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# Comparison of Different Approaches of Mathematical Modelling of Ackerman Steered Car-like System

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**Abstract** The article deals with the issue of mathematical modelling of nonholonomic system. The introductory part of article contains theory about different approaches of mathematical modelling that we used. Further we explained the basic principle of Ackerman steered car-like system. Then it contains the determination of mathematical model for Ackerman steered car-like system where we consider ideal source of velocity. In the last part we compared and showed individual advantages and disadvantages of approaches of mathematical modelling.

***Keywords:*** *Lagrangian mechanics, geometric mechanics, Ackerman, mathematical model, vector fields*

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## 1. Introduction

Mechanical systems have traditionally provided a fertile area of study for researchers interested in nonlinear control, due to the inherent nonlinearities and the Lagrangian structure of these systems. Recently, a great deal of emphasis has been placed on studying systems with nonholonomic (non-integrable) constraints, including mobile wheeled robots and multiple-trailer vehicles, where the wheels provide a no-slip velocity constraint. For the purposes of controls, however, these systems are very often treated as kinematic systems, i.e. the dynamics of these mechanical systems are assumed to be inverted out. Very often this assumption is quite valid, but frequently it is not [1]. There are also a growing number of systems in which this type of assumption is not even approximately valid. We focus in this paper on two method of mathematical modelling: *Lagrangian mechanics* and *geometric mechanics*. Making use of modern advances in geometric mechanics, researchers have made great progress in analyzing the mechanics of locomotion. This problem asks the fundamental question of how does system use its control inputs to effect motion from one place to another. By utilizing the inherent mathematical structure found in these types of problems, one can formulate the dynamics of a wide variety of locomotion problems in a very intuitively appealing and insightful manner. Doing so leads to a stronger comprehension of the mechanics of locomotion, which in turn provides a foundation for studying issues of control for such systems. It is clear [2,3,4,5] that the geometric tools used to formulate the mechanics can also provide a basis for unlocking the answers to many of the questions regarding the control theoretic issues involved. A general introductory review to some of the ideas of geometric mechanics in motion control is given in [6].

## 2. Lagrangian Mechanics

Lagrangian mechanics is a reformulation of classical mechanics, introduced by the Italian-French mathematician and astronomer Joseph-Louis Lagrange in 1788.

In Lagrangian mechanics, the trajectory of a system of particles is derived by solving the Lagrange equations in one of two forms, either the Lagrange equations of the first kind, which treat constraints explicitly as extra equations, often using Lagrange multipliers, or the Lagrange equations of the second kind, which incorporate the constraints directly by judicious choice of generalized coordinates. In each case, a mathematical function called the Lagrangian is a function of the generalized coordinates, their time derivatives, and time, and contains the information about the dynamics of the system.

No new physics is introduced in Lagrangian mechanics compared to Newtonian mechanics. Newton's laws can include non-conservative forces like friction, however, they must include constraint forces explicitly and are best suited to Cartesian coordinates. Lagrangian mechanics is ideal for systems with conservative forces and for bypassing constraint forces in any coordinate system. Dissipative and driven forces can be accounted for by splitting the external forces into a sum of potential and non-potential forces, leading to a set of modified Euler-Lagrange equations. Generalized coordinates can be chosen by convenience, to exploit symmetries in the system or the geometry of the constraints, which may simplify solving for the motion of the system. Lagrangian mechanics also reveals conserved quantities and their symmetries in a direct way, as a special case of Noether's theorem [7].

When we work with mechanical systems, we will assume the existence of Lagrangian function $L(q,\dot{q})$. In the absence of constraints, dynamic equations can be derived from Lagrange's equations [7]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^i}\right)-\frac{\partial L}{\partial q^i}-\tau_i=0, \quad (1)$$

where $\tau_i$ is input torque. In general apply, that some type of interaction with environment we will model as *a constraint*. We will devote to constraints, which are linear in velocities. Let we have $k$ constraints, which we can write as set of $k$ vector equations:

$$\omega_j^i(q)\dot{q}^j=0,\ pre\ i=1,\ldots,k. \quad (2)$$

This class of constraints includes most commonly investigated nonholonomic constraints. The constraints can be incorporated into the dynamics through the use of *Lagrange multipliers* $\lambda$. That is, equation (1) is modified by adding a force of constraint with an unknown multiplier $\lambda$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^i}\right)-\frac{\partial L}{\partial q^i}+\lambda_j\omega_j^i-\tau_i=0. \quad (3)$$

## 3. Geometric Mechanics

Geometric mechanics represents the application of differential geometry to problems in classical mechanics. By studying locomotion with geometric tools, we can make rigorous statements about systems' motion capabilities.

