

Geometric Transformations in the Design of Assembly Systems

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Abstract The design of the assembly systems is an important role optimizing the spatial resolution. An important tool for the problem solving is approaches to mathematics based modeling. The article presents the basic geometric transformations to identify the location.

Keywords: modeling, assembly systems, scaling

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1. Introduction

The optimization of the spatial resolution is very important in the design of assembly systems. An important tool for the problem solving approaches is based on the mathematical modeling. For the creation and design assembly systems is significant systemic approach.

When exploring a systems approach involves the modeling of object decomposition into elemental components, examine their functional interaction and integration in the structure of the system [1,2].

The system approach is promising for identification of complex, interrelated functions of system elements, the elements of the system are treated as parts of an integrated whole system approach is an effective method for analysis and design solutions to complex problems. There are

applied two basic strategies for addressing of the problem in the systems approach: analysis and synthesis [3].

The analysis process is thought to spread to the whole or part of its conceptual distinction between the individual properties.

The system is divided into subsystems. In the process of fusion is to merge components phenomena objects, their properties and relations, solving partial problems are compiled into a functional whole [4]. Analysis and synthesis of integration allow you to create interfaces with various disciplines.

The current mathematics has developed a rich description of the apparatus for quantitative and qualitative relationships between variables. In addressing placement relationship is most often used mathematical theories listed in Table 1.

Table 1. The mathematical theory

Name	Areas of application
Graph theory	<ul style="list-style-type: none"> • Modeling of the economical and the technical systems • Description of the transport and utilities • Description of the material flow systems • Modeling the behavior of parallel discrete systems
Theory of sets	<ul style="list-style-type: none"> • Description of the technical systems - axiomatic theory of sets • Description of the terms with some degree of uncertainty - Fuzzy Sets
Measurement theory	<ul style="list-style-type: none"> • Quantification of qualitative characteristics of the aggregate valuation and the variations of individual awards • Problems objectification of the subjective information

The creating geometric models of technical objects are based of the basic geometric elements and their mathematical expression.

Topology is a branch of geometry that studies the properties of geometric shapes which are retained for each unique two-sided continuous mapping. The examining topological relationships of structural elements assembly systems can be advantageously used tools of the analytic geometry.

The analytic geometry using algebraic tools defines and studies different types of spaces (Euclidean space E, the affine space A, projective space P), which approximates the different meaning of actual physical space. The analytical geometry examines particular geometric properties of the spatial units of the analysis method, i.e. the spatial units expressed mathematically means (eg by coordinates).

2. Basic Geometric Transformations

Translation - characterized by a vector displacement, displacement of point X is the displacement vector (x, y), for new coordinates valid: $x_N = x + x_P$

$$y_N = y + y_P$$

expression of matrix transformation:

$$(x_N, y_N, 1) = (x, y, 1) * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_P & y_P & 1 \end{pmatrix} \quad (1.1)$$

$$(x_N, y_N, 1) = (x + x_P, y + y_P, 1).$$

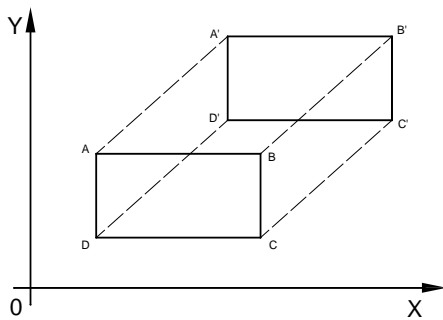


Figure 1. Translation

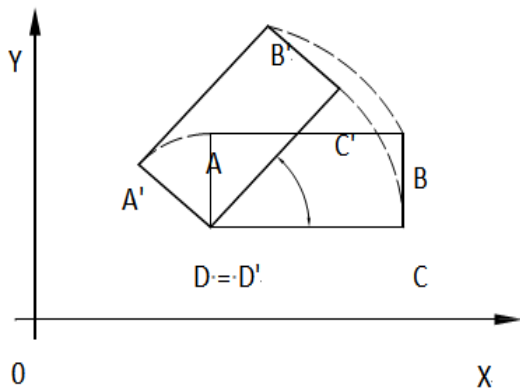


Figure 2. Rotation

Rotation - given center of the rotation and the angle of the rotation (increasing counterclockwise), for new coordinates valid:

$$x_N = x \cdot \cos \alpha - y \cdot \sin \alpha \quad y_N = x \cdot \sin \alpha + y \cdot \cos \alpha$$

expression of matrix transformation:

$$(x_N, y_N, 1) = (x, y, 1) * \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.2)$$

$$(x_N, y_N, 1) = (x \cdot \cos \alpha - y \cdot \sin \alpha, x \cdot \sin \alpha + y \cdot \cos \alpha, 1)$$

(* - matrix multiplication)

If we want to implement the rotation around any point, then we have to do the moving to the point O, then turning and pushing back into the original center of the rotation.

