

# A New and Efficient Proposed Approach to Find Initial Basic Feasible Solution of a Transportation Problem

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**Abstract** In this research, a new and efficient approach of finding an initial basic feasible solution to transportation problems is proposed. The proposed approach is named "Inverse Coefficient of Variation Method (ICVM)", and the method is illustrated with seven numerical examples. Six existing methods; North West Corner Method (NWCN), Column Minimum Method (CMM), Least Cost Method (LCM), Row Minimum Method (RMM), Vogel's Approximation Method (VAM), and Allocation Table Method (ATM) were compared with the proposed approach. It can be said conclusively that the proposed Inverse Coefficient of Variation Method (ICVM) provides an improved Initial Basic Feasible Solution to all the transportation problems used in the experiment. Further, the new method leads to the optimal solution to many of the problems considered.

**Keywords:** transportation problem, inverse coefficient of variation method, initial basic feasible solution, optimal solution, proposed algorithm

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## 1. Introduction

Transportation problem is popular in operations research for its real life wide application. This is a special kind of network optimization problems which deals with the determination of a minimum-cost schedule for transporting a single commodity from a number of sources (warehouses) to a number of destinations (markets). This class of problem which is basically a linear programming problem can be extended to some practical applications such as inventory control, staff assignment, job scheduling, cash flow etc. The transportation algorithm is based on the assumption that the model is balanced, meaning that the total demand equals the total supply. If the model is unbalanced [5,13], we can always augment it with a dummy source or a dummy destination to restore balance.

The basic transportation problem was originally developed by Hitchcock in 1941. Efficient methods for obtaining solution were developed, primarily by Dantzig in 1951 [3], followed by Charnes *et al* in 1953 [2].

The optimal solution to the classical transportation problem requires the determination of a number of units of commodities to be transported from each origin to various destinations, satisfying source availability and destination demands so that the total cost of transportation is

minimum. The available amounts at the supply points and the amounts required at the demand points are the parameters of the transportation problem [4].

The transportation problem has a lot of special structure. For instance, each variable appears in exactly two constraints (with a non-zero coefficient). When a variable has a non-zero coefficient, the coefficient is either plus or minus. Due to this special structure, two possible things turn out to be true. The first is that, there are alternative methods of solving transportation problems that are more efficient than the standard simplex algorithm. This turns out to be important in practice, because real-world transportation problems have enormous numbers of variables. The Second is that, because of the special structure, it is possible to solve the transportation problem in whole numbers. That is, if the data of the problem (supplies, demands, and costs) are all whole numbers, then there is a whole number solution. The significance of this property is that you do not need to impose the difficult to handle integer constraints in order to get a solution that satisfies the constraints.

There are basically three stages for the solution procedure for the transportation problem:

Stage 1: Mathematical formulation of the transportation problem,

Stage 2: Finding an initial basic feasible solution,

Stage 3: Optimize the initial basic feasible solution which is obtained in Stage 2.

Our focus in this study is on stage 2: obtaining an improved initial basic feasible solution for the transportation problems.

## 2. Related Literature Review

Megha *et al* [10] worked on a Comparative Study of Transportation Problem Using C++ and Mat-lab. The study was about solving transportation problem using Operation Research (OR) approach in analysis and design phases and they tried to use C++ programming language and Mat lab to model the problem. The results obtained from the method were compared in order to make analysis and prove the object-oriented model correctness. They tried to make the results obtained to be identical and have the same results when solving the problem using the five methods: Northwest Corner Method, Minimum Cost Method, Row Minimum Cost Method, Column Minimum Cost Method, and Vogel's Approximation Method. The researchers recommended that in future, these problems should be compared for speed and accuracy using C + + and Mat-lab.

Opara *et al* [12] carried out an experimental study of the existing methods of solving linear transportation problems, comparing them with a new approach proposed by Mollah *et al* [11]. The existing methods, North West Corner Method, Least Cost Method, and Vogel's Approximation Method were compared to the newly proposed algorithm known as Allocation Table Method (ATM). Real life data were collected from Dangote Flour Mills Plc, Calabar, Cross River State Nigerian, and two numerical examples were collected from the exercise of Inyama [9]. The Allocation Table Method and the three existing methods were used to solve the problems. From the analysis carried out in the study, it was observed that the allocation table method did not yield comparatively better result. Several number of cost minimizing transportation problems were solved using ATM and it was found that the Allocation Table Method did not yield better results compared to the existing methods. It was therefore recommended that Mollah *et al* [11] should re-visit their proposed algorithm to probably know the lacuna in it, and future researchers should also look into a similar study to make their comments as regards to this contradiction.

