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The Role of Induction for Human Reasoning and Science

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Abstract The present paper investigates the valuable role of induction for human reasoning and in particular for the scientific method of thinking, which is analyzed in detail. It is made clear that the possible error of induction is transferred to the inference of deduction, which, therefore, is not an infallible method, although it is always a valid method. Aristotle's bivalent logic disputes the effectiveness of inductive reasoning, which, however, finds its real value within a multi-valued logic, completing and extending the traditional logic. The unreasonable effectiveness of mathematics for the natural sciences, known as the Winger's enigma, is also studied and the paper closes with some remarks on the concept of randomness.

Keywords: *human reasoning, induction, deduction, absolute words, scientific method, bivalent logic, multi-valued logics, Winger's enigma, randomness, free will*

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1. Introduction

Aristotle (384-322 B.C), the founder of the *bivalent logic (BL)*, gave the following paradigm for deductive reasoning [1]:

- All humans are mortal.
- Socrates is a human

- Therefore, Socrates is mortal.

The motive for starting to write this paper was our will to give a convincing answer to the following question: Why is the previous Aristotle's paradigm used, even nowadays, as the main example of deductive reasoning in almost all books of Logic around the world?

This investigation, however, marked out several significant issues, which have been included in this work and which are:

- The valuable role of induction for human reasoning and in particular for the scientific method of thinking.
- The importance of the correct use of the natural language in the science of Logic and in particular of a small group of words, which are called here "absolute words".
- The necessity of completing and extending the traditional logic by introducing a multi-valued logic, in which the neglected by the BL induction finds its real position.
- The unreasonable effectiveness of mathematics for the natural sciences.
- Some remarks for the concept of randomness.

The rest of the paper is organized as follows: Section 2 presents a rough description of the anatomy of the human brain, which is the center of logical reasoning, and proceeds to the definition of the science of Logic. Section 3 recalls basic issues for the inductive and deductive reasoning. Section 4 refers to the use of the "absolute words" and the necessity of the introduction of a multi-valued logic for completing and extending the traditional BL. Section 5 analyzes the usefulness of induction for the scientific method of thinking, while Section 6 deals with the unreasonable effectiveness of mathematics for the natural sciences. Some remarks for the concept of randomness are included in Section 7 and the paper closes with the final conclusions and some hints for future research contained in its last Section 8.

2. Human Reasoning and Logic

The recent findings of the magnetic tomography verified that the human brain consists of three parts (Figure 1).

The first part (black colored in Figure 1) is the center of the *instincts* and the *automated functions* of the human body and corresponds to the brain of the first reptiles that moved from the sea to the earth.

The second part (grey colored in Figure 1), referred to as the *cerebral cortex*, is the center of the *emotions* and corresponds to a second cortex created in the brain of the upper class of mammals.

The third part (white colored in Figure 1) is the *neo-cortex*, which is the center of *human logical reasoning*.

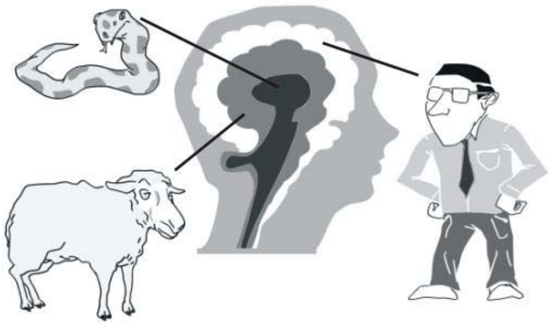

Figure 1. A simplified representation of the anatomy of the human brain

This is, however, only a simplified representation of the anatomy of the human brain [2]. In reality, its three parts are not strictly separated from each other, which explains why some elements of simple logical reasoning can be traced in certain kinds of animals, like monkeys, dogs, etc.

Logic is the science that deals with the rules and principles of the correct reasoning [3]. It studies arguments which consist of a set of premises together with an inference and plays a central role in multiple fields, like mathematics, computer science, linguistics, philosophy, etc.

3. Inductive and Deductive Reasoning

Induction and *deduction* are the main types of human reasoning. The former is the process of going from the specific to general, whereas the latter is the opposite process of going from the general to the specific.