When studying locomotion systems, it is important to note two additional characteristics that can be used to simplify the equations of motion.

First, the configuration spaces of these systems can be decomposed into the structure [7]:

$$Q=G\times M, \quad (4)$$

where $G$ is *Lie group* (often used to describe position and orientation) and $M$ is any manifold (often called *the shape space*, as it is used to describe the internal shape of a robot). In many cases, the shape space is assumed to be controlled by suitable inputs. The calculation of the reduced equations is important in justifying this assumption and designing appropriate controls.

Second, Lagrangians of these systems are *invariant* with respect to the group action of $G$. That is, these systems exhibit a *symmetry*. In these cases, it can be shown that the equations of motion can be put in the following simplified form (Figure 1):

$$g^{-1}\dot{g}=-A(r)\dot{r}+\tilde{I}^{-1}(r)p, \quad (5)$$

$$\dot{p}=\frac{1}{2}\dot{r}^T\sigma_{\dot{r}\dot{r}}(r)\dot{r}+p^T\sigma_{p\dot{r}}(r)\dot{r}+\frac{1}{2}p^T\sigma_{pp}(r)p, \quad (6)$$

$$M\ddot{r}+\dot{r}^TC(r)\dot{r}+N(r,\dot{r},p)=\tau. \quad (7)$$

Equation (5) we call as *reconstruction equation* and express the dependence fiber velocities $\dot{g}$ on base velocities $\dot{r}$ and momentum $p$. *Momentum equation* (6) describes evolution of momentum and *reduced base dynamic equation* (7) describes evolution of base variables for given generalized fiber, where $M$ is *reduced mass-inertia matrix*, $C$ contains *Coriolis and centripetal force terms* and $N$ includes *conservative forces* and any additional *external forces* such as *friction*.

The *reconstruction equation* (5) and the *momentum equation* (6) represent set of $l$ first order differential equations, where $l$ is dimension of the fiber space $G$. The *reduced base dynamic equation* (7) is set of $m$ second order differential equations, where $m$ is dimension of the base space $M$. Thus, we reduce original set of $n=(m+l)$ second order dynamic equation of motion obtained from equation (4) to set of $2l$ first order differential equations and $m$ second order differential equations [7].

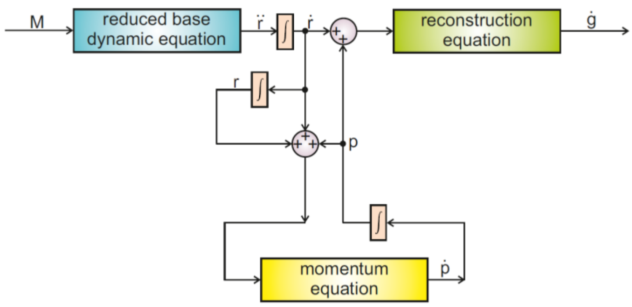


**Figure 1**. Representation of reduced base dynamic equation, reconstruction equation, momentum equation and their relation expressed using block diagram

### 3.1. Reconstruction Equation

In this part we will show the form of reconstruction equations for individual type of mechanical systems.

