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Truth Values in t-norm based Systems Many-valued FUZZY Logic

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Abstract In t-norm based systems many-valued logic, valuations of propositions form a non-countable set: interval [0,1]. In addition, we are given a set E of truth values p, subject to certain conditions, the valuation v is v=V(p), V reciprocal application of E on [0,1]. The general propositional algebra of t-norm based many-valued logic is then constructed from seven axioms. It contains classical logic (not many-valued) as a special case. It is first applied to the case where E=[0,1] and V is the identity. The result is a t-norm based many-valued logic in which contradiction can have a nonzero degree of truth but cannot be true; for this reason, this logic is called quasi-paraconsistent.

Keywords: *contradiction, denier, logic coordinations, propositions, truth value*

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1. Introduction

Many-valued logic (Béziau, 1997; Cignoli et al, 2000; Malinowski, 2001; Miller and Thornton, 2008) differs from classical logic by the fundamental fact that it allows for partial truth. In classical logic, truth takes on values in the set {0, 1}, in other words, only the value 1 or 0, meaning "*Yes, it's true*," or "*No, it's not*," respectively. The authors propose a t-norm based systems Many-valued logics as their natural extension take on values in the interval [0,1]. Per definition, t-norm based systems are many-valued if the set of valuations is not countable and this set is the interval [0,1].

Let p be the truth value of a proposition or utterance P, P false $p=0$, P true $p=1$; P approximated E = [0,1] = $\{p \in R \mid 0 \leq p \leq 1\}$.

We design for a non-countable set F whose algebraic structure is at least that of a semi-ring. We lay the following definitions and fundamental axioms:

Axiom 1: *Any proposition P has a truth value p, element of a set E which is a part, not countable and stable for multiplication of the set F.*

These systems are basically determined by a strong conjunction connective Λ_V which has as corresponding truth degree function a *t-norm* V, i.e. a binary operation V in the unit interval which is associative, commutative, non-decreasing, and has the degree 1 as a neutral element. Let p_1, p_2, p_3 be three truth values.

Then:

Axiom 2: *Any proposition P is endowed with a valuation* $v \in [0,1]$ *such that* $v = V(p)$, *V reciprocal application of E on* $[0,1]$ *subject to the following conditions:*

1). $V^{-1}(0) = 0$

2). $V(p_1, p_2) = V(p_1)V(p_2)$

Axiom 3: $V(p_1, V(p_2, p_3)) = V(V(p_1, p_2), p_3)$,

Axiom 4: $V(p_1, p_2) = V(p_2, p_1)$, $p_1 \leq p_2 \Rightarrow V(p_1, p_3) \leq V(p_2, p_3)$,

Axiom 5: $V(p_1, 1) = p_1$.

For all those t-norms which have the *sup-preservation property* $V(p, \sup_i p_i) = \sup_i V(p, p_i)$, there is a standard way to introduce a related implication connective $\underset{V}{\longrightarrow}$ with the truth degree function $p_1 \underset{V}{\longrightarrow} p_2 = \sup\{p \mid V(p_1, p) \leq p_2\}$. This implication connective is connected with the t-norm V by the crucial *adjointness condition* $V(p_1, p_2) \leq p_3 \Leftrightarrow p_1 \leq (p_2 \underset{V}{\longrightarrow} p_3)$, which determines $\rightarrow_V$ uniquely for each V with sup-preservation property.

The language is further enriched with a negation connective, $\neg_V$, determined by the truth degree function $\neg_V p = p \underset{V}{\longrightarrow} 0$. We have a conjunction ∧ and a disjunction ∨ with truth degree functions.

$$p_1 \wedge p_2 = \min\{p_1, p_2\},$$
$$p_1 \vee p_2 = \max\{p_1, p_2\}.$$

For t-norms which are continuous functions these additional connectives become even definable. Suitable definitions are

$$\min\{p_1, p_2\} = V(p_1, (p_1 \underset{V}{\longrightarrow} p_2)),$$

$$\max\{p_1, p_2\} = \min\left\{\left(p_1 \underset{V}{\longrightarrow} p_2\right) \underset{V}{\longrightarrow} p_2, \left(p_2 \underset{V}{\longrightarrow} p_1\right) \underset{V}{\longrightarrow} p_1\right\}.$$

For a t-norm V their sup-preservation property is the left-continuity of this binary function V. And the continuity of such a t-norm V can be characterized through the algebraic *divisibility condition* Λ_V $(p_1 \underset{V}{\longrightarrow} p_2) = p_1 \wedge p_2$.

