American Journal of Mechanical Engineering, 2024, Vol. 12, No. 3, 26-34
Available online at http://pubs.sciepub.com/ajme/12/3/1
Published by Science and Education Publishing
DOI:10.12691/ajme-12-3-1

Formulation and Simulation for Speed Control Humps

Fenghui Shi*

Department of Mechanical Engineering, National Institute of Technology, Akashi College,
Japan 679-3 Nishioka, Uozumi-cho, Akashi, Hyogo, 674-8501 Japan
*Corresponding author: shi@akashi.ac.jp

Received August 05, 2024; Revised September 07, 2024; Accepted September 13, 2024

Abstract In this study, mathematical models for conventional speed control humps, including circular, sinusoidal, and modified sinusoidal humps with lengths of 4m, 6.5m, and 9.5m, have been established. Additionally, mathematical models for circular, sinusoidal, normal distribution, trapezoidal, and flat-top sign humps applicable to any length and height are proposed. Simulations and comparisons have been conducted using five types of vehicles to evaluate the speed control humps and the maximum accelerations experienced by vehicles as they pass over them. The results demonstrate that the mathematical models for both conventional speed control humps and the proposed formulations—circular, sinusoidal, normal distribution, trapezoidal, and flat-top sign profiles—are effective for designing speed control humps of any length and height.

Keywords: *speed control hump, conventional, sinusoidal, normal distribution, trapezoidal, flat top sign, circular, round-top hump*

Cite This Article: Fenghui Shi, "Formulation and Simulation for Speed Control Humps." *American Journal of Mechanical Engineering*, vol. 12, no. 3 (2024): 26-34. doi: 10.12691/ajme-12-3-1.

1. Introduction

Traffic control and safety is still a serious problem for almost all the countries in the world. As well known, one of the main accidents reason is excessive speed, thousands of people die or have severe injuries by traffic accidents each year. According to the report of the Department for Transport in the UK (2010), the risk increases slowly until crash speeds of around 30mph. Above this speed, risk increases rapidly, between 3.5 and 5.5 times from 30mph to 40mph [1]. Reference [2,3] and [4] have shown that there is an important statistical relationship between speed variance and accident rate. Thus controlling the travelling speed of vehicles not to be excessive is very important.

Speed control humps are known as the most popular traffic calming devices, which are the raised area from the pavement, installed transversely to the traffic flow, are used for resident area for reducing the speed of vehicles which are traveling faster than the designated limit. According the hump profiles, speed control humps can be classified round-top and flat-top speed humps. In the round-top, there are circular, sinusoidal, parabolic, cycloidal and harmonic profile road humps. The original work on speed reducing road humps was carried out in the 1970's at TRRL [5] on circular profile (round-top) humps of various dimensions. The oldest design shape is the Watts profile or called circular speed hump. It is circular section of a 3.7m long and a height of 0.075 to 0.1m extending over the whole width of the road [6].

Accordingly, another design developed in the United States is the Seminole profile or called flat-top hump. The design is characterized by the addition of a 3-meter flat section into the Watts profile hump with a total length of 6.7m [6]. In the UK, the Road Hump Regulations [7], allow a maximum hump height of 100mm. Experience with round-top humps indicates that sinusoidal hump heights of greater than 100mm would cause grounding problems for cars with long wheelbases and low ground clearances when crossing humps of 3.7m or less in length [7]. The Danish guidelines for speed reducing measures in urban traffic areas show a wide variety of different road hump profiles: round-top, dome-shaped and flat-top (Danish Road Directorate, 1991). For round-top and dome shaped humps, the recommended height is 100mm and the length is 3.0m, 4.0m, 6.5m and 9.5m for use on roads where the desired speeds are 20kph, 30kph, 40kph and 50kph respectively. Sinusoidal humps have also been tried in Denmark [8]. Sinusoidal humps have also been used in New Zealand where it was reported [9] that 'the crossing of the hump whilst causing a significant reduction in speed appeared to be more comfortable than the 'Watts' profile round-top hump'.

About of designing speed control humps, there are many researches focused on the optimization of speed humps profiles [10,11,12,13,14]. Maemori and Sakamoto [12] applied multi-objective optimization methods with different car base lengths and various suspension systems. Maemori and Ando [10] determined the optimal circular hump dimensions in order to reduce the excessive shocks experienced by drivers of heavy vehicles passing current humps at normal speed and to compensate for the too few shocks experienced by light vehicles with semi-active suspensions passing over the same humps at excessive speeds. In an attempt to meet the intended objectives of these devices, many researchers have developed new optimal designs of speed control humps.

Other innovative solutions which can enhance the current speed humps could be found in the TER report (2004). Some of the solutions are smart speed bumps [15], intelligent speed humps [16], humps injected with smart materials (Barron and Gostout, 2013), and virtual humps [17,18] proposed an optimization method of speed hump.

Experiments reported in Moinat, [19] and Jarvis[20], have shown that as the hump length increases, peak accelerations tend to occur at higher speeds, and more linear dynamic effects are created. It has been concluded that longer humps exhibit better characteristics in terms of speed reduction.

Other studies have focused on the significance of the vehicle type and suggested that it should be taken into consideration in the course of hump profile design. As an instance, it has been mentioned that humps designed for transit buses at 25km/h would allow automobiles to cross them at 35km/h with no irritating shock [21].

The sinusoidal hump profile was based on a study by Delft Technical University using field experiments and a computer model that simulated the behaviour of a car and driver. The results indicated that traffic humps for 85th percentile crossing speeds of 30kph must be 4-5m long and that the sinusoidal hump profile was the most preferable of those tested [22]. Sinusoidal humps have been used in Dutch 30kph zones and are the recommended road hump profile (height 120mm, length 4.5 to 5m) for use on cycle routes because they are less 'annoying' for cyclists than the round-top or flat-top profiles [23].

