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An Interval-Valued Fuzzy Portfolio Decision Model with Entropy and VaR Constraints

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Abstract To deal with portfolio problem with interval-valued fuzzy return, this paper firstly employs interval-valued fuzzy possibilistic variance and information entropy to measure the risk and dispersion of portfolio, and uses value-at-risk (VaR) to measure the maximum loss under a given confidence level. Then an interval-valued fuzzy portfolio decision model is constructed with information entropy and VaR constraints. Secondly, we convert the uncertain risk objective function of portfolio model into a simple quadratic function and obtain the optimal portfolio strategy by LINGO optimization software. Finally, the effectiveness of the proposed portfolio decision model is empirically analyzed through real stock investment data, and the impacts of different entropy and VaR on portfolio strategies are tested.

Keywords: *interval-valued fuzzy number, possibilistic variance, value-at-risk, entropy, portfolio*

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1. Introduction

Since Markowitz [1] proposed the mean-variance portfolio model, some scholars have extended it to mean-absolute deviation model [2], mean-semi-variance model [3] and other classical portfolio models. The traditional portfolio models only use the mean of historical returns to estimate the expected return. However, the expected return of security is not only affected by historical data, but also involved in investors' subjective experience and psychological emotion, so it is necessary to explore the portfolio decision problem with interval and fuzzy uncertainty.

For the portfolio problem under fuzzy environment, Zhang [4] proposed mean-absolute deviation fuzzy portfolio optimization model with entropy constraints. Zhu [5] investigated fuzzy multi-objective portfolio decision-making method. Since the value-at-risk (VaR) can effectively measure the potential downside risk of portfolio, Li [6], Wang [7], and Zhang [8] have studied on fuzzy portfolio optimization models with value-at-risk constraints. In addition, Wang [9], Sun [10], Sui [11], Moghadam [12] discussed interval portfolio decision models, and Kumar [13] designed an interval mean-VaR portfolio optimization model.

Combining the advantages of interval and fuzzy numbers in dealing with uncertain information, Chen [14] applied interval-valued fuzzy numbers to analyze uncertain risk decision problems. Since interval-valued fuzzy number (IvFN) can flexibly represent the expected return and risk, it has also been applied to portfolio decision-making problem. Yin [15] presented a stock portfolio optimization decision model based on triangular interval-valued fuzzy return. Khalifa [16] investigated the multi-objective portfolio model based on interval-valued fuzzy linear programming method. Wu [17] studied the multi-attribute group decision-making method for portfolio in interval fuzzy environment. However, the above existing interval fuzzy portfolio models did not consider the influence of information entropy and value-at-risk on portfolio decision.

In the real investment market, investors not only hope to obtain high return and low risk, but also ensure that the maximum loss of the portfolio is lower than a certain VaR with some confidence degree. Therefore, in order to overcome the drawbacks of the existing portfolio models [15,16,17], this paper aims to employ IvFN to assess the uncertain return of security and use the possibilistic variance of IvFN to measure portfolio risk. And we establish an interval-valued fuzzy portfolio decision model with the constraints of information entropy and VaR interval, which can effectively improve the diversification of portfolio and control the maximum risk of loss. Since the proposed interval fuzzy portfolio model is a nonlinear optimization model, we utilize the LINGO nonlinear optimization software to solve the optimal portfolio strategy. Finally, a numerical example of stock investment from Chinese stock market is given to demonstrate the efficiency of the presented portfolio model.

