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Kinematic Model and Control Algorithm for the Path Tracking of Nonholonomic Mobile Robots

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Abstract Provided in this article is a general overview of nonholonomic mobile robots' modelling. Emphasis is given to the structural characteristics of kinematic models, taking into account the mobility restrictions caused by various links. Another problem of nonholonomic mobile robots is tracking of the prescribed path. The classic "tracking controllers" are not appropriate for this type of tasks, because they do not guarantee that the robot remains on the prescribed path. The aim of this paper is to propose and to verify, by means of computer simulation, the method of control, which ensures that the "output" of the robot will move along the prescribed path.

Keywords: *mobile robot, path tracking, kinematic model*

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1. Introduction

Research, development and education in the mobile robotics require verification of properties of different systems. [1]. Position of the kinematic model allows describing the behaviour of wheeled mobile robots. At an early design analysis is necessary a complete the knowledge of all input parameters and demands of function along with environment in which the system shall work. Wheeled undercarriage is one of most often chosen forms of locomotion in the present [2].

Position of the kinematic model allows us to describe the behaviour of wheeled mobile robots. It also provides a simple state model, which gives a global description of wheeled mobile robots [3,4].

Chassis of a mobile service robot is essentially a system of bodies interconnected by links that can move against each other with a certain degree of freedom. For a description of these relative motions it is appropriate to establish a set of linked coordinate systems for individual elements of the chassis, of which one is the main one (frame).Coordinate system of a mobile service robot is firmly connected most often with the centre of gravity of chassis of the robot and we use it to apply the position and the orientation of the robot in its space relative to the general coordinate system [5].

Mobile robots can be remotely operated or autonomous; the former are used to access hazardous or inaccessible places, hostile to people, allowing a human operator to determine and control the robot's movements. Autonomous robots, not having the knowledge provided by an operator, must be able to selflocate and maneuver in an automatic way, if they are intended to perform their tasks effectively in accordance with previously defined control criteria. Therefore, they must have wide flexibility and adaptability to operate when faced with many situations and stochastic environments not previously considered in the design. [7]

Traditionally, the tasks of the robots are specified by time parametrization of the path in the working space, which the robot tracks at each point of time. However, in many applications, the time parameter of the path is not important compared to the coordination and requirements for the synchronization between the various degrees of freedom.

The path-tracking component of the vehicle controller has the mission of generating the vehicle's steering to track a previously defined path, by taking into account the vehicle's actual position and orientation and the constraint imposed by the the vehicle and its lower-level motion controller. [8]

2. Wheel Kinematic

During the motion of a mobile robot the plane of each wheel remains vertical and the wheel rotates about its (horizontal) axis, whose orientation relative to the chassis can be fixed or variable [1].

Differentially controlled robot has two wheels of radius r and each with a separate drive. If r, l and ψ are defined and the velocity of wheels rotation, is also defined, then the kinematic model presumes overall velocity of the robot in the global reference system as the following:

$$\dot{\xi} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix} = f\left(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2\right) \tag{1}$$

Motion in direction of axis y is simpler, because neither of the wheels has a velocity component in this direction of motion. Therefore with regard to the reference system $\dot{y}_r = 0$.

Furthermore it is possible to state:

$$\omega_1 = \frac{r\dot{\varphi}_1}{2l} \qquad \omega_2 = -\frac{r\dot{\varphi}_2}{2l} \tag{2}$$

By combining these parts of kinematic model of a differentially controlled robot it is possible to state:

$$\dot{\xi} = R(\theta)^{-1} = \begin{bmatrix} \frac{r\dot{\varphi}_1}{2l} + \frac{r\dot{\varphi}_2}{2l} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} - \frac{r\dot{\varphi}_2}{2l} \end{bmatrix} \tag{3}$$

Two groups of ideal wheels exist: controlled wheels and mecanum wheels [10].

