as 'Schwarzschild interaction'.
5) Strength of 'Schwarzschild interaction' can be assumed to be unity.
6) Strength of any other interaction can be defined as the ratio of operating force magnitude and the classical or astrophysical force magnitude $\left(c^4/G\right)$.
7) If one is willing to represent the magnitude of the operating force as a fraction of $\left(c^4/G\right)$ i.e X times of $\left(c^4/G\right)$, where $X << 1$, then

$$\frac{X \text{ times of } \left(c^4/G\right)}{\left(c^4/G\right)} \cong X \rightarrow \text{Effective } G \Rightarrow \frac{G}{X} \quad (6)$$

If X is very small, $\frac{1}{X}$ becomes very large. In this way, X can be called as the strength of interaction. Clearly speaking, strength of any interaction is $\frac{1}{X}$ times less than the 'Schwarzschild interaction' and effective G becomes $\frac{G}{X}$.

4. Basic Concepts on Final Unification

The following concepts and relations can be given a chance in final unification program.

1) With reference to the elementary charge and with mass similar to the Planck mass, a new mass unit can be constructed in the following way.

$$\left.\begin{aligned} (M_S)^{\pm} &\cong \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} \cong 1.859272\times10^{-9}\text{ kg} \\ M_S c^2 &\cong \sqrt{\frac{e^2 c^4}{4\pi\varepsilon_0 G}} \cong 1.042975\times10^{18}\text{ GeV}\end{aligned}\right\} \quad (7)$$

It can be called as the Stoney mass [21]. It is well known that e, c, G play a vital role in fundamental physics. With these 3 constants, space-time curvature concepts at a charged particle surface can be studied. It was first introduced by the physicist George Johnstone Stoney. He is most famous for introducing the term 'electron' as the 'fundamental unit quantity of electricity'. In unification program, with this mass unit and with a suitable proportionality ratio- characteristic mass of any elementary charge can be generated.

2) Avogadro number is an absolute number and it is having no units like 'per mole'. Independent of system of units, universally 'gram mole' can be called as 'molar mass constant' meaning 0.001 kg or 1 gram per mole [22] and can be universally represented by $M_u \cong 0.001$ kg.mole^{-1} $\cong$ 1gram.mole^{-1} and number of entities in one mole can be represented by $n \cong N_A$ mole^{-1} and there is no need to consider units for the Avogadro number.
3) Atomic gravitational constant can be considered as the as the squared Avogadro number times the Newtonian gravitational constant.

$$G_A \cong N_A^2 G \quad (8)$$

4) Independent of system of units and without considering the Avogadro number, unified atomic mass unit [23,24] can be fitted as follows.

$$m_u c^2 \cong \left(\sqrt{m_n m_p}\, c^2 - B_a\right) + m_e c^2 \quad (9)$$

where m_u is the unified atomic mass unit and B_a is the average binding energy per nucleon. If $B_a \cong (7.90 \text{ to } 8.0)$ MeV, obtained magnitude of $m_u \cong 931.4295$ to 931.5295 MeV/c^2. Thus it can be suggested that, accuracy of m_u depends only on the accurate 'average binding energy per nucleon' and it can be estimated from the semi empirical mass formula.

4.1. Semi Empirical Numbers and Semi Empirical Relations

There exist two new numbers (x, y). They can be called as the 'primordial unified back ground numbers' [9]. They can also be called as the 'back ground analytical numbers' using by which micro-macro physical constants can be interlinked qualitatively and quantitatively.

Semi empirical applications of (x, y):

Electron rest mass can be expressed in the following way.

$$m_e \cong x^{\frac{1}{2}} y \sqrt{\frac{e^2}{4\pi\varepsilon_0 G_A}} \quad (10)$$

With (x, y), proton rest mass can be expressed in the following way.

$$m_p \cong x^{\frac{3}{2}} y^2 \sqrt{\frac{e^2}{4\pi\varepsilon_0 G_A}} \quad (11)$$

Thus,

$$\frac{m_p}{m_e} \cong xy \tag{12}$$

$$\text{Let,} \quad \beta \cong x^{\frac{1}{2}} y \tag{13}$$

where β can be called as the electron mass index. It can be estimated as:

$$\beta \cong x^{\frac{1}{2}} y \cong \sqrt{\frac{4\pi\varepsilon_0 N_A^2 G m_e^2}{e^2}} \cong 295.0509223 \tag{14}$$

With this number β, electron, muon and tau rest masses can be fitted with the semi empirical relation.

$$\left(m_{lepton}\right)_n c^2 \cong \left[\beta^3 + \left(n^2 \beta\right)^n \sqrt{N_A}\right]^{\frac{1}{3}} \sqrt{\frac{e^2 F_X}{4\pi\varepsilon_0}} \cong \left[\beta^3 + \left(n^2\beta\right)^n \sqrt{N_A}\right]^{\frac{1}{3}} 0.001732 \text{ MeV} \tag{15}$$

where $n = 0,1,2$. Obtained rest energies are 0.511 MeV, 105.95 MeV and 1777.4 MeV respectively [24]. New heavy charged lepton at $n = 3$ may be predicted close to 42262 MeV. From above relations,

$$\left.\begin{aligned} x &\cong \left(\frac{1}{\beta}\frac{m_p}{m_e}\right)^2 \cong 38.72787108 \\ y &\cong \left(\frac{1}{x}\frac{m_p}{m_e}\right) \cong 47.41166036 \end{aligned}\right\} \tag{16}$$

If so, Reduced Planck's constant [23,24,25,26] can be expressed in the following way.

$$\left(e^x\right)^{-\frac{1}{6}} \left(\frac{G_A m_e^2}{c}\right) \cong \hbar \cong 1.053946635 \times 10^{-34} \text{ J.sec} \tag{17}$$

Characteristic nuclear radii like rms radius of proton [23,24,27,28,29], nuclear charge radius etc can be expressed in the following way.

$$\left.\begin{aligned} \left(e^x\right)^{-\frac{1}{2}} \left(\frac{G_A m_p}{c^2}\right) &\cong 1.753816617 \times 10^{-15} \text{ m} \\ \frac{1}{2}\left(e^x\right)^{-\frac{1}{2}} \left(\frac{G_A m_p}{c^2}\right) &\cong 0.8769083083 \times 10^{-15} \text{ m} \\ \frac{1}{\sqrt{2}}\left(e^x\right)^{-\frac{1}{2}} \left(\frac{G_A m_p}{c^2}\right) &\cong 1.240135623 \times 10^{-15} \text{ m} \end{aligned}\right\} \tag{18}$$

If so, from relations (17) and (18), the Reduced Planck's constant can be expressed as follows.

$$\hbar \cong \left(\frac{2R_p c^2}{G_A m_p}\right)^{\frac{1}{3}} \left(\frac{G_A m_e^2}{c}\right) \tag{19}$$

where R_p is the rms radius of proton. Two new numbers (γ, κ) can be introduced with the following semi empirical relation.

$$\left.\begin{aligned} (\gamma,\kappa) &\cong \left(\frac{\beta}{2}\right) \pm 2\ln\left(\frac{\beta}{2}\right) \pm \frac{1}{2} \\ &\text{and} \\ \gamma &\cong \left(\frac{\beta}{2}\right) + 2\ln\left(\frac{\beta}{2}\right) + \frac{1}{2} \cong 158.01345 \\ \kappa &\cong \left(\frac{\beta}{2}\right) - 2\ln\left(\frac{\beta}{2}\right) - \frac{1}{2} \cong 137.03746 \end{aligned}\right\} \tag{20}$$