In this work we develop a many-valued logic: known as quasi-paraconsistent because the contradiction cannot be true, but can be approximated, $E=[0,1]$ (Newton da Costa, in Susana Nuccetelli, Ofelia Schutte, and Otávio Bueno, 2010; Bueno, 2010; Carnielli and Marcos, 2001; Fisher, 2007; Priest and Woods, 2007). In 2013, Castiglioni and Ertola Biraben provide some results concerning a logic that results from propositional intuitionistic logic when dual negation is added in certain way, producing a paraconsistent logic that has been called da Costa Logic.

Axiom 6: *If $\neg P$ truth value p^* denotes the negation or contradiction of P, we must have:* $V\left(p+p^*\right)=1$.

Let $\mathbf{P_i}$ be n propositions, i= 1,2,…, n of p_i and p_i^* be the truth values of their contradictories. Then:

Definition 1: *A compound proposition (or logical coordination or logical expression) of order n is a proposition whose truth value c is a function f_n of p_i and p_i^*.*

$$c = f_n\left(p_1, p_1^*, p_2, p_2^*, \ldots p_n, p_n^*\right)$$

f_n values in F; it determines a truth value if $c \in E$ .The condition of existence of a compound proposition defined by f_n is $c \in E$, or what is equivalent, $V(c) \in [0,1]$.

Axiom 7: *f_n is a polynomial in which each index 1, 2, ...,n must be at least once and that all coefficients are equal to unity.*

Condition $V(p_1, p_2) = V(p_1)V(p_2)$ requires the stability of E for multiplication because $[0,1]$ possesses the stability and function V is reciprocal.

If e designates the neutral element of the multiplication of truth values, we have $V(e,e)=V(e)$ and for Axiom 2, $V(e)V(e)=V(e)$. Solution $V(e)=0$ is to reject because for Axiom 2 would lead to $e=0$, therefore remains $V(e)=1$.

We apply Axiom 7 and n = 2. Among polynomials f_2 are the monomial p_1p_2 and p_1+p_2 polynomial.

Definition 2: *The conjunction of two propositions P_1 and P_2 is compound proposition, denoted $P_1 \wedge P_2$ whose truth value is $p_1 p_2$.*

For definition 2 and Axiom 2, $v(P_1 \wedge P_2) = V(p_1 p_2) = V(p_1)V(p_2)$.

From Axiom 2 the conjunction is commutative. Stability of E and $[0,1]$ for the multiplication result that $\forall p_1, \forall p_2, \exists(P_1 \wedge P_2)$.

As in classical logic:

$$v(P_1) = v(P_2) = 1 \Leftrightarrow v(P_1 \wedge P_2) = 1$$
$$v(P_i) = 0 \Rightarrow v(P_1 \wedge P_2) = 0, i = 1,2$$

For definition 2, $v(P \wedge P) = V(pp) = V\left(p^2\right) = \left|V(p)\right|^2$.

In general, unless $v(P)=0$ or $v(P)=1$, the conjunction is not idempotent in many-valued logic.

Definition 3: *The complementarity of two propositions P_1 and P_2 is compound proposition, denoted $P_1 \Im P_2$ whose truth value is $p_1 + p_2$.*

For axiom 1, the complementarity is commutative. It exists only if $p_1 + p_2 \in E$. When $p_1 \neq 0$ and $p_2 \neq 0$, so that $v(P_1 \Im P_2) = 0$, it is necessary that $p_1 + p_2 = 0$, so that the addition of the truth values admit opposed.

When n = 1, among the functions f_1, there are polynomial $p+p^*$ and monomial pp^*. For axiom 6 and definition 3, $v(P \Im \neg P) = V\left(p+p^*\right) = 1$; complementarity of the two contradictories is always true.

Let $p + p^* = u$ be.

Definition 4: *u is a denier of the proposition P if the following three conditions are fulfilled:*

a) $u \in E$

b) $V(u)=1$; *u unitary truth value (from axiom 6)*

c) $u - p = p^* \in E$ *(from axiom 1)*

In general, these three conditions can be satisfied by a set of deniers of P, then the contradictory ¬P has a priori, once fixed p, a set of truth values $p^*(u)$; so the choice of a denier who, in a problem of applied logic, will determine the truth value of the contradictory.