The typical form of a deductive argument is the following:

- ALL the elements of a set S have the property P.
- X is an element of S.

- Therefore, X has the property P.

Thus, a deductive argument, starting from a first premise and based on a second one, reaches a logical inference.

On the contrary, an inductive argument starts from a series of observations and reaches an inference by making an imperfect generalization of these observations. The typical form of an inductive argument is the following:

- The elements x_1, x_2, ….., x_n,…. of a set S have the property P.

- Therefore, ALL the elements of the set S have the property P.

If both the premises of a deductive argument are true, then its inference is also true. This is expressed by saying that a deductive argument is always *valid*. On the contrary, an inductive argument is never valid, even if its inference is true. This happens because the fact that a number of elements of a set S have a certain property does not guarantee that all the elements of S have this property.

It is of worth noting that, if at least one of the premises of a deductive argument is false, then its inference may also be false. For example, “All animals have four feet”, “A chicken is an animal”, therefore “A chicken has four feet”. This argument, however, remains valid as a logical procedure, in the sense that “If all animals had four feet, then a chicken would have four feet”. Another characteristic example for this case is related to the function of computers, where the popular in the early days of computing credo “Rubbish in, rubbish out” remains still timely.

The main concern of the BL is the validity and not the truth of an argument. In this sense, a deductive argument is always compatible with the principles of the BL, regardless if its inference is true or false, whereas a deductive argument is not. In other words, BL focuses on the value of deduction and underestimates the value of induction.

People, however, in their everyday life activities, always want to know the truth in order to organize better, or even to protect their lives. Consequently, in such cases the truth of an argument is much more important than its validity.

For psychological reasons, therefore, a strong paradigm for deductive reasoning must be not only valid, but also true. But, as we said before, this happens only if there is no doubt that its premises are true, as it happens in the case of the proposition “All humans are mortal”, included in Aristotle’s paradigm, which was presented in our Introduction.

Unfortunately, however, only a few propositions are known that could remain true forever, the most of them being in the area of Mathematics, which is considered to be the most “solid” science (e.g. the arithmetic of natural numbers, the axioms of the Euclidian Geometry on the level, etc.). This is due to the fact that, despites the enormous progress of science and technology during the last years, human knowledge remains limited (see also Section 5), and explains why Aristotle’s paradigm continues, even nowadays, to be used as the main paradigm of deductive reasoning in almost all books of BL around the world.

Even the truth of the first premise of the Aristotle’s paradigm, however, which is generally acceptable nowadays, can be disputed! In fact, nobody can exclude the possibility that humans, as a result of the continuous scientific progress, could find, in the remote future, the way to become immortal.

This could be parallelized with the widely known story concerning the color of the swans. Namely, before the discovery of Australia it was believed that all swans are white, until a species of swans (cygnus atratus) was discovered in the “new continent” in 1697, which are black [4].

4. “Absolute Words” and Multi-Valued Logics

Observe now that the word “all’, written, for emphasis, with capital letters, appears in the first premise of the typical form of a deductive argument, but also in the inference of an inductive one. This happens, because it is necessary to express in our natural language the generalizations involved in both arguments.

The word “all” belongs to a small family of words of our natural language, which we call here *absolute words*, because of their absolute character. These words can be grouped in pairs as follows:

- Always – Never (for events connected to time)
- Everywhere - Nowhere (for space)
- All – None (for living or lifeless objects)
- Correct – False (for propositions), etc.

The absolute words cover space, time, materials, as well as spiritual concepts and ideas. No logical argument can exist which does not contain at least one absolute word. In everyday life, however, these words must be used very carefully, because they could lead to inaccuracies or misunderstandings (like the color of the swans, for example).

In a more general context, one could argue that Logic is strictly connected to the correct use of our natural language, since the correct use of the language is necessary for the correct statement of the logical arguments [5]. It is not a random fact that the word Logic comes from the Greek word ΛΟΓΟΣ, which has a double meaning. It means speech, but it also means reason. In fact, in the case of a logical argument its premises are the reason and its inference is the result.

Let us now restate the typical form of an inductive argument as follows:

- The elements x_1, x_2, ….., x_n,…. of a set S have the property P.

