American Journal of Industrial Engineering, 2019, Vol. 6, No. 1, 19-25
Available online at http://pubs.sciepub.com/ajie/6/1/3
Published by Science and Education Publishing
DOI:10.12691/ajie-6-1-3

Review of Rotor Balancing Techniques

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Received November 10, 2019; Revised December 11, 2019; Accepted December 22, 2019

Abstract This paper reviews the history of rotor balancing researches and techniques to avoid balancing problem and its consequences. The discussion in site and factory methods includes influence coefficient, conventional software, no phase and frequency response methods to balance rotors. The survey present theory used in computational algorithms related to Eigensystem realization algorithm (ERA) and its application to develop FRF.

Keywords: *balancing, single plan balancing, four run methods, frequency response function, self-balancing systems*

Cite This Article: Ahmed A. Ibraheem, Nouby M. Ghazaly, and G. T. Abd el-Jaber, “Review of Rotor Balancing Techniques.” *American Journal of Industrial Engineering*, vol. 6, no. 1 (2019): 19-25. doi: 10.12691/ajie-6-1-3.

1. Introduction

Balancing represent 35% of rotating machines mechanical problems, so it’s critical to know how to eliminate this problem and the common techniques to achieve reliable operation. The general concept to solve unbalance problem is to added or remove mass at angular positions which contribute to balance the centrifugal forces effect on the system. It’s a process requires skills, time and money so a long research history try to find effective techniques to eliminate this unbalance problem.

A rotor is said to be unbalanced when its mass center does not lie on the axis of rotation [1]. According the International Organization of Standardization (ISO) as seen in [2] defines unbalance as that condition which exists in a rotor when a vibratory force or motion is imparted to its bearings as a result of centrifugal forces. Unbalance is the uneven distribution of mass about a rotor’s rotating centerline. In other words, rotational unbalance results when the axis of rotation of a rotor system is not coincident with the principal axis of inertia. This eccentricity occurs whenever there is geometric, material and property asymmetry about a rotor’s rotational axis.

Also, unbalance might originate during the manufacturing of rotor where additional mass could be present or removed at a location of rotating shaft. Due to erosion between parts there could be loss of material leading again to an unbalanced condition. They are compensated during commissioning by placing balancing weights. It is not possible to completely balance a rotating system, as there is a small amount of residual unbalance. The system can only be brought to acceptable. Unbalance in rotating machinery causes dynamic forces that bring about vibration and intensification of stresses at the bearing and other receivers.

The literature on balancing is large. Parkinson [3] provides a comprehensive review including more than 60 references on the unbalance problem till 1990.

W.C. Foiles et al [4] provide a wide review containing more than 160 references prior to 1998, where he reviews the literature concerning the balancing of rotors including the origins of various balancing techniques including ones that use influence coefficient, modal, unified, no phase, and no amplitude methods to balance. The survey covers the computational algorithms as well as the physical concepts involved in balancing rotating equipment.

2. Field Balancing Considerations

S. Edwards et al [5] mentioned that field balancing of rotors with unknown foundation dynamics was more often than not a persevering, time consuming and expensive process. They argued a new method to identify both the excitation and flexible support parameters of a rotor bearings foundation system had been verified experimentally in their paper. In addition to mass unbalance, the excitation due to a bent rotor had been included in the method, which had great potential in the field, since it allows balancing to be performed using data obtained from just a single run-up or run-down. Using this single-shot balancing technique, vibration levels of an experimental rotor rig were successfully reduced to less than one-tenth of their original levels. Their bent rotor geometry had been also accurately identified and it was shown that including bend identification in those cases where only unbalance forcing was present in no way detracted from the accuracy of the estimated unbalance or foundation parameters. The identification of the flexible foundation parameters was generally successful, with measured and estimated parameters matching very closely in most cases. Their method was tested for a wide range of conditions and proved suitably robust to changes in system modeling error.

There are many different concerns with balancing. Balancing with a limited number of trials (or sometimes none) was important in many cases.

F. Fujisawa et al [6] found an improved balancing method which can reduce the correction masses was proposed. In this balancing method, both residual vibration and correction mass were adopted as parameters of the performance function. They compared their technique with the method of calculation the correction masses for balancing the rigid rotor by the least squares method and they used a performance function defined as a sum of squares of a residual vibration were often undesirable values. In the actual correction process, the magnitude of the correction masses which could be put on or cut off from the rotor was constrained by the structure and the shape of the rotor or the method of the process. Their results show a good effectiveness for the proposed balancing method was confirmed by computer simulation and experiment using the rotating cylinder unit. Computing suitable unbalance corrections subject to bounding constraints on the unbalance corrections is now perceived to be an issue.