Theorem 1: *If u is a denier of P, it is also a denier of ¬P.*

Proof

Indeed, $u \in E$; $V(u)=1$ $u - p^* = p \in E$

Definition 5: *The conjunction of a proposition and its contradictory is called contradiction.*

In paraconsistent logic, a contradiction is not necessarily false. It may be true, then:

$$v(P \wedge \neg P) = 1 \Leftrightarrow v(P) = v(\neg P) = 1$$

2. ALGEBRA OF t-norm based Systems Many-valued FUZZY Logic

Definition 6: *A compound proposition of order n is called normal and polynomial P_n^P that determines its truth value is called normal if is homogeneous polynomial of degree p, if in any of its monomials there is repetitions of index, and no monomial is repeated.*

Definition 7: *Normal polynomial is said to be complete and denoted $\bar{P}_n^P$ if includes all monomials of degree p allowed by combinatorial analysis.*

It is easy to see that the complete normal polynomials can be formed from the complementarities of contradictory $P_i \Im \neg P_i$ of truth value $p_i + p_i^* = u_i$, $i = 1, 2, ..., n$. Indeed:

$$\bar{P}_n^1 = \sum_i \left(p_i + p_i^* \right) = \sum_i u_i$$

$$\bar{P}_n^2 = \sum_{ij} \left(p_i + p_i^* \right)\left(p_j + p_j^* \right) = \sum_{ij} u_i u_j$$

where ij refers to a combination of the two indices; $\bar{P}_n^2$ includes $2^2 C_n^2$ monomials of degree 2. Similarly:

$$\bar{P}_n^3 = \sum_{ijk} \left(p_i + p_i^* \right)\left(p_j + p_j^* \right)\left(p_k + p_k^* \right) = \sum_{ijk} u_i u_j u_k$$

where ijk means a combination of three indexes; $\bar{P}_n^3$ includes $2^3 C_n^3$ monomials of degree 3. And so on until:

$$\bar{P}_n^n = \left(p_1 + p_1^* \right)\left(p_2 + p_2^* \right)....\left(p_n + p_n^* \right) = u_1 u_2 ... u_n$$

which includes 2^n monomials of degree n.

Any normal compound proposition has equal truth value either one of the monomials of a complete normal polynomial, or a combination of several of these monomials.

Definition 8: *A family p of normal compound propositions of order n contains all those derived from complete normal polynomial* $\bar{P}_n^p$.

Within the same family, the propositions can be classified into groups according to the number of monomials of $\bar{P}_n^p$ composing P_n^p .

2.1. Normal Binary Propositions: Family 2, Group 1

Group 1 is that of binary propositions whose truth value is the sum of an odd number of monomials $\bar{P}_2^2$.

$$\bar{P}_2^2 = p_1 p_2 + p_1 p_2^* + p_1^* p_2 + p_1^* p_2^* = u_1 u_2 .$$

The monomials of $\bar{P}_2^2$ are the respective truth values of the conjunctions $P_1 \wedge P_2, P_1 \wedge \neg P_2, \neg P_1 \wedge P_2, \neg P_1 \wedge \neg P_2$ that exist unconditionally.

Definition 9: *Polynomial* $p_1 p_2^* + p_1^* p_2 + p_1^* p_2^* = u_1 u_2 - p_1 p_2$ *is the truth value of the proposition called incompability of P_1 and P_2 and denoted* $\neg P_1 \vee \neg P_2$.

There is incompatibility if $P_1 \wedge P_2$ admits $u_1 u_2$ as denier. Then, after the axiom 6, definition 3 and definition 4, we have:

$$v\left(\neg P_1 \vee \neg P_2 \right) = v\left[\neg \left(P_1 \wedge P_2 \right) \right]$$

and the truth value of $\neg P_1 \vee \neg P_2$ is fixed, once fixed $p_1 p_2$, by the denier $u_1 u_2$. $\neg P_1 \vee \neg P_2$ is commutative.

Definition 10: $p_1 p_2 + p_1 p_2^* + p_1^* p_2 = u_1 u_2 - p_1^* p_2^*$ *is the truth value of the compound proposition called disjunction of P_1 and P_2 and denoted* $P_1 \vee P_2$.

There is disjunction if $\neg P_1 \wedge \neg P_2$ admits $u_1 u_2$ as denier. Then:

$$v\left(P_1 \vee P_2 \right) = v\left[\neg \left(\neg P_1 \wedge \neg P_2 \right) \right]$$

and the truth value of $P_1 \wedge P_2$ is fixed, once fixed $p_1^* p_2^*$, by the denier $u_1 u_2$.