Abdul *et al* [1] worked on a comparative study of initial basic feasible solution methods for transportation problems. In their work, three methods were used to find an initial basic feasible solution for the balanced transportation model. They used a new method of Minimum Transportation Cost Method (MTCM) to find the initial basic feasible solution for the solved problem by Hakim [7]. Hakim used Proposed Approximation Method (PAM) to find initial basic feasible solution for balanced transportation model and then compared the results with Vogel's Approximation Method (VAM). The results of both methods were noted to be the same but in their study, they took the same transportation model and used MTCM to find its initial basic feasible solution and compared the result with PAM and VAM. It was noted that the MTCM process provided not only the minimum transportation

cost but also an optimal solution.

Mollah *et al* [11] carried out a research on a New Approach to Solve Transportation Problems. Solution of transportation problems was proposed. Efficiency of allocation table method was tested by solving several number of cost minimizing transportation problems and it was found that the allocation table method yields comparatively a better result. Finally it can be claimed that the allocation table method may provided a remarkable Initial Basic Feasible Solution by ensuring minimum transportation cost. According to the researcher, it will help to achieve the goal to those who want to maximize their profit by minimizing the transportation cost.

Sankar and Gurupada [16] researched on minimizing cost and time through single objective function in multi-choice interval valued transportation problem. The study explored the study of Transportation Problem (TP) under the light of multi-choice environment with interval analysis. The parameters of TP followed multi-choice interval-valued type so the form of TP is called Multi-Choice Interval Transportation Problem (MCITP). Introduction of time was an important notion in TP of the paper. Transportation time and cost, both were minimized through single objective function of TP, which was the main aim of the study. A procedure was shown for converting from MCITP to deterministic TP and then solved it. A case study was included to illustrate the usefulness of the study. Finally, concluding remarks and an outlook for future study were presented to the study.

In the present work we experiment with a new transportation method that leads to an improved initial feasible solution to transportation problems.

## 3. Linear Transportation Model Problem

Transportation is an example of network optimization problem. It deals with the efficient distribution (transportation) of product (goods) and services from several supply locations (sources) with limited supply, to several demand locations (destinations) with a specified demand with the objective of minimizing total distribution cost.

This objective is achieved under the following constraints;

1. Each demand point receives its requirement.
2. Distributions from supply points do not exceed its available capacity.

This goal is achieved contingent on availability and requirements constraints. Transportation problem therefore assumes that the transportation cost on a given route is directly proportional to the number of units of the commodity transported [9].

## 4. Model Formulation

The formulation of the transportation model employs double – subscripted variables of the form  $x_{ij}$ . Thus, the general formulation of the transportation problem with supply (sp), demand (d), n sources and m destinations, is given by

$$\left. \begin{aligned} & \text{Minimize } \sum_{i=1}^n \sum_{j=1}^m C_{ij}x_{ij} \\ & \text{Subject to } \sum_{i=1}^n (sp)_i = sp \quad i = 1, 2, 3, \dots, n \\ & \sum_{j=1}^m d_j = d \quad j = 1, 2, 3, \dots, m \end{aligned} \right\}$$

The general formulation of the transportation problem reveals that m supply constraints and n demand constraints translate into m + n total constraints. The flow chart in Figure 1 illustrates the various phases leading to the optional solution of a transportation problem

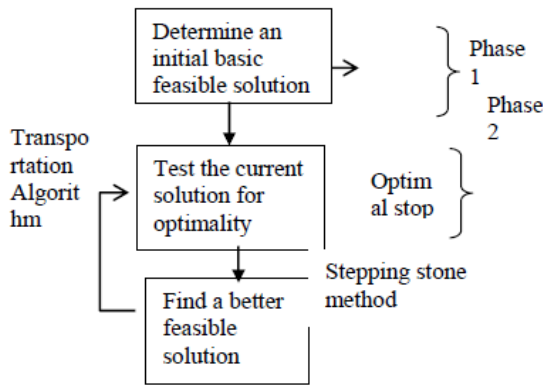


Figure 1.

### 5. Transportation Tableau

The transportation tableau is a unique tabular representation of the transportation problem. The  $x_{ij}$  variable gives the number of units transported from source i to destination j (which is to be solved for) while the unit cost for the transportation from i to j, denoted by  $C_{ij}$ , is recorded in a small box in the upper – right – hand corner of each cell. Table A is the form of the general transportation tableau.

