A Trapezoidal Fuzzy Method for Assessing Students' Mathematical Modeling Skills

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Abstract   Fuzzy logic due to its nature of characterizing each case with multiple values offers a rich field of resources covering the assessment of situations characterized by a degree of vagueness and/or uncertainty. In this paper, we apply a new trapezoidal fuzzy assessment model (TRFAM) for measuring students' mathematical modeling (MM) skills. TRFAM is actually a variation of the commonly used in fuzzy logic centre of gravity (COG) defuzzification technique, which we have properly adapted and utilized in earlier papers as an assessment model. Our COG and TRFAM models focus on students' quality performance by assigning greater coefficients to the higher scores. A classroom experiment is also presented, illustrating our results in practice. Through this experiment the above two models are compared to each other and also with two traditional assessment methods based on principles of bivalent logic (the calculation of means and of the GPA index) and some useful conclusions are drawn.

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1. Introduction

The assessment of a system's effectiveness (i.e. of the degree of attainment of its targets) with respect to an activity performed within the system (e.g. problem-solving, decision making, learning process, etc) is a very important task that enables the correction of the system's weaknesses resulting to the improvement of its general performance.

The assessment methods that are commonly used in practice are based on principles of the bivalent logic (yes-no). However, there are cases where a crisp characterization is not probably the proper one for an assessment. In the area of Education, for example, a teacher is frequently not sure about a particular numerical grade characterizing a student's performance. *Fuzzy logic*, due to its nature of characterizing a case with multiple values, offers wider and richer resources for covering such kind of cases than the bivalent logic does.

The several methods being in use for assessing a system's performance focus on different targets: Some of them measure the *mean system's performance*, while others measure its *quality performance* by assigning greater coefficients (weights) to the higher scores. Therefore, one who wants to obtain a comprehensive view of a system's performance should make use of more than one assessment methods for this purpose.

In this paper we apply a new fuzzy model for assessing students' mathematical modelling (MM) skills. The rest of the paper is organized as follows: In section 2 we attempt a brief description of the MM process (circle of modelling) as a method for teaching mathematics. In section 3 we develop our new *trapezoidal fuzzy assessment model* (TRFAM), which is actually a variation of the *centre of gravity* (COG) defuzzification technique that we have properly adapted and used in earlier papers as an assessment method. An application on MM is presented in section 4 illustrating our results in practice. Through this application the COG and TRAMF methods are compared to each other and with other traditional assessment methods (calculation of the means and of the GPA index). We close by stating, in section 5, our final conclusions and our future plans for further research on the subject.

For general facts on fuzzy sets we refer to the book [9].
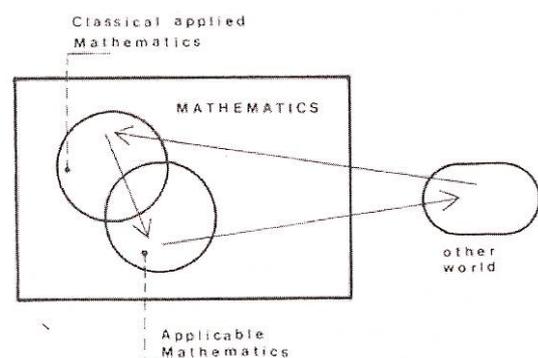
2. The circle of Modelling

Figure 1. The circle of modelling

Until the middle of 1970's mathematical modelling (MM) was mainly a tool in hands of scientists and engineers for solving real world problems related to their disciplines (physics, industry, constructions, economics, etc). One of the first who described the process of MM in such a way that it could be used for teaching mathematics was Pollack [11]. He represented the interaction between mathematics and the real world with the scheme shown in Figure 1, which is known as the *circle of modelling*.