Definition 11: $p_1 p_2 + p_1 p_2^* + p_1^* p_2^* = u_1 u_2 - p_1^* p_2$ *is the truth value of the proposition called implication of P_2 by P_1 denoted* $\neg P_1 \vee P_2$ *or* $P_1 \Rightarrow P_2$.

There is implication if $\neg P_1 \wedge P_2$ admits $u_1 u_2$ as denier. Then:

$$v\left(P_1 \Rightarrow P_2 \right) = v\left[\neg \left(\neg P_1 \wedge P_2 \right) \right]$$

and the truth value of $P_1 \Rightarrow P_2$ is fixed, once fixed $p_1^* p_2$, by the denier $u_1 u_2$.

Condition 1 of existence: *Let P_1, P_2 be two propositions, $\exists u_1$ denier of P_1 and $\exists u_2$ denier P_2 such that $u_1 u_2$ is a denier of* $P_1 \wedge P_2$.

Indeed, if E and function V can satisfy this condition, then $\neg P_1 \vee \neg P_2$ exists, but as u_1 is also a denier of $\neg P_1$ and u_2 of $\neg P_2$, other disjunctions may also exist.

2.2. Normal Binary Propositions: Family 2, Group 2

The truth value of a proposition of this group is the sum of an even number of monomials of $\bar{P}_2^2$.

Definition 12: $p_1 p_2 + p_1^* p_2^*$ *is the truth value of the compound proposition called concordance and denoted* $P_1 \Xi P_2$.

Definition 13: $p_1 p_2^* + p_1^* p_2$ *is the truth value of the compound proposition called discordance and denoted* $P_1 \mathrm{X} P_2$.

Condition 2 of existence: $P_1 \Xi P_2$ *exists if $\exists u_1$ denier of P_1 and $\exists u_2$ denier P_2 such that* $p_1 p_2 + p_1^* p_2^* \in E$.

Condition 3 of existence: $P_1 \mathrm{X} P_2$ *exists if $\exists u_1$ denier of P_1 and $\exists u_2$ denier P_2 such that* $p_1 p_2^* + p_1^* p_2 \in E$.

If conditions 2 and 3 are satisfied, then:

$$v\left(P_1 \Xi P_2 \right) = v\left[\neg \left(P_1 \mathrm{X} P_2 \right) \right]$$

$$v\left(P_1 \mathrm{X} P_2 \right) = v\left[\neg \left(P_1 \Leftrightarrow P_2 \right) \right]$$

We leave aside the coordination of this group whose respective truth values are: $p_1 p_2 + p_1 p_2^* = u_2 p_1$, $p_1 p_2 + p_1^* p_2 = u_1 p_2$, $p_1 p_2^* + p_1^* p_2^* = u_1 p_2^*$ and $p_1^* p_2 + p_1^* p_2^* = u_2 p_1^*$ whose degrees of truth are determined by the truth value of only one of the propositions $P_1, \neg P_1, P_2, \neg P_2$.

2.3. Normal Binary Propositions: Family 1

$$\bar{P}_2^1 = p_1 + p_2 + p_1^* + p_2^*$$

We have defined the complementarity.

Definition 14: *$p_1 + p_2^*$ is the truth value of the inverse complementarity of P_1 and P_2 denoted $\neg P_1 \Im \neg P_2$.*

Condition 4 of existence: *$\exists u_1$ denier of P_1 and $\exists u_2$ denier P_2 such as if $p_1 + p_2 \notin E$ then $p_1^* + p_2^* \in E$.*

Note that in Condition 4 the truth values intervene and not just the deniers**.** Condition 4 satisfied if $P_1 \Im P_2$ does not exist, and then $\neg P_1 \Im \neg P_2$ exists and vice versa.

Definition 15: *$p_1 + p_2^*$ is the truth value of the compound proposition called equivalence and denoted $P_1 \wp P_2$.*

The denomination equivalence is due to that $p_1 = p_2 \Rightarrow v(P_2 \wp P_1) = v(P_1 \wp P_2) = 1$. Indeed, $p_1 + p_2^* = p_1 - p_2 + u_2$ and $p_1^* + p_2 = p_2 - p_1 + u_1$, $p_1 = p_2 \Rightarrow v(P_2 \wp P_1) = V(u_2) = 1$ and $v(P_1 \wp P_2) = V(u_1) = 1$.