2. Interval-Valued Fuzzy Possibilistic Mean and Variance

Definition 1 [14]. $\tilde{A}=\left[\tilde{A}^L,\tilde{A}^U\right]=\left(a,b;\alpha,\alpha',\beta,\beta'\right)$ is called a trapezoidal interval-valued fuzzy number, if the membership functions of upper fuzzy number $\tilde{A}^U=\left(a,b;\alpha',\beta'\right)$ and lower fuzzy number $\tilde{A}^L=\left(a,b;\alpha,\beta\right)$ have the following forms:

$$\mu_{\tilde{A}^U}(x)=\begin{cases}1-\dfrac{a-x}{\alpha'} & ,a-\alpha'\le x\le a\\ 1 & ,a\le x\le b\\ 1-\dfrac{x-b}{\beta'} & ,b\le x\le b+\beta'\\ 0 & ,else\end{cases},\quad \mu_{\tilde{A}^L}(x)=\begin{cases}1-\dfrac{a-x}{\alpha} & ,a-\alpha\le x\le a\\ 1 & ,a\le x\le b\\ 1-\dfrac{x-b}{\beta} & ,b\le x\le b+\beta\\ 0 & ,else\end{cases}$$

where α' and β' are the left and right widths of the upper fuzzy number respectively, α and β are the left the right widths of the lower fuzzy number respectively, and $0\le\alpha\le\alpha'$, $0\le\beta\le\beta'$.

Corollary 1. For any $\lambda\in(0,1]$, the λ cut set of lower fuzzy number $\tilde{A}^L$ is $\tilde{A}^L_\lambda=\left[a_L(\lambda),b_L(\lambda)\right]=\left[a-\alpha(1-\lambda),b+\beta(1-\lambda)\right]$, and the λ cut set of upper fuzzy number $\tilde{A}^U$ is $\tilde{A}^U_\lambda=\left[a_U(\lambda),b_U(\lambda)\right]=\left[a-\alpha'(1-\lambda),b+\beta'(1-\lambda)\right]$.

Definition 2 [18]. The upper and lower possibilistic mean of IvFN $\tilde{A}$ are, respectively, defined as

$$M\left(\tilde{A}^U\right)=\int_0^1\lambda\left[a_U(\lambda)+b_U(\lambda)\right]d\lambda=\frac{a+b}{2}+\frac{\beta'-\alpha'}{6},$$
$$M\left(\tilde{A}^L\right)=\int_0^1\lambda\left[a_L(\lambda)+b_L(\lambda)\right]d\lambda=\frac{a+b}{2}+\frac{\beta-\alpha}{6}.$$

The possibilistic mean of IvFN $\tilde{A}$ can be computed by

$$M\left(\tilde{A}\right)=\frac{\left[M\left(\tilde{A}^L\right)+M\left(\tilde{A}^U\right)\right]}{2}=(a+b)/2+\left[\beta+\beta'-(\alpha+\alpha')\right]/12 \tag{1}$$

Definition 3. The upper and lower possibilistic variance of interval-valued fuzzy number $\tilde{A}$ are, respectively, defined as:

$$Var\left(\tilde{A}^U\right)=\frac{1}{2}\int_0^1\lambda\left[b_U(\lambda)-a_U(\lambda)\right]^2d\lambda=(b-a)^2/4+(b-a)(\alpha'+\beta')/6+(\alpha'+\beta')^2/24,$$

$$Var\left(\tilde{A}^L\right)=\frac{1}{2}\int_0^1\lambda\left[b_L(\lambda)-a_L(\lambda)\right]^2d\lambda=(b-a)^2/4+(b-a)(\alpha+\beta)/6+(\alpha+\beta)^2/24.$$

The possibilistic variance of IvFN $\tilde{A}$ is calculated by

$$Var\left(\tilde{A}\right)=\frac{\left[Var\left(\tilde{A}^L\right)+Var\left(\tilde{A}^U\right)\right]}{2}=\frac{(b-a)^2}{4}+\frac{(b-a)(\alpha+\beta+\alpha'+\beta')}{12}+\frac{[(\alpha+\beta)^2+(\alpha'+\beta')^2]}{48} \tag{2}$$