R. E. L. Bishop and G.M.L. Gladwell [7] introduce what was generally thought of as modal balancing. In their investigation they show the inadequateness of low speed balancing (rigid rotor balancing) for high speed flexible rotors, analyze the effects of shaft bow, and study the effects of the rotor's weight.

3. Rigid Rotor Balancing Method

3.1. Influence Coefficient

L. J. Everett [8] developed an influence coefficient model for a rigid rotor and relates measured vibration to unbalance and trial mass magnitude. By spin the rotor multiple times at a constant speed (the speed at which we want to balance) and measure vibration magnitudes each time. He reached that for the 30 uniformly distributed random systems, the non-optimal result had a mean and standard deviation of 19.7% and 12.0%, whereas the optimal had a mean and standard deviation of 15.8% and 10.2%. sing these statistics and a cumulative normal distribution chart, one can conclude with 96% confidence that the optimal method of balancing produces a better residual measure than does the non-optimal technique.

R. Ambur, S. Rinderknecht [9] state that faults were detected with self-sensing piezoelectric actuators where time domain methods are used. Because of computational complexity and requirement of some very accurate model time domain methods are not preferred. Instead faults are detected in frequency domain in the present article.

M. Chouksey et al [10] attempted experimental studies in finite element model updating of an actual rotor system mounted on ball bearings by using Inverse Eigen Sensitivity Method (IESM). Their IESM was applied on state space representation of equations of motion and is used to identify bearing stiffness, damping and shaft material damping parameters. Non-proportional viscous damping model was used to model the bearing and shaft material damping. They work on experimental identification of viscous coefficient of shaft material damping was not found in the available and they tried to put the deceleration. Their updated model was validated for its accuracy by comparing the predicted frequency response with that obtained from the experiments. The final results shown that the updated finite element model of the rotor system could be efficiently used to predict the unbalance in the rotor.

C. C. Ozoegwu et al [11] mentioned that there was no single cause of undesirable vibrations occurring in rotating machinery. Poor operating conditions like loose mechanical parts, faulty impellers, faulty bearings, faulty gears, unbalanced machine elements, whirling and unbalanced shafts intensify vibration in rotating machines. In their work rotational unbalance was singled out as a cause of vibration and its nature, causes, effects and remedies explored and explained. Analytical equations were derived for both cases of single and double plane balancing of experimentally determined unbalance. They introduce some novel derived balancing equations for double plane in their work. They believe that the operator could avoid the difficult to understand, difficult to use and error prone graphical approach. They use a modern world's industrial set-up productivity was improved by integrating high speed computers into the process of production based on this need a general MATLAB program was written for quick solution of experimental double plane balancing problems. Three exercises drawn from a standard text was used to illustrate the usefulness of the derived equations. They finally, highlighted for their study that rotational unbalance cannot be eliminated. There would be amount of unfavorable cost to try dealing with rotational unbalance by investing heavily in achieving high precision in manufacture of machine parts.

S.H. Weaver [12] realized that the balance weights and imbalances act as forces to the system. Weaver was aware that the forces at the bearing locations for a rigid rotor changed with the magnitude and phase of the imbalance weights. Later balancers would develop the notion of influence coefficients, though at first, they would not use this name.

Numerous solutions have been proposed aiming at minimizing damage effects of unbalance. The most widely known balancing techniques are the following: modal balancing, four-run without phase, combined techniques, and the popular influence coefficients method. Although widely used, the rotor balancing technique using influence coefficients (IC) presents some adverse points that encourage the search for alternative balancing approaches. As many of the balancing approaches, IC considers a linear relation between the unbalance excitation and the resulting vibration. However, if the structure presents nonlinearities the results obtained, regarding the correction weights and their corresponding angular positions, are not satisfactory.

Additionally, IC method requires trial weights (known masses that are positioned at specific locations along the rotor) in order to determine the unbalance response sensitivity for constant rotation speed. Aiming at overcoming the limitations faced by the IC technique, a model based balancing methodology was developed. This methodology does not require a linear relationship between unbalance and vibration responses; besides, trial weights are not necessary. However, a reliable model of the rotating machine was required [13, 14].

A. Silva el [15] reported that the unbalance was one of the most common problems found in rotating machines in the context of industrial plants. They present an alternative balancing methodology for rotating machines, aiming at overcoming the limitations faced by the frequently used methods. This alternative technique first identifies the model of the machine, and then the unbalance was determined by solving a typical inverse problem through an optimization method by considering the inherent uncertainties that affect the balancing performance. The robust balancing methodology was based on a multi-objective fuzzy optimization procedure, in which the uncertainties are treated as fuzzy variables. The robust optimum was determined by using an objective function which minimizes a predefined robustness metric. Finally, the numerical investigation was applied to a rotor composed by a horizontal flexible shaft, two rigid discs, and two ball bearings. The results indicate the effectiveness of the proposed technique.