Condition 4 satisfied, at least one of two equivalences $P_2 \wp P_1$ and $P_1 \wp P_2$ exists since $P_2 \wp P_1 \equiv P_1 \Im \neg P_2$ and $P_1 \wp P_2 \equiv \neg P_1 \Im P_2$.

We make a summary of the main coordination in the following table (Table 1):

Table 1. Table of principal normal binary propositions

Notation	Name	Truth value
$P_1 \wedge P_2$	Conjunction	$p_1 p_2$
$\neg P_1 \vee \neg P_2$	Incompatibility	$u_1 u_2 - p_1 p_2$
$P_1 \vee P_2$	Disjunction	$u_1 u_2 - p_1^* p_2^*$ $= u_2 p_1 + u_1 p_2 - p_1 p_2$
$P_1 \Rightarrow P_2$	Implication	$u_1 u_2 - p_1 p_2^*$ $= u_1 u_2 - p_1 (u_2 - p_2)$
$P_1 \Xi P_2$	Concordance	$p_1 p_2 + p_1^* p_2^*$ $= u_1 u_2 - (u_2 p_1 + u_1 p_2) + 2 p_1 p_2$
$P_1 X P_2$	Discordance	$p_1 p_2^* + p_1^* p_2$ $= u_2 p_1 + u_1 p_2 - 2 p_1 p_2$
$P_1 \Im P_2$	Complementarity	$p_1 + p_2$
$\neg P_1 \Im \neg P_2$	Inverse complementarity	$p_1^* + p_2^*$
$P_2 \wp P_1$	Equivalence	$p_1 + p_2^* = p_1 + u_2 - p_2$
$P_1 \wp P_2$	Inverse equivalence	$p_1^* + p_2 = u_1 - p_1 + p_2$

2.4. Normal propositions of order n: 1 and n families

We consider only those two families and only a few of coordination within them.

$$\bar{P}_n^1 = p_1 + p_2 + \ldots + p_n + p_1^* + p_2^* + \ldots + p_n^*$$

The sum of truth values is associative (axiom 1), but the complementarity is not in general. We write in all cases $P_1 \Im P_2 \Im P_3$ the compound proposition whose veracity is $r = p_1 + p_2 + p_3$, $r \in E$. Iff $P_1 \Im P_2$ exists $p_1 + p_2 \in E$ and $P_2 \Im P_3$ exists $p_1 + p_3 \in E$ there is associativity, resulting from the addition of the truth values; $(P_1 \Im P_2) \Im P_3$, $P_1 \Im (P_2 \Im P_3)$, $P_1 \Im P_2 \Im P_3$ then have the same truth value $p_1 + p_2 + p_3$.

The complementarity of the n propositions P_i, commutative, is the compound proposition $P_1 \Im P_2 \Im \ldots \Im P_3$ whose truth value is $p_1 + p_2 + \ldots + p_n$.

It will be even the inverse complementarity of n propositions, associative but not commutative in general, with respect to $\neg P_i$, denoted $\neg P_1 \Im \neg P_2 \Im \ldots \Im \neg P_3$ and truth value $p_1^* + p_2^* + \ldots + p_n^*$.

If set E and function V satisfy Condition 4, so when the complementarity does not exist, there is the inverse complementarity.

$$\bar{P}_n^n = (p_1 + p_1^*)(p_2 + p_2^*) \ldots (p_n + p_n^*) = u_1 u_2 \ldots u_n$$

Multiplication of truth values is associative; the conjunction is also because it is not subject to any condition of existence.

$$P_1 \wedge (P_2 \wedge P_3),\ (P_1 \wedge P_2) \wedge P_3,\ P_1 \wedge P_2 \wedge P_3$$

always same truth value $p_1 p_2 p_3$.

Conjunction of n propositions P_i, commutative and associative, is the compound proposition denoted $P_1 \wedge P_2 \wedge \ldots \wedge P_n$ whose truth value $p_1 p_2 \ldots p_n$ is one of the monomials P_n^n.