Surprising observation is that, $\kappa \cong 137.03746$ is very close to the presently believed inverse of the Fine structure ratio. Considering these two numbers, stable nuclear mass number range can be fitted as expressed here. With reference to Up and Down quark masses, this relation can be understood more clearly as expressed in relation (41).

$$\left.\begin{aligned} A_{up} &\cong 2Z + \left[\frac{Z}{\kappa}\right]^2 \\ A_{low} &\cong 2Z + \left[\frac{Z}{\gamma}\right]^2 \end{aligned}\right\} \tag{21}$$

where $\left(A_{up}, A_{low}\right)$ seem to represent upper and lower stable mass numbers of Z respectively. For example, $\left(A_{low}, A_{up}\right)$ of $Z = 53$ seem to be 123.78 and 126.5. Note that, relation (20) is free from the reduced Planck's constant. With trial-error it is noticed that,

$$\left.\begin{aligned} &\left(\frac{m_n - m_p}{m_e}\right) \cong \ln\sqrt{\gamma} \\ &\to m_n c^2 - m_n c^2 \cong 1.293512066 \text{ MeV} \\ &\Rightarrow m_n c^2 \cong 939.5655581 \text{ MeV} \end{aligned}\right\} \tag{22}$$

where m_n, m_p and m_e represent rest masses of neutron, proton and electron respectively.

Average binding energy per nucleon and maximum binding energy per nucleon [29,30,31,32] can be expressed as follows.

$$\left.\begin{aligned} \left(B_a\right)_{\text{ave}} &\cong \sqrt{x}\sqrt{m_p c^2 \sqrt{\frac{e^2 F_X}{4\pi\varepsilon_0}}} \cong 7.933 \text{ MeV} \\ \left(B_a\right)_{\max} &\cong \sqrt{y}\sqrt{m_p c^2 \sqrt{\frac{e^2 F_X}{4\pi\varepsilon_0}}} \cong 8.7745 \text{ MeV} \end{aligned}\right\} \tag{23}$$

$$\frac{\left(B_a\right)_{\text{ave}}}{\left(B_a\right)_{\max}} \cong \sqrt{\frac{x}{y}} \tag{24}$$

If so, unified atomic mass unit can be expressed as follows.

$$m_u c^2 \cong \left\{\begin{aligned} &\sqrt{m_p m_n c^2} \\ &-\sqrt{x}\sqrt{m_p c^2 \sqrt{\frac{e^2 F_X}{4\pi\varepsilon_0}}} \end{aligned}\right\} + m_e c^2 \tag{25}$$

5. To Fit the Magnitude of the Gravitational Constant Up to 10 Digits

In astronomy, the only one available characteristic empirical physical constant is the gravitational constant. Its value has been measured in the lab only within a range of 1 cm to a few meters. Until one measures the value of the gravitational constant with microscopic physical constants, the debate of final unification cannot be stopped up. In this context,

G. Rosi et al say [33]: "There is no definitive relationship between G and the other fundamental constants, and there is no theoretical prediction for its value, against which to test experimental results. Improving the precision with which we know G has not only a pure metrological interest, but is also important because of the key role that G has in theories of gravitation, cosmology, particle physics and astrophysics and in geophysical models".

L.L. Williams says [34]: "A theory which unifies gravity and electromagnetism is expected to provide a theoretical or analytical value for G which depends on electro-dynamic and/or atomic constants. However, because such expressions are not unique, and because many such expressions can come within the accuracy to which G is measured, an accurate expression of this sort would only be a necessary condition, not a sufficient condition, to prove the validity of the underlying theory".

George T Gillies says [35]: "Improvements in our knowledge of the absolute value of the Newtonian gravitational constant, G, have come very slowly over the years. Most other constants of nature are known (and some even predictable) to parts per billion, or parts per million at worst. However, G stands mysteriously alone, its history being that of a quantity which is extremely difficult to measure and which remains virtually isolated from the theoretical structure of the rest of physics. Several attempts aimed at changing this situation are now underway, but the most recent experimental results have once again produced conflicting values of G and, in spite of some progress and much interest, there remains to date no universally accepted way of predicting its absolute value".

In general, 'Unification' means:

A) Understanding the origin of the rest mass of atomic elementary particles.

B) Finding and understanding the critical compositeness of the elementary physical constants

C) Minimizing the number of elementary physical constants.

D) Merging different branches of physics with possible and suitable physical concepts.

Considering the proposed concepts and relations accurate values of Gravitational constant [33-41] and Avogadro number [22,23,24,42,43] can be estimated from elementary atomic physical constants. For the time being (i.e until a perfect model is developed), if one is willing to consider the revolving electron's angular momentum as a compound physical constant and depends on the proton-electron rest masses, characteristic nuclear charge radius and the proposed discrete force $\left(c^4/N_A^2 G\right)$, it paves a path for coupling and interconnecting the micro-macro elementary physical constants in a consistent manner. Thus it is possible to couple Avogadro number and Gravitational constant in the following way.

$$\left.\begin{array}{l} x \cong \ln\left(\dfrac{G_A m_e^2}{\hbar c}\right)^6, \\ y \cong \left(\dfrac{m_e}{m_p}\right)\left(\dfrac{4\pi\varepsilon_0 G_A m_e^2}{e^2}\right) \text{ and} \\ xy - \left(\dfrac{m_p}{m_e}\right) \cong 0 \end{array}\right\} \quad (26)$$

By considering

$N_A \cong 6.022141293\times10^{23}$,

$m_e \cong 9.109382914\times10^{-31}$ kg

$m_p \cong 1.672621777\times10^{-27}$ kg,

$\hbar \cong 1.054571726\times10^{-34}$ J.sec

$c \cong 2.99792458\times10^{8}$ m/sec,

$\mathrm{e} \cong 1.602176565\times10^{-19}$ C

$\varepsilon_0 \cong 8.854187817\times10^{-12}$ J.m

and by assuming,

$G \cong 6.674378868\times10^{-11}$ m^{3}kg^{-1}sec^{-2}

obtained values of x and y are,

$x \cong 38.72479081$ and $y \cong 47.41543166$

Thus relation (26) can be considered as the characteristic semi empirical unified relation. This assumed value of G may not be absolute but can be given some consideration in unification program for further analysis This entire procedure depends on the two proposed new numbers (x, y) and needs further research. So far there is no verifying procedure for the measured or estimated magnitude of G. With this kind of procedure, like other physical constants, value of G can be fixed up to 10 digits. From relation (25) the unified atomic mass unit can be fitted to be $m_u c^2 \cong 931.4969569$ MeV.