Definition 4. Let $\tilde{A}_i=\left(a_i,b_i;\alpha_i,\alpha_i',\beta_i,\beta_i'\right)$ and $\tilde{A}_j=\left(a_j,b_j;\alpha_j,\alpha_j',\beta_j,\beta_j'\right)$ be two IvFNs, the upper and lower possibilistic covariance between them are, respectively, defined as:

$$cov\left(\tilde{A}_i^U,\tilde{A}_j^U\right)=\frac{(b_i-a_i)(b_j-a_j)}{4}+\frac{(b_i-a_i)(\alpha_j'+\beta_j')}{12}+\frac{(b_j-a_j)(\alpha_i'+\beta_i')}{12}+\frac{(\alpha_i'+\beta_i')(\alpha_j'+\beta_j')}{24},$$

$$cov\left(\tilde{A}_i^L,\tilde{A}_j^L\right)=\frac{(b_i-a_i)(b_j-a_j)}{4}+\frac{(b_i-a_i)(\alpha_j+\beta_j)}{12}+\frac{(b_j-a_j)(\alpha_i+\beta_i)}{12}+\frac{(\alpha_i+\beta_i)(\alpha_j+\beta_j)}{24}$$

Corollary 2. Let $\tilde{A}_i=\left(a_i,b_i;\alpha_i,\alpha_i',\beta_i,\beta_i'\right)$, $\tilde{A}_j=\left(a_j,b_j;\alpha_j,\alpha_j',\beta_j,\beta_j'\right)$ be two IvFNs, then the following properties hold.

1). $\left(\tilde{A}_i+\tilde{A}_j\right)$ is an IvFN, and $\left(\tilde{A}_i+\tilde{A}_j\right)=\left(a_i+a_j,b_i+b_j;\alpha_i+\alpha_j,\alpha_i'+\alpha_j',\beta_i+\beta_j,\beta_i'+\beta_j'\right)$;

2). $\left(\tilde{A}_i-\tilde{A}_j\right)$ is an IvFN, and $\left(\tilde{A}_i-\tilde{A}_j\right)=\left(a_i-b_j,b_i-a_j;\alpha_i+\beta_j,\alpha_i'+\beta_j',\beta_i+\alpha_j,\beta_i'+\alpha_j'\right)$;

3). For any $k\ge 0$, $k\tilde{A}_i$ is an IvFN, and $k\tilde{A}_i=\left(ka_i,kb_i;k\alpha_i,k\alpha_i',k\beta_i,k\beta_i'\right)$.

According to Definitions 1 - 4, it is easy to deduce the following properties:

Property 1. Let $\left\{\tilde{A}_i=\left(a_i,b_i;\alpha_i,\alpha_i',\beta_i,\beta_i'\right)\right\}|\,i=1,\ldots,K$ be a series of IvFNs, for any real number x_i, then

$$M\left(\sum_{i=1}^K x_i\tilde{A}_i\right)=\sum_{i=1}^K x_iM\left(\tilde{A}_i\right). \tag{3}$$

Property 2. Let $\{\tilde{A}_i = (a_i, b_i; \alpha_i, \alpha_i', \beta_i, \beta_i')\} \mid i = 1, \ldots, K$ be a series of IvFNs, for any non-negative real number x_i, then

$$Var\left(\sum_{i=1}^{K} x_i \tilde{A}_i\right) = \sum_{i=1}^{K} x_i^2 Var\left(\tilde{A}_i\right) + 2 \sum_{i=1}^{K}\sum_{j>i}^{K} x_i x_j \, cov\left(\tilde{A}_i, \tilde{A}_j\right). \quad (4)$$

3. Construction of Interval-Valued Fuzzy Portfolio Decision Model

Suppose an investor with initial wealth of 1 wants to invest in n risky assets $\{S_1, S_2, \ldots, S_n\}$ and a risk-free asset. Assume x_i is the capital ratio invested in asset $i(i = 1, \ldots, n)$, and x_0 is the capital ratio invested in the risk-free asset. Due to the subjective experience and investor's emotion, the expected return of risk asset $i(i = 1, \ldots, n)$ is assessed by IvFN $\tilde{r}_i = (a_i, b_i; \alpha_i, \alpha_i', \beta_i, \beta_i')$, and the return of risk-free asset r_0 is denoted by a constant.

So, the possibilistic mean of the expected return of portfolio can be calculated as $M(\tilde{R}_p) = \sum_{i=1}^{n} x_i M(\tilde{r}_i) + r_0 x_0$.