For controlled /standard/ wheels the point contact between the wheel and the surface is assumed if two conditions are met, namely clear rolling and non-slippage of wheels in motion. That means that the velocity in the point of contact of the wheel and the surface is equal to zero. Both components of velocity, i.e. parallel to plane of the wheel and perpendicular to this plane are equal to zero. The basic motion of a wheel is rotation about its axis (center of the gravity of a wheel), by which upon the contact with the surface (while meeting the condition of rolling) translational motion of the wheel on the surface and at the same time of all the bodies firmly connected to it (chassis) happens. For its relative simplicity and wide scope of use it is currently the most commonly used functional principle of a wheel.

2.1. Fixed Standard Wheel

A fixed wheel has one degree of freedom (1° of freedom – rotation about the axis parallel to the surface and perpendicular to the direction of translation, the orientation of the coordinate system towards the coordinate system of the robot is fixed [5]. The middle of a fixed wheel, denoted as A, is a fixed point of the chassis, as is illustrated on Figure 1. Position of point A during the motion of the chassis is characterized by polar coordinates, i.e. the distance l from point A to point P and the angle α. [3].

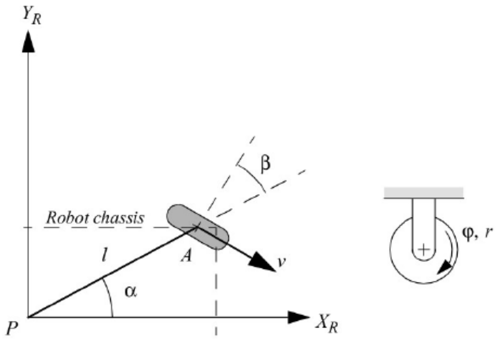

Figure 1. Fixed standard wheel

Fixed standard wheel does not allow control in the vertical axis. This renders the angle of chassis β fixed and chassis motion is restricted to motion forward and backward along the plane of the wheel. Motion in the direction of the wheel is equal to the size of the wheel roll assuming clear rolling and point contact of the wheel. Wheel position is characterized by four parameters: α, β, l, r and its motion is characterized by the angle changing in time $\varphi(\mathrm{t})$. Using this description the components of velocity in a point can be easily calculated and it is possible to derive two following constraints:

$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l.\cos\beta\end{bmatrix} R(\theta)\dot{\xi} - r\dot{\varphi} = 0 \tag{4}$$

$$\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & l.\sin\beta\end{bmatrix} R(\psi)\dot{\xi} = 0 \tag{5}$$

2.2. Steered Standard Wheel

Steered standard wheel has two degrees of freedom (2° of freedom - rotation about an axis parallel to the surface and rotation about an axis perpendicular to the surface). Rotation about a perpendicular axis is often limited to a certain range, so called control angle β.

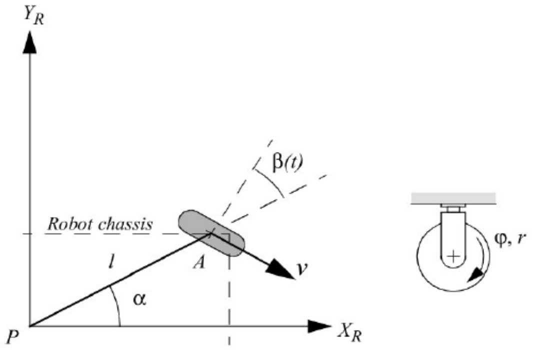

Figure 2. Steered standard wheel

Description of the position is the same as with the fixed wheel, however now the angle β is not constant but time-variable. Position of the wheel is described by three variables: α, l, r and its movement relative to the chassis by two angles, which are a function of time: $\beta(\mathrm{t})$ and $\varphi(\mathrm{t})$.

$$\begin{bmatrix}\sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l.\cos\beta\end{bmatrix} R(\theta)\dot{\xi} - r\dot{\varphi} = 0 \tag{6}$$

$$\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta) & l.\sin\beta\end{bmatrix} R(\theta)\dot{\xi} = 0 \tag{7}$$

2.3. Castrol wheel

Castrol wheel, same as controlled wheel, has two degrees of freedom, while the rotation about an axis perpendicular to the pad is misaligned by some distance d. It is a passive, uncontrolled and unpowered wheel. This misalignment during wheel trailing causes formation of additional moment, which facilitates its orientation relative to the chassis during motion of the mobile service robot. Vertical axis does not pass through the center of gravity of the wheel, as is shown on Figure 3. For a description of the wheel more parameters are needed.