6. Understanding 'Gram Mole' with the Atomic Gravitational Constant

With reference to the assumed atomic gravitational constant, and with reference to the 'one gram mole' [22,23,24,42,43] it is noticed that,

$$G_A m_u^2 \cong G M_u^2 \quad (27)$$

where, m_u is the unified atomic mass, G_A is the assumed atomic gravitational constant, G is the gravitational constant and M_u is the 'gram mole' expressed in kg.

$$M_u \cong \sqrt{\frac{G_A}{G}}\cdot m_u \cong \sqrt{\frac{N_A^2 G}{G}}\cdot m_u \cong N_A \cdot m_u \cong 0.001 \text{ kg} \cong 1 \text{ gram.} \quad (28)$$

If G and M_u and m_u are known, G_A can be estimated as follows.

$$G_A \cong \left(\frac{M_u}{m_u}\right)^2 \cdot G \tag{29}$$

Thus it can be suggested that,

1) Magnitude of the Avogadro number remaining the same, in CGS system of units, obtained $M_u \cong 1$ gram and in SI system of units, obtained $M_u \cong 0.001$ kg.
2) One gram =0.001 kg of hydrogen constitutes N_A number of atoms.
3) One gram of any atom having A number of nucleons will constitute, $(M_u/Am_u) \cong (N_A/A)$ number of atoms.
4) A grams of any atom having A number of nucleons will constitute, $(AM_u/Am_u) \cong N_A$ number of atoms.
5) Independent of system of units, from now onwards universally 'gram mole' can be called as 'molar mass constant' meaning 0.001 kg or 1 gram per mole and can be universally represented by $M_u \cong 0.001\ \text{kg.mole}^{-1} \cong 1\text{gram.mole}^{-1}$ and number of entities in one mole can be represented by $n \cong N_A\ \text{mole}^{-1}$ and there is no need to consider units for the Avogadro number.

7. Estimating 'Atomic Radii' with the Atomic Gravitational Constant

Dulal C Ghosh et al say [44]: "According to quantum mechanical view, the atoms and the ions do not have any rigid shape or size and hence the question of atomic and ionic radii simply does not arise in the true sense of the term. However, chemical experience suggests that the atoms and ions do have an effective size because, the atoms and ions cannot approach each other beyond certain limiting distance under the influence of forces encountered in chemical interactions. The determination of the empirical atomic and ionic radii, on the basis of hard sphere approximation model, has a history stretching back to the work of Bragg in 1920's to the work of Slater in the year 1964. Attempts of computing theoretical atomic and ionic radii within the scope of Self Consistent Field (SCF) theory are also reported".

With reference to the assumed atomic gravitational constant, let

$$R_g \cong \left(\frac{2G_A m_u}{c^2}\right) \cong 8.944\times10^{-7}\ \text{m} \tag{30}$$

where R_g can be called as the atomic gravitational radius of atom and m_u is the unified atomic mass unit. In analogy with the famous Rutherford's Alpha-scattering experiments [27], for any atom having A number of nucleons,

$$\left(R_g\right)_A \cong A^{\frac{1}{3}}\left(\frac{2G_A m_u}{c^2}\right) \cong A^{\frac{1}{3}} R_g \tag{31}$$

With reference to the nuclear charge radius 1.25 fermi and experimental atomic radii it is noticed that,

$$\left(R_\odot\right)_A \cong \sqrt{\left(A^{\frac{1}{3}}R_c\right)\left(A^{\frac{1}{3}}R_g\right)} \cong A^{\frac{1}{3}}\sqrt{R_c R_g} \cong A^{\frac{1}{3}}\times 3.344\times10^{-11}\ \text{m} \cong A^{\frac{1}{3}}\times 0.334\ \text{Angstroms} \tag{32}$$

where $\left(R_\odot\right)_A$ can be considered as the atomic radius, $R_c \cong 1.25$ fm can be considered as the characteristic nuclear charge radius. See the following Table 1 for the estimated Vander Waal's radii of atoms [44,45].

Table 1. Comparative study of estimated radii and Pauling's experimental Vander Waal's radii of atoms

Atom	A	Calculated Atomic radii in Angstroms	Experimental Vander Waal's radii in Angstroms
Hydrogen	1	0.334	1.2
Nitrogen	14	0.806	1.5
Oxygen	16	0.842	1.4
Phosphorous	31	1.05	1.9
Sulphur	32	1.06	1.85
Chlorine	35	1.093	1.8
Arsenic	75	1.41	2.0
Bromine	80	1.44	1.95
Iodine	127	1.68	2.15

From this data, one can see and confirm the grand unified role of $G_A \cong N_A^2 G$.

8. Strong Coupling Constant, Down and Up Quark Masses and Nucleon Rest Masses

Note that, in the earlier published paper the authors proposed a 'super symmetry' based simple method for estimating the six quark masses [16,17]. Proposed up quark mass is $m_u \cong 4.4\ \text{MeV}/c^2$ and its current estimate is 2.15 MeV. Proposed down quark mass is $m_d \cong 9.47\ \text{MeV}/c^2$ and its current estimate is 4.70. These proposed magnitudes are roughly two times higher than the current quark estimates [46,47] and the very interesting thing is that, current estimated up and down quark mass ratio is 0.46 (5) and is almost matching with the authors' basic assumption in estimating the quark masses [16].

The currently believed strong coupling constant can be fitted in the following way [47].

$$k_1 \cong \frac{1}{\alpha_s} \cong 1+\frac{y}{\sqrt{x}} \cong 1+7.6194771 \cong 8.6194771 \rightarrow \alpha_s \cong 0.11601632 \tag{33}$$

With this fitting, Down quark mass can be fitted as follows.

$$m_d c^2 \cong e^{\left(\frac{1}{\alpha_s}\right)}\sqrt{\frac{e^2c^4}{4\pi\varepsilon_0 G_A}} \cong e^{k_1}\sqrt{\frac{e^2c^4}{4\pi\varepsilon_0 G_A}} \cong 9.59\ \text{MeV} \tag{34}$$

$$k_1 \cong \frac{1}{\alpha_s} \cong \ln\left[m_d c^2 \Big/ \sqrt{\frac{e^2c^4}{4\pi\varepsilon_0 G_A}}\right] \cong \ln\left[\sqrt{\frac{4\pi\varepsilon_0 G_A m_d^2}{e^2}}\right] \tag{35}$$

Cleary speaking, physical meaning of 'inverse of strong coupling' constant can be considered as follows.

1) The natural logarithm of ratio of down quark mass and $0.001732\ \text{MeV}/c^2$.
2) The natural logarithm of square root of ratio of gravitational and electromagnetic force ratio of down quark mass where the operating or effective gravitational constant is $G_A \cong N_A^2 G$.

Down quark and up quark mass ratio can be fitted in the following form.

$$\left.\begin{aligned} & k_2 \cong \frac{m_d c^2}{m_u c^2} \cong \ln\left(\frac{1}{\alpha_s}\right) \cong \ln(k_1) \cong 2.1540244 \\ & \rightarrow \frac{m_u c^2}{m_d c^2} \cong \left[\ln\left(\frac{1}{\alpha_s}\right)\right]^{-1} \cong \left[\ln(k_1)\right]^{-1} \cong 0.4642473 \end{aligned}\right\} \quad (36)$$

Up quark mass can be fitted in the following way.

$$m_u c^2 \cong \frac{m_d c^2}{k_2} \cong \frac{9.59\ \text{MeV}}{k_2} \cong 4.45\ \text{MeV} \quad (37)$$

Now proton rest mass can be fitted in the following relation.

$$\left.\begin{aligned} & m_p c^2 \cong \left[\left(\frac{1}{\alpha}-1\right)+\left(\frac{1}{\alpha_s}-1\right)\right]\sqrt{m_u m_d}\, c^2 \\ & \cong \left[\left(\frac{1}{\alpha}-1\right)+(k_1-1)\right]\sqrt{m_u m_d}\, c^2 \cong 938.394\ \text{MeV} \end{aligned}\right\} \quad (38)$$

Neutron and proton rest mass difference can be fitted in the following relation.