- Therefore, it is POSSIBLE that all the elements of the set S have the property P.

In this form the inductive argument becomes a valid logical argument! The problem, however, is that this form is not compatible with BL, because its inference contradicts with its fundamental principle of the “*excluded middle*” (every proposition is either true or false). The necessity, therefore, to introduce a *multi-valued logic* for completing and extending the traditional BL becomes evident.

Despites the fact that human reasoning, and therefore the progress of science and the growth of human civilization, were based for more than 2300 years on the principles of Aristotle’s BL, opposite views also appeared early in human history, starting from the 6th century BC, by Heraclitus, Buddha, Plato, by the Marxist philosophers much later, etc., supporting the idea of the existence of intermediate truth values between correct and false. Integrated propositions, however, for introducing multi-valued logics appeared only in the beginning of the 20th century, mainly by Lukasiewicz and Tarski [6].

Zadeh**,** based on the concept of *fuzzy set (FS)* [7], introduced in 1973 the infinite-valued *fuzzy logic (FL)* [8] on the purpose of tackling mathematically the existing in real life partial truths. FL, in which the truth values (*membership degrees*) are expressed by numbers of the unit interval [0,1], satisfies Lukasiewicz’s *“principle of valence”*, according to which propositions may have intermediate values between true and false.

A little later, when membership degrees were reinterpreted as *possibility* distributions, FSs and FL were also used for tackling the existing in real world *uncertainty* [9,10]. Uncertainty is understood to be the shortage of knowledge regarding some situation. The reduction of uncertainty due to a new evidence (e.g. receipt of an additional message) indicates a gain of an equal amount of information. The main types of the existing in real life uncertainty include *vagueness, imprecision, ambiguity* and *inconsistency* [11].

Zadeh [9] clarified the relationship between *probability* and possibility by stating that whatever is probable must be primarily possible. Note that probability theory has been also proposed by Jaynes [12] as a means for introducing a multi-valued logic (*probabilistic logic*). Probability, however, was proved to be sufficient only for tackling the uncertainty which is due to *randomness* and not the other forms of the existing in real life uncertainty [13]. Probabilistic logic, therefore, is subordinate to FL.

Several other theories have been introduced during the last years for completing and extending Zadeh’s seminal theories of FSs and FL, on the purpose of tackling more efficiently all the forms of the existing in real world uncertainty. The main among those theories are briefly reviewed in [14]. Although none of these theories is sufficient to tackle alone all kinds of uncertainty, all together form an efficient framework for this purpose.

In conclusion, although an inductive argument is not valid according to the standards of the BL, it becomes valid according to the standards of FL, or of any other multi-valued logic.

5. The Importance of Induction for the Scientific Method of Thinking

As we have seen in Section 3, the sufficient condition for the truth of a deductive argument is that both its premises are true. The question is, therefore, how one could be helped to set out a deductive argument with true premises? The answer is that this can be succeeded starting from a true inductive argument.

In fact, if a great number of elements of a given set S have a certain property, it is possible that all the elements of S have this property. Using, therefore, the inference of such an inductive argument as the first premise of a deductive argument, one produces a valid logical conclusion, which is also very possible to be true. It becomes evident, however, that in this case the possible error of induction is transferred to the inference of the corresponding deductive argument.

The “collaboration” of induction and deduction for obtaining true and simultaneously valid logical inferences is illustrated below by using again Aristotle’s paradigm of Section 1, as follows:

- *Induction*: In the whole human history people in Europe, Asia, America, Africa and Australia used to be and remain mortal. Therefore, all humans are mortal
- *Deduction*: All humans are mortal, Socrates is a human and therefore Socrates is mortal.

This way of reasoning is actually the basis of the *scientific method (SM)* of thinking. In fact, the SM of thinking is nothing else but a continuous "Ping-Pong" between induction and deduction or, mathematically speaking, an algorithm the successive steps of which consist of deductive arguments using as premises possible truths obtained intuitively by induction.

The term SM was introduced during the 19th century, when significant terminologies appeared establishing clear boundaries between science and non-science. Aristotle, however, is considered as the founder of the SM due to his refined analysis of the principles of the BL. According to the existing historical evidence, the first book written on the basis of the principles of the SM is the "Elements" of Euclid (365-300 BC), addressing the axiomatic foundation of Geometry.