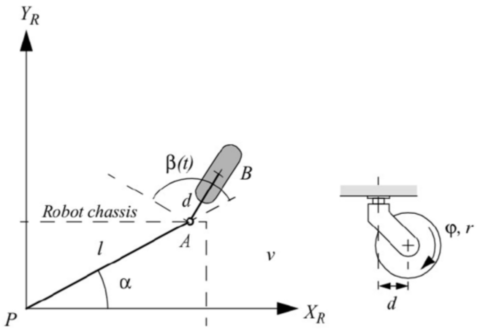

Figure 3. Castrol wheel

Wheel position is described by four variables: α , l, r, d, wheel motion by two time-variable angles $\beta(\mathrm{t})$ and $\varphi(\mathrm{t})$.

$$\left[\sin(\alpha+\beta)\ \ -\cos(\alpha+\beta)\ \ -l.\cos\beta\right]R(\theta)\dot{\xi}-r\dot{\varphi}=0 \quad (8)$$

$$\left[\cos(\alpha+\beta)\ \ \sin(\alpha+\beta)\ \ d+l.\sin\beta\right]R(\theta)\dot{\xi}+d\dot{\beta}=0 \quad (9)$$

3. Motion Control of Nonholonomic Mobile Robots in Low Velocity

Automatic control of nonholonomic robots has attracted the attention of scientific and research community since the early 80-ies of the last century. At the beginning of this activity of researchers Brockett [4] already defined the necessary conditions for such systems, in which he concluded that the said systems are in the case of open control system controllable, but cannot be stabilized using smooth time-invariant feedback. There are particular restrictions for chassis motion which result from chassis parameters (wheels diameter, wheel base and geometry of wheels layout). Chassis motion should be stable and fluent to avoid slipping between wheels and terrain and to avoid mechanical shock resulting from rapid changes in chassis motion [6]. To stabilize such systems a number of time-variant feedback control algorithms have been developed.

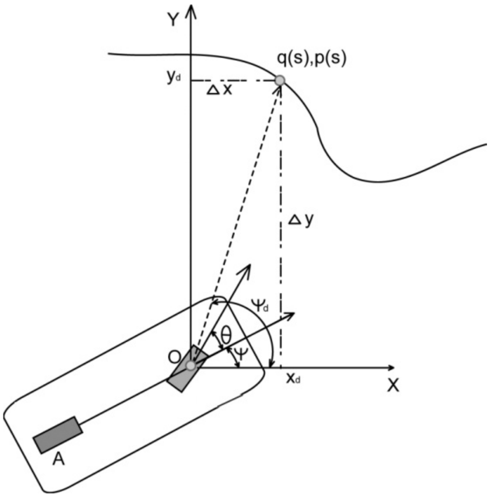

Figure 4. Distance and deviation angle from the target point

Kinematics of nonholonomic mobile robots with controlled front wheels is described by following equations:

$$\begin{aligned}\dot{x}&=v.\cos(\psi)\\ \dot{y}&=v.\sin(\psi)\\ \dot{\psi}&=\omega=\frac{v}{l}\sin(\theta)\end{aligned} \quad (10)$$

where ψ is the orientation angle of the robot relative to the x axis (Figure 4), and v is the forward speed of the robot, l is the distance between the axes of rotation of the front and back wheels.

If inertia of the wheels is not negligible, the dynamics of mobile robot can be described by following equation:

$$M(q,\psi,\theta)\dot{q}+b(q,\psi,\theta)=u(t)+z(t) \quad (11)$$

Coordinates X and Y can by expressed using coordinates x , y and ψ .

The problem is to find time parameterized curve $\mathrm{s}(\mathrm{t})$ and control angle $\theta(\mathrm{t})$, which ensure fulfillment of inequality (6) [7,8,9].

The control algorithm is based on the feedback deviation control of the position and orientation of the robot. One advantage of the algorithm is the substantiality considering the measurement errors and disturbance variables. If these errors are within certain limits, the mobile robot moves along the required path [9,11].

The main task of the control by means of the virtual vehicle is to determine the actuating variables for the control of lateral and longitudinal movement of the robot, which aims at following the smooth path, where the required point on the path is called a virtual vehicle [12,13].