**Table 1. List of types of mechanical systems and their reconstruction equation [7]**

| System type | Condition | Reconstruction equation of motion |
|---|---|---|
| mixed | $0 < k < l$ | $\xi = -\mathbf{A}(r)\dot{r} + \Gamma(r)p^T$ |
| principally kinematic | $l = k$ | $\xi = -\mathbb{A}(r)\dot{r}$ |
| purely mechanic | $k = 0$ and $p(t) = 0$ | $\xi = -A(r)\dot{r}$ |
| purely dynamic | $m = 1$ | $\xi = \Gamma(r)p^T$ |

#### 3.1.1. Mixed Systems

Mixed systems are systems that have at least one non-holonomic constraint and at most one less nonholonomic constraints than the dimension of the fiber space, that is $0<k<l$ where $l$ is the dimension of the fiber space and $k$ is the number of nonholonomic velocity constraints acting on the system. The reconstruction equation for mixed systems has form:

$$\xi=-\mathbf{A}(r)\dot{r}+\Gamma(r)p^T. \quad (8)$$

#### 3.1.2. Principally Kinematic Systems

Principally kinematic systems are systems that have just enough nonholonomic constraints to fully span the fiber space, that is, $k = l$ where $l$ is the dimension of the fiber space and $k$ is the number of nonholonomic velocity constraints acting on the system. The reconstruction equation for principally kinematic systems has form:

$$\xi = -\mathbb{A}(r)\dot{r}. \tag{9}$$

### 3.1.3. Purely Mechanical Systems

Purely mechanical systems are systems that have no nonholonomic constraints acting on the system, that is, $k = 0$ where $k$ is the number of nonholonomic velocity constraints action on the system. Thus, for such purely mechanical systems, the reconstruction equation simplifies to:

$$\xi = -A(r)\dot{r}. \tag{10}$$

### 3.1.4. Purely Dynamic Systems

Purely dynamic are a special family of the mixed type of systems whose base space is one-dimensional, that is, $m = 1$, where $m$ is the dimension of the base space $M$. The reconstruction equation for purely dynamic systems reduces to:

$$\xi = \Gamma(r)p^T. \tag{11}$$

## 4. Ackerman Steered car-like System

One of the most interesting features on a wheeled vehicle is the steering system. The steering system consists of all the parts necessary to make the front wheels turn in the direction we wish to go. These parts include a steering wheel, a gearbox, and all linkages and levers needed to control the front wheels.

Steering systems are carefully designed so that the driver can, without too much effort, keep the vehicle going straight ahead or turn it to the right or left. The driver must be able to easily overcome the tendency of the front wheels to go to the right or left as a result of striking holes in the road, rocks, stumps, or other obstructions. Obstructions try to stop the wheel that strikes them, while the other front wheel tries to keep rolling, which causes the vehicle to turn in the direction of the obstruction. This is called road shock. Road shock tries to jerk the steering wheel out of the driver's hands. Hitting obstructions makes it difficult to control the vehicle, and steering systems are designed to reduce the shock caused by striking obstructions.

Another feature of the steering system is the front-wheel alignment, which is referred to as steering geometry. Front-wheel alignment can be defined as the proper positioning of the front wheels to make them easy to turn to the right or left and to reduce the tendency of the tires to scuff or wear unevenly. Proper alignment also reduces the tendency of the front wheels to wander or shimmy and makes it much easier to control the vehicle.

Ackerman steering (Figure 2) is used for three- and four-wheeled vehicles. Four-wheeled vehicle must have in part of rear axle mechanical or electric differential. Intersection of wheels of front axle and rear axle at turning of directional wheels determines *instantaneous center of rotation (ICR)*. Angle of rotation of control directional wheel is determined depending on relative angle of rotation of inner wheel $\Phi_i$, outer wheel $\Phi_o$ and parameters of wheelbase $l$ and track $d$:

$$\cot\Phi_i - \cot\Phi_o = \frac{d}{l}. \tag{12}$$

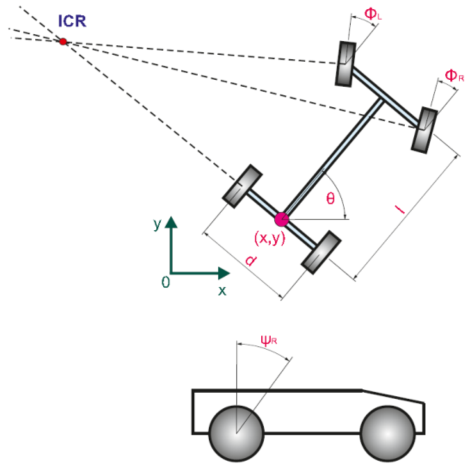


**Figure 2**. Model of Ackerman steering car

This type of steering is widely useful and has appropriate parameters of maneuverability for movement in standard and complicated terrain.