$$\left.\begin{aligned} & (m_n - m_p)c^2 \cong \frac{1}{k_2^2}\left(\frac{2m_u m_d}{(m_u + m_d)}\right) \cong 1.31\ \text{MeV (Or)} \\ & (m_n - m_p)c^2 \cong \ln\left(\frac{\sqrt{m_u m_d}}{m_e}\right) m_e c^2 \cong 1.302\ \text{MeV} \end{aligned}\right\} \quad (39)$$

9. Role of Down and Up Quark Masses in Understanding Nuclear Stability, Nuclear Binding Energy and Nucleon Rest Masses

Despite the protons' mutual electromagnetic repulsion, a stronger attractive force was postulated to explain how the atomic nucleus was bound together. This hypothesized force was called the 'strong force', which was believed to be a fundamental force that acted on nucleons: the protons and neutrons that make up the nucleus. It was later discovered that protons and neutrons were not fundamental particles, but were made up of constituent particles called 'quarks' [48]. In particle physics, the 'strong interaction' is the mechanism responsible for the strong nuclear force. It is approximately 100 times stronger than electromagnetism, a million times stronger than the weak force interaction. It ensures the stability of ordinary nuclear matter that constitutes observable neutrons and protons. It is also believed that, 'residual strong force' plays a key role in the context of binding protons and neutrons together to form atoms. Clearly speaking, it is the residuum of the strong interaction between the up and down quarks that make up the protons and neutrons. In this paper, by considering the up and down quark masses, the authors proposed a very simple relation for understanding the nuclear binding energy. If B_U is the characteristic unified nuclear binding energy constant, it is possible to show that, ***close to stable mass number, nuclear binding energy is very close to*** $3kZB_U$ ***where*** $k \cong 0.637$ at $Z = 2$ and $k \cong 1$ at $Z >= 30$ ***and seems to be connected with*** Z **.** This proposal seems to be simple in understating and compact in presentation. Now the basic questions to be answered are: How to fit/estimate the stable mass number of Z? How to fit/estimate the characteristic nuclear binding energy constant? and How to fix/estimate the magnitude of k **?**

In the earlier published paper the authors proposed a 'super symmetry' based simple method for estimating the six quark masses [16,17]. Proposed up quark mass is $m_u \cong 4.4\ \text{MeV}/c^2$ and its current estimate is 2.15 MeV. Proposed down quark mass is $m_d \cong 9.47\ \text{MeV}/c^2$ and its current estimate is 4.70. These proposed magnitudes are roughly two times higher than the current quark estimates [46,47] and the very interesting thing is that, current estimated up and down quark mass ratio is 0.46 (5) and is almost matching with the authors' basic assumption in estimating the quark masses [16]. With these two mass units, nuclear stability and nuclear binding can be understood in a very simplified and compact form. It is quite interesting and seems to be unique at fundamental level.

In the semi empirical mass formula [29,30,31,32], by maximizing $B(A,Z)$ with respect to Z, we find the number of protons Z of the stable nucleus of atomic weight A as,

$$Z \approx \frac{A}{1+(a_c/2a_a)A^{2/3}}. \quad (40)$$

This is roughly $A/2$ for light nuclei, but for heavy nuclei there is an even better agreement with nature. By substituting the above value of Z back into B one obtains the binding energy as a function of the atomic weight, $B(A)$. Maximizing $B(A)/A$ with respect to A gives the nucleus which is most strongly bound and most stable. Considering the up and down quark masses, and without considering the semi empirical mass formula it is also possible to show that [4],

$$\left.\begin{aligned} & A_{up} \cong 2Z + \left[Z\left(\frac{2m_u m_d}{m_e(m_u+m_d)}\right)^{-1}\right]^2 \cong 2Z + \left[\frac{Z}{11.897}\right]^2 \\ & A_{mean} \cong 2Z + \left[Z\left(\frac{\sqrt{m_u m_d}}{m_e}\right)^{-1}\right]^2 \cong 2Z + \left[\frac{Z}{12.783}\right]^2 \\ & A_{low} \cong 2Z + \left[Z\left(\frac{m_u+m_d}{2m_e}\right)^{-1}\right]^2 \cong 2Z + \left[\frac{Z}{13.736}\right]^2 \end{aligned}\right\} \quad (41)$$

where, m_e is rest mass of electron and $(A_{low}, A_{mean}, A_{up})$ represent the lower, mean and upper stable mass numbers of Z respectively. It can be applied to super heavy elements also. Note that, from relation (20), κ is close to $(11.897)^2$ and γ is close to $(12.783)^2$. See Table 2 for fitting the stable nucleon number with its corresponding proton number.

Table 2. To fit the stable mass numbers of Z

Z	A_{low}	A_{mean}	A_{up}	Z	A_{low}	A_{mean}	A_{up}
2	4.0	4.0	4.0	53	120.9	123.2	125.8
3	6.0	6.1	6.1	54	123.5	125.8	128.6
4	8.1	8.1	8.1	55	126.0	128.5	131.4
5	10.1	10.2	10.2	56	128.6	131.2	134.2
6	12.2	12.2	12.3	57	131.2	133.9	137.0
7	14.3	14.3	14.3	58	133.8	136.6	139.8
8	16.3	16.4	16.5	59	136.4	139.3	142.6
9	18.4	18.5	18.6	60	139.1	142.0	145.4
10	20.5	20.6	20.7	61	141.7	144.8	148.3
11	22.6	22.7	22.9	62	144.4	147.5	151.2
12	24.8	24.9	25.0	63	147.0	150.3	154.0
13	26.9	27.0	27.2	64	149.7	153.1	156.9
14	29.0	29.2	29.4	65	152.4	155.9	159.9
15	31.2	31.4	31.6	66	155.1	158.7	162.8
16	33.4	33.6	33.8	67	157.8	161.5	165.7
17	35.5	35.8	36.0	68	160.5	164.3	168.7
18	37.7	38.0	38.3	69	163.2	167.1	171.6
19	39.9	40.2	40.6	70	166.0	170.0	174.6
20	42.1	42.4	42.8	71	168.7	172.9	177.6
21	44.3	44.7	45.1	72	171.5	175.7	180.6
22	46.6	47.0	47.4	73	174.2	178.6	183.7
23	48.8	49.2	49.7	74	177.0	181.5	186.7
24	51.1	51.5	52.1	75	179.8	184.4	189.7
25	53.3	53.8	54.4	76	182.6	187.3	192.8
26	55.6	56.1	56.8	77	185.4	190.3	195.9
27	57.9	58.5	59.2	78	188.2	193.2	199.0
28	60.2	60.8	61.5	79	191.1	196.2	202.1
29	62.5	63.1	63.9	80	193.9	199.2	205.2
30	64.8	65.5	66.4	81	196.8	202.2	208.4
31	67.1	67.9	68.8	82	199.6	205.1	211.5
32	69.4	70.3	71.2	83	202.5	208.2	214.7
33	71.8	72.7	73.7	84	205.4	211.2	217.9
34	74.1	75.1	76.2	85	208.3	214.2	221.0
35	76.5	77.5	78.7	86	211.2	217.3	224.3
36	78.9	79.9	81.2	87	214.1	220.3	227.5
37	81.3	82.4	83.7	88	217.0	223.4	230.7
38	83.7	84.8	86.2	89	220.0	226.5	234.0
39	86.1	87.3	88.7	90	222.9	229.6	237.2
40	88.5	89.8	91.3	91	225.9	232.7	240.5
41	90.9	92.3	93.9	92	228.9	235.8	243.8
42	93.3	94.8	96.5	93	231.8	238.9	247.1
43	95.8	97.3	99.1	94	234.8	242.1	250.4
44	98.3	99.8	101.7	95	237.8	245.2	253.8
45	100.7	102.4	104.3	96	240.8	248.4	257.1
46	103.2	104.9	107.0	97	243.9	251.6	260.5
47	105.7	107.5	109.6	98	246.9	254.8	263.9
48	108.2	110.1	112.3	99	249.9	258.0	267.2
49	110.7	112.7	115.0	100	253.0	261.2	270.7
50	113.3	115.3	117.7	101	256.1	264.4	274.1
51	115.8	117.9	120.4	102	259.1	267.7	277.5
52	118.3	120.5	123.1	103	262.2	270.9	281.0