To make the portfolio $X = (x_0, x_1, \ldots, x_i, \ldots, x_n)$ more diversified, we can set the following information entropy constraint $En(X) = -\sum_{i=0}^{n} x_i \ln x_i \geq H$, where H represents the minimum proportional entropy threshold of portfolio.

Also, considering the expected maximum loss value of portfolio at a given confidence level $\gamma \in (0,1)$, we set the VaR constraint $Pos\left(\sum_{i=1}^{n} x_i \tilde{r}_i \leq -VaR\right) \leq 1-\gamma$, where Pos represents the possibility and VaR is a positive number.

Based on the above analysis, we can construct the following interval-valued fuzzy portfolio decision model (P1):

$$(P1)\ min\, \sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_{\tilde{r}_i}^2 + 2\sum_{i=1}^{n}\sum_{j>i}^{n} x_i x_j \, cov\left(\tilde{r}_i, \tilde{r}_j\right)$$

$$s.t.\begin{cases} \sum_{i=1}^{n} x_i M(\tilde{r}_i) + r_0 x_0 \geq \bar{r} \\ -\sum_{i=1}^{n} x_i \ln x_i \geq H \\ Pos\left(\sum_{i=1}^{n} x_i \tilde{r}_i \leq -VaR\right) \leq 1-\gamma \\ \sum_{i=1}^{n} x_i = 1 \end{cases}$$

where $\bar{r} \geq 0$ is the minimum desired return level of portfolio.

If the expected return of risk asset i is evaluated by IvFN $\tilde{r}_i$, then the portfolio return of n risky assets is $\sum_{i=1}^{n} x_i \tilde{r}_i = \left[\sum_{i=1}^{n} x_i \tilde{r}_i^L, \sum_{i=1}^{n} x_i \tilde{r}_i^U\right] = \left(\sum_{i=1}^{n} x_i a_i, \sum_{i=1}^{n} x_i b_i; \sum_{i=1}^{n} x_i \alpha_i, \sum_{i=1}^{n} x_i \alpha_i', \sum_{i=1}^{n} x_i \beta_i, \sum_{i=1}^{n} x_i \beta_i'\right)$, and its interval-valued fuzzy membership function is $\mu_{\sum_{i=1}^{n} x_i \tilde{r}_i}(t) = \left[\mu^L_{\sum_{i=1}^{n} x_i \tilde{r}_i}(t), \mu^U_{\sum_{i=1}^{n} x_i \tilde{r}_i}(t)\right]$. Assume the value-at-risk $VaR = \left[VaR^L, VaR^U\right]$ is an interval number, the VaR constraint $pos\left(\sum_{i=1}^{n} x_i \tilde{r}_i \leq -VaR\right) = 1-\gamma$ in model (P1) is equivalently transformed to the following:

$$Pos\left(\sum_{i=1}^{n} x_i \tilde{r}_i^L \leq -VaR^U\right) = \sup_{t \leq -VaR^U}\left\{\mu^L_{\sum_{i=1}^{n} x_i \tilde{r}_i}(t)\right\} = \begin{cases} 1 - \dfrac{\sum_{i=1}^{n} x_i a_i + VaR^U}{\sum_{i=1}^{n} x_i \alpha_i'} & , \sum_{i=1}^{n} x_i a_i > -VaR^U \\ 1 & , \sum_{i=1}^{n} x_i a_i \leq -VaR^U \end{cases} \leq 1-\gamma;$$

$$Pos\left(\sum_{i=1}^{n} x_i \tilde{r}_i^U \leq -VaR^L\right) = \sup_{t \leq -VaR^L}\left\{\mu^U_{\sum_{i=1}^{n} x_i \tilde{r}_i}(t)\right\} = \begin{cases} 1 - \dfrac{\sum_{i=1}^{n} x_i a_i + VaR^L}{\sum_{i=1}^{n} x_i \alpha_i'} & , \sum_{i=1}^{n} x_i a_i > -VaR^L \\ 1 & , \sum_{i=1}^{n} x_i a_i \leq -VaR^L \end{cases} \leq 1-\gamma;$$