According to the above scheme, in the "universe" of mathematics *classical applied mathematics* and *applicable mathematics* are two intersected but not equal to each other sets. In fact, there are topics from classical mathematics with great theoretical interest, but without any visible applications (although such applications it is possible to be found in future), while at the same time they are branches of mathematics with many practical applications, which are not characterized by the experts as classical mathematics (e.g. statistics, fuzzy logic, fractals, etc). But the most important feature in Pollack's scheme is the direction of the arrows, representing a looping between the other (real) world, (including all the other sciences and the human activities of everyday life) and the "universe" of mathematics: Starting from a problem of the real world we transfer to the other part of the scheme, where we use or develop suitable mathematics for its solution. Then we return to the real world interpreting and testing the mathematical results obtained. If these results are not giving a satisfactory solution to the problem, then we repeat the same circle again one or more times.

From the time that Pollack presented this scheme in ICME-3 (Karlsruhe, 1976) until nowadays much effort has been placed to analyze in detail the process of MM as a method of teaching mathematics (Barry & Davies [2], Edwards & Hauson [5], Blomhψj & Jensen [3], Greefrath [7], Blum & Leib [4], Voskoglou [17], etc). A brief but comprehensive account of the different models used for the description of the MM process can be found in Chaines & Crouch [8] including our stochastic model [[16,17,18] (section 5.2)].

As a result of all these research efforts it is more or less acceptable nowadays that the process of MM basically involves the following stages:

S₁: *Analysis* of the problem (understanding the statement and recognizing the restrictions and requirements of the real system.

S₂: *Mathematization* (formulation of the problem and construction of the model).

S₃: *Solution* of the model.

S₄: *Validation* (control) of the model, which is achieved by reproducing, through the model, the behavior of the real system under the conditions existing before the construction of the model. In case of a system having no previous "history" an extra (usually simulation) model can be used for this purpose.

S₅: *Interpretation* of the final mathematical results and implementation of them to the real system, in order to give the "answer" to the real world problem.

In [17] we have treated the MM circle as a Markov chain process dependent upon the transition between the discrete stages of the MM process. The arrows in Figure 2 are showing the possible transitions between stages.

Some authors consider further stages in the MM process; e.g. some of them divide mathematization to the stages of *formulation* of the real problem in a way that it will be ready for mathematical treatment and of *construction* of the model, others divide the validation to the stages of *interpretation* and *evaluation* of the model, others add the stage of *refining* the model, etc (e.g. see [8]). However, all these minor variations do not change the general idea that we nowadays have about the circle of MM.
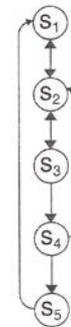

Figure 2. The flow-diagram the MM process

MM appears today as a dynamic tool for teaching and learning mathematics, because it connects mathematics with our everyday life giving the possibility to students to understand its usefulness in practice and therefore increasing their interest about mathematics. In other words, according to the Polya's [12] terminology (axioms of learning), MM works as a *best motivation* for learning mathematics. But we must be careful! The process of MM could not be considered as a general and therefore applicable in all cases method for teaching mathematics. In fact, such a consideration could lead to far-fetched situations, where more emphasis is given to the search of the proper application rather, than to the consolidation of the new mathematical knowledge!

3. Development of the TRFAM

The COG (or centroid) technique is a commonly used in fuzzy logic defuzzification method (e.g. see [15]). In earlier papers (e.g. [13,20,21], etc) the COG technique has been properly adapted to be used as an assessment method.

In fact, let G be a group of individuals participating in a certain activity and let U={A, B, C, D, F} be a set of linguistic labels (grades) characterizing the individuals' performance with respect to this activity as follows: A=excellent, B=very good, C=good, D=moderate (satisfactory) and F=unsatisfactory. Then, we can express G as a *fuzzy set* in U in the form G = {(x, m(x)), x ∈ U}, where $y=m(x)$ is the corresponding membership function.

In this case, according to our results (e.g. see section 2 of [21]) the group's performance can be measured by the coordinates (x_c, y_c) of the COG of the level's section contained between the graph of the membership function $y=m(x)$ (which in our case turns out to be a bar graph consisting of five rectangles; e.g. see Figure 1 of [21]) and the OX axis, which are calculated by the formulas

$$x_c = \frac{1}{2}\left(y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5\right) \text{ and}$$

$$y_c = \frac{1}{2}\left(y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2\right)$$

(1)

with $y_1=m(F)$, $y_2=m(D)$, $y_3=m(C)$, $y_4=m(B)$, $y_5=m(A)$ and $\sum_{i=1}^{5} y_i = 1$ (the values of the y_i's can be normalized by dividing each of them by their sum). The ideal case appears when all the individuals of the group demonstrate excellent performance, i.e. when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. In this case formulas (1) give that the COG is the point $(\frac{9}{2}, \frac{1}{2})$.