B. XU el [16] argued that for most balancing techniques the used test weights and runs were required for the calculation of correction masses. they developed a new rotor balancing method without test runs, which uses the balancing objective of influence coefficient method and the initial phase point of Holo-spectrum. By calculating theoretical unbalance responses and measuring original unbalance vibrations, a new type of intelligent optimization technique, genetic algorithm, was applied to optimize the correction masses to minimize residual vibrations at selected measurement locations and balancing speeds. Their practical field balancing experiment on rotating rotor was balanced by employing the new method. in which average fluid oil coefficients within the balancing speeds were used in the calculation of unbalance responses, and the optimization correction masses were compared with these of the influence coefficient method. Both the simulation and experiment results show that the new method could reduce the residual vibrations effectively.

Although, the rotor balancing technique using influence coefficients (IC) widely used presents some adverse points that encourage the search for alternative balancing approaches. As many of the balancing approaches, IC considers a linear relation between the unbalance excitation and the resulting vibration. However, if the structure presents nonlinearities the results obtained, regarding the correction weights and their corresponding angular positions, are not satisfactory. Additionally, IC method requires trial weights (known masses that were positioned at specific locations along the rotor) in order to determine the unbalance response sensitivity for constant rotation speed. Aiming at overcoming the limitations faced by the IC technique, a model based balancing methodology was developed by M.V. Saldarriaga, V. Steffen, J. Der Hagopian and J. Mahfoud, [17]. Their methodology does not require a linear relationship between unbalance and vibration responses; besides, trial weights are not necessary. However, a reliable model of the rotating machine was required. Thus, from a reliable mathematical model, the proposed method shows up to be well adapted for industrial applications in which the non-producing time dispensed to reduce the synchronous vibration of the rotor to acceptable levels should be as small as possible.

3.2. Balancing Using Amplitude Only

Sometimes, it may become necessary to balance a rotating machine or part under conditions where a phase measurement was either impossible or it was difficult to obtain the phase of the 1X vibration accurately. Although the vibration should be filtered to 1X rotation often the overall vibration amplitude was used sometimes it was measured using a mechanical indicator, vibrometer. Techniques were developed to balance using only the amplitude of the vibration this practice continues even today.

G.B. Karelitz [18], in the Research Department of Westinghouse Electric & Manufacturing Company, used a three-trial weight to balance turbine generators. This graphical technique used an unbalance finder to locate the mass imbalance; the unbalance finder consisted of four transparent strips held together with a pivot at one end. The method could be used with trial weights of unequal magnitudes.

F. Ribary [19] presented a graphical construction that balanced using only the amplitude taken from an initial run and three trial weight runs.

I.J. Somervaille [20] considerably simplified the graphical construction of Ribary [19]. Somervaille's construction was also known as the four-circle method of balancing without phase. The four-circle method, as it was generally used now, can be found in C. Jackson [21].

L.E. Barrett, D.F. Li, and E.J. Gunter [22] adapted the technique to balance a rotor through two modes using modal balance weights; E.J. Gunter, H. Springer, and R.R. Humphris [23] used modal balancing without phase to balance a rotor through three modes. So, the main problems of using of rotor balancing using amplitude only are:

- The number of runs required for a balance using amplitude only makes their method inherently less efficient than an equivalent influence coefficient method.
- After completing such a balance, one has no information that would help to trim balance or in the future perform one-shot balancing.
- A trim balance requires another four runs.

3.3. Balancing Using Phase Only

Phase data can be obtained by directly marking the shaft as was done on some of the early balancing machines from the 1800's as described by N.F. Rieger [24] and F. Ribary [19].

C. Jackson [21, 25] described methods of obtaining phase using a pencil to mark the shaft and using orbit analysis; Jackson then incorporated the physics of the rotor, whether it was above, below, or near a critical speed, to balance. Their technique could require some iteration to find a solution depending upon the knowledge and experience of the balance practitioner.

K.R. Hopkirk [26] derived a technique for two planes balancing using only phase information. Hopkirk's method comprises a two plane exact-point balance, and the procedure required five trial runs including the initial one. I.J. Somervaille [20] presented a graphical means to solve for unbalance on a disc (single plane) using only the phase information.

