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Studying Three Types of Integrals with Maple

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Abstract This paper uses the mathematical software Maple for the auxiliary tool to study three types of integrals. We can obtain the infinite series forms of these three types of integrals by using integration term by term theorem. In addition, we propose some examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions using Maple.

Keywords: *integrals, infinite series forms, integration term by term theorem, Maple*

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1. Introduction

In calculus and engineering mathematics courses, we learnt many methods to solve the integral problems including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, and so on. In this paper, we mainly study the following three types of integrals which are not easy to obtain their answers using the methods mentioned above.

$$\int \exp(ax)\cdot\exp(\lambda \exp bx)dx \quad (1)$$

$$\int \exp(ax)\cdot\sin(\lambda \exp bx)dx \quad (2)$$

$$\int \exp(ax)\cdot\cos(\lambda \exp bx)dx \quad (3)$$

where a,b,λ are real numbers, and $b,\lambda \neq 0$. We can obtain the infinite series forms of these three types of integrals by using integration term by term theorem; these are the major results of this paper (i.e., Theorems 1-3). The study of related integral problems can refer to [1-17]. In addition, we provide some integrals to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

2. Main Results

Firstly, we introduce some formulas used in this study.

2.1. Formulas

2.1.1. The exponential function $\exp y = \sum_{n=0}^{\infty}\frac{1}{n!}y^n$, where y is any real number.

2.1.2. The sine function $\sin y = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}y^{2n+1}$, where y is any real number.

2.1.3. The cosine function $\cos y = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}y^{2n}$, where y is any real number.

Next, we introduce an important theorem used in this paper.

2.2. Integration Term by Term Theorem ([18])

Suppose $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on an inteval I. If $\sum_{n=0}^{\infty}\int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n = \sum_{n=0}^{\infty}\int_I g_n$.

The following is the first result of this study, we obtain the infinite series form of the integral (1).

2.3. Theorem 1

Assume a,b,λ are real numbers, $b,\lambda \neq 0$, and C is a constant. If $-\frac{a}{b}$ is not a non-negative integer. Then the integral

$$\int \exp(ax)\cdot\exp[\lambda \exp(bx)]dx = \sum_{n=0}^{\infty}\frac{\lambda^n}{n!(bn+a)}\exp[(bn+a)x]+C \quad (4)$$

Proof Because

$$\exp(ax)\cdot\exp[\lambda\exp(bx)]$$
$$=\exp(ax)\cdot\sum_{n=0}^{\infty}\frac{1}{n!}[\lambda\exp(bx)]^n$$
(By Formula 2.1.1) (5)
$$=\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\exp[(bn+a)x]$$

It follows that

$$\int\exp(ax)\cdot\exp[\lambda\exp(bx)]dx$$
$$=\int\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\exp[(bn+a)x]dx$$
$$=\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\int\exp[(bn+a)x]dx$$
(By integration term by term theorem)
$$=\sum_{n=0}^{\infty}\frac{\lambda^n}{n!(bn+a)}\exp[(bn+a)x]+C$$

Next, we determine the infinite series form of the integral (2).

2.4. Theorem 2

If the assumptions are the same as Theorem 1. If $-\frac{b+a}{2b}$ is not a non-negative integer, then the integral

$$\int\exp(ax)\cdot\sin[\lambda\exp(bx)]dx=\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{2n+1}}{(2n+1)!(2bn+b+a)}\exp[(2bn+b+a)x]+C \quad (6)$$

Proof Because

$$\exp(ax)\cdot\sin[\lambda\exp(bx)]$$
$$=\exp(ax)\cdot\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}[\lambda\exp(bx)]^{2n+1}$$
(ByFormula 2.1.2) (7)
$$=\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{2n+1}}{(2n+1)!}\exp[(2bn+b+a)x]$$

We have

$$\int\exp(ax)\cdot\sin[\lambda\exp(bx)]dx$$
$$=\int\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{2n+1}}{(2n+1)!}\exp[(2bn+b+a)x]dx$$
$$=\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{2n+1}}{(2n+1)!}\int\exp[(2bn+b+a)x]dx$$
$$=\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{2n+1}}{(2n+1)!(2bn+b+a)}\exp[(2bn+b+a)x]+C$$

The following is the third result in this study, we obtain the infinite series form of the integral (3).