The above two constraints are equivalent to the following two VaR inequalities:

$$VaR^L \geq \gamma \sum_{i=1}^{n} x_i \alpha_i' - \sum_{i=1}^{n} x_i a_i \quad (5)$$

$$VaR^U \geq \gamma \sum_{i=1}^{n} x_i \alpha_i - \sum_{i=1}^{n} x_i a_i \quad (6)$$

Hence, with formulas (1) - (4) and inequality constraint (5), model (P1) is converted to the following portfolio model (P2):

$$(P2)\,min\ \sigma_p^2 = \frac{1}{4}\left[\sum_{i=1}^{n} x_i\left(b_i - a_i\right)\right]^2$$
$$+\frac{1}{12}\left[\sum_{i=1}^{n} x_i\left(b_i - a_i\right)\right]\left[\sum_{i=1}^{n} x_i\left(\alpha_i + \beta_i + \alpha_i' + \beta_i'\right)\right]$$
$$+\frac{1}{48}\left[\sum_{i=1}^{n} x_i\left(\alpha_i + \beta_i\right)\right]^2 + \frac{1}{48}\left[\sum_{i=1}^{n} x_i\left(\alpha_i' + \beta_i'\right)\right]^2$$

$$s.t.\begin{cases} \sum_{i=1}^{n} x_i\left[\frac{a_i + b_i}{2} + \frac{\beta_i + \beta_i' - \left(\alpha_i + \alpha_i'\right)}{12}\right] + r_0 x_0 \ge \bar{r} \\ -\sum_{i=0}^{n} x_i \ln x_i \ge H \\ \sum_{i=1}^{n} x_i\left(\gamma \alpha_i' - a_i\right) \le VaR^L \\ \sum_{i=0}^{n} x_i = 1 \end{cases}$$

Similarly, if the VaR constraint of model (P2) is replaced by an inequality constraint (6), the corresponding interval fuzzy portfolio model (P3) can be obtained (The specific form of model P3 is omitted for space).

4. Empirical Analysis of Portfolio Decision Model

To verify the effectiveness of the proposed interval-valued fuzzy portfolio decision model, we select Jiuan Medical (002432), Mingde Biological (002932), BYD (002594), Gree Electric Appliances (000651), Kweichow Moutai (600519) and SAIC Motor (600104) as risky assets from China's CSI 300 index. Bank saving deposit is selected as risk-free investment product, and the average weekly risk-free return rate r_0 is 0.0003208. We collect the weekly return rate of the sample stocks from January 2018 to November 2023, and evaluate the interval-valued fuzzy return of stock i as $\tilde{r}_i = \left(a_i, b_i; \alpha_i, \beta_i, \alpha_i', \beta_i'\right)$ by Vercher's statistical method [19], where a_i is P_{40} percentile and b_i is P_{60} percentile of the return rate of sample stocks, $\alpha_i = P_{40} - P_5$, $\alpha_i' = P_{40} - P_3$, $\beta_i = P_{95} - P_{60}$, $\beta_i' = P_{97} - P_{60}$. The evaluation of interval-valued fuzzy returns for all the risky assets are shown in Table 1.

In the portfolio model (P2) the information entropy threshold H is set as 1.2, confidence level γ is 0.9, and VaR^L is 0.081. By using LINGO nonlinear optimization software we can solve the optimal portfolio strategies under different desired returns as shown in Table 2. From Table 2 one can see that the risk of portfolio increases with the growth of desired return. When the desired return is greater than 0.006, the portfolio is mainly concentrated in risky assets. When the desired return is less than 0.006, the risky assets of portfolio are relatively diversified; and when the desired return rises by 0.001, the risk of portfolio increases less. Therefore, it is appropriate for a neutral investor to set the minimum desired return at 0.006.

Next, we will discuss the impact of different information entropy, VaR and confidence levels on the portfolio.