Table A. Transportation Tableau

To (i) From (j)	DESTINATIONS							
	1	2	...	J	...	M	Supply	
SOURCES	1	$\begin{matrix} C_{11} \\ x_{11} \end{matrix}$	$\begin{matrix} C_{12} \\ x_{12} \end{matrix}$	...	$\begin{matrix} C_{1j} \\ x_{1j} \end{matrix}$	...	$\begin{matrix} C_{1m} \\ x_{1m} \end{matrix}$	(sp) <sub>1</sub>
	2	$\begin{matrix} C_{21} \\ x_{21} \end{matrix}$	$\begin{matrix} C_{22} \\ x_{22} \end{matrix}$	...	$\begin{matrix} C_{2j} \\ x_{2j} \end{matrix}$	...	$\begin{matrix} C_{2m} \\ x_{2m} \end{matrix}$	(sp) <sub>2</sub>
	...	...	...	...	...	...	...	...
	i	$\begin{matrix} C_{i1} \\ x_{i1} \end{matrix}$	$\begin{matrix} C_{i2} \\ x_{i2} \end{matrix}$	...	$\begin{matrix} C_{ij} \\ x_{ij} \end{matrix}$	...	$\begin{matrix} C_{im} \\ x_{im} \end{matrix}$	(sp) <sub>i</sub>
	...	...	...	...	...	...	...	...
	n	$\begin{matrix} C_{n1} \\ x_{n1} \end{matrix}$	$\begin{matrix} C_{n2} \\ x_{n2} \end{matrix}$	...	$\begin{matrix} C_{nj} \\ x_{nj} \end{matrix}$	...	$\begin{matrix} C_{nm} \\ x_{nm} \end{matrix}$	(sp) <sub>n</sub>
Demand	d <sub>1</sub>	d <sub>2</sub>	...	d <sub>j</sub>	...	d <sub>m</sub>	$\sum (sp)_i = \sum d_j$	

### 6. Methods for Finding Initial Basic Feasible Solution

The first phase of the algorithm for solving a transportation problem for optimal solution involves

finding the initial basic feasible solution. An initial feasible solution is a set of arc flows that satisfies each demand requirement without supplying more from any origin node than the supply available. This paper compares only six heuristics method for developing an initial solution to transportation problem, with the new proposed approach; which are;

- i. The Northwest Corner Method (NWCM)
- ii. The Least Cost Method (LCM)
- iii. The Vogel’s Approximation Method (VAM)
- iv. Row Minimum Method (RMM)
- v. Column Minimum Method (CMM)
- vi. Allocation Table Method (ATM)

#### 6.1. Algorithm for Inverse Coefficient of Variation Method (ICVM)

Our algorithm for the method for determining the initial basic feasible solution to transportation problem of the proposed approach is stated as follows:

**Step 1:** Construct a balance Transportation Table (TT) from the given transportation problem.

**Step 2:** Determine the mean ( $\bar{x}_i$ ),  $\bar{x}_i = \frac{\sum_{i=1}^n x_i}{n}$  and

standard deviation ( $s_i$ ),  $s_i = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$  for each row. Then obtain the respective inverse of coefficient of variation  $[(CV)^{-1}]$ ,  $(CV)^{-1} = \frac{\bar{x}_i}{s_i}$

**Step 3:** Determine the mean ( $\bar{x}_j$ ),  $\bar{x}_j = \frac{\sum_{j=1}^m x_j}{m}$  and

standard deviation ( $s_j$ ),  $s_j = \sqrt{\frac{\sum_{j=1}^m (x_j - \bar{x})^2}{m-1}}$  for each column. Then obtain the respective inverse coefficient of variation  $[(CV)^{-1}]$ ,  $(CV)^{-1} = \frac{\bar{x}_j}{s_j}$ .

Ignore any row or column whose standard deviation is zero.

**Step 4:** Identify the row or column with the smallest  $(CV)^{-1}$  among the rows and columns (breaking ties arbitrarily). Locate the cell with the lowest cost in the selected row or column and assign as many units as possible to it.

**Step 5:** Reduce the row supply and column demand by the number of units assigned to the cell and cross out the row supply or column demand that is satisfied then form a new tableau. If a row and a column are satisfied simultaneously, only one of them is crossed out and the remaining column (or row) is assigned a zero demand (or supply). Again, any column or row with zero demand or supply should not be used in computing subsequent  $(CV)^{-1}$ .

**Step 6:** Recalculate the row and column  $(CV)^{-1}$  for the reduced transportation tableau, as indicated in **steps 2 & 3**, and then go to **steps 4 & 5**. Continue until all the requirements (demands and supplies) are satisfied.

**Step 7:** Finally calculate the total transportation cost of the TT. This calculation is the sum of the product of cost and corresponding assigned value of the TT.