W.C. Foiles and D.E. Bently [27] found both analytical and graphical solutions for single-plane and multi- plane balancing using only phase information; their solution used a type of influence coefficient applicable to balancing using this partial information. Whereas single plane balancing without phase requires three trial weights; their technique uses just two trial weight runs. Methods were developed for both single plane and multi-plane balancing. Their paper presented both analytical and graphical solutions for a single plane balance (or the influence coefficients for a multi-plane balance), and the authors applied the technique to a cooling tower fan.

Similar to the techniques that use only amplitude, these methods require additional balance runs compared to influence coefficient methods that use full in formation both amplitude and phase.

Also, phase balancing only has shortages:

- After a balance, one has no useful residual data for trim balancing.
- Can't use one shot balancing in the future efforts.

These deficiencies result in serious inefficiencies for the techniques that use only partial information in balancing, either only amplitude or only phase.

3.4. Modal Balancing

Balancing generally assumes that the rotor system has planar modes of vibration. Balancing one mode should not affect any other mode; although higher modes that are not being considered may be adversely affected. Often the procedure can be quite efficient.

J.R. Lindsey [28] used static and couple balance weights combined with a sensitivity factor and a high spot number. The modal component of the vibration was determined by graphical means using vibration data from each end of a machine in one plane. The high spot number relates to the phase angle. Sensitivities and high spot numbers have been developed over time for a variety of machines. The strength of this technique derives from using this historical data, and the method was most often used as a one-shot balance method.

The desire with this method was arrive at an adequate balance usually not the best achievable balance in an efficient manner. A weakness of the method involves its lack of concern with the cross effect of a static weight on the couple vibration and the effect of a couple weight on the static component of the vibration. Lindsey stated, "When extensive coupling exists among unbalances in several rotors, the methods errs significantly." This method also, has difficulties in differentiate between one and three loop modes in the first and second critical speeds.

R.E.D. Bishop [29] in 1959 formulated equations for the displacement amplitudes of a circular rotor with distributed mass and elasticity. His solution for a rotor's vibration looks like a power series of Jeffcott rotors.

3.5. Automatic Balancing of Rigid Rotors

D. Garvey et al [30] discusses some practical methods for Robust balancing for rotating machines and how such information might be extracted. The definition of the cost function as a matrix quadratic form provides potentially valuable information about the necessary number and the optimal location of balance planes on a given rotor, and methods for determining an optimal set of balance planes were outlined. They considered the effect of variability within the foundations of a rotating machine on the optimum balance corrections. The parameter variability may be given in terms of a probability distribution or in terms of fixed limits. Unbalance corrections had been derived in both cases. OF fundamental importance was the concept of distributions of significant residual unbalance, which determines the optimum balance planes for a given machine and parameter variability and plays a similar role to the machine modes in modal balancing.

T. Majewski et al [31] analyzed the stability and efficiency of using of the automatic balancing of a rigid disk mounted on an elastic shaft. Their balancing system consists of two drums at a variable distance from the disk and free balls (or rollers) inside the disk. The balls in the system were able to change positions with respect to the rotor and compensate the rotor unbalance. They present the equations of motion for the disk as wet al l as for the balls during balancing. It was shown that the balls can compensate a part or all of the rotor unbalance depending on the positioning of the drums. There are vibratory forces that push the balls to new positions; these are responsible for the behavior of the balls and the final results. The vibratory forces were defined as a function of the system's parameters and they determine the position of equilibrium of the balls.

D.J. Rodrigues et al [32] addressed an analysis of a two-plane automatic balancing device for rigid rotors. Ball bearings, which are free to travel around a race, were used to eliminate imbalance due to shaft eccentricity or misalignment. The rotating frame was used to derive autonomous equations of motion and the symmetry breaking bifurcations of their system were investigated. Stability diagrams in various parameter planes show the coexistence of a stable balanced state with other less desirable dynamics alignment. However, the balancing process works only for sufficiently high rotation speeds above the second critical frequency.

D.J. Rodrigues et al [33] promoted an experimental investigation of a single plane automatic balancer that was fitted to a rigid rotor. They used two balls, which were free to travel around a circular race, are used to compensate for the mass imbalance in the plane of the device. Their experimental test rig possessed both cylindrical and conical rigid body modes and the performance of the automatic balancer was assessed for a variety of different level of imbalance. A non-planar mathematical model that also includes the observed effect of support anisotropy was developed and numerical simulations are compared with the experimental findings. In the highly supercritical frequency range, the balls act to balance the rotor and a good quantitative match was found between the modal and the experimental data. However, during the rigid body resonances the dynamics of the ball balancer was highly nonlinear and for that speed range the agreement between theory and experiment was mainly qualitative. Nevertheless, their modal was able to successfully reproduce many of the solution types that are found experimentally.