Further, the following criterion holds for comparing the performance of two (or more) groups:

Between two groups, the group with the greater value of x_c performs better. Also, if the value of x_c is equal for both groups, then, if $x_c \geq 2.5$, the group with the greater value of y_c performs better, while, if $x_c < 2.5$, then the group with the smaller value of y_c performs better.

The TRFAM is a recently developed variation of the COG method [14]. The important novelty of this approach is in the replacement of the rectangles appearing in the graph of the membership function of the COG method by isosceles trapezoids sharing common parts. In the TRFAM's scheme (Figure 3) we have five trapezoids, corresponding to the above defined grades F, D, C, B and A respectively of the individuals' performance. Without loss of generality and for making our calculations easier we consider isosceles trapezoids with bases of length 10 units lying on the OX axis. The height of each trapezoid is equal to the percentage of individuals who achieved the corresponding grade for their performance, while the parallel to its base side is equal to 4 units. We allow for any two adjacent trapezoids to have 30% of their bases (3 units) belonging to both of them. In this way we cover the ambiguous cases of individuals' scores being at the boundaries between two successive grades. For students' assessment, for example, it is a very common approach to divide the interval of the specific grades in three parts and to assign the corresponding grade using + and -. For example, 75 – 77% = B-, 78 – 81% = B, 82 – 84% = B+. However, this consideration does not reflect the common situation, where the teacher is not sure about the grading of the students whose performance could be assessed as marginal between and close to two adjacent grades; for example, something like 84 - 85 being between B+ and A-. The TRFAM fits this situation.
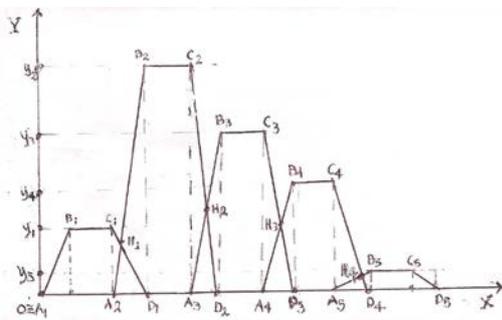

Figure 3. The TRAFM's scheme

An individuals'' group can be represented, as in the COG method, as a fuzzy set in U, whose membership function $y=m(x)$ has as graph the line $OB_1C_1H_1B_2C_2H_2B_3C_3H_3B_4C_4H_4B_5C_5D_5$ of Figure 2, which is the union of the line segments OB_1, B_1C_1, C_1H_1,.., B_5C_5,

C_5D_5. However, in case of the TRFAM the analytic form of $y = m(x)$ is not needed for calculating the COG of the resulting area. In fact, since the boundary cases of the individuals' scores are considered as common parts for any pair of the adjacent trapezoids, it is logical to count these parts twice; e.g. placing the ambiguous cases B+ and A- in both regions B and A. In other words, the COG method, which uses the analytic form of $y = m(x)$ for calculating the coordinates of the COG of the area between the graph of the membership function and the OX axis, thus considering the areas of the "common" triangles $A_2H_1D_1$, $A_3H_2D_2$, $A_4H_3D_3$ and $A_5H_4D_4$ only once, is not the proper one to be applied in the above situation.

Therefore, in this case we represent each one of the five trapezoids of Figure 3 by its COG F_i, i=1, 2, 3, 4, 5 and we consider the entire area, i.e. the sum of the areas of the five trapezoids, as the system of these points-centers. More explicitly, the steps of the whole construction of the TRFAM are the following:

1. Let y_i, i=1, 2, 3, 4, 5 be the percentages of the individuals' whose performance was characterized by F, D, C, B, and A respectively; then $\sum_{i=1}^{5} y_i = 1$ (100%).