**6.2. Numerical Problems**

We shall use seven numerical examples to illustrate the proposed algorithm. Examples one to three were extracted from Opara *et al* [12]; examples four to six were gotten from Mollah *et al* [11]; while the seventh example was gotten from Example 9.3 of Inyama [9].

**Example 1**

Consider a Transportation problem with four markets and four warehouses. The market demands are 10, 4, 6, and 14 while the warehouse capacities are 6, 9, 7, and 12. The cell entries represent unit cost of transportation, and the table is shown in Table 1.

**Table 1. Data of example 1**

Warehouse	Markets				Supply
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	
W <sub>1</sub>	2	5	6	3	6
W <sub>2</sub>	9	6	2	1	9
W <sub>3</sub>	5	2	3	6	7
W <sub>4</sub>	7	7	2	4	12
<b>Demand</b>	<b>10</b>	<b>4</b>	<b>6</b>	<b>14</b>	<b>34</b>

We shall obtain the means ( $\bar{x}_i$ ), standard deviations ( $S_i$ ) and the inverse of the coefficient of variation  $(CV)^{-1}$  as shown in Table 2. For instance, to calculate the average, standard deviation, and  $(CV)^{-1}$  for warehouse one, we

$$\text{have, } \bar{x}_1 = \frac{\sum_{i=1}^n x_i}{n} = \frac{16}{4} = 4.0,$$

$$s_1 = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x}_1)^2}{n-1}} = \sqrt{\frac{[(2-4)^2 + (5-4)^2 + (6-4)^2 + (3-4)^2]}{4-1}} = 1.826$$

$$\text{and } (CV)^{-1} = \frac{\bar{x}_1}{s_1} = \frac{4}{1.826} = 2.191$$

The remaining warehouses 2, 3, and 4 were done in the same manner as well as markets 1, 2, 3, and 4 to obtain the averages, standard deviations and inverse coefficient of variations and the results are presented in Table 2. However, for easier computation, an electronic calculator or Microsoft excel can be used to compute  $(\bar{x}_i)$ ,  $S_i$ , and  $(CV)^{-1}$ .

**Table 2. Iteration 1**

Warehouse	Markets				Supply	$\bar{X}_i$	$S_i$	$(CV)^{-1}$
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>				
W <sub>1</sub>	2	5	6	3	6	4.0	1.826	2.191
W <sub>2</sub>	9	6	2	1	9	4.5	3.697	1.217
W <sub>3</sub>	5	2	3	6	7	4.0	1.826	2.191
W <sub>4</sub>	7	7	2	4	12	5.0	2.449	2.042
<b>Demand</b>	<b>10</b>	<b>4</b>	<b>6</b>	<b>14</b>	<b>34</b>			
$\bar{X}_i$	5.75	5	3.25	3.5				
$S_i$	2.986	2.160	1.893	2.082				
$(CV)^{-1}$	1.926	2.315	1.717	1.681				

The minimum  $(CV)^{-1}$  is 1.217 which occurs in row 2. The least cost in this row is ~~1~~1 which corresponds to column 4. Hence;

$X_{24} = \text{minimum}(a_2, b_4) = \text{minimum}(9, 14) = 9$ . Row 2 is satisfied and crossed out, while column 4 remains  $14 - 9 = 5$ . Thus, the summary of the adjustment is shown in Table 3.

**Table 3. Iteration 2**

Warehouse	Markets				Supply	$\bar{X}_i$	$S_i$	$(CV)^{-1}$
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>				
W <sub>1</sub>	2	5	6	3	6	4.0	1.826	2.191
W <sub>2</sub>	0	0	0	(9)	0	-	-	-
W <sub>3</sub>	5	2	3	6	7	4.0	1.826	2.191
W <sub>4</sub>	7	7	2	4	12	5.0	2.449	2.042
<b>Demand</b>	<b>10</b>	<b>4</b>	<b>6</b>	<b>5</b>	<b>25</b>			
$\bar{X}_i$	4.67	4.67	3.67	4.33				
$S_i$	2.517	2.517	2.082	1.528				
$(CV)^{-1}$	1.855	1.855	1.763	2.834				

The minimum  $(CV)^{-1}$  is 1.763 which occurs in column 3. The least cost in this column is ~~2~~2 which corresponds to row 4. Hence;

$X_{43} = \text{minimum}(a_4, b_3) = \text{minimum}(12, 6) = 6$ . Column 3 is satisfied and crossed out, while row 4 remains  $12 - 6 = 6$ . Thus, the summary of the adjustment is displayed in Table 4.