S. ZHOU and J. SHI [34] adaptive an active balancing strategy for a rotor system which can reduce the imbalance-induced vibration efficiently. By using their strategy, the imbalance induced vibration lees during the acceleration period can be efficiently suppressed.

They had two challenges in their study. The first one was the estimation of the system imbalance during the acceleration period; this problem was solved by using an ordinary recursive least-squares estimation method based on a time domain modal of the rotor system.

The second problem was also related to the acceleration. Since there were two or more vibration modes during acceleration in general, an optimal balancing strategy was required to suppress all the modes. This problem was solved based on an analytical expression of the imbalance response of a Jeffcott rotor during acceleration. Based on that expression, the optimal balancing strategy was found to be a simple step function.

3.6. Other Balancing Methods

G. A. Hassan Mohammed [35] suggest a new approach named it the black box based on running a minimum number of balancing trials and using regression analysis to ret al ate the vibration amplitude at one or two rotor bearings to the balancing variables (weight, mass and location angle). A powerful optimization technique was used to minimize an objective function combining the vibration levet al s at both bearings with pre-assigned weighting constants subjected to constraints on the balancing variables. He applied the technique on both static and dynamic balancing of rigid rotors and a computer program was outlined to facilitate the application of the suggested approach. The results were reduction in the vibration amplitude in static balancing had decreased by 83·7%, while in dynamic balancing it had decreased by 25% at bearing 1 and 76·4% at bearing 2.

L. Sperling et al [36] present a device that could be efficiently compensate rigid rotor unbalanced forces and unbalanced moments in the range of rotational speeds above the higher resonant speed. Their analytical and numerical investigations of a two-plane automatic balancing device for equilibration of rigid rotor unbalance. It includes the derivation of the full system of equations of motion, the stability analysis on the basis of an analytical approximation and results of numerical simulations. The influence of system parameters, such as rotational speed, damping in supports and resistance to the motion of the compensation balls on the operation of device was analyzed.

They claim that to do the work successfully, it required a correct choice of the ball damping coefficient and sufficient support damping. The numerical simulations were performed on the basis of partially linearized equations of motion. They confirmed the results of our analytical approximation for the existence and stability conditions.

4. Early Fault Detection and Prediction Vibration Response Methods

R. Ambur and S. Rinderknecht [37] focus on rotating machine with active bearings equipped with piezoelectric actuators. As machines which were developed and highly automated due to using mechatronic systems. To ensure their reliable operation, fault detection and isolation (FDI) was an important feature along with a better control. Their research work aimed to achieve and integrate both these functions with minimum number of components in a mechatronic system. There was an inherent coupling between their electrical and mechanical properties because of which they could also be used as sensors. Mechanical deflection could be reconstructed from those set al f-sensing actuators from measured voltage and current signals. These virtual sensor signals were utilized to detect unbalance in a rotor system. The used parameters of unbalance such as its magnitude and phase were detected by parametric estimation method in frequency domain. Unbalance location had been identified using hypothesis of localization of faults. Robustness of the estimates against outliers in measurements was improved using weighted least squares method. In their experimental test bench apart from simulation unbalances were detected using the model. Experiments were performed in stationary as wet al l as in transient case. As a further step unbalances are estimated during simultaneous actuation of actuators in closed loop with an adaptive algorithm for vibration minimization. Their strategy could be used in systems which aim for both fault detection and control action.

Z. Wang and P. Zhu [38] shows a general method using in-situ frequency response functions (FRFs) was proposed for predicting operational responses of modified mechanical systems. The method responses of modified mechanical systems could be calculated by using the det al ta dynamic stiffness matrix, the subsystem FRF matrix and responses of the original system, even though operational forces are unknown. The proposed method was derived theoretically in some general form Six specific scenarios. The six scenarios correspond respectively to:

(a) modifications made on the mass; (b) changes made on the stiffness values of the link between a degree-of-freedom (DOF) and the ground; (c) the fully rigid link between a DOF and the ground; (d) changes made on the stiffness values of the link between two DOFs; (e) the null link between two DOFs and (f) the fully rigid link between two DOFs. It was found that scenarios (a), (b) and (d) the details dynamic stiffness matrix was required when predicting responses of the modified mechanical system. But for scenarios (c), (e) and (f), no det al ta dynamic stiffness matrix was required and the new system responses could be calculated solet al y using the subsystem FRF matrix and responses of the original system. They focused on the structural modification stage for troubleshooting noise and vibration problems. A general method for predicting responses of the modified mechanical systems has been proposed.