10. Nuclear Binding Energy with Up and Down Quark Masses

In this section the authors proposed a very simple method for understanding the nuclear binding energy with one energy constant. Let the unified nuclear binding energy constant be:

$$B_U \cong \sqrt{m_u m_d}\, c^2 \cong 6.53 \text{ MeV} \quad (42)$$

With 6.53 MeV as a characteristic single binding energy coefficient or potential, a one term nuclear binding energy formula can be developed. Here the authors would like to stress the fact that, with further research and analysis, qualitatively a further simplified and unified method/formula can be developed. Close to the mean stable mass number, it is possible to show that,

$$B \cong 3ZkB_U \cong 3kZ * 6.53 \text{ MeV} \quad (43)$$

where k is a coefficient that seems to be related with proton number. For the observed data it can be suggested that,

$$\left.\begin{array}{l} \underline{\text{Case-1}}: Z \cong 2 \text{ to } 30,\ \text{k} \cong \left(\dfrac{Z}{30}\right)^{\frac{1}{6}} \\ \underline{\text{Case-2}}: Z >= 30,\ \text{k} \cong 1.0 \end{array}\right\} \quad (44)$$

One important point to be noted here is that, as per the quark model, proton constitutes 3 quarks as (U,U,D).

1) For Z numbers of protons, one can expect $3Z$ number of (U,U,D) bound complex quark systems. Number of up quarks can be $2Z$ and number of down quarks can be Z.
2) At the stable mass number, it is also possible to show that,

$$\left.\begin{aligned} B &\cong k\left[(2Z)m_u c^2 + \left(1+\frac{1}{k_1}\right)Zm_d c^2\right] \\ &\cong k\left[(2Z)m_u c^2 + (1+\alpha_s)Zm_d c^2\right] \cong 3kZ\sqrt{m_u m_d}\,c^2 \end{aligned}\right\} \quad (45)$$

Note that,

$$\begin{cases} 3\sqrt{m_u m_d}\,c^2 \cong 19.59 \text{ MeV} \\ \cong m_p c^2/y \cong 938.272/47.41532 \cong 19.79 \text{ MeV.} \end{cases}$$

Authors are working in this new direction. It is very strange and very interesting to say that, at the stable mass number, nuclear binding energy is approximately equal to the sum of rest energy of $2Z$ up quarks and $Z(1+\alpha_s)$ down quarks. Another interesting point is that, binding energy near to stable mass number is practically independent of mass number! See the following Table 3 for nuclear binding energy close to the mean stable mass number of Z.

In the following Table 3, column-1 represents proton number, column-2 represents stable estimated (mean) stable mass number with the proposed relation (41), column-3 represents the estimated value of k, column-4 represents the neutron number, column-5 represents the mass number, column-6 represents binding energy calculated from SEMF with current energy coefficients and column-7 represents binding energy calculated with proposed relations (43,44). ***Note that relations (43 and 44) both are no way linked with the mass number***.

Table 3. To fit the nuclear binding energy near the estimated mean stable mass number of Z

Proton number	Estimated mean stable mass number	Estimated value of k	Neutron number	Mass number	Binding energy in MeV form SEMF	Proposed binding energy in MeV
2	4	0.6368	2	4	22.0	24.9
3	6	0.6813	3	7	40.0	40.0
4	8	0.7148	4	8	52.9	56.0
5	10	0.7418	5	10	62.3	72.7
6	12	0.7647	6	12	87.4	89.9
7	14	0.7846	7	14	98.8	107.6
8	16	0.8023	8	16	123.2	125.7
9	18	0.8182	9	18	135.7	144.3
10	21	0.8327	11	21	167.5	163.1
11	23	0.8460	12	23	186.1	182.3
12	25	0.8584	13	25	204.7	201.8
13	27	0.8699	14	27	223.2	221.5
14	29	0.8807	15	29	241.6	241.5
15	31	0.8909	16	31	260.0	261.8
16	34	0.9005	18	34	290.8	282.3
17	36	0.9097	19	36	305.1	303.0
18	38	0.9184	20	38	327.2	323.8
19	40	0.9267	21	40	341.5	344.9
20	42	0.9347	22	42	363.2	366.2
21	45	0.9423	24	45	389.6	387.6
22	47	0.9496	25	47	407.5	409.3
23	49	0.9567	26	49	425.2	431.1
24	52	0.9635	28	52	454.6	453.0
25	54	0.9701	29	54	468.9	475.1
26	56	0.9764	30	56	489.6	497.3
27	58	0.9826	31	58	503.7	519.7
28	61	0.9886	33	61	532.5	542.2
29	63	0.9944	34	63	549.7	564.9
30	66	1.0000	36	66	577.9	587.7
31	68	1.0000	37	68	592.0	607.3
32	70	1.0000	38	70	611.7	626.9
33	73	1.0000	40	73	636.6	646.5
34	75	1.0000	41	75	653.3	666.1
35	78	1.0000	43	78	677.9	685.7
36	80	1.0000	44	80	697.0	705.2
37	82	1.0000	45	82	710.7	724.8
38	85	1.0000	47	85	737.6	744.4
39	87	1.0000	48	87	753.7	764.0
40	90	1.0000	50	90	780.2	783.6
41	92	1.0000	51	92	793.6	803.2
42	95	1.0000	53	95	819.8	822.8
43	97	1.0000	54	97	835.5	842.4
44	100	1.0000	56	100	861.2	862.0
45	102	1.0000	57	102	874.4	881.6
46	105	1.0000	59	105	899.8	901.1
47	108	1.0000	61	108	922.7	920.7
48	110	1.0000	62	110	940.2	940.3
49	113	1.0000	64	113	962.8	959.9
50	115	1.0000	65	115	977.9	979.5