Table 1. Evaluation of the Expected Interval-valued Fuzzy Returns of Stocks

Assets	$\tilde{r}_i = (a_i, b_i; \alpha_i, \beta_i, \alpha_i', \beta_i')$	Possibilistic Mean	Possibilistic Variance
1	(-0.0103,0.0099,0.0845, 0.1169,0.0986,0.1901)	0.0101	0.0035
2	(-0.0131,0.0124,0.1031, 0.1525,0.1157,0.2290)	0.0132	0.0053
3	(-0.0109,0.0087,0.0844, 0.1030,0.0997,0.1589)	0.0054	0.0030
4	(-0.0095,0.0088,0.0526, 0.0720,0.0653,0.1041)	0.0045	0.0015
5	(-0.0012,0.0160,0.0627, 0.0581,0.0720,0.1035)	0.0096	0.0014
6	(-0.0080,0.0104,0.0598, 0.0547,0.0697,0.0849)	0.0020	0.0013

4.1. The Impact of Information Entropy Thresholds on Portfolio Strategy

The degree of diversification of a portfolio can be characterized by information entropy threshold and the larger is entropy value the more diversified is portfolio. So, we keep parameters $\bar{r} = 0.006$, $VaR^L = 0.081$, $\gamma = 0.9$, and set different information entropy thresholds to solve the corresponding portfolio strategies as shown in Table 3.

Table 2. The Optimal Portfolio Strategy for Model (P2) with Different Desired Return

$\bar{r}$	0.004	0.005	0.006	0.007	0.009
$\sigma_{\tilde{R}}^2$ (%)	0.0342	0.0490	0.0689	0.0944	0.1691
x_1	0.0383	0.0412	0.0463	0.0554	0.0895
x_2	0.0281	0.0351	0.0454	0.0629	0.1478
x_3	0.0123	0.0085	0.0070	0.0067	0.0079
x_4	0.0476	0.0372	0.0318	0.0291	0.0251
x_5	0.2833	0.3866	0.4784	0.5538	0.6169
x_6	0.0263	0.0163	0.0119	0.0097	0.0071
x_0	0.5642	0.4751	0.3793	0.2824	0.1058

Table 3. The Optimal Portfolio Strategy for Model (P2) with Different Information Entropy thereshold

H	1	1.1	1.2	1.3	1.4
$\sigma_{\tilde{R}}^2$ (%)	0.0617	0.0651	0.0689	0.0735	0.0849
x_1	0.0268	0.0364	0.0463	0.0566	0.0670
x_2	0.0264	0.0358	0.0454	0.0552	0.0649
x_3	0.0023	0.0042	0.0070	0.0107	0.0157
x_4	0.0162	0.0235	0.0318	0.0411	0.0515
x_5	0.5362	0.5082	0.4784	0.4469	0.4136
x_6	0.0045	0.0077	0.0119	0.0173	0.0242
x_0	0.3875	0.3842	0.3793	0.3723	0.3629

From Table 3 one can see that the portfolio strategy becomes more diversified as the information entropy threshold increases. If H is less than 1.2 the investment is more concentrated. If H is greater than 1.2 the investment becomes more diversified, but the total funds invested in risky assets increases, resulting in a small growth of portfolio risk. Thus, it is appropriate for investor to set the information entropy threshold as 1.2 in model (P2).

4.2. The Impact of Different Lower Bounds of Value-at-Risk on Portfolio Strategy

The impact of the value-at-risk constraint on the portfolio strategy is mainly determined by the value of VaR and confidence level. If we keep $\bar{r}=0.006$, $\gamma=0.9, H=1.2$ and set different values of VaR^L, we can solve model (P2) to get the corresponding optimal portfolio strategies as shown in Table 4. From Table 4, it can be shown that the funds invested in all the risky assets become smaller as the value of VaR^L decreases, resulting in the reduction of portfolio risk.

Also, if we keep other parameters in model (P2) unchanged, i.e. $\bar{r}=0.006$, $VaR^L=0.081$, $H=1.2$, and set different confidence level we can also solve the corresponding optimal portfolio strategies as shown in Table 5. From Table 5, one can see that the risk and return of portfolio decrease with the increasing of confidence level, which conforms to the basic investment principle.