2. We consider the isosceles trapezoids with heights equal to y_i, i=1, 2, 3, 4, 5, in the way that has been illustrated in Figure 2.

3. We calculate the coordinates (x_{c_i}, y_{c_i}) of the COG F_i, i=1, 2, 3, 4, 5, of each trapezoid as follows: It is well known that the COG of a trapezoid lies along the line segment joining the midpoints of its parallel sides *a* and *b* at a distance *d* from the longer side *b* given by $d = \frac{h(2a+b)}{3(a+b)}$, where *h* is its height (e.g. see [23]).

Therefore in our case we have

$$y_{c_i} = \frac{y_i(2*4+10)}{3*(4+10)} = \frac{3y_i}{7}$$

Also, since the abscissa of the COG of each trapezoid is equal to the abscissa of the midpoint of its base, it is easy to observe that $x_{ci}=7i-2$.

4. We consider the system of the COG's F_i, i=1, 2, 3, 4, 5 and we calculate the coordinates (X_c, Y_c) of the COG F_c of the whole area *S* considered in Figure 2 by the following formulas, derived from the commonly used in such cases definition (e.g. see [24]):

$$X_c = \frac{1}{S}\sum_{i=1}^{5} S_i x_{c_i}, Y_c = \frac{1}{S}\sum_{i=1}^{5} S_i y_{c_i} \qquad (2)$$

In formulas (2) S_i, i= 1, 2, 3, 4, 5 denote the areas of the corresponding trapezoids. Thus, $S_i = \frac{(4+10)y_i}{2} = 7y_i$ and $S = \sum_{i=1}^{5} S_i = 7\sum_{i=1}^{5} y_i = 7$. Therefore, from formulas (2) we finally get that

$$X_c = \frac{1}{7}\sum_{i=1}^{5} 7y_i(7i-2) = (7\sum_{i=1}^{5} iy_i) - 2 \text{ and}$$

$$Y_c = \frac{1}{7}\sum_{i=1}^{5} 7y_i(\frac{3}{7}y_i) = \frac{3}{7}\sum_{i=1}^{5} y_i^2 \qquad (3)$$

5. We determine the area where the COG F_c lies as follows: For i, j=1, 2, 3, 4, 5, we have that $0 \le (y_i - y_j)^2 = y_i^2 + y_j^2 - 2y_iy_j$, therefore $y_i^2 + y_j^2 \ge 2y_iy_j$, with the equality holding if, and only if, $y_i=y_j$. Thus,

$$1 = (\sum_{i=1}^{5} y_i)^2 = \sum_{i=1}^{5} y_i^2 + 2 \sum_{\substack{i,j=1, \\ i \ne j}}^{5} y_i y_j$$

$$\le \sum_{i=1}^{5} y_i^2 + 2 \sum_{\substack{i,j=1, \\ i \ne j}}^{5} (y_i^2 + y_j^2) = 5\sum_{i=1}^{5} y_i^2 \text{ or } \sum_{i=1}^{5} y_i^2 \ge \frac{1}{5}[$$

(4)

with the equality holding if and only if $y_1 = y_2 = y_3 = y_4 = y_5 = \frac{1}{5}$. In the case of equality the first of formulas (3) gives that $X_c = 7(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{5}{5}) - 2 = 15$. Further, combining the inequality (4) with the second of formulas (3) one finds that $Y_c \ge \frac{3}{35}$ Therefore the unique minimum for Y_c corresponds to the COG $F_m(15, \frac{3}{35})$. The ideal case is when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. Then from formulas (3) we get that $X_c = 33$ and $Y_c = \frac{3}{7}$. Therefore the COG in this case is the point $F_i(33, \frac{3}{7})$. On the other hand, the worst case is when $y_1=1$ and $y_2 = y_3 = y_4 = y_5=0$. Then from formulas (3), we find that the COG is the point $F_w(5, \frac{3}{7})$. Therefore the area where the COG F_c lies is the area of the triangle $F_w F_m F_i$ (see Figure 4).
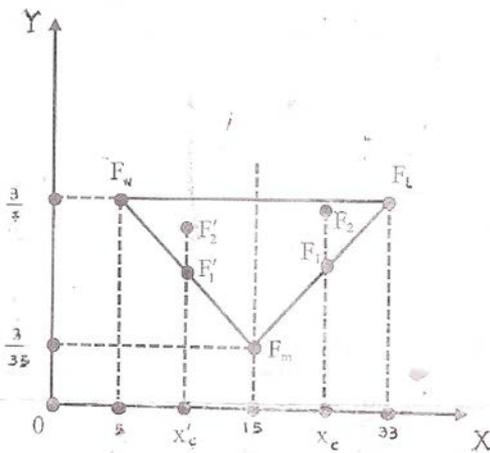