To simplify the model differential is modeled as a single drive wheel located along the medial axis of the vehicle, as shown in Figure 3. This imaginary wheel rotates about its axle by an angle $\psi$. The steering angle of each wheel is controlled by a single steering wheel, with a mechanical linkage coupling each front wheel. In the simple model, a single front wheel, with a steering angle $\Phi$, is used to represent the steering input. Because the distance $l$ between contact points has not changed and *ICR* has not changed, the motion of the two-wheeled vehicle is kinematically equivalent to the motion of the four-wheeled car-like vehicle [8].

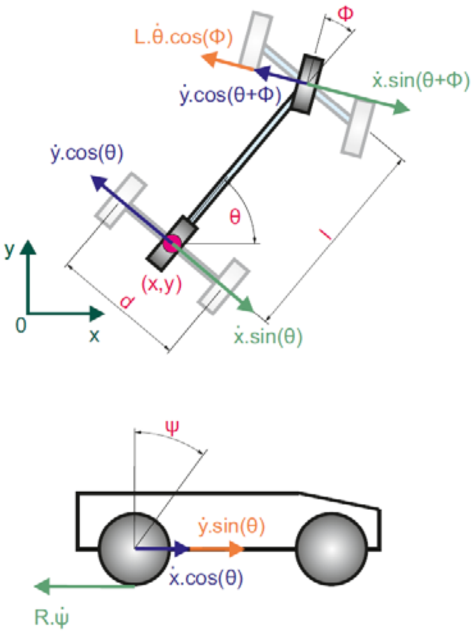


**Figure 3**. Simplified model of Ackerman steering car with shown nonholonomic constraints

# 5. Mathematical Modelling of Ackerman Steered car-like System

In following part we will show to create a mathematical model of Ackerman steered car using Lagrangian mechanics and geometric mechanics.

## 5.1. Lagrangian Mechanics

First, we write Lagrangian of model of Ackerman steered car. We will use the form of simplified Lagrangian, where using $m$ we refer *total weight of Ackerman steered car* including *the wheels*, $J$ will be *moment of inertia of Ackerman steered car* around the center of mass and $J_k$ will be *moment of inertia of wheel*. The simplified Lagrangian has form:

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) + J\dot{\theta}^2 + J\dot{\theta}\dot{\Phi} + \frac{1}{2}J\dot{\Phi}^2 + J_k\dot{\psi}^2. \quad (13)$$

Second, for Ackerman steered car we have *3* nonholonomic constraints and we will expresses these constraints with respect to coordinate *x* and *y*:

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0, \quad (14)$$

$$\dot{x}\sin(\theta+\Phi) - \dot{y}\cos\left(\theta+\Phi\right) - l\dot{\theta}\cos\Phi = 0, \quad (15)$$

$$\dot{x}\cos\theta + \dot{y}\sin\theta - \mathrm{R}\dot{\psi} = 0. \quad (16)$$

According to equation (3) we obtain equations of motion for Ackerman steered car:

$$m\ddot{x} + \lambda_1\sin\theta + \lambda_2\sin(\theta+\Phi) + \lambda_3\cos\theta = 0, \quad (17)$$

$$m\ddot{y} - \lambda_1\cos\theta - \lambda_2\cos(\theta+\Phi) + \lambda_3\sin\theta = 0, \quad (18)$$