51	118	1.0000	67	118	1000.2	999.1
52	121	1.0000	69	121	1024.6	1018.7
53	123	1.0000	70	123	1039.4	1038.3
54	126	1.0000	72	126	1063.4	1057.9
55	129	1.0000	74	129	1085.1	1077.5
56	131	1.0000	75	131	1099.6	1097.0
57	134	1.0000	77	134	1121.1	1116.6
58	137	1.0000	79	137	1144.4	1136.2
59	139	1.0000	80	139	1158.6	1155.8
60	142	1.0000	82	142	1181.7	1175.4
61	145	1.0000	84	145	1202.6	1195.0
62	148	1.0000	86	148	1225.3	1214.6
63	150	1.0000	87	150	1237.2	1234.2
64	153	1.0000	89	153	1259.7	1253.8
65	156	1.0000	91	156	1280.0	1273.4
66	159	1.0000	93	159	1302.1	1292.9
67	161	1.0000	94	161	1315.6	1312.5
68	164	1.0000	96	164	1337.5	1332.1
69	167	1.0000	98	167	1357.3	1351.7
70	170	1.0000	100	170	1378.8	1371.3
71	173	1.0000	102	173	1398.3	1390.9
72	176	1.0000	104	176	1419.5	1410.5
73	179	1.0000	106	179	1438.8	1430.1
74	182	1.0000	108	182	1459.6	1449.7
75	184	1.0000	109	184	1470.7	1469.3
76	187	1.0000	111	187	1491.4	1488.8
77	190	1.0000	113	190	1510.2	1508.4
78	193	1.0000	115	193	1530.5	1528.0
79	196	1.0000	117	196	1549.0	1547.6
80	199	1.0000	119	199	1569.1	1567.2
81	202	1.0000	121	202	1587.4	1586.8
82	205	1.0000	123	205	1607.2	1606.4
83	208	1.0000	125	208	1625.2	1626.0
84	211	1.0000	127	211	1644.7	1645.6
85	214	1.0000	129	214	1662.4	1665.2
86	217	1.0000	131	217	1681.7	1684.7
87	220	1.0000	133	220	1699.2	1704.3
88	223	1.0000	135	223	1718.2	1723.9
89	226	1.0000	137	226	1735.5	1743.5
90	230	1.0000	140	230	1761.2	1763.1
91	233	1.0000	142	233	1778.2	1782.7
92	236	1.0000	144	236	1796.7	1802.3
93	239	1.0000	146	239	1813.4	1821.9
94	242	1.0000	148	242	1831.6	1841.5
95	245	1.0000	150	245	1848.2	1861.1
96	248	1.0000	152	248	1866.1	1880.6
97	252	1.0000	155	252	1887.5	1900.2
98	255	1.0000	157	255	1905.2	1919.8
99	258	1.0000	159	258	1921.3	1939.4
100	261	1.0000	161	261	1938.8	1959.0

See figure 1 for nuclear binding energy near to estimated mean stable mass number. In the figure, red color line represents the estimated binding energy and blue color line represents the nuclear binding energy calculated from the semi empirical mass formula with the current recommended energy coefficients. Above and below the stability line, approximately nuclear binding energy can be expressed in the following way.

$$\left.\begin{array}{l} If\ \left(A >= A_{mean}\right), \\ B \cong \left(\dfrac{A}{A_{mean}}\right)^{2/3} (3kZ) B_U \cong \left(\dfrac{A}{A_{mean}}\right)^{2/3} (3kZ) * 6.53\ MeV \\ If\ \left(A < A_{mean}\right), \\ B \cong \left(\dfrac{A}{A_{mean}}\right)^{4/3} (3kZ) B_U \cong \left(\dfrac{A}{A_{mean}}\right)^{4/3} (3kZ) * 6.53\ MeV \end{array}\right\} \quad (46)$$

With reference to quark soup model, understanding nuclear binding energy is a very critical and very interesting issue. Authors are working in this direction. With further research and analysis, like even-odd corrections etc, it is certainly possible to develop a compact and accurate relation for binding energy above and below the stable mass number. For example, estimated mean stable mass number of $Z = 50$ is $A_{mean} = 115.3$ and after rounding off, its even mean stable mass number can be taken as $A_{mean} = 114$. With this corrected mean stable mass number-attempt is made to fit the nuclear binding energy with relation (46). See Table 4 and also see the figures 2 to 6. In the figures, red color curve represents the estimated binding energy from relation (45) (only round off and no even-odd correction for A_{mean}) and blue color curve represents the nuclear binding energy calculated from the semi empirical mass formula with the current recommended energy coefficients.

11. To Fit and Understand the Semi Empirical Mass Formula Binding Energy Coefficients with Up and Down Quark Masses

With usual notation, the semi empirical mass formula energy coefficients can be fitted and understood in the following way.

$$\left.\begin{array}{l}
\left(\frac{a_c}{B_U}\right) \cong \frac{1}{k_1} \cong \alpha_s \rightarrow a_c \cong 0.7576 \text{ MeV}, \\
\left(\frac{a_s}{a_c}\right) \cong \frac{6}{2}k_1 \cong \frac{6}{2}\left(\frac{1}{\alpha_s}\right) \rightarrow a_a \cong 19.59 \text{ MeV}, \\
\left(\frac{a_v}{a_c}\right) \cong \frac{5}{2}k_1 \cong \frac{5}{2}\left(\frac{1}{\alpha_s}\right) \rightarrow a_v \cong 16.325 \text{ MeV}, \\
\left(a_v + a_s\right) \cong 35.915 \text{ MeV} \cong \left(a_a + a_p\right) \\
a_p \cong \frac{1}{3}\left(a_v + a_s\right) \cong 11.972 \text{ MeV}, \\
a_a \cong 2a_p \cong \frac{2}{3}\left(a_v + a_s\right) \cong 23.943 \text{ MeV} \\
\left(\frac{a_p}{a_c}\right) \cong \frac{11}{6}k_1 \cong \frac{11}{6}\left(\frac{1}{\alpha_s}\right); \left(\frac{a_a}{a_c}\right) \cong \frac{11}{3}k_1 \cong \frac{11}{3}\left(\frac{1}{\alpha_s}\right)
\end{array}\right\} \quad (47)$$

Note that,

1) Strong coupling constant seems to be the ratio of Coulombic energy coefficient and $\sqrt{m_u m_d}c^2 \cong 6.53$ MeV. It is an important point to be considered here.
2) For the medium and heavy atomic nuclides, if shell corrections are significant, then

$$B \cong a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \quad (48)$$

3) For the medium and heavy atomic nuclides, if shell corrections are not significant, then

$$B \cong a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \quad (49)$$

4) Thus by considering the two forms of Coulombic energy terms, the binding energy band can be obtained and error can be minimized with reference to shell corrections. See the Table 5 for the comparison of current and proposed SEMF energy coefficients.

Table 4. To fit the SEMF binding energy of Z=50

Proton number	Mean stable mass number	Assumed value of k	Neutron number	Mass number	Binding energy from SEMF	Binding energy calculated with relation (45)
50	114	1	50	100	809.3	822.5
50	114	1	51	101	822.3	833.5
50	114	1	52	102	837.2	844.5
50	114	1	53	103	849.2	855.6
50	114	1	54	104	863.2	866.6
50	114	1	55	105	874.5	877.8
50	114	1	56	106	887.6	888.9
50	114	1	57	107	898.1	900.1
50	114	1	58	108	910.4	911.4
50	114	1	59	109	920.1	922.6
50	114	1	60	110	931.8	933.9
50	114	1	61	111	940.7	945.3
50	114	1	62	112	951.6	956.7
50	114	1	63	113	960.0	968.1
50	114	1	64	114	970.2	979.5
50	114	1	65	115	977.9	985.2
50	114	1	66	116	987.5	990.9
50	114	1	67	117	994.6	996.6
50	114	1	68	118	1003.6	1002.3
50	114	1	69	119	1010.1	1007.9
50	114	1	70	120	1018.5	1013.6
50	114	1	71	121	1024.4	1019.2
50	114	1	72	122	1032.3	1024.8
50	114	1	73	123	1037.8	1030.4
50	114	1	74	124	1045.1	1036.0
50	114	1	75	125	1050.1	1041.5
50	114	1	76	126	1056.9	1047.1
50	114	1	77	127	1061.4	1052.6
50	114	1	78	128	1067.8	1058.1
50	114	1	79	129	1071.8	1063.6
50	114	1	80	130	1077.7	1069.1
50	114	1	81	131	1081.4	1074.6
50	114	1	82	132	1086.9	1080.1
50	114	1	83	133	1090.1	1085.5
50	114	1	84	134	1095.2	1091.0
50	114	1	85	135	1098.0	1096.4
50	114	1	86	136	1102.7	1101.8
50	114	1	87	137	1105.1	1107.2
50	114	1	88	138	1109.4	1112.6
50	114	1	89	139	1111.5	1117.9
50	114	1	90	140	1115.5	1123.3
50	114	1	91	141	1117.2	1128.6
50	114	1	92	142	1120.8	1134.0
50	114	1	93	143	1122.3	1139.3
50	114	1	94	144	1125.6	1144.6