Y. Yang et al [39] concerned a general method to predict unbalance responses for a geared rotor system, which contains n shafts and at least n- 1 spur and/or helical gear meshes. They built a finite element modal s of spur gear pairs and helical gear pairs are developed with arbitrary orientation angles, respectively. Then, a general method with a strict mathematical proof was proposed to calculate maximum axis radii of the unbalance response orbit for a geared rotor system. Their proposed method was validated through three applications, i.e., a spur geared two shaft rotor system, a spur geared multi-shaft rotor system and a helical geared multi-shaft rotor system. A classical modal synthesis was also employed as a comparison. Results show that proposed analytical solutions are consistent with the numerical results and had advantages over the modal synthesis. The investigation of their results was summarized as follows:

The method mirrors numerical methods wet for spur/helical geared rotor systems, even for complicated systems with different shaft angles and some gear parameters. The error percent of this method were very limited and the average errors of three applications were only 0.102%, 0.139% and 0.867%, respectively. Particularly, within a large range of rotating speeds, errors approximate to zero. Although slight differences exist in localized regions, these differences were negligible without prejudice to the reliability.

The study saved plenty of processing time significantly. The computer processing time of the proposed method was far less than the numerical methods and only about half of the modal synthesis.

Their proposed method was applicable to some cases in which unbalance responses couldn't be calculated with the previous method. Meanwhile, it was suitable for geared rotor systems including more than two shafts. Hence, the proposed method had the potential and advantages to be applied in engineering generally.

D. E. Bently and C. T. Hatch [40] mention in their leading book that when unbalance appears to be in a single axial plane such that the principal axis of rotation was displaced parallel to the geometric centerline, static unbalance results. Many causes result in mass unbalance, such as unreasonable structure design, material inhomogeneity, manufacturing tolerance, unqualified balancing and mechanical looseness or damage during operations. Generally, excessive unbalance accelerates bearing wear, makes harmful noises and threatens the smooth running of the system. Hence, it was necessary to reduce the residual unbalance to acceptable levels by rotor balancing. Unbalance response calculations can be helpful in evaluating the sensitivity of the rotor to unbalance and choosing the best locations for balancing planes. Engineers deal with unbalance by addition or removal of matter such that the geometric axis of rotation approaches the principal axis of rotation in alignment and proximity. Static unbalance was corrected by addition or removal of correction weight in the plane of the unbalance while dynamic unbalance was corrected by addition of two appropriate weights in two arbitrarily chosen planes. Thus, predicting amplitudes of synchronous vibration due to mass unbalance was one of the main objectives of rotor dynamics analysis.

5. Theoretical Methods to Define Frequency Response Function (FRF)

Several methods used in defining the modal characteristics according to the input excitation and response measurements. Juang *et al.* [8] suggested the eigensystem realization algorithm (ERA) the foundation of this method depends mainly on two assumptions:

Linearity of the structure is a time invariant (LTI) system. Un-correlation between the excitation and the response. A continuous research investigates the ERA trying to enhance its application.

Juang et al [41] explain how to minimize the noise effect on rotor balancing process.

Pappa et al [42] studied indicators to assessing the consistency of modal parameters identified with the ERA. Also, [43,44] investigate and compared the effectiveness of four types of modal identification algorithms.

Tom Irvine [45,46] solve the steady-state transfer function for the absolute response displacement via Laplace transforms which het al p in derivation of the Transfer functions quantities indicated in Table 1.

Table 1. FRF display quantities:

	Response/ Impact force	Impact force /Response
Acceleration (g's)	Accelerance (m/s^2.N)	Effective mass (s^2.N /m)
Velocity (mm/sec)	Mobility (m/ N.s)	Mechanical impedance (N.s/m)
Displacement (µm)	Dynamic compliance (µm/N)	Dynamic stiffness (N/ µm)

All the mentioned FRF mentioned in Table 1used in vibration analysis and modal testing to identify system natural frequency, damping ratio, mode shape representation and the new suggested use to het al p in defining the unbalance mass and its location on the rotor using FRF magnitude and phase. The generated FRF is constant for the system at each frequency. Velocity and acceleration transfer function in equations:

$$\left|\frac{V(\omega)}{F(\omega)}\right| = \left[\frac{1}{m}\right]\left[\frac{\omega}{\sqrt{\left(\omega n^2-\omega^2\right)^2+\left(2\xi\omega\omega n\right)^2}}\right]$$

$$\left|\frac{A(\omega)}{F(\omega)}\right| = \left[\frac{1}{m}\right]\left[\frac{-\omega^2}{\sqrt{\left(\omega n^2-\omega^2\right)^2+\left(2\xi\omega\omega n\right)^2}}\right].$$

References

[1] H.H. Jeffcott, The lateral vibration of loaded shafts in the neighborhood of a whirling speed the effect of want of balance, Philosophical Magazine, 1919, Series 37, PP 304-314.