$$2J\ddot{\theta} + J\ddot{\Phi} - \lambda_2 l\cos\theta = 0, \quad (19)$$

$$J\ddot{\Phi} + J\ddot{\theta} = \tau_1, \quad (20)$$

$$J_k\ddot{\psi} - \lambda_3 R = \tau_2. \quad (21)$$

## 5.2. Geometric Mechanics

### 5.2.1. Reconstruction Equation

The configuration space of Ackerman steered car has two parts. Locally the body pose is given by *(x, y, θ)*, where *(x, y)* is the location of the midpoint of the rear axle with respect to a global coordinate and $\theta$ is the orientation of the body with respect to the x-axis. The body pose evolves on the *SE(2)* manifold. The configuration of the drive wheel is $\psi \in S$ and that of the steering wheel is $\Phi \in R = \left(-\Phi_{max}, \Phi_{max}\right)$. Thus globally the configuration space is:

$$Q = G \times M = SE\left(2\right) \times S \times \mathrm{R}. \quad (22)$$

Ackerman steered car has three degrees of freedom given by variables $\left(x, y, \theta\right)$ and *2* shape (control) variables $\psi$, $\Phi$. This means, that car-like robot belongs to *under-actuated nonholonomic mechanical systems first order*.

For this type car-like system we have *3 nonholonomic constraints*, $k = 3$ and thus we get 3 equations of motion and dimension of *SE (2)* group is also $l = 3$, (*l=dim{SE (2)}=3*). Because of $k = l = 3$ three-link kinematic snake belongs to the category of *principally kinematic systems*, where to build the reconstruction equation fully suffice nonholonomic constraints. In this subchapter we will consider situation where the base variables represent ideal source of velocity.

At creating of the mathematical model – reconstruction equation we will use approach where we work with nonholonomic constraints expressed in world coordinates. Nonholonomic constraints (14), (15) and (16) we will express in *Pfaffian form*:

$$\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}\sin\theta & -\cos\theta & 0 & 0 & 0\\ \sin(\theta+\Phi) & -\cos(\theta+\Phi) & -l\cos\theta & 0 & 0\\ \cos\theta & \sin\theta & 0 & 0 & -\mathrm{R}\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\\ \dot{\Phi}\\ \dot{\psi}\end{bmatrix}. \quad (23)$$

After multiplying of the previous equation and separating group from shape variables we obtain *the reconstruction equation* in the form:

$$\dot{g} = \underbrace{\begin{bmatrix}0 & R\cos\theta\\ 0 & R\sin\theta\\ 0 & \frac{R}{l}\tan\Phi\end{bmatrix}}_{\mathbb{A}}\dot{r}, \quad (24)$$

Now we express reconstruction equation using local velocities:

$$\begin{bmatrix}\xi^x\\ \xi^y\\ \xi^\theta\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{g}_x\\ \dot{g}_y\\ \dot{g}_\theta\end{bmatrix}, \quad (25)$$

$$\xi = \underbrace{\begin{bmatrix}0 & R\\ 0 & 0\\ 0 & \frac{R}{l}\tan\Phi\end{bmatrix}}_{\mathbb{A}}\dot{r}. \quad (26)$$

### 5.2.2. Representation of Local Connection by Vector Fields

Expressions for local connections $\mathbb{A}(r)$ are quite complicated and do not provide deep insight into the behavior of the system. However, their geometric view using visualization means provides deeper insight. One is *the representation of the local connection by vector fields*. Each row of the local connection $\mathbb{A}(r)$ can be considered as a definition of the vector field $\mathbb{A}(r)^i$ on the shape (base) space whose scalar product with the shape velocity creates a corresponding component of body velocity.

The local connection of Ackerman steered car we have viewed by vector fields for $R = 0.15$ *mm* and $l = 1.1835$

*mm* using program MATLAB R2013b, shown in Figure 4, Figure 5 and Figure 6. Into vector fields were also at the same time generated three line segments at intervals for one second that represent the relation between the shape variables $\Phi$ and $\psi$ by this way:

- the first line segment represents situation when rear wheels are rotated about $2\pi$ and front wheels are not steered,
- the second line segment represents situation when rear wheels are rotated about $4\pi$ and front wheels are steered about $\pi/2$,
- the third line segment represents situation when rear wheels are rotated about $4\pi$ and front wheels are steered about $-\pi/2$.