**Table 4. Iteration 3**

Warehouse	Markets				Supply	$\bar{X}_i$	$S_i$	$(CV)^{-1}$
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>				
W <sub>1</sub>	2	5	0	3	6	3.33	1.528	2.179
W <sub>2</sub>	0	0	0	(9)	0	-	-	-
W <sub>3</sub>	5	2	0	6	7	4.33	2.082	2.080
W <sub>4</sub>	7	7	(6)	4	6	6	1.732	3.464
<b>Demand</b>	<b>10</b>	<b>4</b>	<b>0</b>	<b>5</b>	<b>19</b>			
$\bar{X}_i$	4.67	4.67	-	4.33				
$S_i$	2.517	2.517	-	1.528				
$(CV)^{-1}$	1.855	1.855	-	2.834				

The minimum  $(CV)^{-1}$  is 1.855 (breaking ties arbitrarily) which occurs in column 1. The least cost in this column is ~~2~~ which corresponds to row 1. Hence  $X_{12} = \text{minimum}(a_1, b_1) = \text{minimum}(6, 10) = 6$ . Row 1 is satisfied and crossed out, while column 1 remains  $10 - 6 = 4$ . We now adjust the table to obtain the result in Table 5.

Table 5. Iteration 4

Warehouse	Markets				Supply	$\bar{X}_i$	$S_i$	$(CV)^{-1}$
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>				
W <sub>1</sub>	<del>2</del> (6)	5 0	6 0	3 0	0	-	-	-
W <sub>2</sub>	9 0	6 0	2 0	1 (9)	0	-	-	-
W <sub>3</sub>	5 0	2 (4)	3 0	6 6	7	4.33	2.082	2.080
W <sub>4</sub>	7 0	7 0	2 (6)	4 (5)	6	6	1.732	3.464
Demand	10	4	0	5	13			
$\bar{X}_i$	6	4.5	-	5				
$S_i$	1.414	3.536	-	1.414				
$(CV)^{-1}$	4.243	1.273	-	3.536				

The minimum  $(CV)^{-1}$  is 1.273 which appears in column 2. The least cost in this column is ~~2~~ which corresponds to row 3. Hence  $X_{32} = \text{minimum}(a_3, b_2) = \text{minimum}(7, 4) = 4$ . Column 2 is satisfied and crossed out, while row 3 is left with  $7 - 3 = 3$ . We now adjust the table to get the result in Table 6.

Table 6. Iteration 5

Warehouses	Markets				Supply	$\bar{X}_i$	$S_i$	$(CV)^{-1}$
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>				
W <sub>1</sub>	2 (6)	5 0	6 0	3 0	0	-	-	-
W <sub>2</sub>	9 0	6 0	2 0	1 (9)	0	-	-	-
W <sub>3</sub>	5 0	2 (4)	3 0	6 (6)	3	5.5	0.707	7.779
W <sub>4</sub>	7 0	7 0	2 (6)	4 (5)	6	5.5	2.121	2.593
Demand	4	0	0	5	9			
$\bar{X}_i$	6	-	-	5				
$S_i$	1.414	-	-	1.414				
$(CV)^{-1}$	4.243	-	-	3.536				

The minimum  $(CV)^{-1}$  is 2.593 which appears in row 4. The least cost in this row is ~~4~~ which corresponds to column 4. Hence  $X_{44} = \text{minimum}(a_4, b_4) = \text{minimum}(6, 5) = 5$ . Hence column 4 is satisfied and crossed out, while row 4 is left with  $6 - 5 = 1$ . We now adjust the table to get the result in Table 7.

The only  $(CV)^{-1}$  remaining is 4.243 which appears in column 1 with a least cost of ~~5~~, corresponding to row 3. Hence  $X_{31} = \text{minimum}(3, 4) = 3$ . Hence row 3 is satisfied and crossed out, while column 1 remains  $4 - 3 = 1$ . After adjustment, only cell  $X_{41}$  will be left with a supply and demand of 1. This finally gives the summary result in Table 8.

Table 7. Iteration 6

Warehouses	Markets				Supply	$\bar{X}_i$	$S_i$	$(CV)^{-1}$
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>				
W <sub>1</sub>	<del>2</del> (6)	5 0	6 0	3 0	0	-	-	-
W <sub>2</sub>	9 0	6 0	2 0	1 (9)	0	-	-	-
W <sub>3</sub>	5 (4)	2 0	3 0	6 6	3	5.5	0.707	7.779
W <sub>4</sub>	7 0	7 0	2 (6)	4 (5)	1	5.5	2.121	2.593
Demand	4	0	0	0	4			
$\bar{X}_i$	6	-	-	-				
$S_i$	1.414	-	-	-				
$(CV)^{-1}$	4.243	-	-	-				

Table 8. Iteration 7

Warehouses	Markets			
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>
W <sub>1</sub>	<del>2</del> (6)	5 0	6 0	3 0
W <sub>2</sub>	9 0	6 0	2 0	1 (9)
W <sub>3</sub>	5 (3)	2 (4)	3 0	6 (6)
W <sub>4</sub>	7 (1)	7 0	2 (6)	4 (5)

Hence, the total transportation cost is  $6(2) + 9(1) + 3(5) + 4(2) + 1(7) + 6(2) + 5(4) = \text{N}83$ .