Figure 4. The area where the COG lies

6. We formulate our criterion for comparing the performances of two (or more) different groups' as follows: From elementary geometric observations (see Figure 3) it follows that for two groups the group having the greater X_c performs better. Further, if the two groups have the same $X_c \ge 15$, then the group having the COG which is situated closer to Fi is the group with the greater Y_c. Also, if the two groups have the same $X_c<15$, then the group having the COG which is situated farther to Fw is the group with the smaller Y_c. Based on the above considerations we obtain the following criterion:

- *Between two groups the group with the greater value of X_c demonstrates the better performance.*
- *If two groups have the same $X_c \ge 15$, then the group with the greater value of Y_c demonstrates the better performance.*
- *If two groups have the same $X_c < 15$, then the group with the smaller value of Y_c demonstrates the better performance.*

Once developed, the TRFAM is very easy to be applied in practice, because it needs no complicated calculations in its final step. The same also holds for the COG technique.

Observing the first of formulas (1) and (3) we can see that both of the above fuzzy assessment methods assign for the abscissa of the COG greater coefficients to the higher scores. Therefore, since the value of the COG's abscissa measures in both cases the group's performance (see the above criterion), we conclude that these fuzzy assessment methods focus on the *quality performance* rather than on the *mean performance* of the corresponding group.

4. An Application to MM

Exploratory investigations have demonstrated how exposure to computers enhances the way students approach MM problems (Asiala et al. [1], Weller et al. [22], Lewandowski et al. [10], Yadav et al. [25], Voskoglou & Buckley [19], etc).

In our will to explore further the effect of the use of computers as a tool for solving MM problems we performed the following classroom experiment with subjects students of the School of Technological Applications (prospective engineers) of the Graduate Technological Educational Institute (T. E. I.) of Western Greece attending the course "Higher Mathematics I" of their first term of studies, which involves Differential and Integral Calculus in one variable, Elementary Differential Equations and Linear Algebra. The students, who had no previous experience on computers apart from the basics learned in secondary education, were divided in two equivalent groups according to their grades obtained in the Panhellenic maths exam for entrance in higher education.

For the control group the lectures were performed in the classical way on the board, followed by a number of exercises and examples connecting mathematics with real world applications and problems. The students participated in solving these problems. The difference for the experimental group was that about the 1/3 of the lectures and exercises were performed in a computer laboratory. There the instructor using the computers presented the corresponding mathematical topics in a more "live" and attractive way, while the students themselves, divided in small groups and making use of known mathematical software packages solved the problems with the help of computers. Notice that the teaching schedule of the course involved six hours per week for both groups including the time spent in the computer laboratory for the experimental group.

At the end of the term all students participated to the final written exam of the course for the assessment of their progress. The exam involved a number of general theoretical questions and exercises covering all the topics

taught and three simplified real world problems (see Appendix) requiring MM techniques for their solutions. We marked the students' papers in a scale from 0 to 100, separately for the questions and exercises and separately for the problems. Further, we characterized their performance as follows: A (90-100) = Excellent, B (75-89) = Very Good, C (60-74) = Good, D (50-59) = Satisfactory and E (0-49) = Unsatisfactory.