If condition 1 is satisfied by $u_2 u_3$ and $u_1 u_2 u_3$, the incompatibility $\neg P_1 \vee (\neg P_2 \vee \neg P_3)$ exists, then it is the negation by denier $u_1(u_2 u_3) = u_1 u_2 u_3$ of the conjunction $P_1 \wedge (P_2 \wedge P_3)$, identical to $P_1 \wedge P_2 \wedge P_3$. If $u_1 u_2$ satisfies condition 1, the incompatibility $(\neg P_1 \vee \neg P_2) \vee \neg P_3$ exists and it is also the negation by denier $u_1 u_2 u_3$ of the conjunction $P_1 \wedge P_2 \wedge P_3$.It is seen that if the condition 1 is satisfactory wherever it is necessary, there is an incompatibility of P_1, P_2 and P_3, commutative and associative, which is denoted $\neg P_1 \vee \neg P_2 \vee \neg P_3$ and will have as truth value $u_1 u_2 u_3 - p_1 p_2 p_3$.More generally, it may be a commutative and associative incompatibility of n propositions P_i, denoted as $\neg P_1 \vee \neg P_2 \vee \ldots \vee \neg P_n$ and will have as truth value $u_1 u_2 \ldots u_n - p_1 p_2 \ldots p_n$.But we can also, as the complementarity, noted in all cases $\neg P_1 \vee \neg P_2 \vee \ldots \vee \neg P_n$ the compound proposition, non-associative in general, of veracity $u_1 u_2 \ldots u_n - p_1 p_2 \ldots p_n$. Similarly, through a suitable choice of the deniers, it may be a disjunction of n propositions P_i, commutative and associative sometimes, denoted $P_1 \vee P_2 \vee ... \vee P_n$ and truth value $u_1 u_2 ... u_n - p_1^* p_2^* ... p_n^*$.

3. t-norm based Systems Many-valued Logic

The set of truth values E is interval $[0,1]$.; function V is the identity: the degree of truth is equal to the truth value. V does satisfy axiom 2, $V(u)=1$ and $V(u)=u \Rightarrow u=1$; therefore this logic does not know a single denier, number 1.

Axiom 6 is written $p+p^*=1$. Contradiction has a value $v(P \wedge \neg P) = p(1-p)$.It cannot be true, it is false if P is false or if P is true; it is approximated if P is approximated but the maximum degree of truth is 0.25, achieved when p = 0.50.

3.1. Normal Binary Propositions. Conditions: 1-4

$$\bar{P}_2^2 = p_1 p_2 + p_1 p_2^* + p_1^* p_2 + p_1^* p_2^* = 1$$

The four terms of $\bar{P}_2^2$ belongs to $[0,1]$. Therefore $(1-p_1 p_2) \in [0,1]$ and $(1-p_1 p_2^*), (1-p_1^* p_2), (1-p_1^* p_2^*) \in [0,1]$. Condition 1 is filled by the denier 1. The sum of four terms being 1, $(p_1 p_2 + p_1^* p_2^*) \in [0,1]$ and $(p_1 p_2^* + p_1^* p_2) \in [0,1]$: conditions 2 and 4 are also always met. Finally, if $p_1 + p_2 \geq 1$ then $p_1^* + p_2^* = 2 - (p_1 + p_2) \leq 1$ because $p_1 + p_2 \leq 2$: condition 4 is fulfilled too.

Truth values of the main normal binary coordinations are the following:

1. *Conjunction*: $v(P_1 \wedge P_2) = p_1 p_2$

2. *Incompatibility*: $v(\neg P_1 \vee \neg P_2) = 1 - p_1 p_2$
$\neg P_1 \vee \neg P_2 = \neg(P_1 \wedge P_2)$

3. Disjunction: $v(P_1 \vee P_2) = 1 - p_1^* p_2^* = p_1 + p_2 - p_1 p_2$
$P_1 \vee P_2 = \neg(\neg P_1 \wedge \neg P_2)$

4. Implication: $v(P_1 \Rightarrow P_2) = 1 - p_1 p_2^* = 1 - p_1(1-p_2)$
$P_1 \Rightarrow P_2 = \neg(P_1 \wedge \neg P_2) = \neg P_1 \vee P_2$

5. Concordance: $v(P_1 \Xi P_2) = p_1 p_2 + p_1^* p_2^*$
$= 1 - (p_1 + p_2) + 2 p_1 p_2$

6. *Discordance:* $v(P_1 \mathrm{X} P_2) = p_1 p_2^* + p_1^* = p_1 + p_2 - 2 p_1 p_2$
$P_1 \mathrm{X} P_2 = \neg(P_1 \Xi P_2)$