We now improve the initial feasible solution obtained by first calculating the shadow costs  $C_{ij} = U_i + V_j$  where  $U_i$  and  $V_j$  ( $i = 1, 2, 3, 4; j = 1, 2, 3, 4$ ) are the multipliers. That is;

For  $X_{11} : U_1 + V_1 = 2$  For  $X_{24} : U_2 + V_4 = 1$

For  $X_{31} : U_3 + V_1 = 5$  For  $X_{32} : U_3 + V_2 = 2$

For  $X_{41} : U_4 + V_1 = 7$  For  $X_{43} : U_4 + V_3 = 2$

For  $X_{44} : U_4 + V_4 = 4$

Setting  $U_1 = 0$  and solving produces;

$U_1 = 0, U_2 = 2, U_3 = 3, U_4 = 5; V_1 = 2, V_2 = -1, V_3 = -3, V_4 = -1$ .

We now proceed by calculating the potential benefits ( $L_{kl}$ ) of each of the 9 non-basic variables by using  $L_{kl} = C_{kl} - U_k - V_l$ . That is;

For  $X_{12} : L_{12} = C_{12} - U_1 - V_2 = 5 - 0 - (-1) = 6$

For  $X_{13} : L_{13} = C_{13} - U_1 - V_3 = 6 - 0 - (-3) = 9$

For  $X_{14} : L_{14} = C_{14} - U_1 - V_4 = 3 - 0 - (-1) = 4$

For  $X_{21} : L_{21} = C_{21} - U_2 - V_1 = 9 - 2 - 2 = 5$

For  $X_{22} : L_{22} = C_{22} - U_2 - V_2 = 6 - 2 - (-1) = 5$

For  $X_{23} : L_{23} = C_{23} - U_2 - V_3 = 2 - 2 - (-3) = 3$

For  $X_{33} : L_{33} = C_{33} - U_3 - V_3 = 3 - 3 - (-3) = 3$

For  $X_{34} : L_{34} = C_{34} - U_3 - V_4 = 6 - 3 - (-1) = 4$

For  $X_{42} : L_{42} = C_{42} - U_4 - V_2 = 7 - 5 - (-1) = 3$

Since none of the potential benefit is ( $L_{kl}$ ) is negative, we stop; the present initial feasible solution is optimal. Hence the optimal solution is  $X_{11} = 6$ ,  $X_{24} = 9$ ,  $X_{31} = 3$ ,  $X_{32} = 4$ ,  $X_{41} = 1$ ,  $X_{43} = 6$ ,  $X_{44} = 5$  and the minimum cost for this transportation problem is  $6(2) + 9(1) + 3(5) + 4(2) + 1(7) + 6(2) + 5(4) = 1183$ .

If there is negative value in the potential benefits ( $L_{kl}$ ), then we select the cell with the most negative  $L_{kl}$  value. The next step is assign the quantity "0" to the selected cell and find a group of cells which are presently in use (that is, variables  $X_{ij}$ ) which when modified by the "0" value will keep  $a_i$  and  $b_j$  values unchanged. In other words, a closed loop for the current entering variable will be constructed (the loop starts and ends with the entering variable). The loop consists of successive horizontal and vertical connected lines whose end points must be basic variables except the end points associated with the entering variable. The loop can be traced in clockwise or anti-clockwise direction for a given basic feasible solution; only one unique loop is possible for each non-basic variable. Then adjust the corner basic variable of the loop by 0. The last step is to set "0" to be as large as possible. Redraw tableau and go to the step of calculating the multipliers  $U_i$  and  $V_j$ . Hence, the method employed here to obtain the optimal solution is the Modified Distribution (MODI) Method.

The remaining six examples shall be presented in their last tableau without illustration.

**Example 2**

Dangote Flour Mills Plc is a manufacturing company located in Calabar. The company produces Bread Flour (BF), Confectionery Flour (CF), Penny Semolina (PS) and Wheat Offals (WO). These products are supplied to the following states (locations) Bayelsa, Anambra, Rivers, Kano, Abia, Enugu, Akwa-Ibom etc. For the purpose of this study, only four (4) of these demand points shall be considered; Enugu, Akwa-Ibom, Anambra and Rivers. The estimated supply capacities of the four products, the demand requirements at the four sites (states) and the transportation cost per bag of each product are given in Table 9.