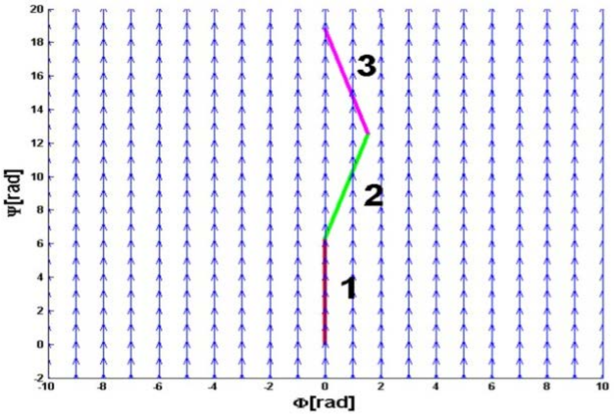


**Figure 4**. Vector field of local connection $\mathbb{A}^{\xi x}$

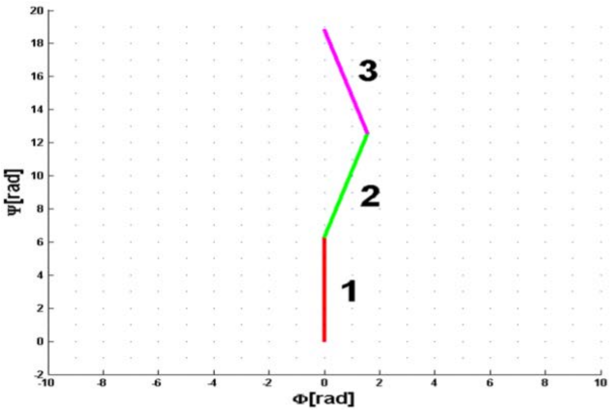


**Figure 5**. Vector field of local connection $\mathbb{A}^{\xi y}$

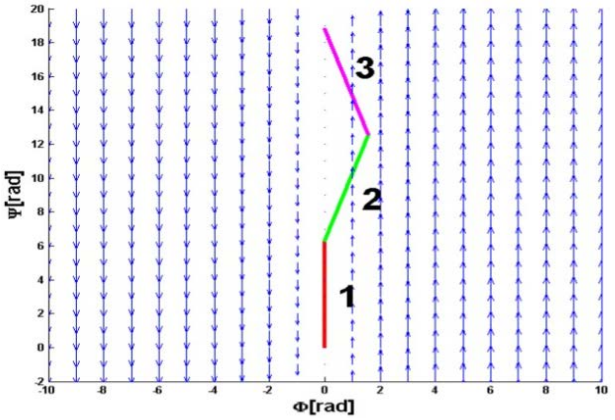


**Figure 6**. Vector field of local connection $\mathbb{A}^{\xi \theta}$

The analysis of the shape of vector field towards the shape of generated line segments we can obtain deeper insight to the issue of the movement with nonholonomic constraints. From Figure 4, Figure 5 and Figure 6 we can deduce how car-like system car will move:

- the first line segment is parallel with the vector field of local connection $\mathbb{A}^{\xi x}$ and perpendicular with the vector field of local connection $\mathbb{A}^{\xi \theta}$, the vector field of local connection $\mathbb{A}^{\xi y}$ is zero vector field $\rightarrow$ Ackerman steered car moves forward,
- the second line segment is approvingly oriented with the vector field of local connection $\mathbb{A}^{\xi x}$ and disapprovingly oriented with the vector field of local connection $\mathbb{A}^{\xi \theta}$, the vector field of local connection $\mathbb{A}^{\xi y}$ is zero vector field $\rightarrow$ Ackerman steered car moves forward and at the same time rotates counterclockwise,
- the third line segment is approvingly oriented with the vector field of local connection $\mathbb{A}^{\xi x}$ and approvingly oriented with the vector field of local connection $\mathbb{A}^{\xi \theta}$, the vector field of local connection $\mathbb{A}^{\xi y}$ is zero vector field $\rightarrow$ Ackerman steered car moves reverses and at the same time rotates counterclockwise,

The courses of individual group variables of car-like system we can see in the following figure (Figure 7).