2.5. Theorem 3

Let the assumptions be the same as Theorem 1. If $-\frac{a}{2b}$ is not a non-negative integer, then

$$\int\exp(ax)\cdot\cos[\lambda\exp(bx)]dx=\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{2n}}{(2n)!(2bn+a)}\exp[(2bn+a)x]+C \quad (8)$$

Proof Because

$$\exp(ax)\cdot\cos[\lambda\exp(bx)]$$
$$=\exp(ax)\cdot\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}[\lambda\exp(bx)]^{2n}$$
(ByFormula 2.1.3) (9)
$$=\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{2n}}{(2n)!}\exp[(2bn+a)x]$$

It follows that

$$\int\exp(ax)\cdot\cos[\lambda\exp(bx)]dx$$
$$=\int\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{2n}}{(2n)!}\exp[(2bn+a)x]dx$$
$$=\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{2n}}{(2n)!}\int\exp[(2bn+a)x]dx$$
$$=\sum_{n=0}^{\infty}\frac{(-1)^n\lambda^{2n}}{(2n)!(2bn+a)}\exp[(2bn+a)x]+C$$

3. Examples

In the following, for the three types of integrals in this study, we provide three integrals and use Theorems 1-3 to determine their infinite series form. On the other hand, we evaluate some related definite integrals and employ Maple to calculate the approximations of these definite integrals and their solutions for verifying our answers.

3.1. Example 1

By Theorem 1, we obtain the following integral

$$\int\exp(-6x)\cdot\exp[5\exp(4x)]dx=\sum_{n=0}^{\infty}\frac{5^n}{n!(4n-6)}\exp[(4n-6)x]+C \quad (10)$$

Thus, we can evaluate the related definite integral from $x=-5$ to $x=-2$,

$$\int_{-5}^{-2}\exp(-6x)\cdot\exp[5\exp(4x)]dx=\sum_{n=0}^{\infty}\frac{5^n}{n!(4n-6)}[\exp(-8n+12)-\exp(-20n+30)] \quad (11)$$

We use Maple to verify the correctness of (11).

```
>evalf(int(exp(-6*x)*exp(5*exp(4*x)),x=-5.-2),20);
```

$$1.7810791247247284432.10^{12}$$

>evalf(sum(5^n/(n!*(4*n-6))*(exp(-8*n+12)-exp(-20*n+30)),n=0.infinity),20);

$$1.7810791247247284432 \cdot 10^{12} + 0.I$$

The above answers obtained by Maple appears I ($=\sqrt{-1}$), it is because Maple calculates by using special functions built in. But the imaginary part of the above answer is zero, so can be ignored.

3.2. Example 2

Using Theorem 2, we can determine the integral

$$\int \exp(2x) \cdot \sin[7\exp(-6x)]dx = \sum_{n=0}^{\infty} \frac{(-1)^n 7^{2n+1}}{(2n+1)!(-12n-4)} \exp[(-12n-4)x] + C \quad (12)$$

Therefore, the definite integral

$$\int_{-1}^{5} \exp(2x) \cdot \sin[7\exp(-6x)]dx = \sum_{n=0}^{\infty} \frac{(-1)^n 7^{2n+1}}{(2n+1)!(-12n-4)} [\exp(-60n-20) - \exp(12n+4)] \quad (13)$$

>evalf(int(exp(2*x)*sin(7*exp(-6*x)),x=-1.5),24);

0.647591256320565998061

>evalf(sum(((-1)^n*7^(2*n+1))/((2*n+1)!*(-12*n-4))*(exp(-60*n-20)-exp(12*n+4)),n=0.infinity),24);

0.647591256320565998068

3.3. Example 3

By Theorem 3, we obtain

$$\int \exp(4x) \cdot \cos[8\exp(-3x)]dx = \sum_{n=0}^{\infty} \frac{(-1)^n 8^{2n}}{(2n)!(-6n+4)} \exp[(-6n+4)x] + C \quad (14)$$

Hence, the definite integral

$$\int_{-3}^{2} \exp(4x) \cdot \cos[8\exp(-3x)]dx = \sum_{n=0}^{\infty} \frac{(-1)^n 8^{2n}}{(2n)!(-6n+4)} \exp[(-12n+8) - \exp(18n-12)] \quad (15)$$

>evalf(int(exp(4*x)*cos(8*exp(-3*x)),x=-3..2),18);

737.407836945388

>evalf(sum((-1)^n*8^(2*n)/((2*n)!*(-6*n+4))*(exp(-12*n+8)-exp(18*n-12)),n=0..infinity),18);

737.407836945389

4. Conclusion

As mentioned, the integration term by term theorem plays a significant role in the theoretical inferences of this study. In fact, the applications of this theorem are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. On the other hand, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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