**Table 9. Data of Example 2**

	Enugu	Akwa-Ibom	Anambra	Rivers	Supply
BF	45	52	63	57	15500
CF	58	48	56	54	12000
PS	52	55	62	58	14400
WO	65	48	44	54	11600
Demand	12600	12500	13000	15400	53500

To obtain the initial basic feasible solution of Example 2, using the proposed algorithm, the final result is summarized in Table 10.

Hence, the total transportation cost is  $12600(45) + 2900(52) + 9600(48) + 1400(56) + 1000(54) + 14400(58) + 11600(44) = 2,656,600$ .

**Table 10. Initial Basic Feasible Solution According to ICVM**

	Enugu	Akwa-Ibom	Anambra	Rivers
BF	(12600) 45	(2900) 52	63	57
CF	58	(9600) 48	(1400) 56	(1000) 54
PS	52	55	62	(14400) 58
WO	65	48	(11600) 44	54

**Example 3**

Consider a Transportation problem with four markets and three warehouses. The market demands are 10, 6, 8, and 12 while the warehouse capacities are 12, 14, and 10. The cell entries represent unit cost of transportation, and the table is shown in Table 11.

**Table 11. Data of Example 3**

Warehouses	Markets				Supply
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	
W <sub>1</sub>	5	7	9	6	12
W <sub>2</sub>	6	7	10	5	14
W <sub>3</sub>	7	6	8	1	10
Demand	10	6	8	12	36

The final result using the proposed algorithm is displayed in Table 12.

**Table 12. Initial Basic Feasible Solution According to ICVM**

Warehouses	Markets			
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>
W <sub>1</sub>	(10) 5	7	(2) 9	6
W <sub>2</sub>	6	(6) 7	(6) 10	(2) 5
W <sub>3</sub>	7	6	8	(10) 1

Total transportation cost =  $10(5) + 2(9) + 6(7) + 6(10) + 2(5) + 10(1) = 190$ .

**Example 4**

A company manufactures motor cars and it has three factories F<sub>1</sub>, F<sub>2</sub> and F<sub>3</sub> whose weekly production capacities are 300, 400 and 500 pieces of cars respectively. The company supplies motor cars to its four showrooms located at D<sub>1</sub>, D<sub>2</sub>, D<sub>3</sub> and D<sub>4</sub> whose weekly demands are 250, 350, 400 and 200 pieces of cars respectively. The transportation costs per piece of motor cars are given in the transportation Table 13. Find out the schedule of shifting of motor cars from factories to showrooms with minimum cost:

**Table 13. Data of Example 4**

Factories	Showrooms				
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
F <sub>1</sub>	3	1	7	4	300
F <sub>2</sub>	2	6	5	9	400
F <sub>3</sub>	8	3	3	2	500
Demand	250	350	400	200	



The final result using the proposed algorithm is displayed in Table 14.

**Table 14. Initial Basic Feasible Solution According to ICVM**

Warehouses	Markets			
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>
W <sub>1</sub>	3	11 (300)	7	4
W <sub>2</sub>	6 (250)	7	10 (150)	9
W <sub>3</sub>	7	6 (50)	8 (250)	2 (200)

Total transportation cost = 300(1) + 250(2) + 150(5) + 50(3) + 250(3) + 200(2) = 2850.

**Example 5**

Consider the following transportation problem (Transportation Table 15) involving four sources and four destinations. The cell entries represent the cost of transportation per unit. Obtain an initial basic feasible solution.

**Table 15. Data of Example 5**

Sources	Destinations				Supply
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
S <sub>1</sub>	7	5	9	11	30
S <sub>2</sub>	4	3	8	6	25
S <sub>3</sub>	3	8	10	5	20
S <sub>4</sub>	2	6	7	3	15
Demand	30	30	20	10	

The final result using the proposed algorithm is displayed in Table 16.

**Table 16. Initial Basic Feasible Solution According to ICVM**

Sources	Destinations			
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>
S <sub>1</sub>	7 (5)	5 (5)	9 (20)	11
S <sub>2</sub>	4	3 (25)	8	6
S <sub>3</sub>	3 (20)	8	10	5
S <sub>4</sub>	2 (5)	6	7	3 (10)

Total transportation cost = 5(7) + 5(5) + 20(9) + 25(3) + 20(3) + 5(2) + 10(3) = 415.

**Example 6**

Consider the following transportation problem (Transportation Table 17) involving three origins and three destinations. The cell entries represent the cost of transportation per unit. Obtain an initial basic feasible solution.