Table 5. Existing and proposed SEMF binding energy coefficients

Existing energy coefficients	Proposed energy coefficients
$a_v \cong 15.78$ MeV	$a_v \cong 16.325$ MeV
$a_s \cong 18.34$ MeV	$a_s \cong 19.59$ MeV
$a_c \cong 0.71$ MeV	$a_c \cong 0.7576$ MeV
$a_a \cong 23.21$ MeV	$a_a \cong 23.943$ MeV
$a_p \cong 12.0$ MeV	$a_p \cong 11.972$ MeV

See Table 6 for nuclear binding energy estimated with proposed SEMF energy coefficients.

Table 6. To fit the measured binding energy with the proposed SEMF energy coefficients

Z	A	Measured $(BE)_{meas}$ in MeV	$(BE)_{cal}$ in MeV from relation (48)	$(BE)_{cal}$ in MeV from relation (49)
26	56	492.258	493.5	488.36
44	100	861.928	868.35	861.17
50	116	988.684	995.44	987.67
70	170	1378.13	1387.70	1378.11
82	208	1636.43	1636.43	1626.1
92	238	1801.69	1816.95	1805.70

12. Conclusion

So far no model succeeded in coupling and understanding the unified concepts of gravity and atomic interactions. In this context, it can be suggested that,

1) With reference to the final unification and Schwarzschild interaction, if one is willing to consider elementary charge, speed of light, gravitational constant, atomic gravitational constant and Avogadro number as elementary physical constants, then electron rest mass, nucleon rest masses, nuclear unit radii, the reduced Planck's constant, Planck's constant, Fine structure ratio and strong coupling constant can be considered as compound physical constants.
2) Avogadro number is a pure number and there is no need to assign units. Magnitude of atomic gravitational constant seems to be N_A^2 times the gravitational constant. In finding the secrets of unification, the proposed grand unified back ground numbers (x, y) and their various combinations can be given some fundamental significance.
3) Nuclear stability, nuclear binding energy, Up and Down quark masses, strong coupling constant etc can be understood with the proposed (x, y) numbers and unified potential 0.001732 MeV.
4) Fitted magnitude of the gravitational constant is $G \cong 6.674378868 \times 10^{-11}$ m^{3}kg^{-1}sec^{-2} can be given significance in unification program.

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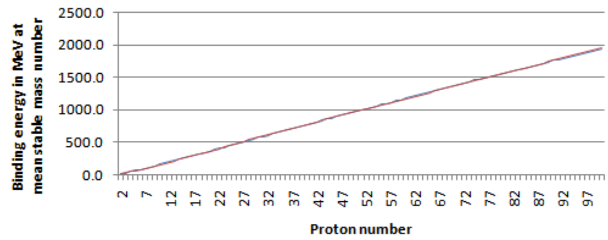

Figure 1. Binding energy at mean stable mass numbers of Z=2 to 100

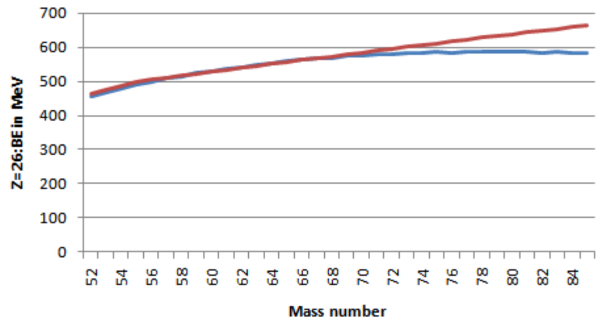

Figure 2. Binding energy of isotopes of Z=26

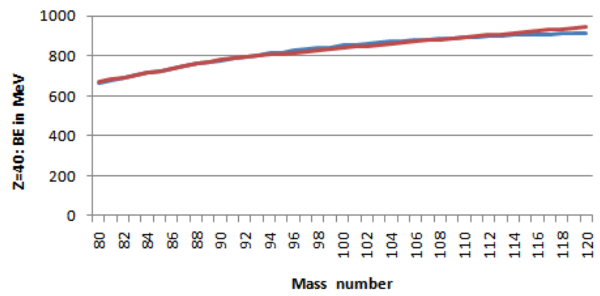

Figure 3. Binding energy of isotopes of Z=40

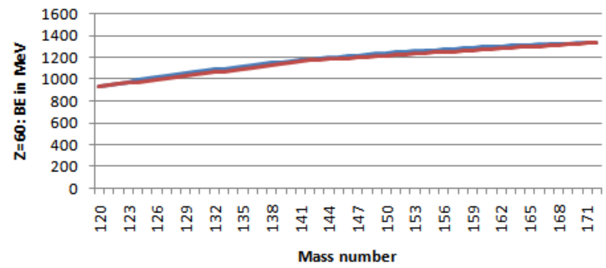

Figure 4. Binding energy of isotopes of Z=60

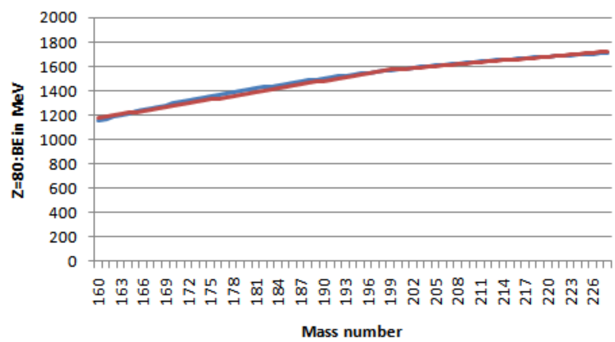

Figure 5. Binding energy of isotopes of Z=80

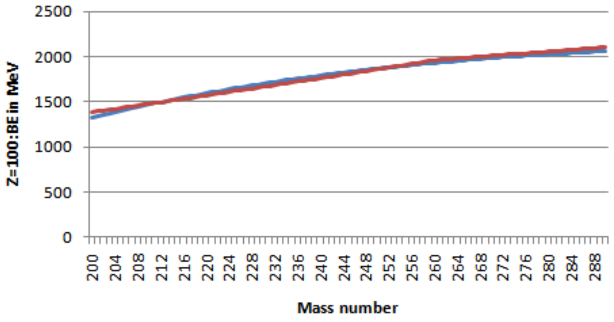

Figure 6. Binding energy of isotopes of Z=100

References

[1] P. A. M. Dirac, The cosmological constants. Nature, 139, 323, (1937).

[2] Witten, Edward. Search for a realistic Kaluza-Klein theory. Nuclear Physics B 186 (3): 412-428. (1981).

[3] David Gross, Einstein and the search for Unification. Current science, Vol. 89, No. p 12. (2005).

[4] Abdus Salam. Einstein's Last Dream: The Space-Time Unification of Fundamental Forces, Physics News, 12 (2): 36, (1981).