The process which is followed in SM is graphically represented in Figure 2 [15].

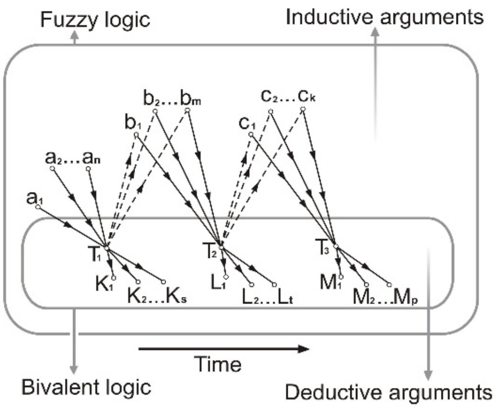

Figure 2. A graphical representation of the scientific method

In Figure 2, α_1, α_2, α_3, α_4,…, α_n represent observations of the real world that led by induction to the empirical formulation of the theory T_1, which was verified afterwards deductively. Then, additional deductive inferences K_1, K_2,…, K_n were obtained as consequences of the theory T_1. A new series of observations b_1, b_2, b_3,..., b_m follows and, if some of them are not compatible to the laws of theory T_1, a new theory T_2 is formed by induction to replace or extend T_1. The deductive verification of T_2 is based on premises partially or even completely different to those of the theory T_1 and new deductive inferences L_1, L_2,…, L_t follow as consequences of theory T_2. The same process could be repeated (observations c_1, c_2, ..., c_k, theory T_3, deductive inferences M_1, M_2,…M_p, etc.) one or more times. In each case the new theory extends or rejects completely the previous one approaching more and more the absolute truth. According to Ladyman, however, the relationship between observations and theory is much more composite and complicated than it looks at first glance [16].

For example, the *geocentric theory* (Almagest) of Ptolemy of Alexandria (100-170), although it was able to predict satisfactorily the movements of the planets and the moon, was finally proved to be wrong and was replaced by the *heliocentric theory* of Copernicus (1473-1543) [17]. Also Newton's *theory of gravitation* was extended to cover the existing in the Universe (outside the earth) strong gravitational forces, by Einstein's *general theory of relativity*, otherwise known as the new theory of gravitation. The soundness of Einstein's theory was verified experimentally first by the irregularity of Hermes' orbit around the sun and definitely by the magnitude of the divergence of the light's journey, calculated during the eclipse of the sun on 05/29/1919 [18].

It becomes evident that the scientific method is highly based on the process of *trial and error*, characterized by repeated attempts, continued until the final success or until the subject stops trying [19]. Popper, one of the 20th century's most influential philosophers of science, proposed the *principle of falsification*, according to which a proposition can be characterized as scientific only if it includes criteria for its control, i.e. if it could be falsified [20]. Popper's principle, however, which gives emphasis to the second component of the process of trial and error, received critiques reporting that it decreases the importance of induction for the SM [16].

In conclusion, the analysis performed in this Section reveals the necessity of induction for the SM, but it also makes clear that the possibly existing inductive error is transferred to the final deductive conclusions of the SM. This disputes the view of many philosophers that deduction is an infallible method, a view which is wrongly based on the doubtless fact that it is always a consistent method. It also brings more light to the fact that there are not many propositions known that could remain true for ever (see Section 3). Thus, adapting a view of the Scottish philosopher David Hume (1711-1776) to the nowadays existing data, one may claim that without inductive reasoning no further scientific progress could be achieved.

6. The Winger's "Enigma" for Mathematics

Mathematics is unique among the sciences which does not always follow the general lines of the SM. In fact, mathematical theories are developed in two different ways, the *energetic* and the *passive* one.

In the former case, which follows the lines of the SM, a mathematical theory is developed to fit to the existing empirical observations. In the latter case, however, although the axioms introduced for the study of a mathematical topic are sometimes based on intuitional criteria or beliefs, the method which is followed is purely deductive. The results obtained are considered to be correct, if the mathematical manipulation is correct and regardless whether or not they have any physical or practical meaning. The amazing thing, however, is that these completely abstract mathematical theories, without any visible applications at the time of their creation, are utilized in unsuspicious time for the construction of physical models or for other practical applications! For this reason, the famous astrophysicist and bestselling author M. Livio wondered: "Is God a mathematician?" [21].