[2] International Organization of Standardization, Mechanical vibration - Balance quality requirements for rotors in a constant (rigid) state - Part 1: Specification and verification of balance tolerances, ISO 1940-1:2003 (E).

[3] A. G. Parkinson, Balancing of rotating machinery, Proc. Instn Mechanical Engineers, Part C, Journal of Mechanical Engineering Science, 1991, V 205(C2), PP 53-66.

[4] W. C. Foiles, P. E. Allaire and E. J. Gunter, Rotor balancing. Shock and Vibration, 1998, V 5(5-6), PP 325-336.

[5] S. Edwards, A. W. Lees and M. I. Friswell, Experimental identification of excitation and support parameters of a flexible rotor bearings foundation system from a single run down, Journal of Sound and vibration, 2000, V 232(5), PP 963-992.

[6] F. Fujisawa, T. Kojima, T. Terayama, K. Shiohata and H. Yokota, Balancing method for a rigid rotor considering constraint of the correction mass, Transport Japan Society Mechanical Engineers, Part C, 1993, V 59 (558), PP 78-83.

[7] R.E.D. Bishop and G.M.L. Gladwell, The vibration and balancing of an unbalanced rotors, Journal Mechanical Engineering Science, 1959, V 1 (1), PP 66-77.

[8] L.J. Everett, Optimal two plane balance of rigid rotors, Journal of sound and Vibration, 1997, V 208 (4), PP 656–663.

[9] R. Ambur, S. Rinderknecht, Self-sensing techniques of piezoelectric actuators in detecting unbalance faults in a rotating machine, Procedia Engineering Journal, 2016, V 144, PP 833-840.

[10] M. Chouksey, J.K. Dutt, S.V. Modak, Model updating of rotors supported on ball bearings and its application in response prediction and balancing, journal of Measurement, 2013, V 46 (10), PP 4261-4273.

[11] C. C. Ozoegwu, C. C. Nwangwu, C. F. Uzoh, A. V. Ogunoh, Analytical Approach to Rotational Balancing with computer aids, Journal of Safety Engineering, 2012, V 1(4), PP 50-56.

[12] S.H. Weaver, Balancing of rotors in factory and at installation, General Electric Review, 1928, V 31 (10), PP542-545.

[13] Fedoseev, Y. N., Grebennikov, A. S. and Rymalova, S. I. Use of multi criterion optimization method for machine balancing, Strength and Reliability Problems of Machines Journal, 1992, V 2, PP 10-14.

[14] F. Fujisawa, M. Sasaki, Y. Hori, K. Shiohata, T. Kohno and H. Ohsawa, Experimental investigation on rigid rotor balance containing error in influence coefficient, Japan Society of Mechanical Engineers, 1995, Part C, V 61(584), PP 1443-1449.

[15] A. D. G. Silva, Al. Cavalini and V. Steffen, Model Based Robust Balancing Approach for Rotating Machines, A Conference and Exposition on Structural Dynamics, 2016 , Chapter 24, V 3, 34th IMAC.

[16] B. Xu, L.Qu and Sun, R. Optimization technique-based balancing of flexible rotors without test runs, Journal of Sound Vibration., 2000, V 238(5), PP877-892.

[17] M.V. Saldarriaga, V. Steffen, J. Der Hagopian, J. Mahfoud, Balancing of flexible rotating machines by using an inverse problem approach, Journal of Vibration Control, 2011, V 17(7), PP 1021-1033.

[18] G.B. Karelitz, Field balancing rotors at operating speed, Power 67(7) (1928), 286-289.

[19] F. Ribary, The balancing of masses in rotating bodies, Brown Boveri Review 23 (1936), PP 186-192.

[20] I.J. Somervaille, Balancing a rotating disc with simple graphical construction, Engineering, 1954, PP 241-242.

[21] C. Jackson, the Practical Vibration Primer, Gulf Publishing Company, Houston, Texas, 1979.

[22] L.E. Barrett, D.F. Li and E.J. Gunter, Second mode balancing without phase measurement using the three-point method, in: Selected Papers on Field Balancing of Rotating Machinery Advanced Theory and Techniques, E.J. Gunter, ed. No. UVA/643092/MAE81/137, ROMAC Report 80, Charlottesville, VA, 1978.

[23] E.J. Gunter, H. Springer and R.R. Humphris, Balancing of multi-mass flexible rotor bearing system without phase measurements, in: 11th Biennial Conf. on Mechanical Vibration and Noise, Boston, MA, 1982.