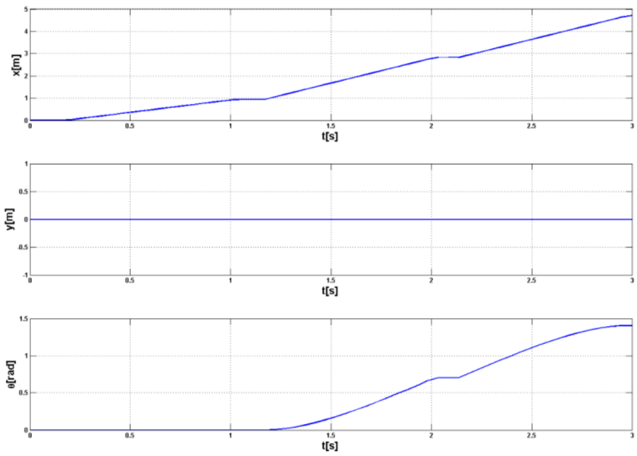


**Figure 7**. Courses of individual components $x$, $y$ and $\theta$

### 5.2.3. Reduced base Dynamic Equation

In this subchapter we will consider situation where the base variables do not represent ideal source of velocity. The Ackerman steered car will be controlled using torque and so we have to use also *reduced base dynamic equation*, whose solution are base-shape variables. Then these base variables we substitute to reconstruction equation and obtain fiber velocity $\dot{g}$ or $\xi$.

Form of simplified Lagrangian is *invariant* and this special form is called as *reduced Lagrangian* $l$. This reduced Lagrangian is depend only on body velocities $\xi$ and shape variables $r$, $\dot{r}$ and we can express in matrix form:

$$l = L = \frac{1}{2}\begin{bmatrix} \xi^x \\ \xi^y \\ \xi^\theta \\ \dot{\Phi} \\ \dot{\psi} \end{bmatrix}^T \begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & 2J & 2J & 0 \\ 0 & 0 & 2J & J & 0 \\ 0 & 0 & 0 & 0 & J_k \end{bmatrix} \begin{bmatrix} \xi^x \\ \xi^y \\ \xi^\theta \\ \dot{\Phi} \\ \dot{\psi} \end{bmatrix}. \quad (27)$$

Then we use relation to determine of reduced mass-inertia matrix [6]:

$$M(r) = m(r) - A(r)^T I(r) A(r), \quad (28)$$

$$M(r) = \begin{bmatrix} J & 0 \\ 0 & J_k - R^2 m - \frac{2JR^2}{l^2}\tan^2\Phi \end{bmatrix}. \quad (29)$$

Term $C$ is defined by [6]:

$$C_{ijk}\dot{r}^j\dot{r}^k = \frac{1}{2}\left(\frac{\partial M_{ij}}{\partial r^k} + \frac{\partial M_{ji}}{\partial r^k} - \frac{\partial M_{jk}}{\partial r^i}\right)\dot{r}^j\dot{r}^k, \quad (30)$$

and $C$ for our kinematic snake has form:

$$C(r) = \begin{bmatrix} 0 \\ -\frac{2JR^2}{l^2}\dot{\Phi}\dot{\psi}\tan\Phi\left(\tan^2\Phi + 1\right) \end{bmatrix}. \quad (31)$$

The last term $N$ we determine according to relation [6]:

$$N(r) = \left\langle \frac{\partial l}{\partial \xi}; d\mathbb{A}(\dot{r},\delta r) + \left[\xi, \mathbb{A}(\delta r)\right] \right\rangle, \quad (32)$$

$$N(r) = \frac{JR}{l^2}\dot{\Phi}\left(l\dot{\Phi} + 2R\dot{\psi}\tan\Phi\right)\left(\tan^2\Phi + 1\right)\delta\psi, \quad (33)$$

where $d\mathbb{A}$ is exterior derivative of connection defined as [6]:

$$d\mathbb{A}(\dot{r},\delta r) = \left(\frac{\partial \mathbb{A}_i}{\partial r^j} - \frac{\partial \mathbb{A}_j}{\partial r^i}\right)\dot{r}^i\delta r^j, \quad (34)$$

$$d\mathbb{A}(\dot{r},\delta r) = \begin{bmatrix} 0 \\ 0 \\ \frac{R}{l}\left(\tan^2\Phi + 1\right)\dot{\Phi}\delta\psi \end{bmatrix}. \quad (35)$$