Table 4. The Optimal Portfolio Strategy for Model (P2) with Different Lower Bounds of Value-at-risk

VaR^L	0.081	0.071	0.062	0.053	0.046
σ_R^2 (%)	0.1932	0.1493	0.1165	0.0874	0.0699
x_1	0.0108	0.0056	0.0024	0.0045	0.0352
x_2	0.0022	0.0006	0.0002	0.0005	0.0219
x_3	0.3830	0.1260	0.0489	0.0302	0.0138
x_4	0.0119	0.0272	0.0243	0.0295	0.0397
x_5	0.3347	0.4558	0.5149	0.5455	0.5123
x_6	0.2572	0.3637	0.2989	0.1614	0.0284
x_0	0.0002	0.0212	0.1104	0.2284	0.3486

Table 5. The Optimal Portfolio Strategy for Model (P2) with Different Confidence Level

γ	0.8	0.85	0.9	0.95
σ_R^2 (%)	0.2412	0.2150	0.1932	0.1746
x_1	0.0433	0.0222	0.0108	0.0077
x_2	0.0266	0.0078	0.0022	0.0011
x_3	0.5903	0.4962	0.3830	0.2673
x_4	0.0163	0.0133	0.0119	0.0150
x_5	0.1713	0.2639	0.3347	0.3899
x_6	0.1521	0.1965	0.2572	0.3178
x_0	0.0001	0.0001	0.0002	0.0012

4.3. The Impact of Different Upper Bounds of Value-at-Risk on Portfolio Strategy

Assume the value-at-risk constraint in model (P3) is inequality (6), $\sum_{i=1}^{n} x_i(\gamma\alpha_i - a_i) \le VaR^U$, and keep other parameters unchanged, $\bar{r}=0.006$, $\gamma=0.9$, $H=1.2$, then we can set different values of VaR^U and solve the corresponding optimal portfolio strategies as shown in Table 6.

From Table 6 one can see that the total capital invested in risky assets is close to 1 under the constraint of VaR^U. When the value of VaR^U increases, the risk of the portfolio also increases, which is also consistent with the investment principle.

Table 6. The Optimal Portfolio Strategy for Model (P3) with Different Upper Bounds of Value-at-risk

VaR^U	0.084	0.074	0.065	0.056	0.049
σ_R^2 (%)	0.2852	0.2145	0.1637	0.1231	0.0974
x_1	0.0457	0.0000	0.0116	0.0077	0.0068
x_2	0.1666	0.0471	0.0291	0.0077	0.0057
x_3	0.5521	0.4459	0.1401	0.0518	0.0301
x_4	0.0095	0.0000	0.0034	0.0087	0.0097
x_5	0.0135	0.2562	0.4048	0.4972	0.5331
x_6	0.2126	0.2508	0.4077	0.3372	0.2249
x_0	0.0001	0.0000	0.0033	0.0896	0.1897

Table 7. The Optimal Portfolio Strategy for Model (P3) with Different Confidence Level

γ	0.8	0.85	0.9	0.95
σ_R^2 (%)	0.3759	0.3241	0.2850	0.2535
x_1	0.0817	0.0559	0.0457	0.0030
x_2	0.4464	0.2740	0.1666	0.1276
x_3	0.3854	0.5235	0.5521	0.5186
x_4	0.0089	0.0077	0.0095	0.0000
x_5	0.0086	0.0089	0.0135	0.1024
x_6	0.0689	0.1300	0.2126	0.2484
x_0	0.0000	0.0000	0.0001	0.0000

Besides, if parameters $\bar{r}=0.006$, $VaR^U=0.084, H=1.2$ remain unchanged and given different confidence levels, we can solve the corresponding optimal portfolio strategies of model (P3) as shown in Table 7. From Table 7 one can see that the risk of portfolio decreases as the confidence level increases, which is also in line with the investment principle.