In this study, mathematical models for conventional speed control humps, including circular, sinusoidal, and modified sinusoidal profiles with lengths of 4m, 6.5m, and 9.5m, have been developed.

Mathematical models for circular, sinusoidal, normal distribution, trapezoidal, and flat-top sign speed control humps for any length and height have been established.

Simulations and comparisons have been conducted using five types of vehicles to analyze speed control humps.

2. Conventional Speed Control Humps

In this section, nine types of humps, as shown in Table 1 and Figure 1, are used for formulation and simulation to compare the performance of speed control humps as vehicles (listed in Table 2) pass over them. Figure 1 illustrates the profiles of conventional round-top speed control humps listed in Table 1. All the speed humps have the same height of 0.1m, with lengths of 4m, 6.5m, and 9.5m.

We address speed control humps such as circular humps (dotted lines in Figure 1), sinusoidal humps (solid lines in Figure 1), and modified sinusoidal humps (dashed lines in Figure 1) to compare the performance of these three types through simulation.

2.1. Formulation of the Conventional Speed Control humps

The formulations of the conventional speed control humps are proposed approximate equations (1) to (6). The circular speed control humps are Quadratic Polynomial shown in equations (1), (4) and (7) for 4m, 6.5m and 9.5m long respectively. The sinusoidal and modified sinusoidal speed control humps are expressed as Quadratic Fourier Transform shown in equations (2) and (3) for 4m long, (5) and (6) for 6.5m long, (8) and (9) for 9.5m long humps. The height H is the function of the length for all kind of the humps. The formulations are quantitative approximate for the conventional speed control humps.

Circular speed control humps (L=4m, H=0.1m for 30kph)

$$H = -2.501\times10^{-2} L^2 + 1.000\times10^{-1} L + 1.078\times10^{-4} \tag{1}$$

Sinusoidal speed control humps (L=4m, H=0.1m for 30kph)

$$\begin{aligned} H &= 4.027\times10^{-2} - 2.018\times10^{-3}\cos(0.8354L) \\ &+ 2.013\times10^{-2}\sin(0.8354L) \\ &- 3.825\times10^{-2}\cos(1.671L) \\ &- 7.749\times10^{-3}\sin(1.671L) \end{aligned} \tag{2}$$

Modified Sinusoidal speed control humps (L=4m, H=0.1m for 30kph)

$$\begin{aligned} H &= 5.673\times10^{-2} - 5.026\times10^{-2}\cos(1.515L) \\ &+ 5.588\times10^{-3}\sin(1.515L) \\ &- 5.715\times10^{-3}\cos(3.030L) \\ &+ 1.287\times10^{-3}\sin(3.030L) \end{aligned} \tag{3}$$

Circular speed control humps (L=6.5m, H=0.1m for 40kph)

$$H = -4.521\times10^{-3} L^2 + 4.250\times10^{-2} L - 1.497\times10^{-4} \tag{4}$$

Sinusoidal speed control humps (L=6.5m, H=0.1m for 40kph)

$$\begin{aligned} H &= 5.138\times10^{-2} - 5.049\times10^{-2}\cos(0.9557L) \\ &- 6.116\times10^{-4}\sin(0.9557L) \\ &- 6.986\times10^{-4}\cos(1.911L) \\ &- 1.693\times10^{-5}\sin(1.911L) \end{aligned} \tag{5}$$

Modified Sinusoidal speed control humps (L=6.5m, H=0.1m for 40kph)

$$\begin{aligned} H &= 5.801\times10^{-2} - 4.995\times10^{-2}\cos(0.9207L) \\ &+ 5.170\times10^{-3}\sin(0.9207L) \\ &- 6.897\times10^{-3}\cos(1.841L) \\ &+ 1.443\times10^{-3}\sin(1.841L) \end{aligned} \tag{6}$$

Table 1. Half conventional hump profile dimensions (TRL report 377)Length (m)

	L=4m, *H*=0.1m *for* 30kph			*L*=6.5m, *H*=0.1m *for* 40kph			*L*=9.5m, *H*=0.1m *for* 50kph		
	Circular	Sinusoidal	Modified Sinusoidal	Circular	Sinusoidal	Modified Sinusoidal	Circular	Sinusoidal	Modified Sinusoidal
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.100	0.010	0.001	0.003	0.006	0.000	0.002	0.004	0.000	0.001
0.200	0.019	0.002	0.007	0.012	0.001	0.004	0.008	0.000	0.002
0.300	0.028	0.006	0.012	0.018	0.002	0.007	0.012	0.001	0.003
0.400	0.036	0.010	0.018	0.023	0.004	0.010	0.016	0.002	0.005
0.500	0.044	0.015	0.025	0.028	0.006	0.013	0.020	0.003	0.007
0.600	0.051	0.021	0.033	0.033	0.009	0.017	0.024	0.004	0.009
0.700	0.058	0.028	0.041	0.038	0.012	0.021	0.028	0.005	0.011
0.800	0.064	0.035	0.049	0.043	0.015	0.025	0.032	0.007	0.014
0.900	0.070	0.042	0.057	0.048	0.018	0.030	0.035	0.009	0.017
1.000	0.075	0.050	0.065	0.052	0.022	0.035	0.038	0.011	0.020
1.100	0.080	0.058	0.072	0.056	0.026	0.040	0.041	0.013	0.023
1.200	0.084	0.065	0.079	0.060	0.030	0.045	0.044	0.015	0.026
1.300	0.088	0.072	0.085	0.064	0.034	0.050	0.047	0.017	0.029
1.400	0.091	0.079	0.090	0.068	0.039	0.055	0.050	0.019	0.033
1.500	0.094	0.085	0.094	0.071	0.044	0.060	0.053	0.022	0.037
1.600	0.096	0.090	0.097	0.074	0.049	0.065	0.056	0.025	0.041
1.700	0.098	0.094	0.099	0.077	0.054	0.070	0.059	0.028	0.045
1.800	0.099	0.097	0.100	0.080	0.059	0.074	0.062	0.031	0.049
1.900	0.100	0.099	0.100	0.083	0.064	0.078	0.065	0.034	0.053
2.000	0.100	0.100	0.100	0.085	0.069	0.082	0.067	0.037	0.056
2.100				0.087	0.073	0.085	0.068	0.040	0.059
2.200				0.089	0.077	0.088	0.071	0.043	0.062
2.300				0.091	0.081	0.091	0.073	0.046	0.065
2.400				0.093	0.085	0.093	0.075	0.050	0.068
2.500				0.095	0.088	0.095	0.077	0.054	0.071
2.600				0.096	0.091	0.097	0.079	0.057	0.074
2.700				0/097	0.094	0.098	0.081	0.060	0.077
2.800				0.098	0.096	0.099	0.083	0.063	0.079
2.900				0.099	0.098	0.100	0.085	0.066	0.081
3.000				0.100	0.099	0.100	0.087	0.069	0.083
3.100				0.100	0.100	0.100	0.089	0.072	0.085
3.200				0.100	0.100	0.100	0.090	0.075	0.087
3.300				0.100	0.100	0.100	0.091	0.078	0.089
3.400							0.092	0.081	0.091
3.500							0.093	0.083	0.092
3.600							0.094	0.085	0.093
3.700							0.095	0.087	0.094
3.800							0.096	0.089	0.095
3.900							0.097	0.091	0.096
4.000							0.098	0.093	0.097
4.100							0.099	0.095	0.098
4.200							0.099	0.096	0.099
4.300							0.099	0.097	0.100
4.400							0.099	0.098	0.100
4.500							0.100	0.099	0.100
4.600							0.100	0.100	0.100
4.700							0.100	0.100	0.100