After substituting individual components of $M(r)$, $C(r)$ and $N(r)$ to equation (7) we get reduced base dynamic equations in general form:

$$J\ddot{\Phi} = \tau_1, \quad (36)$$

$$\left(J_k - R^2 m - \frac{2JR^2}{l^2}\tan^2\Phi\right)\ddot{\psi} - \frac{2JR^2}{l^2}\dot{\Phi}\dot{\psi}\tan\Phi\left(\tan^2\Phi + 1\right) + \frac{JR}{l^2}\dot{\Phi}\left(l\dot{\Phi} + 2R\dot{\psi}\tan\Phi\right)\left(\tan^2\Phi + 1\right) = \tau_2. \quad (37)$$

When we will integrate previous equations (36), (37) and base velocities substitute to equation (26), we get desired body velocities of center of mass of Ackerman steered car.

## 6. Comparison of Different Approaches to Mathematical Modelling

Disadvantages of Lagrangian mechanics:

- the mathematical model contains $n$ second order differential equations and $l$ first order differential equations, where $n$ is dimension of configuration space (number of degrees of freedom) and $l$ is number of nonholonomic constraints,
- we do not see to the structure of differential equations from the point of view control.

Advantages of *geometric mechanics*:

- the mathematical model contains $n-l$ second order differential equations (*reduced base dynamic equations*) and $l$ first order differential equations (*reconstruction equation*),
- original differential equations were divided to two groups with following interaction – reconstruction equation does not influence to reduced base dynamic equation and is affected by this dynamic equation through torque,
- the representation of local connection by vector fields.

## 7. Conclusion

The aim of paper was to introduce the use different approaches of mathematical modelling of nonholonomic systems. At first we discussed about theory of different modelling process based on Lagrangian mechanics and geometric mechanics that we used to create mathematic model of Ackerman steered car-like system. In the case of Lagrangian mechanics we deduced mathematic model that consists of five second order differential equations. But in the case of geometric mechanics we obtained three first order differential equations when we consider ideal source of velocity as input. When we cannot reach ideal source of velocity we have to add two second order differential equations called as reduced base dynamic equations. The main advantage of reconstruction equation is existence of connection, where individual components of connection we can show using vector fields and then we can analyze the shape of vector fields to the shape of gait by which we obtain the ability to predict how designed system will move in the plane.

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## References

[1] LEWIS A. D.: When is a mechanical control system kinematic? IEEE Conf. on Decision and Control, December 1999.

[2] BULLO F., LEONARD N. E. and LEWIS A. D.: Controllability and motion algorithms for underactuated Lagrangian systems on Lie groups, IEEE Transactions on Automatic Control, February 1998.

[3] KELLY S. D. and MURRAY R. M.: Geometric phases and locomotion. J. Robotic Systems, 12(6): 417-431, June 1995.

[4] LEONARD N. E. and KRISHNAPRASAD P. S.: Motion control of drift-free, left-invariant systems on Lie groups, IEEE Trans. on Automatic Control, 40(9): 1539-1554, September 1995

[5] OSTROWSKI J. P. and BURDICK J. W.: Controllability tests for mechanical systems with symmetries and constraints, J. Applied Mathematics and Computer Science, 7(2): 305-331, June 1997.

[6] MARSDEN J. E. and OSTROWSKI J. P.: Symmetries in motion: Geometric foundations of motion control, in Nonlinear Science Today, January 1998.

[7] OSTROWSKI J. P.: The Mechanics and Control of Undulatory Robotic Locomotion [thesis]. California Institute of Technology Pasadena, September 19, 1995.

[8] CONNER David: Integrating Planning and Control for Constrained Dynamical Systems [thesis]. Robotic Institute, Carnegie Mellon University, Pittsburgh, Pennsylvania, December, 2007.