A characteristic example is the non- Euclidean Geometries of Lobachevsky (1792-1856) and Riemann (1826-1866), which were developed on a purely theoretical basis by replacing the fifth Euclid's axiom with the statement that at least two (none respectively) parallel lines can be drawn through a given point lying outside a given straight line. Approximately fifty years later, in 1915, Einstein used these Geometries to prove theoretically his general relativity theory (see also Section 5).

The non-Euclidean form of the 4-dimensional Einstein's time-space is physically explained by its distortion created by the presence of mass or of an equivalent amount of energy, which appears analogous to the distortion created by a ball of bowling on the level of a trampoline. The theoretical foundation of the general relativity theory made Einstein to state with excitement: "How is it possible for mathematics to fit so eminently to the natural reality?"

The Nobel prize winner E. P. Winger, in his famous Richard Courant lecture at the New York University on 05-11-1959, characterized the success of mathematics for describing the natural laws and the architecture of the Universe as the "unreasonable effectiveness of mathematics in the natural sciences" [22]. Since then, this phenomenon is referred to as the *Winger's enigma*.

It is worth adding here a personal experience of the first author of the present work, related to the Winger's enigma. In fact, his earlier research interests were focused on skew polynomial rings [23,24], a purely theoretical topic of the abstract algebra, which, however, has found recently important applications to the theory of Quantum Groups [25] and to Cryptography [26].

The Winger's enigma is one of the main arguments of the philosophical school of *mathematical realism*, supporting Plato's (428-348 BC) idea for the existence of an eternal and unchanged "universe" of mathematical forms, hence arguing that mathematics is discovered and not invented by humans.

7. Some Remarks about Randomness

As we have seen in Section 4, in a logical argument there exists always a reason expressed by its premises and a result expressed by its inference. There are, however, events in real life the evolution of which is not predictable. Such kind of events are characterized as random events, the best example being the games of chance.

Randomness, therefore, can be defined as the lack of pattern or predictability in information. But why does it happen? We believe that this is due to the human weakness to determine in each case the reason of the evolution of a random event. To put it in other words, we believe that each event involves always a reason, which, however, for certain events cannot be determined by humans. In this sense, randomness is a word used in our language to "cover" our weakness to determine the reasons of the evolution of certain events.

In terms of the scientific method of thinking, which was analyzed in Section 5, this means that humans have not invent yet theories for explaining the reasons of the evolution of the existing random events. The only "tool" in hands of the experts towards this direction is the theory of probability. In fact, if the *probability distribution* of the evolution of a random event is known, the frequency over repeated trials is predictable, although the sequence of the different outcomes remains unknown.

8. Conclusions and Hints for Further Research

The discussion performed in this work leads to the following conclusions:

- The role of induction is valuable for human reasoning in general and for the scientific way of thinking in particular. Inductive reasoning, however, finds its real position only within a multi-valued logic, since it is not a valid logical argument within the narrow frames of the traditional BL.
- The careful use of the natural language and especially of a small family of words, called here "absolute words", plays an important role for the correct formulation of the logical arguments.
- Mathematics is the unique science which does not always follow the typical process of the scientific method of thinking. The amazing thing, however, is that frequently abstract mathematical theories find, in unsuspicious time, important practical applications (Winder's enigma).
- It is argued here that all events of the real world have a reason and a result. Thus, randomness is interpreted as the humans' weakness to determine the reasons of the evolution of certain events. Probability theory, however, makes predictable over repeated trials the frequency of the unknown (random) sequence of the different outcomes.

The previous discussion leaves also a number of questions, manly of philosophical character, open for further study and investigation. Such a question is, for example, if mathematics was discovered or invented by humans. Also, adopting the expressed here view that all the events of the real world involve a known or not known yet to humans reason, and combining it with the hypothesis that humans have the ability to affect the evolution of all the events with known reason, one may raise the question if people have free will or not, etc. Such questions give hints for future research related to the present subject.

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