[5] Salam A. and Sivaram C. Strong Gravity Approach to QCD and Confinement. Mod. Phys. Lett.,v. A8 (4), 321-326. (1993).

[6] Recami E. Elementary Particles as Micro-Universes, and "Strong Black-holes": A Bi-Scale Approach to Gravitational and Strong Interactions. Preprint NSF-ITP-02-94. posted in the arXives as the e-print physics/0505149, and references therein.

[7] Dine, Michael. Supersymmetry and String Theory: Beyond the Standard Model. Cambridge University Press. (2007)

[8] Roberto Onofrio. On Weak Interactions as Short-Distance Manifestations of Gravity. Modern Physics Letters A, Vol. 28, No. 7 (2013) 1350022.

[9] U. V. S. Seshavatharam and S. Lakshminarayana. Two Background Unified Numbers & Their Possible Role. Prespace time journal, Vol 5, issue 13, pp 1338-1353. (November 2014).

[10] U. V. S. Seshavatharam and S. Lakshminarayana. On the plausibility of final unification with Avogadro Number. Prespace time journal, Vol 5, issue 10, pp 1028-1041. (2014).

[11] U. V. S. Seshavatharam and S. Lakshminarayana. Nucleus in Strong nuclear gravity. Proceedings of the DAE Symp. On Nucl. Phys. 56: 302, 2011.

[12] U. V. S. Seshavatharam and S. Lakshminarayana, Role of Avogadro number in grand unification. Hadronic Journal. Vol-33, No 5, (2010) October. p 513.

[13] U. V. S. Seshavatharam and S. Lakshminarayana, To confirm the existence of atomic gravitational constant. Hadronic journal, Vol-34, No 4, p 379 (2011).

[14] U. V. S. Seshavatharam and S. Lakshminarayana. Logic Behind the Squared Avogadro Number and SUSY. International Journal of Applied and Natural Sciences. Vol. 2, Issue 2, 23-40 (2013).

[15] U. V. S. Seshavatharam and S. Lakshminarayana. Integral charge SUSY in Strong nuclear gravity. Proceedings of the DAE Symp. on Nucl. Phys. 56 (2011) p. 842.

[16] U. V. S. Seshavatharam and S. Lakshminarayana. Super Symmetry in Strong and Weak interactions. Int. J. Mod. Phys. E, Vol. 19, No. 2, (2010), p. 263-280.

[17] U. V. S. Seshavatharam and S. Lakshminarayana. SUSY and strong nuclear gravity in (120-160) GeV mass range. Hadronic journal, Vol-34, No 3, (2011) June, p. 277-300

[18] U. V. S. Seshavatharam and S. Lakshminarayana. New concepts and semi empirical fittings in understanding SUSY and the four cosmological interactions. Prespace time journal, Vol 4, issue 11, (December 2013), pp 1027-1038.

[19] Roger Penrose. Chandrasekhar, Black Holes, and Singularities. J. Astrophys. Astr. (1996) 17, 213-231.

[20] Subrahmanyan Chandrasekhar. On Stars, Their Evolution and Their Stability', Nobel Prize lecture, December 8, 1983.

[21] G..J. Stoney, On the Physical Units of Nature. Phil. Mag. 11 (1881) 381-390.

[22] Martin J.T. Milton. A new definition for the mole based on the Avogadro constant: a journey from physics to chemistry. Phil. Trans. R. Soc. A 369, 3993-4003. (2011).

[23] P.J. Mohr, B.N. Taylor, and D.B. Newell. CODATA Recommended Values of the Fundamental Physical Constants: 2010" by in Rev. Mod. Phys. 84, 1527 (2012) http://pdg.lbl.gov/2014/reviews/rpp2014-rev-phys-constants.pdf

[24] K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014).

[25] N. Bohr. On the Constitution of Atoms and Molecules. (Part-1) Philos. Mag. 26, 1913, p 1.

[26] N. Bohr. On the Constitution of Atoms and Molecules. Systems containing only a Single Nucleus. (Part-2) Philos. Mag. 26, 476, (1913).

[27] Geiger H and Marsden E. On a diffuse reaction of the particles. Proc. Roy. Soc., Ser. A 82: 495-500, (1909).

[28] Michael O. Distler et al. The RMS Charge Radius of the Proton and Zemach Moments. Phys. Lett.B. 696: 343-347, (2011).

[29] Roberto Onofrio. Proton radius puzzle and quantum gravity at the Fermi scale. EPL, 104 (2013) 20002

[30] Chowdhury, P.R. et al. Modified Bethe-Weizsacker mass formula with isotonic shift and new driplines. Mod. Phys. Lett. A20 p. 1605-1618. (2005).

[31] W.D. Myers and W.J. Swiatecki. Table of Nuclear Masses according to the 1994 Thomas-Fermi Model. LBL-36803. (1994).

[32] G. Audi and A.H. Wapstra. The 1993 atomic mass evolution.(I) Atomic mass table. Nuclear physics, A 5 65, p1-65 (1993).

[33] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli and G. M. Tino1. Precision measurement of the Newtonian gravitational constant using cold atoms. Nature 510, 518-521. (2014)

[34] L.L. Williams. Analytical Expressions for the Gravitational Constant. (August 2009) http://www.konfluence.org/CalculatingG.pdf

[35] George T Gillies. The Newtonian gravitational constant: recent measurements and related studies. Rep. Prog. Phys. 60 151

[36] J Stuhler et al. MAGIA—using atom interferometry to determine the Newtonian gravitational constant. J. Opt. B: Quantum Semiclass. Opt. 5 (2003) S75-S81.

[37] Terry Quinn, Harold Parks, Clive Speake and Richard Davis. An uncertain big G. Phys. Rev. Lett. 112.068103. (2013)

[38] J. B. Fixler; G. T. Foster; J. M. McGuirk; M. A. Kasevich. Atom Interferometer Measurement of the Newtonian Constant of Gravity, Science 315 (5808): 74-77, (2007).

[39] Brandenburg, J.E., 1992, IEEE Transactions on Plasma Science, v20, n6, p 944. Unification of Gravity and Electromagnetism in the Plasma Universe.

[40] Jun Luo and Zhong-Kun Hu. Status of measurement of the Newtonian gravitational constant G. Class. Quantum Grav. 17 (2000) 2351-2363

[41] St. Schlamminger et al. Determination of the Gravitational Constant Using a Beam Balance. (July 2002) http://www.schlammi.com/pdf/prl02.pdf

[42] B. Andreas et al. An accurate determination of the Avogadro constant by counting the atoms in a 28Si crystal. Phys. Rev. Let. 106 (3): 030801 (2011).

[43] B P Leonard. On the role of the Avogadro constant in redefining SI units for mass and amount of substance. Metrologia 44, 82-86 (2007).

[44] Dulal C Ghosh, rakha Biswas. Theoretical Calculation of Absolute Radii of Atoms and Ions. Part 1. The Atomic Radii. Int. J. Mol. Sci. 3, 87-113. (2002).

[45] Slater, J.C. Quantum Theory of Moleculs and Solids; McGraw-Hill Book: New York; Vol. 2.

[46] J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012).

[47] A.V. Manohar and C.T. Sachrajda. Quark masses. Updated Jan 2014. Pages 1-20. (pdg.lbl.gov/2014/reviews/rpp2014-rev-quark-masses.pdf)

[48] Halzen, F.; Martin, A. D. Quarks and Leptons: An Introductory Course in Modern Particle Physics. John Wiley & Sons. (1984).