7. *Equivalence*[1]*:* $v(P_2 \wp P_1) = p_1 + p_2^* = p_1 + 1 - p_2$
$v(P_1 \wp P_2) = p_1^* + p_2 = 1 - p_1 + p_2$
$v(P_1 \Im P_2) = p_1 + p_2$

8. Complementarity[2]: $v(\neg P_1 \Im \neg P_2) = p_1^* + p_2^*$
$= 2 - (p_1 + p_2)$

3.2. Normal Propositions of n Order

We have, for example, the following degrees of truth:

1. *Conjunction:* $v(P_1 \wedge P_2 \wedge ... \wedge P_n) = p_1 p_2 ... p_n$

2. *Incompatibility:* $v(\neg P_1 \vee \neg P_2 \vee ... \vee \neg P_n)$
$= 1 - p_1 p_2 ... p_n$
$\neg P_1 \vee \neg P_2 \vee ... \vee \neg P_n$
$= \neg(P_1 \wedge P_2 \wedge ... \wedge P_n)$

3. *Disjunction:* $v(P_1 \vee P_2 \vee ... \vee P_n) = 1 - p_1^* p_2^* ... p_n^*$
$P_1 \vee P_2 \vee ... \vee P_n = \neg(\neg P_1 \wedge \neg P_2 \wedge ... \neg P_n)$

4. *Complementarities:*

$$v(P_1 \Im P_2 \Im ... \Im P_n) = p_1 + p_2 + ... + p_n$$
$$iff \quad (p_1 + p_2 + ... + p_n) \in [0,1]$$
$$v(\neg P_1 \Im \neg P_2 \Im ... \Im P_n) = n - (p_1 + p_2 + ... + p_n)$$
$$iff \quad (p_1 + p_2 + ... + p_n) \geq n-1$$

4. BOOLEAN REDUCTION OF t-norm Based Systems Many-valued FUZZY Logic

The Boolean reduction of many-valued logic is to reduce the set of valuations $[0,1]$ to the set $\{0,1\}$. Accordingly, the set E of truth values retains its neutral element 0, since $V^{-1}(0)=0$ and the set of unitary truth values. The result is a Boolean logic, not many-valued since $\{0,1\}$ is countable.

The classical propositional algebra can be deduced from the evaluation of negation and that of conjunction, by defining from it and little by little other compound propositions. We already know that if the valuations of P_1 and P_2 are Boolean that $P_1 \wedge P_2$ is the same in many-valued logic in classical logic.

Regarding the negation, it follows from $p+p^*$, $V^{-1}(0)=0$, $V(0)=0$ and $V(u)=1$ that:

$$\forall u, v(P) = 0 \Rightarrow v(\neg P) = 1$$
$$\forall u, v(\neg P) = 0 \Rightarrow v(P) = 1$$

If $v(P)=1$ may be selected $u=p$ as denier because $p \in E$, $v(p)=1$ and $u-p=0 \in E$ so that:

[1] Only $v(P_2 \wp P_1)$ exists when $p_2 \geq p_1$, and only $v(P_1 \wp P_2)$ exists when $p_1 \geq p_2$. We can define in quasi-paraconsistent logic a unique commutative coordination known as equivalence: $v(P_2 \wp P_1) = v(P_1 \wp P_2) = 1 - |p_1 - p_2|$ which is true iff $p_1 = p_2$.

[2] Only $v(P_1 \Im P_2)$ exists when $(p_1 + p_2) \in [0,1]$, and only $v(\neg P_1 \Im \neg P_2)$ exists when $p_1 + p_2 \geq 1$.

$$v(P)=1 \Rightarrow v(\neg P)=0 \text{ provided that } u = p$$
$$v(\neg P)=1 \Rightarrow v(P)=0 \text{ provided that } u = p.$$

The evaluation of many-valued negation may well be made identical to that of classical negation, choosing specific deniers.

Consider some remarkable links between quasi-paraconsistent logic and classical logic.