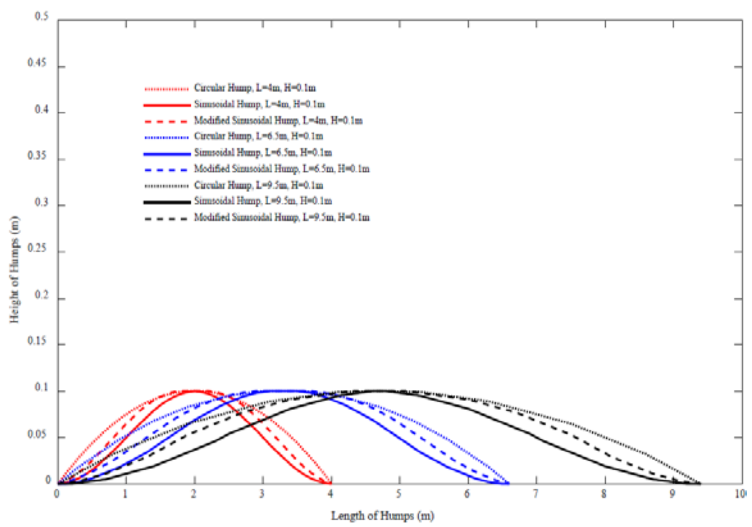

Figure 1. The profiles of Speed Control Hump

Circular speed control humps (L=9.5m, H=0.1m for 50kph)

$$H = -4.521\times10^{-3}L^2 + 4.250\times10^{-2}L - 1.497\times10^{-4} \quad (7)$$

Sinusoidal speed control humps (L=9.5m, H=0.1m for 50kph)

$$\begin{aligned} H &= 5.801\times10^{-2} - 4.995\times10^{-2}\cos(0.9207L) \\ &+ 5.170\times10^{-3}\sin(0.9207L) \\ &- 6.897\times10^{-3}\cos(1.841L) \\ &+ 1.443\times10^{-3}\sin(1.841L) \end{aligned} \quad (8)$$

Modified Sinusoidal speed control humps (L=9.5m, H=0.1m for 50kph)

$$\begin{aligned} H &= 5.642\times10^{-2} - 4.982\times10^{-2}\cos(0.6411L) \\ &+ 6.441\times10^{-3}\sin(0.6411L) \\ &- 6.749\times10^{-3}\cos(1.282L) \\ &+ 1.775\times10^{-3}\sin(1.282L) \end{aligned} \quad (9)$$

3. General Formulation of Speed Control Humps

The profile, length, and height of speed control humps are crucial factors in their design. The commonly used profiles for round-top speed control humps include sinusoidal, circular, and parabolic shapes. Each profile requires the optimization of two parameters: length L and height H as illustrated in Figure 1.

In order to do simulation and optimization of speed control hump easily, the formalization of all types hump need to be established. In this study, the qualitative formalizations are established as the equations (16) to (22). They are expressed by the functions of the hump length L and the height H. If input the data of L and H, the speed control hump with any length or height will be obtained. Example, if input L=4m, H=0.1m to equations (16), (17) and (18), the approximate circular hump profile with L=4m, H=0.1m will be expressed. The errors will be discussed in section 4.2.

3.1. Formulation of Circular Hump

Approximate formula of the circular hump shown in Figure 2 is expressed as Equation (10) and (11)

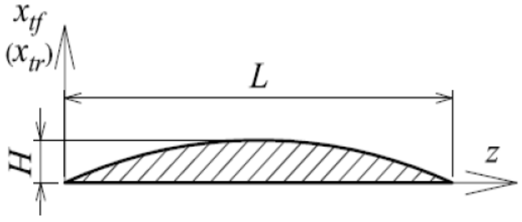

Figure 2. The profiles of Circular Hump

$$x_{tf} = \sqrt{-(-z+L/2)^2 + R^2} - L^2/(8H) + H/2 \quad (10)$$

$$x_{tr} = \sqrt{-\{-(z-L_w)+L/2\}^2 + R^2} - L^2/(8H) + H/2 \quad (11)$$

$$R = L^2/(8H) + H/2$$

Where x_{tf} and x_{tr} are the front and rear wheel vertical displacements, respectively, z is the forward displacement of vehicles, L and H are the length and height of the hump (Figure 6), L_w is the wheel base of vehicle (Figure 6). Those symbols are used in next section 3.2 to 3.5 as the same means.