No significant differences were found between the two groups concerning the theoretical questions and exercises, their overall performances were almost identical. Below we shall present and evaluate in detail the results of the two groups concerning the MM problems. The scores achieved by the students of the two groups were the following:

Experimental group (G₁): 100(5 times), 99(3), 98(10), 95(15), 94(12), 93(1), 92 (8), 90(6), 89(3), 88(7), 85(13), 82(4), 80(6), 79(1), 78(1), 76(2), 75(3), 74(3), 73(1), 72(5), 70(4), 68(2), 63(2), 60(3), 59(5), 58(1), 57(2), 56(3), 55(4), 54(2), 53(1), 52(2), 51(2), 50(8), 48(7), 45(8), 42(1), 40(3), 35(1).

Control group (G₂) *:* 100(7), 99(2), 98(3), 97(9), 95(18), 92(11), 91(4), 90(6), 88(12), 85(36), 82(8), 80(19), 78(9), 75(6), 70(17), 64(12), 60(16), 58(19), 56(3), 55(6), 50(17), 45(9), 40(6).

The above data is summarized in Table 1:

Table 1. Characterization of the students' performance

Characterizations	G₁	G₂
A	60	60
B	40	90
C	20	45
D	30	45
F	20	15
Total	170	255

The evaluation of the above data was performed in two ways:

I) Traditional methods

a) Calculation of the means: A straightforward calculation gives that the means of the students' scores are approximately 76.006 and 75.09 for the experimental and the control group respectively. This shows that the *mean performances* of both groups were very good (on the boundary), with the performance of the experimental group being slightly better.

b) Calculation of the Great Point Average (GPA) index: We recall that the GPA index is a weighted mean, focusing on the *quality performance* of a student group (e.g. see [6]). Let us denote by n_A, n_B, n_C, n_D and n_E the numbers of students whose performance was characterized by A, B, C, D and F respectively and by n the total number of students of each group.. Then the GPA index is calculated by the formula GPA= $\dfrac{n_D + 2n_C + 3n_B + 4n_A}{n}$.

Obviously we have that

$0 \leq$ GPA ≤ 4 (in the worst case $n=n_F$, while in the ideal case $n=n_A$). In terms of the terminology introduced in the TRFAM (see paragraph 1 of section 3) the above formula can be written in the form:

$$\text{GPA} = y_2 + 2y_3 + 3y_4 + 4y_5 \qquad (5)$$

In our case, applying formula (5) on the data of Table 1 we find that the GPA of both student groups' is equal to

$\dfrac{43}{17} \approx 2.53$. This is a satisfactory value for the GPA index, since it is equal to the 63.25% of its maximal possible value, which is equal to 4. Thus, according to the GPA index, the two student groups demonstrated the same (good) quality performance.

II) Fuzzy logic methods

c) The COG method: Applying the first of formulas (3) on the data of Table 1 one finds that the coordinate x_c of the COG for both G₁ and G₂ is equal to $\dfrac{103}{34} \approx 3.03 > 2.5$. Thus, since the value 3.03 is approximately equal to the 67.3% of the corresponding value $\dfrac{9}{2}$ of the ideal case, both groups demonstrated a good quality performance. Further, from the second of formulas (3) it turns out that the coordinate y_c is equal to $\dfrac{69}{578}$ for G₁ and to $\dfrac{71}{578}$ for G₂. Therefore, according to our criterion for the COG technique stated in section 3, the control group demonstrated a slightly better performance.

d) Application of the TRFAM: Applying the data of Table 1 on the first of formulas (5) one finds that the coordinate X_c of the whole area S considered in Figure 2 is equal to $\dfrac{386}{17} \approx 22.71 > 15$ for both groups. The value 22.71 is approximately equal to the 66.67% of the corresponding value of the ideal case, which shows again that both groups demonstrated a good quality performance. Further, from the second of formulas (3) we find that that the coordinate y_c is approximately equal to 0.102 for G₁ and to 0.107 for G₂. Therefore, according to our criterion for the TRFAM stated in section 3, the control group demonstrated a slightly better performance.

III) Discussion of the experimental results

The application of the above - four in total - methods for assessing students' MM skills resulted to different conclusions. However, this is not embarrassing, since, in contrast to the calculation of the means which focuses on the mean performance of a student group, the GPA the COG and the TRFAM methods focus on its quality performance by assigning weight coefficients to the higher scores achieved by students.