Scale - the scaling factors given in the direction of the axes coordinate system and the reference point (the point

whose position relative to the beginning of the coordinate system remains unchanged),

m - giving a constant size scaling

for new coordinates valid: $x_N = m_x \cdot x$

$$y_N = m_y \cdot y$$

expression of matrix transformation:

$$(x_N, y_N, 1) = (x, y, 1) * \begin{pmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.3)$$

$$(x_N, y_N, 1) = (x \cdot m_x, y \cdot m_y, 1).$$

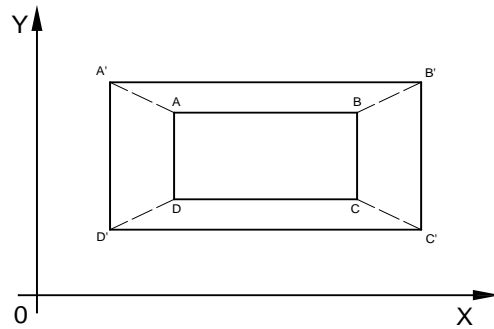


Figure 3. Scale

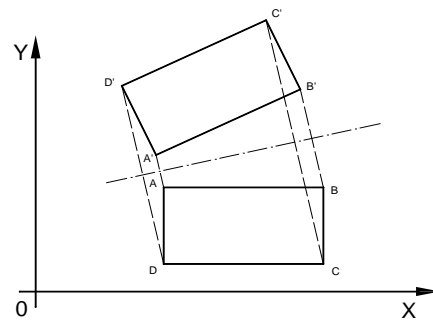


Figure 4. Reflections

Reflections - mirror point X can be carried out with respect to the axis of respect to the axis of or with respect to the beginning of the coordinate system,

for new coordinates valid:

- with regard to o_x : $x_N = x$, $y_N = -y$

- with regard to o_y : $x_N = -x$, $y_N = y$

- with regard to the center: $x_N = -x$, $y_N = -y$

transformation matrix representation with regard to

$$o_x: (x_N, y_N, 1) = (x, y, 1) * \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.4)$$

$$(x_N, y_N, 1) = (x_N, -y_N, 1).$$

transformation matrix representation with regard to

$$o_y: (x_N, y_N, 1) = (x, y, 1) * \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.5)$$

$$(x_N, y_N, 1) = (-x_N, y_N, 1).$$

transform. matrix representation relative to the center:

$$(x_N, y_N, 1) = (x, y, 1) * \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.6)$$

$$(x_N, y_N, 1) = (-x_N, -y_N, 1)$$

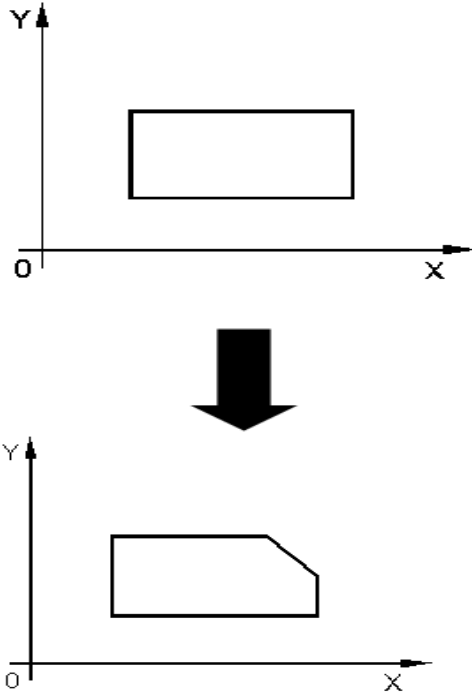


Figure 5. Slope

Slope - skewing point X is the coefficient of skew S, new coordinates for the skew in the x-axis applies:

$$x_N = x + S_x \cdot y \quad y_N = y$$

new coordinates for the skew in the y-axis applies:

$$x_N = x \quad y_N = x \cdot S_y + y$$

expression of matrix transformation:

$$(x_N, y_N, 1) = (x, y, 1) * \begin{pmatrix} 1 & S_y & 0 \\ S_x & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.7)$$

$$(x_N, y_N, 1) = (x + yS_x, x \cdot S_y + y, 1)$$

The combination of that elementary transformation can be expressed mathematically or any change in position scale geometric object.

When combined transformation does not apply to matrix multiplication commutative law. For example:

Translation and rotation:

expression of matrix transformation:

$$(x, y, 1) * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{pmatrix} * \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= ((x + x_p) \cos \alpha - (y + y_p) \sin \alpha, (x + x_p) \sin \alpha + (y + y_p) \cos \alpha, 1)$$

Rotation and translation:

expression of matrix transformation:

$$(x, y, 1) * \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{pmatrix}$$

$$= (x \cos \alpha - y \sin \alpha + x_p, x \sin \alpha + y \cos \alpha + y_p, 1)$$

Translation and scale:

expression of matrix transformation:

$$(x, y, 1) * \begin{pmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{pmatrix}$$

$$= (x \cdot m_x + x_p, y \cdot m_y + y_p, 1)$$

Scale and translation:

expression of matrix transformation:

$$(x, y, 1) * \begin{pmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{pmatrix}$$

$$= (x \cdot m_x + x_p, y \cdot m_y + y_p, 1)$$

3. Summary

The designing assembly systems are a great emphasis on the initial technical work associated with the graphical representation of the process. The selection of the appropriate mathematical tools to solve placement and the zonal problems is usually influenced by the chosen design method and concept solutions [5,6]. The advantage of using a mathematical expression that position leads to a shortening of the time needed to conclusively establish the location of places of the assembly machinery.

The methodical progresses of automated designing are based on the utilizing of topological models of constructed equipments of assembled systems. These progresses go out these assumptions:

- The model of raster grid is assigned to reference area for location assembled system. This system is laid out capacitive calculation of required surface (volume).
- Reference area of assembled system is located to the reference area by the assembled activity.
- The models of topological relations of the constructed equipments of assembled models are solved in the individual zones.
- The calculation-interactive mode of the solution of topological relations is based on the utilizing 2D or 3D models of constructed equipments of topological relations.
- The concrete 2D and 3D models are chosen from graphic database and inserted into calculated reference points.
- The optimal variant of structure of the assembled model is chosen on the base of interactive decision progress.

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