3.2. Formulations of Sinusoidal Hump

Approximate formula of the sinusoidal hump is expressed as Equation (12) and (13)

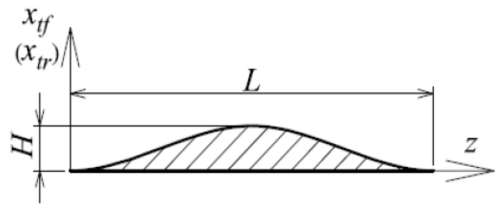

Figure 3. The profiles of Sinusoidal Hump

$$x_{tf} = \frac{H}{2}\left(1-\cos\frac{2\pi z}{L}\right) \quad (12)$$

$$x_{tr} = \frac{H}{2}\left\{1-\cos\frac{2\pi(z-L_w)}{L}\right\} \quad (13)$$

3.3. Formulations of Normal Distribution Hump

Approximate formulations of the normal distribution hump is expressed as Equation (14) and (15)

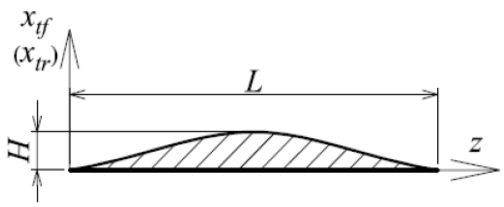

Figure 4. The profiles of Normal Distribution Hump

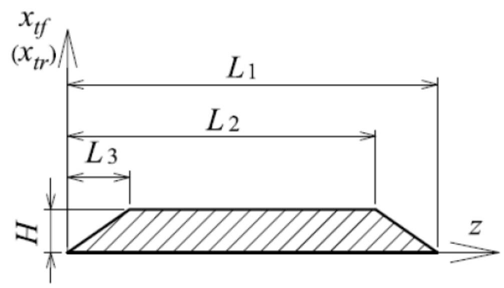

Figure 5. The profiles of Trapezoidal Hump

$$x_{tf} = \frac{1}{\sqrt{2\pi H}}\exp\left\{-\frac{1}{2}\left(\frac{z-L/2}{H}\right)^2\right\} - \frac{1}{\sqrt{2\pi H}}\exp\left\{-\frac{1}{2}\left(\frac{L}{2H}\right)^2\right\} \quad (14)$$

$$x_{tr} = \frac{1}{\sqrt{2\pi H}} \exp\left\{-\frac{1}{2}\left(\frac{(z-L_w)-L/2}{H}\right)^2\right\} - \frac{1}{\sqrt{2\pi H}} \exp\left\{-\frac{1}{2}\left(\frac{L}{2H}\right)^2\right\} \quad (15)$$

3.4. Formulations of Trapezoidal Hump

Approximate formulations of the Trapezoidal hump is expressed as Equation (16) and (17)

Table 2. Symbols and values of half vehicles parameter

Parameters of Vehicles	Standard Vehicle	Vehicle A	Vehicle B	Vehicle C	Vehicle D
The half mass of the vehicle body M_b (kg)	875(1750/2)	415(820/2)	765(1530/2)	1165(2530/2)	5050(10100/2)
Front unsprung mass M_f (kg)	55	35	43	75	149
Rear unsprung mass M_r (kg)	60	40	45	80	180
Inertia moment of the body around the center of gravity I_b (kgm^2)	1600	1000	1283.83	2400	20900
Stiffnesses of the front suspensions K_{sf} (N/m)	15000	12000	18606	25000	80500
Stiffnesses of the rear suspensions K_{sr} (N/m)	15000	12000	18606	25000	90500
Effective vertical stiffness of the front tires K_{tf} (N/m)	238180	180000	201021	250000	590000
Effective vertical stiffness of the rear tires K_{tr} (N/m)	238180	180000	201021	250000	590000
Damping coefficient of the front suspensions C_{sf} (Ns/m)	1776	1050	1189	3000	10950
Damping coefficient of the front suspensions C_{sr} (Ns/m)	1500	850	998	2500	9970
Distance of front wheel from the c.g. L_f (m)	1.370	1.210	1.250	1.350	3.095
Distance of rear wheel from the c.g. L_r (m)	1.470	1.310	1.350	1.500	3.000
Wheel base L_w (m)	2.840	2.520	2.700	2.850	6.095

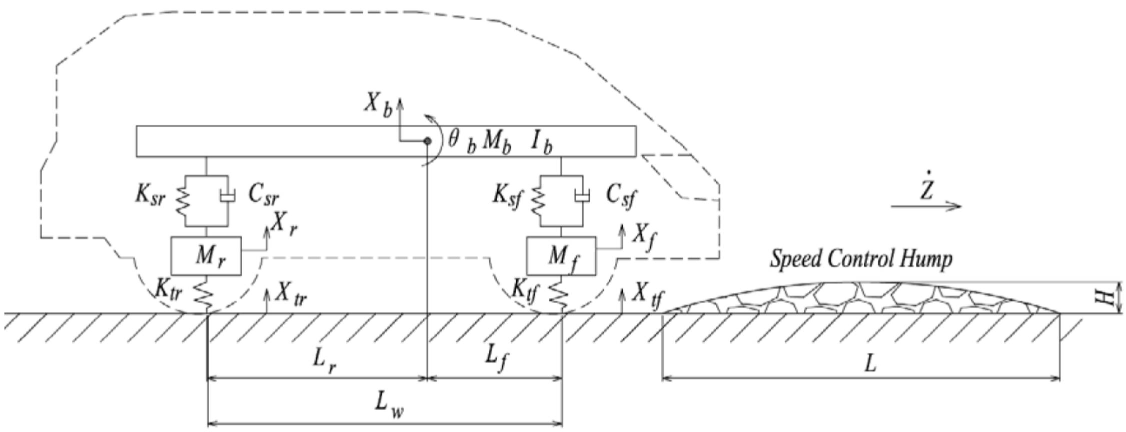

Figure 6. Vehicle dynamic model and Speed Control Hump

$$\begin{cases} x_{tf} = \dfrac{H}{L_1} z & (0 < z < L_1) \\ x_{tf} = H & (L_1 \le z < L_2) \\ x_{tf} = \dfrac{H}{(L_3 - L_2)}(L_3 - z) & (L_2 \le z < L_3) \end{cases} \quad (16)$$