The coefficients attached to the y_i's in the last three methods -see formula (5) and the first of formulas (1) and (3) respectively- are presented in Table 2. From this Table becomes evident that TRFAM assigns greater coefficients to the higher with respect to the lower scores than COG and also COG does the same thing with respect to GPA. In other words TRFAM is more sensitive than COG, and COG is more sensitive than GPA for measuring the quality performance of a student group.

Table 2. Weight coefficients of the y_i's

y_i	GPA	COG (x_c)	TRAFM (X_c)
y_1	0	1/2	7
y_2	1	3/2	14
y_3	2	5/2	21
y_4	3	7/2	28
y_5	4	9/2	35

Concerning the effect of the use of computers for enhancing students MM skills that we wanted to investigate with the above experiment, according to the results obtained it looks to improve the mean performance of the experimental with respect to the control group. On the contrary the GPA index shows that the quality performance of the two groups was the same, while the GOC and TRFAM methods show a slight superiority of the control group. In other words, the use of computers enhanced the performance of the moderate students (lower scores), but it had no effect on the performance of the good students (higher scores). An explanation about this could be that the figures' animation, the quick transformations of the numerical and algebraic representations, the easy and accurate construction of the several graphs especially in the 3-dimensional space, etc, which are comfortably achieved using the computers, increased the moderate students' imagination and helped them in using their intuition more effectively for designing the solutions of the corresponding problems. On the contrary, this had no effect on the good students, who had already developed high MM skills.

5. Final Conclusions and Discussion

In the present paper methods were developed and/or applied for assessing students' model building abilities with emphasis to our new TRFAM model. A classroom experiment, performed recently at the Graduate T. E. I. of Western Greece, was also presented illustrating our results in practice and connecting students' MM skills with the use of computers.

According to the results of the above experiment small differences appeared between the performances of the experimental and the control group giving a rather weak indication about the positive effect of computers for enhancing students' MM skills.. This, combined with the results of older similar researches mentioned in section 4, underlines the need of further experimental investigation for obtaining safer statistical conclusions.

Further, the TRAFM seems to have the potential of a general assessment method that could have many other applications in future on several sectors of human activity (not on Education only) and this is another priority of our future research plans on this topic.

References

[1] Asiala, M., et al., "A framework for research and curriculum development in undergraduate mathematics education", *Research in Collegiate Mathematics Education II, CBMS Issues in Mathematics Education*, 6, 1-32, 1996.

[2] Berry, J. and Davies, A., "Written reports, Mathematics Learning and Assessment: Sharing Innovative Practices", in Haines, C. R. & Dunthorn (Eds.), London, Arnold, 1996.

[3] Blomhψj, M. and Jensen, T. H., "Developing mathematical modelling competence: Conceptual clarification and educational planning", *Teaching Mathematics and its Applications*, 22, 123-129, 2003.

[4] Blum, W. & Leii, "How do students and teachers deal with modelling problems?'. In C. Chaines, P. Galbraith, W. Blum & S. Khan (Eds.): *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics*, Chichester, Horwood Publishing, 222-231, 2007.

[5] Edwards D. & Hamson M. J., *Mathematical Modelling Skills*, London, Macmillan, 1996.

[6] "Grade Point Average Assessment", available in the Web at: http://www.swinburne.edu.au/student-administration/assessment/gpa.html, visited October 15, 2014.

[7] Greefrath, G., *Modellieren lernen mitoffenen realitatsnahen Aufgaben*, Koln, Aulis Verlag, 2007.

[8] Haines, C. & Crouch, R., "Remarks on a Modelling Cycle and Interpretation of Behaviours'. In R.A. Lesh et al. (Eds.): *Modelling Students' Mathematical Modelling Competencies* (ICTMA 13), 145-154, Springer, US, 2010

[9] Klir, G. J. & Folger, T. A.., *Fuzzy Sets, Uncertainty and Information*, Prentice-Hall, London, 1988.