The required trajectory is described parametrically as:

$$\begin{aligned}x_d&=p(s)\\ y_d&=q(s)\end{aligned} \quad (12)$$

where $\mathrm{s}(\mathrm{t})$ is a parameter relating to the trajectory.

The aim of control is to maintain lateral and longitudinal control deviation of the robot with respect to the required point on the path within certain limits:

$$\lim_{t\to\infty}\left(\left|\psi(t)-\psi_d(t)\right|\right)\le d_\psi \quad (13)$$

where $d\psi$ is the maximum angular deviation of the robot with respect to the x axis.

Euclidean distance between the robot and the virtual vehicle, which may be expressed as:

$$\rho(t)=\sqrt{\Delta x^2+\Delta y^2} \quad (14)$$

To control, what was used is the deviation of the current angular orientation of the robot from the required angular orientation, i.e. the control algorithm can be formulated by the following equations:

$$\theta=A\left[\left(\psi_d-\psi\right)\right] \quad (15)$$

4. The Results of Computer Simulations

Simulations were carried out for the purpose of verifying the asymptotic convergence of the path difference between the robot and the virtual vehicle ρ so that the convergence condition is met:

$$\lim_{t\to\infty} \rho(t) \le d_{\rho} \tag{16}$$

The convergence was also monitored for the selected cases of parametric uncertainty of the kinematic model of the robot.

Simulation objective: obtention of information on how the difference between the robot and the virtual vehicle, in case of four-wheeled robot, converges to a predefined value dρ.

Parameter setting:

- Radius of the predefined semicircle curvature:R=5m
- Initial configuration of the virtual vehicle: R=5m, α =-60°
- Distance between the axes of the front and back wheel: l=0,3m

Graphical outputs obtained by simulation are illustrated in Figure 5 and Figure 6.

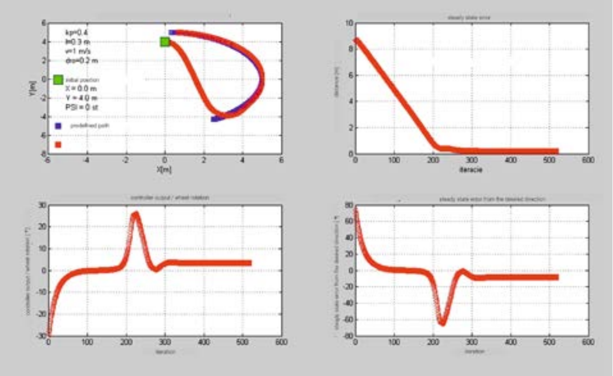

Figure 5. Configuration 0_4_0

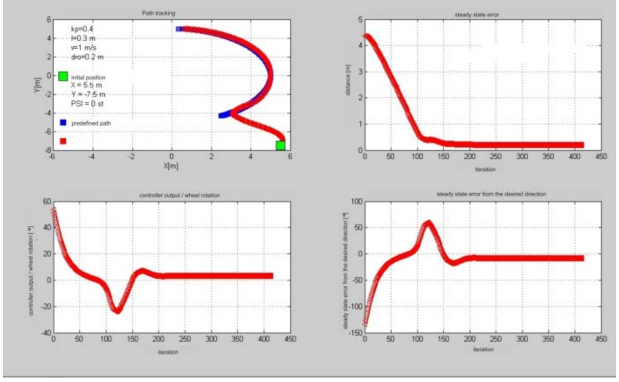

Figure 6. Configuration 5_-7_0

5. Conclusion

Application of methods and techniques for modeling and simulation for the design of robotic systems has its professional and technical requirements, but also professional and social topicality. [5,11] Computer-aided modelling of kinematics and dynamics of robots is an important integrated part within the frame of a modern approach to the design of technical subjects (machines, apparatuses, motor-cars, planes, etc.). [12,13] The article deals with motion control of nonholonomic robot. Furthermore a kinematic model of a mobile robot and kinematic models of wheels are presented. The obtained results confirm the theoretical assumptions that the distance between the robot and the virtual vehicle converges to the predefined value

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