$$\begin{cases} x_{tr} = \dfrac{H}{L_1}(z - L_w) & (L_w < z < L_1 + L_w) \\ x_{tr} = H & (L_1 + L_w \le z < L_2 + L_w) \\ x_{tr} = \dfrac{H}{(L_3 - L_2)}(L_3 - z - L_w) & (L_2 + L_w \le z < L_3 + L_w) \end{cases} \quad (17)$$

3.5. Formulations of Flat Top Sign Hump

Approximate formulations of the flat top sign hump is expressed as Equation (18) and (19)

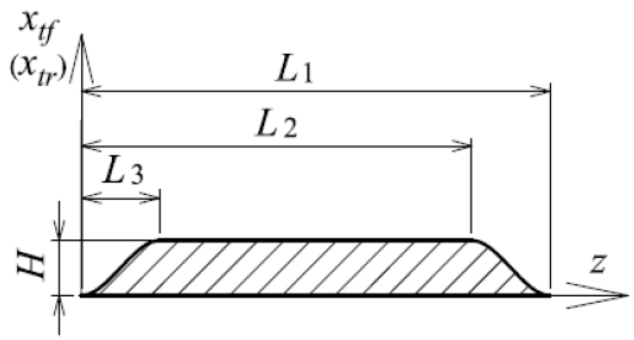

Figure 7. The profiles of Flat Top Sign Hump

$$\begin{cases} x_{tf} = \dfrac{H}{L_1}\left\{1-\cos\left(2\pi\dfrac{z}{L_1}\right)\right\} & (0 < z < L_1) \\ x_{tf} = H \quad (L_1 \le z < L_2) & \\ x_{tf} = \dfrac{H}{2}\left\{1+\cos\left(\pi\dfrac{z-L_2}{L_3-L_2}\right)\right\} & (L_2 \le z < L_3) \end{cases} \tag{18}$$

$$\begin{cases} x_{tr} = \dfrac{H}{L_1}\left\{1-\cos\left(2\pi\dfrac{z-L_w}{L_1}\right)\right\} & (L_w < z < L_1 + L_w) \\ x_{tr} = H \quad (L_1 + L_w \le z < L_2 + L_w) & \\ x_{tr} = \dfrac{H}{2}\left\{1+\cos\left(\pi\dfrac{z-L_w-L_2}{L_3-L_2}\right)\right\} & (L_2 + L_w \le z < L_3 + L_w) \end{cases} \tag{19}$$

4. Dynamic model, Equations of Motion and Parameters of a Half Vehicle

This section provides information on dynamics modeling of vehicle (Figure 6), equations of motion (equation20~24) and parameters (Table 2). In the dynamic model of vehicle (Figure 6), M_b , M_f , and M_r represent the mass of the vehicle body (sprung mass), and the front and rear unsprung masses, respectively. Stiffnesses for the front and rear suspensions and the effective vertical stiffnesses of the front and rear tires are represented by K_{sf} , K_{sr} , K_{tf} and K_{tr} . Damping due to tires is ignored as usual. C_{sf} and C_{sr} are damping coefficients of the front and rear suspensions. Bounce dynamics of the sprung mass, wheel hop dynamics of the front unsprung mass, wheel hop dynamics X_r of the rear unsprung mass, and pitch dynamics of the sprung mass of the half car model. X_{tf} and X_{tr} are the vertical displacements of front and rear tires. The equations of motion are given by equations of motion from equation (20) to (24).

Table 2 is the parameters of five kind of vehicles using in simulations. The parameters of the standard vehicle are using in optimization in the next study.

$$M_b\ddot{X}_b = K_{sf}X_{pf} + C_{sf}\dot{X}_{pf} + K_{sr}X_{pr} + C_{sr}\dot{X}_{pr} \tag{20}$$

$$M_r\ddot{X}_r = -K_{sr}X_{pr} + K_{tr}\left(X_{tr} - X_r\right) - C_{sr}\dot{X}_{pr} \tag{21}$$

$$I_b\ddot{\theta}_b = K_{sf}L_fX_{pf} - K_{sr}L_rX_{pr} + C_{sf}L_f\dot{X}_{pf} - C_{sr}L_r\dot{X}_{pr} \tag{22}$$

$$M_f\ddot{X}_f = -K_{sf}X_{pf} + K_{tf}\left(X_{tf} - X_f\right) - C_{sf}\dot{X}_{pf} \tag{23}$$

$$\begin{cases} X_{pf} = X_f - (X_b + L_f\theta_b) \\ \dot{X}_{pf} = \dot{X}_f - (\dot{X}_b + L_f\dot{\theta}_b) \end{cases} \tag{24}$$

5. Simulations and Discussions

5.1. Simulation of the Conventional speed Control humps

The results of the simulation for conventional speed control humps are shown in Figure 8, Figure 9, and Figure 10. The circular, sinusoidal, and modified sinusoidal humps are represented by solid, dashed, and dotted lines, respectively. The blue, green, red, and pink lines correspond to the simulation results for vehicles A, B, C, and D, respectively. The black lines represent the simulation results for a standard vehicle.