**Table 17. Data of Example 6**

Origins	Destinations			Supply
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	
O <sub>1</sub>	4	3	5	90
O <sub>2</sub>	6	5	4	80
O <sub>3</sub>	8	10	7	100
Demand	70	120	80	

Using the proposed algorithm, the summary result is presented in Table 18.

**Table 18. Initial Basic Feasible Solution According to ICVM**

Origins	Destinations		
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
O <sub>1</sub>	4	3 (90)	5
O <sub>2</sub>	6	5 (30)	4 (50)
O <sub>3</sub>	8 (70)	10	7 (30)

The transportation case = 90(3) + 30(5) + 50(4) + 70(8) + 30(7) = 1390.

**Example 7**

Consider the transportation problem with three markets and four warehouses. The market demands are 16, 10, 14, while the warehouse capacities are 11, 12, 10, 7. The unit cost of transportation is as given in Table 19.

**Table 19. Data of Example 7**

Warehouses	Markets			Supply
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	
W <sub>1</sub>	4	3	4	11
W <sub>2</sub>	10	7	5	12
W <sub>3</sub>	8	8	3	10
W <sub>4</sub>	5	6	6	7
Demand	16	10	14	

The initial basic feasible solution using the proposed algorithm is presented in Table 20.

**Table 20. Initial Basic Feasible Solution According to ICVM**

Warehouses	Markets		
	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>
W <sub>1</sub>	4 (11)	3	4
W <sub>2</sub>	10	7 (8)	5 (4)
W <sub>3</sub>	8	8	3 (10)
W <sub>4</sub>	5 (5)	6 (2)	6

Total transportation cost = 11(4) + 8(7) + 4(5) + 10(3) + 5(5) + 2(6) = 187.

**6.3. Analysis of Result**

Having obtained the initial basic feasible solution using the proposed method with seven numerical data examples, the gotten results are compared with the existing methods.

Looking at Table 21, the transportation cost for the seven numerical examples used in this study, North West Corner Method (NWCN), and Column Minimum Method (LCM) and Row Minimum Method (RMM) provided only one optimal result; Vogel's Approximation Method (VAM) and Allocation Table Method (ATM) provided two optimal results; while the Proposed Inverse Coefficient of Variation Method (ICVM) provided four optimal results. The remaining three examples for the

proposed method which are not optimal are close to optimal. Thus, the proposed Inverse Coefficient of Variation Method (ICVM) provides comparatively a better initial basicfeasible solution than the results obtained by the traditional algorithms which are either optimal or near to optimal. It is obvious that the performance of the

solution varies for other methods which may be applicable in the proposed method, because it is quite difficult to assume a particular method that will result in the best solution in all examples that may arise. However, on the average, some methods are far better than others, which the proposed method is one of them.

**Table 21. Comparison of the Results Obtained by Various Methods with the Proposed Method**

Method	Initial Basic Feasible Solution						
	Example 1	Example 2	Example 3	Example 4	Example 5	Example 6	Example 7
NWCM	149	2,817,000	192	4400	540	1500	230
LCM	83	2,657,000	192	2900	435	1450	199
VAM	92	2,657,000	190	2850	470	1500	199
RMM	92	2,659,400	216	2850	470	1450	199
CMM	88	2,663,500	216	3600	435	1500	187
ATM	112	2,694,000	192	2850	415	1390	199
Proposed Approach (ICVM)	83	2,656,600	190	2850	415	1390	187
Optimal Solution	<b>83</b>	<b>2,655,600</b>	<b>190</b>	<b>2850</b>	<b>410</b>	<b>1390</b>	<b>183</b>

### 7. Conclusion

The primary aim of a transportation model is to provide a strong structure to obtain the best ways to deliver goods from sources to destinations (customers).In this study, a new approach named Inverse Coefficient of Variation Method (ICVM) for finding an initial basic feasible solution of a balanced transportation problem is proposed. Coefficient of Variation Inverse Method has been tested and proven efficient by solving several number of cost minimizing transportation problems and it is discovered that the Inverse Coefficient of Variation Method gives comparatively a better result. Conclusively, it can be said that Inverse Coefficient of Variation Method an improved Initial Basic Feasible Solution by ensuring minimum transportation cost, though it may be a bit greater in some examples, but the good thing for this proposed method is that in most cases, it yields an optimum or close to optimum solution. This study will assist to achieve the aim of maximizing profits by minimizing the cost of transportation. The method employed here to obtain the optimal solution is the Modified Distribution (MODI) Method.

Future study should look at similar research by comparing other existing methods which were not incorporated in this study to the proposed technique. Again, a program code should be written for the proposed technique for ease computation.

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