[10] Lewandowski, G. et al., "Common sense computing (episode 3): Concurrency and concert tickets", *Proceedings of the Third International Workshop on Computing Education Research (ICER)*, 2007.

[11] Pollack H. O., "The interaction between Mathematics and other school subjects", *New Trends in Mathematics Teaching*, Volume IV, Paris: UNESKO, 1979.

[12] Polya, G., "On learning, teaching and learning teaching", *American Mathematical Monthly,* 70, 605-619, 1963.

[13] Subbotin, I. Ya., Badkoobehi, H., Bilotckii, N. N., "Application of fuzzy logic to learning assessment". *Didactics of Mathematics: Problems and Investigations*, 22, 38-41, 2004.

[14] Subbotin, I. Ya., Voskoglou, M. Gr., "Fuzzy Models for Learning Assessment", *arXiv: 1410.4497 [math. OC]*, submitted on October 15, 2014.

[15] van Broekhoven, E. & De Baets, B., "Fast and accurate centre of gravity defuzzification of fuzzy system outputs defined on trapezoidal fuzzy partitions", *Fuzzy Sets and Systems*, 157 (7), 904-918.

[16] Voskoglou, M. Gr. (1994), An application of Markov Chain to the process of modelling, *International Journal of Mathematical Education in Science and Technology*, 25 (4), 475-480, 2006.

[17] Voskoglou, M. Gr.*,* "A stochastic model for the modelling process". In C. Chaines, P. Galbraith, W. Blum & S. Khan (Eds.): *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics*, Chichester, Horwood Publishing, 149-157, 2007.

[18] Voskoglou, M. Gr., *Stochastic and fuzzy models in Mathematics Education, Artificial Intelligence and Management*, Lambert Academic Publishing, Saarbrucken, Germany, 2011. For more details look at http://amzn.com./3846528218).

[19] Voskoglou, M. Gr. & Buckley, S., "Problem Solving and Computers in a Learning Environment", *Egyptian Computer Science Journal*, 36 (4), 28-46, 2012.

[20] Voskoglou, M. Gr. & Subbotin, I. Ya., "Dealing with the Fuzziness of Human Reasoning", *International Journal of Applications of Fuzzy Sets and Artificial Intelligence*, Vol. 3, 91-106, 2013.

[21] Voskoglou, M. Gr., "Assessing the Players' Performance in the Game of Bridge: A Fuzzy Logic Approach, *American Journal of Applied Mathematics and Statistics*, 2 (3), 115-120, 2014.

[22] Weller, K. et al., "Student performance and attitudes in courses based on APOS theory and the ACE teaching style". In A. Selden et al. (Eds.), *Research in collegiate mathematics education V*, 97-13, Providence, RI, American Mathematical Society, 2003.

[23] Wikipedia, "Trapezoid: Other properties", available in the Web at: http://en.wikipedia.org/wiki/trapezoid#other_properties, visited on October 10, 2014.

[24] Wikipedia, "Center of mass: A system of particles", available in the Web at: http://en.wikipedia.org/wiki/Center_of_mass#A_system_of_partic les, visited on October 10, 2014.

[25] Yadav, A. et al., "Introducing Computational Thinking in Education Courses", *Proceedings of the 42nd ACM technical symposium on Computer Science Education 11*, 465-470, March 9-12, 2011.

APPENDIX: List of MM problems given for solution in our experiment

Problem 1: Among all the cylindrical towers having a total surface of 180π m^2, which one has the maximal volume?

Problem 2: Let us correspond to each letter the number showing its order into the alphabet (A=1, B=2, C=3 etc). Let us correspond also to each word consisting of 4 letters a 2X2 matrix in the obvious way; e.g. the matrix $\begin{bmatrix} 19 & 15 \\ 13 & 5 \end{bmatrix}$ corresponds to the word SOME. Using the matrix E=$\begin{bmatrix} 8 & 5 \\ 11 & 7 \end{bmatrix}$ as an encoding matrix how you could send the message LATE in the form of a camouflaged matrix to a receiver knowing the above process and how he (she) could decode your message?

Problem 3: The population of a country is increased proportionally. If the population is doubled in 50 years, in how many years it will be tripled?