As is well known, the maximum vertical accelerations of vehicles increase with the speed at which they pass over a hump. There is a significant difference in maximum vertical accelerations between light (vehicle A) and heavy (vehicle D) vehicles. The maximum vertical accelerations for vehicle A (represented by blue lines) are more than twice as high as those for vehicle D (represented by pink lines) across all hump profiles. However, for all lengths of humps, vehicle D exhibits the smallest maximum vertical accelerations (see Figure 8, Figure 9, and Figure 10, pink lines). In contrast, vehicle A experiences the highest maximum vertical accelerations when passing over modified sinusoidal speed control humps (see Figure 8, Figure 9, and Figure 10, blue dotted lines).

5.2. Simulation of the Formulation Speed Control humps

To evaluate the qualitative formulas for the circular and sinusoidal speed control humps, as shown in equations 16 to 22, the curves of these speed control humps are compared with those of conventional speed control humps in Figure 8 and Figure 10. The red, blue, and black lines represent humps of lengths 4 meters, 6.5 meters, and 9.5 meters, respectively. The red, blue, and black lines correspond to hump lengths of 4 meters, 6.5 meters, and 9.5 meters, respectively. Solid lines represent the simulation results of the qualitative formulations, while dashed lines denote conventional speed control humps (see Table 1). For 4-meter length circular and sinusoidal humps, the profiles match perfectly between the qualitative formulations and conventional profiles (see Figure 11 and Figure 12, red solid and dashed lines). For 6.5-meter length circular and sinusoidal humps, the curves from the qualitative formulations (Figure 11 and Figure 12, blue solid lines) are smaller than the conventional profiles (Figure 11 and Figure 12, blue dashed lines) by 0.021% and 0.038%, respectively, in terms of section area error. For the 9.5m length circular and sinusoidal humps, the curves of the qualitative formulations profile (Figure 11 and Figure 12, black solid lines) are bigger than the conventional (Figure 11 and Figure 12, black broken lines) 0.014% and 0.032% (section area error) respectively. This means that the errors are enough smaller between the qualitative and conventional hump profiles.

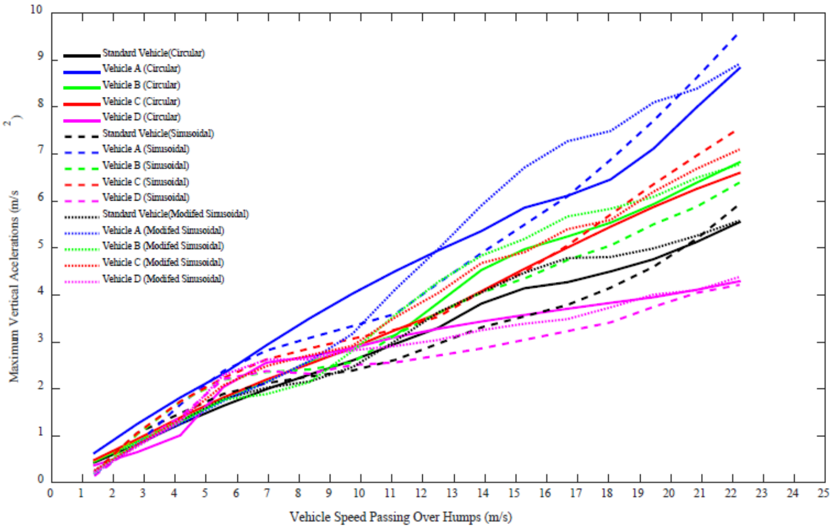

Figure 8. Simulations of Speed Control Hump (*L*=4m, *H*=0.1m)

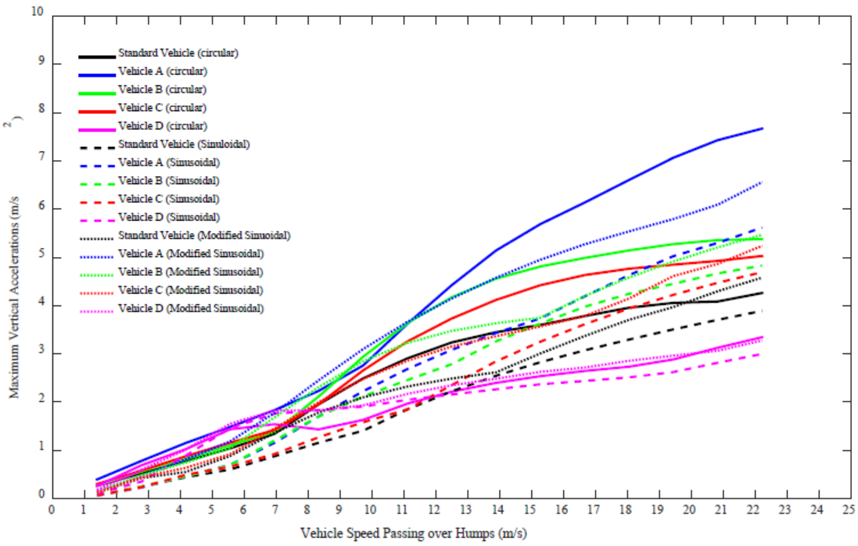

Figure 9. Simulations of Speed Control Hump (*L*=6.5m, *H*=0.1m)

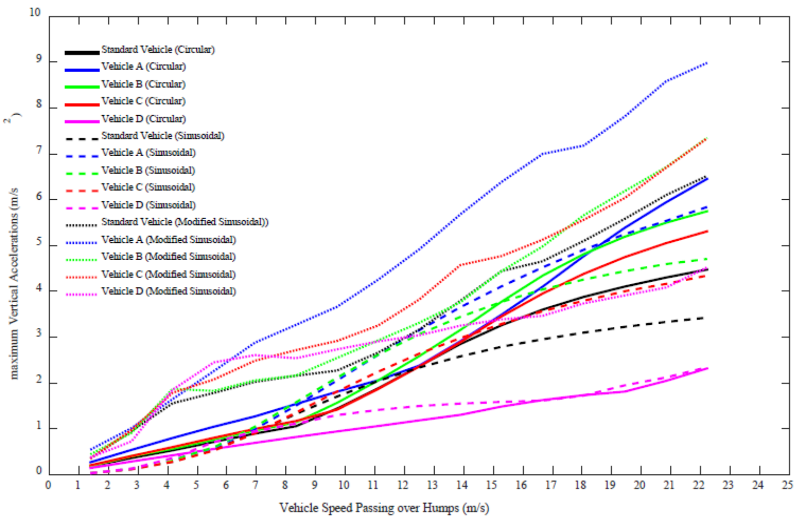

Figure 10. Simulations of Speed Control Hump (*L*=9.5m, *H*=0.1m)