[24] N.F. Rieger, Balancing of rigid and flexible rotors, The Shock and Vibration Information Center, Naval Research Laboratory, Washington DC, 1986.

[25] C. Jackson, Using the orbit to balance, Journal of Mechanical Engineering, 1971, PP 28-32.

[26] K.R. Hopkirk, Notes on methods of balancing, The Engineer, 1940, PP 38-39.

[27] W.C. Foiles and D.E. Bently, Balancing with phase only (single-plane and multiplane), Trans American Society of Mechanical Engineers, Journal of Vibration, Acoustics, Stress, and Reliability in Design, 1988, V 110(2), PP 151-157.

[28] J.R. Lindsey, Significant developments in methods for balancing high-speed rotors, in: Conf. American Society of Mechanical Engineers

[29] R.E.D. Bishop, The vibration of rotating shafts, Journal of Mechanical Engineering Science 1959, V1(1), PP 50-65.

[30] D Garvey, M I Friswell, E J Williams, A W Lees and I D Care, School of Mechanical, Materials, Manufacturing Engineering and Management, Robust balancing for rotating machines, M. Sc. Institute Mech Engineering Science, University of Nottingham, UK, Part C: Journal of Mechanical Engineering Science, 2002, V 216 (11), pp. 1117-1130.

[31] T. Majewski, D. Szwedowicz and M. A. Meraz, Self-balancing system of the disk on an elastic shaft, Journal of Sound and Vibration, 2015, V 359, PP 2-20.

[32] D.J. Rodrigues, A.R. Champneys, M.I. Friswell, R.E. Wilson, Automatic two-plane balancing for rigid rotors, International Journal of Non-Linear Mechanics, 2008, V 43, PP 527-541.

[33] D.J. Rodrigues, A.R. Champneys, M.I. Friswell, R.E. Wilson, Experimental investigation of a single-plane automatic balancing mechanism for a rigid rotor, Journal of Sound and Vibration, 2011, V 330, PP385-403.

[34] S. Zhou and J. Shi, Optimal One-Plane Active Balancing of a Rigid Rotor During Acceleration, Journal of Sound and vibration, 2002, V 249(1), PP 196-205.

[35] G. A. Hassan Mohammed, New approach for computer-aided static and dynamic balancing of Rigid rotors, Journal of Sound and Vibration, 1995, V 179(5), PP 749-76.

[36] L. Sperling, B. Ryzhik, Ch. Linz, H. Duckstein, Simulation of two-plane automatic balancing of a rigid rotor, Journal of Mathematics and Computers Simulation, 2002, V58, PP 351–365.

[37] R. Ambur and S. Rinderknecht, Unbalance detection in rotor systems bearings using self-sensing piezoelectric actuators, Mechanical Systems and Signal Processing Journal, 2018, V 102, PP 72-86.

[38] Z. Wang, P. Zhu, Response prediction for modified mechanical systems based on in-situ frequency response functions: Theoretical and numerical studies, Journal of Sound and Vibration, 2017, V 400, PP 417-441.

[39] Y. Yang, J. Wang, X. Wang and Y. Dai, A general method to predict unbalance responses of geared rigid rotor systems, Journal of Sound and Vibration, 2016, V 381, PP 246-263.

[40] D. E. Bently and C. T. Hatch, Fundamentals of Rotating Machinery Diagnostics, 1st edition, Bently Pressurized Bearing Company, Minden, 2002.

[41] J.-N. Juang and R. S. Pappa, Effects of noise on modal parameters identified by the eigensystem realization algorithm, Journal of Guidance, Control, and Dynamics, vol. 9, pp. 294-303, 1986.

[42] R. S. Pappa, K. B. Elliott, and A. Schenk, A consistent-mode indicator for the eigensystem realization algorithm: National Aeronautics and Space Administration, Langley Research Center, 1992.

[43] J. Lew, J.-N. Juang, and R. W. Longman, Comparison of several system identification methods for flexible structures, Journal of Sound and Vibration, vol. 167, pp. 461-480, 1993.

[44] Nouby M. Ghazaly, G. T. Abd el- Jaber, Nadica Stojanovic, Study Various Defects of Ball Bearings through Different Vibration Techniques, American Journal of Mechanical Engineering, 2019, Vol. 7, No. 3, 146-150

[45] T. Irvine, Total Response of a Single-degree-of-freedom System to a Harmonic Forcing Function, Vibrationdata.com Publications, 1999.

[46] T. Irvine, A. PiersolT. Irvine and T. Paez, Harris, Shock and Vibration Handbook, 6th Edition; V. Bateman, Shock Testing Machines, Chapter 27, New York City: McGraw-Hill, 2010.