Furthermore, when the values of H and γ are set to be 1.2 and 0.9 respectively, we can also use LINGO software to find the efficient frontiers of model (P2) with the lower bound constraint of VaR and model (P3) with the upper bound constraint of VaR, respectively, as shown in Fig. 1. From Fig. 1, one can see that the risk of portfolio strategy obtained by model (P2) is smaller than that of model (P3) if the expected return of portfolio is given, which indicates that the proposed portfolio model (P2) is better than the model (P3) in practical applications.

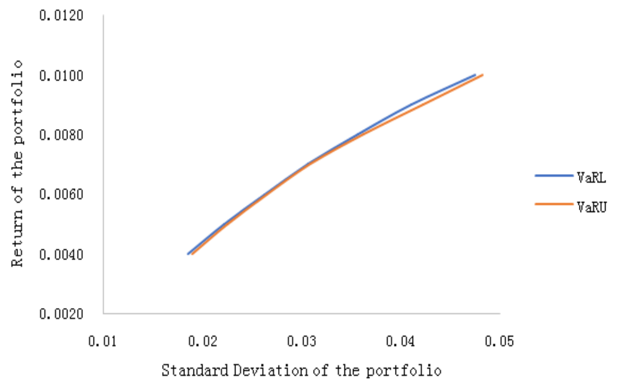

Figure 1. Comparison of efficient frontiers of portfolios with different value-at-risk constraints

5. Comparative Analysis with Existing Model

To further elaborate the superiority of our proposed portfolio model, we substitute the data of the same candidate securities into the existing fuzzy portfolio model proposed by Li [6], and make a comparative analysis of the effect of optimal portfolio strategies.

We transform the interval-valued fuzzy return $\tilde{r}_i = \left(a_i, b_i; \alpha_i, \beta_i, \alpha_i^{'}, \beta_i^{'}\right)$ of the selected stock $i\left(1 \le i \le 6\right)$ into trapezoidal fuzzy return, and then substitute it into the proposed model by Li [6]. Assume the confidence level γ is 0.9, VaR is 0.074, and the capital lower and upper bound vectors of all the alternative assets are $l = (0, 0.03, 0.06, 0, 0.03, 0.3, 0)$ and $u = (0.5, 0.1, 0.3, 0.1, 0.1, 0.5, 0.1)$, respectively, we can solve the optimal portfolio strategies under different desired return levels as shown in Table 8.

Comparing Table 2 and Table 8, one can see that the portfolio strategy obtained by our model (P2) is less risky than that obtained by Li's model [6] under the same desired return. Moreover, the diversification of the obtained portfolio of our model (P2) is higher than that of Li's [6] model because the entropy constraint added in our model can greatly increase the diversification and reduce the risk of portfolio. Additionally, we consider the upper and lower bound constraints of value-at-risk, which can improve the adaptability of the proposed portfolio model.

Table 8. The optimal portfolio strategies obtained by Li's model [6]

$\bar{r}$	0.003	0.004	0.005	0.006
σ_R^2 (%)	0.0339	0.0465	0.0864	0.1428
x_1	0.0300	0.0300	0.0300	0.0371
x_2	0.0600	0.0600	0.1725	0.3000
x_3	0.0000	0.0000	0.0000	0.0000
x_4	0.0300	0.0300	0.0300	0.0300
x_5	0.3174	0.4765	0.5000	0.5000
x_6	0.0626	0.0000	0.0000	0.0000
x_0	0.5000	0.4035	0.2675	0.1329

6. Conclusion

Due to the large amount of uncertain information and subjective emotion in the investment process, this paper employs interval-valued fuzzy numbers to evaluate the uncertain returns of risky assets and construct a new portfolio decision model. Compared with the traditional mean-variance portfolio model, the proposed portfolio model adds the constraints of information entropy and value-at-risk, which makes the portfolio assets more diversified and gets the desired expected return with lower risk value. Empirical analysis shows that the settings of different information entropy threshold, value-at-risk and confidence levels have different impacts on portfolio performance. Therefore, it is necessary to choose appropriate parameters according to investors' risk preferences and psychological emotions in the actual portfolio decision model.

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