4.1. Deductive Equivalence

Concordance, equivalence and reciprocal implication of the quasi-paraconsistent logic just melt in classical logic in a single coordination that is the classic equivalence or equivalence deductive $P_1 \Leftrightarrow P_2$. Indeed, if p_1 and p_2 are Boolean variables

$$v(P_1 \Leftrightarrow P_2)=1-(p_1+p_2)+2p_1p_2=v(P_1 \Xi P_2)$$
$$v(P_1 \Leftrightarrow P_2)=1-|p_1-p_2|=v(P_1 \wp P_2)=v(P_2 \wp P_1)$$

Regarding the reciprocal implication, it has degree of truth in quasi-paraconsistent logic, by defining

$$P_1 \Leftrightarrow P_2=(P_1 \Rightarrow P_2)\wedge(P_2 \Rightarrow P_1)$$
$$v(P_1 \Leftrightarrow P_2)=1-(p_1+p_2)+2p_1p_2 + p_1p_2(1-p_1)(1-p_2)$$

When p_1 and p_2 are Boolean, the third term of the last member is always zero (principle of non-contradiction) so that:

$$v(P_1 \Leftrightarrow P_2)=v(P_1 \Xi P_2).$$

4.2. Mutual Exclusion

Discordance, complementarity and inverse complementarity of quasi-paraconsistent logic blend in classical logic in a single coordination which is the reciprocal exclusion. Adopting the notation of Piaget P_1WP_2 for reciprocal exclusion we have when p_1 and p_2 are Boolean:

$$v(P_1WP_2)=p_1+p_2-2p_1p_2=v(P_1XP_2)$$

$$v(P_1WP_2)=p_1+p_2=v(P_1\Im P_2) \text{ when } p_1+p_2 \le 1$$

$$v(P_1WP_2)=2-(p_1+p_2)=v(\neg P_1\Im\neg P_2)$$
$$\text{when } p_1+p_2 \ge 1.$$

5. Conclusions

The main objective of the authors is to establish a theory of truth-value evaluation for paraconsistent logics, unlike others who are in the literature (Asenjo, 1966; Avron, 2005; Belnap, 1977; Bueno, 1999; Carnielli, Coniglio and Lof D'ottaviano, I.M. 2002; Dunn, 1976: Tanaka et al, 2013), with the goal of using that logic paraconsistent in analyzing ideological, mythical, religious and mystic belief systems (Nescolarde-Selva and Usó-Doménech, 2013[a,b,c,d]; Usó-Doménech and Nescolarde-Selva, 2012).

Quasi-Paraconsistent many-valued fuzzy logic includes the special case of classical logic. In fact, our presentation of the propositional many-valued algebra is developed according to the canons of Aristotelian logic, which borrow from the theory of sets. If classical logic does not was a special case of quasi-paraconsistent many-valued logic, it, mined by a fundamental inconsistency, should be rejected, on the spot.

A statement is analytic for pedagogical need that of quasi-paraconsistent many-valued logic remains rational because the latter is subject to conformity with a logic that it encompasses, in definitive with itself.

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ANNEX A

We will represent in the following table a comparison between two logics: classical (CL), and t-norm based systems many-valued fuzzy logic (MVFL).

Table 2. Truth table of principal normal binary propositions

Notation	Name	MVFL truth values $p_1, p_2 \in \{0,1\}$	QPL truth values $p_1, p_2 \in [0,1]$
$P_1 \wedge P_2$	Conjunction	$p_1 p_2$	$p_1 p_2$
$\neg P_1 \vee \neg P_2$	Incompatibility	$1 - p_1 p_2$	$u_1 u_2 - p_1 p_2$
$P_1 \vee P_2$	Disjunction	$p_1 + p_2 - p_1 p_2$	$u_2 p_1 + u_1 p_2 - p_1 p_2$
$P_1 \Rightarrow P_2$	Implication	$1 - p_1 + p_1 p_2$	$u_1 u_2 - p_1 (u_2 - p_2)$
$P_1 \Leftrightarrow P_2$	Concordance	$1 - p_1 - p_2 + 2 p_1 p_2$	$u_1 u_2 - (u_2 p_1 + u_1 p_2) + 2 p_1 p_2$
$P_1 \Updownarrow P_2$	Discordance	$p_1 + p_2 - 2 p_1 p_2$	$u_2 p_1 + u_1 p_2 - 2 p_1 p_2$
$P_1 \Im P_2$	Complementarity	$p_1 + p_2$	$p_1 + p_2$
$\neg P_1 \Im \neg P_2$	Inverse complementarity	$2 - p_1 - p_2$	$p_1^* + p_2^*$
$P_2 \wp P_1$	Equivalence	$1 + p_1 - p_2$	$p_1 + u_2 - p_2$
$P_1 \wp P_2$	Inverse equivalence	$1 - p_1 + p_2$	$u_1 - p_1 + p_2$