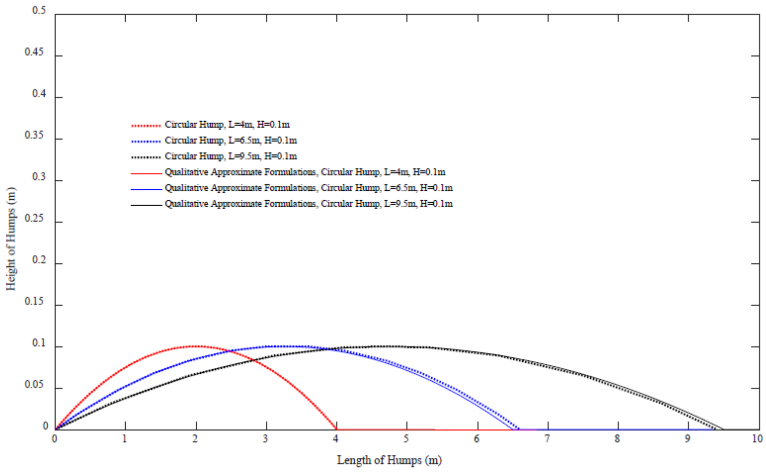

Figure 11. Comparison the Circular Hump profiles of Conventional Speed Control Humps(Table 1) and formula 1, 4 and 7 for formulations

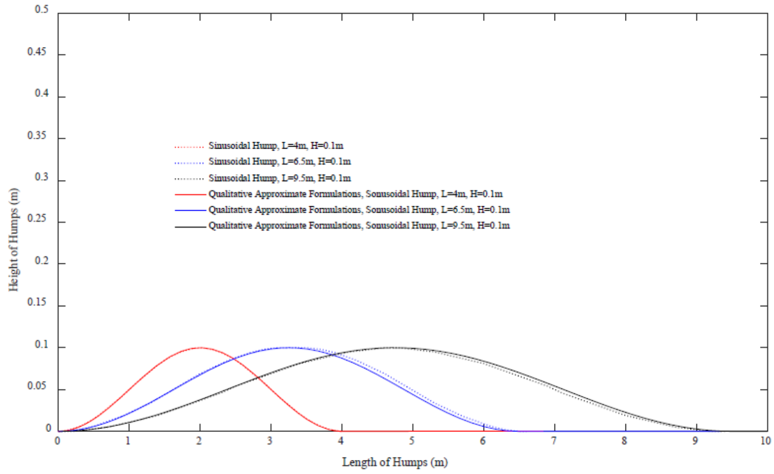

Figure 12. Comparison the Sinusiolar Hump profiles of Conventional Speed Control Humps(Table 1) and formula 2, 5 and 8 for formulations

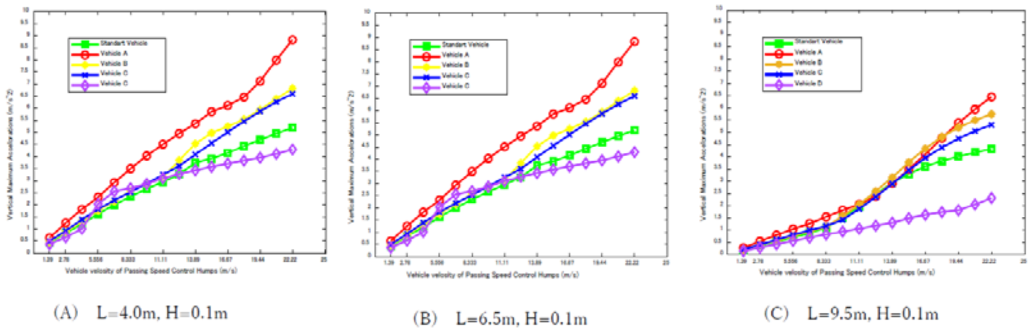

Figure 13. Comparison the Maximum Accelerations of 5 type Vehicles (Table 2) Passing over the Circular Speed Control Humps by formula 10 and 11 for Simulations

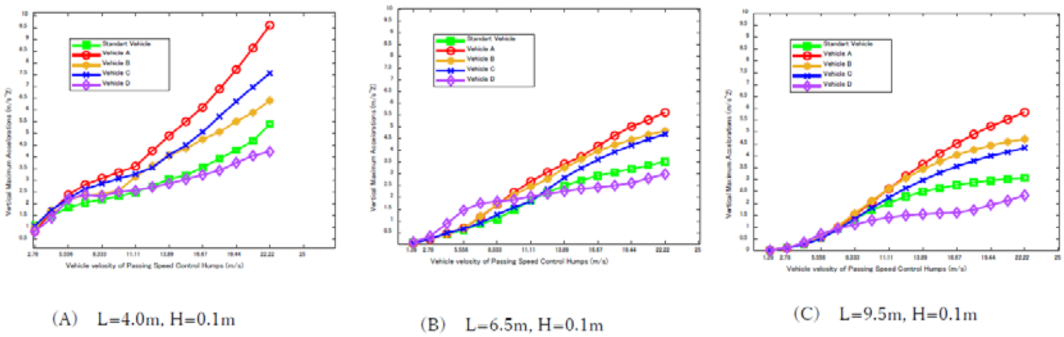

Figure 14. Comparison the Maximum Accelerations of 5 type Vehicles (Table 2) Passing over the Sinusiolar Speed Control Humps by formula 12 and 13 for Simulations

5.3. Simulation of the Vehicles

Figure13 and Figure 14 show the maximum accelerations of five type vehicles pass though the speed control humps with length 4m, 6.5m and 9.5m, the passing speed are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80kph. Figure 13a,13b,13c are the results of the circular hump profiles with 4m, 6.5m and 9.5m length and 0.1m height, respectively. Figure 14a,14b,14c are the results of the Sinusoidal Hump profiles with 4m, 6.5m and 9.5m length and 0.1m height, respectively.

For the circular humps (Figure 13), the accelerations of all the vehicles passing over 4m and 6.5m length speed control humps are bigger than 9.5m length. For the vehicles, the accelerations of the vehicle D is smallest for all the passing over speeds.

For the sinusoidal humps (Figure 14), the accelerations of all the vehicles passing over 4m length speed control humps are greater than 6.5m and 9.5m length. For the vehicles, the accelerations of the vehicle D is smallest for all the passing speeds.

6. Conclusions

A mathematical model for five types of vehicles has been developed to study the dynamics of vehicles crossing speed control humps. Based on the simulation results, the following conclusions and recommendations can be made:

1. The mathematical models for the conventional speed control humps are effective. The section area errors of the curves are smaller than those of the conventional profiles by 0.021% and 0.038%, respectively, for the 6.5-meter length humps. For the 9.5-meter length circular and sinusoidal humps, the section area errors are 0.014% and 0.032%, respectively.
2. Based on the simulations, speed control humps are effective in controlling speeding vehicles. The profile of the humps—whether circular, sinusoidal, normal distribution, trapezoidal, or flat-top sign—have the same effect for controlling vehicle speed.

The mathematical models for circular, sinusoidal, normal distribution, trapezoidal, and flat top speed control humps are effective. These formulas can be used to develop and study speed control humps of various lengths and heights, as well as to analyze the impact on vehicle suspensions when passing over these humps.

References

[1] Richards, D. C., 2010. Relationship between Speed and Risk of Fatal Injury. Pedestrians and Car Occupants.

[2] Pisarski, A.E., 1986. Deep-Six 55. Reason Foundation 17(6), 32-35.

[3] Ministry of Transport and Communications, 1974. Speed and Accidents, A Preliminary.

[4] Garber, N.J. and Gadiraju, R., 1989. Factors Influencing Speed Variance and its Influence.

[5] Watts, G. R., TRL 597, 1973. Road humps for the control of vehicle speeds, Transport and Road Research Laboratory.

[6] Weber, P. A. and Braaksma, J. P., 2000. Towards a North American geometric design standard for speed humps. Institute of Transportation Engineers (ITE) Journal 70, (1), 30–39.

[7] Webster D. C. and Layfield, R. E., 1998. Traffic calming — Sinusoidal, 'H' and 'S' humps". TRL REPORT 377.

[8] Lahrmann, H. and Mathiasen, P., 1992. Bumpudformning, (Hump design). Dansk Vejtidsskrift Nr 9, 16 - 22.

[9] Moses, P. J., 1992. Traffic management in New Zealand -1991 - A perspective. Main Roads Department, East Perth, Western Australia.

[10] Maemori and Ando, 1987. Alternate optimization of speed control hump for automobiles and automobile suspension (reducing excessive shocks to drivers of heavy vehicles by multiobjective optimization of hump). JSME Int. J., Ser. C, 40(1), 17–24.

[11] Maemori, K., 1988. Optimum design of speed control humps for vehicles. Trans. Japan Society of Mechanical Engineers 54(502 C), 1217-1224.

[12] Maemori, K. and Sakamoto, K., 1992. Multiobjective Optimum Design of Circular Humps for Vehicles. Trans. Japan Society of Mechanical Engineers (in Japanese), 58(548C), 1054-1063.

[13] Pedersen, N., 1998. Shape optimization of a vehicle speed control bump. Journal of Structural Mechanics 26(3), 319-342.

[14] Khorshid and Alfares, 2004. A numerical study on the optimal geometric design of speed control humps. Engineering Optimization 36(1), 77-100.

[15] USA today, Retrieved July 15, 2016 from [online]. http://www.usatoday.com/news/world/2009-10-05-.

[16] Shirley, E., 2005. Smart road hump will smooth the way for safe drivers. The Times online, November 11, Retrieved May 2, 2016 from [online] http://www.timesonline.co.uk/tol/news/article588894.ece.

[17] Matthew, M., 2008. Optical illusions used as virtual speed humps. The daily Telegraph, 17 Jun, Retrieved January 30, 2016 from [online]. http://www.telegraph.co.uk/news/newstopics/howaboutthat/2146818/Optical-illusions-used-as-virtual-speed-humps.html#comments.

[18] Caglar Başlamişli, S. and Samim Ünlüsoy, Y., 2009. Optimization of Speed Control Hump Profiles. Journal of Transportation Engineering 135(5).

[19] Moinat, P., Étude, et., Simulation du Passage d'un Véhicule Sur un Dos d'Âne: Prédiction de l'Inconfort, 1991. A Study and Simulation of Vehicle Passage on Speed Humps: Prediction of Discomfort). Master of Engineering Thesis, Université de Sherbrooke, Canada.

[20] Jarvis, J. R. "An investigation of road humps for use on bus routes: final report," No. ARR222, Australian Road Research Board, Melbourne, Australia, 1992, ISBN: 0-86910-544-2.

[21] Weber, P. and Braaksma, J., 2000. Towards a North American geometric design standard for speed humps. Institute of Transportation Engineers, ITE Journal 70(1), 30.

[22] De Wit, T., 1993. Dutch experiences with speed control humps. ITE 63rd Annual Meeting.

[23] Lines, C. J. and Castelijn, H. A., 1991. Translation of Dutch 30 kph zone design manual. Paper PA2046/91. Transport Research Laboratory, Crowthorne.