

The Time-optimal Problems for the Fuzzy R-solution of the Control Linear Fuzzy Integro-Differential Inclusions

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Abstract In this paper, we show some properties of the fuzzy R-solution of the control linear fuzzy integro-differential inclusions and consider the time-optimal problems for it. For such problems we receive necessary conditions of optimality.

Keywords: fuzzy integro-differential inclusions, control problems, time-optimal problems, fuzzy R-solution

1. Introduction

In the last decades, a number of works devoted to problems of optimal control of set-valued trajectories (fuzzy trajectories, trajectory bundles or an ensemble of trajectories) appeared. These works fall into a subdivision of the optimal control theory, namely, the theory of process control under uncertainty and fuzzy conditions. This is caused by the fact that, in actual problems arising in economy and engineering in the course of construction of a mathematical model, it is practically impossible to exactly describe the behavior of an object. This is explained by the following fact. First, for some parameters of the object, it is impossible to specify exact values and laws of their change, but it is possible to determine the domain of these changes. Second, for the sake of simplicity of the mathematical model being constructed, the equations that describe the behavior of the object are simplified and one should estimate the consequences of such a simplification. Therefore, it is possible to divide the articles devoted to this direction into two types characterized by the following distinctive features:

1) there exists an incomplete or fuzzy information on the initial data [1-12];

2) the equations describing the behavior of the object to be controlled are assumed to be inexact, for example, they can contain some parameters whose exact values and laws of variation are unknown but the domain of their values is fuzzy [12-26].

In the second case, fuzzy differential inclusions are frequently used to describe behavior of objects. The reason is that, first this approach is most obvious and, second, theory of fuzzy and ordinary differential inclusions is well found and is rapidly developed at the present time [27-34].

In this article we consider some properties of the fuzzy R-solution of the control linear fuzzy integro-differential inclusions and research the time-optimal problems for it.

2. The Fundamental Definitions and Designations

Let $comp(R^n) \setminus \{conv(R^n)\}$ be a family of all nonempty (convex) compact subsets from the space R^n with the Hausdorff metric

$$h(A, B) = \min_{r \geq 0} \{S_r(A) \supset B, S_r(B) \supset A\}$$

where $A, B \in comp(R^n)$, $S_r(A)$ is r -neighborhood of set A .

Let E^n be the set of all $u: R^n \rightarrow [0, 1]$ such that u satisfies the following conditions:

1) u is normal, i.e. there exists an $x_0 \in R^n$ such that $u(x_0) = 1$;

2) u is fuzzy convex, i.e.

$$u(\lambda x + (1 - \lambda)y) \geq \min\{u(x), u(y)\}$$

for any $x, y \in R^n$ and $0 \leq \lambda \leq 1$;

3) u is upper semicontinuous, i.e. for any $x_0 \in R^n$ and $\varepsilon > 0$ exists $\delta(x_0, \varepsilon) > 0$ such that $u(x) < u(x_0) + \varepsilon$ whenever $\|x - x_0\| < \delta(x_0, \varepsilon)$, $x \in R^n$;

4) the closure of the set $\{x \in R^n : u(x) > 0\}$ is compact.

If $u \in E^n$, then u is called a fuzzy number, and E^n is said to be a fuzzy number space.

The set $\{x \in R^n : u(x) \geq \alpha\}$ is called the α -level $[u]^\alpha$

of a fuzzy number $u \in E^n$ for $0 < \alpha \leq 1$. The closure of the set $\{x \in R^n : u(x) > 0\}$ is called the 0-level $[u]^0$ of a fuzzy number $u \in E^n$.

Then from 1)-4), it follows that the α -level set $[u]^\alpha \in \text{conv}(R^n)$ for all $0 \leq \alpha \leq 1$.

Let θ be the fuzzy mapping defined by $\theta(x) = 0$ if $x \neq 0$ and $\theta(0) = 1$.

Define $D: E^n \times E^n \rightarrow [0, \infty)$ by the relation

$$D(u, v) = \sup_{0 \leq \alpha \leq 1} h([u]^\alpha, [v]^\alpha)$$

Then D is a metric in E^n . Further we know that [35]:

- i) (E^n, D) is a complete metric space,
- ii) $D(u + w, v + w) = D(u, v)$ for all $u, v, w \in E^n$,
- iii) $D(\lambda u, \lambda v) = |\lambda| D(u, v)$ for all $u, v \in E^n$ and $\lambda \in R$.

Definition 1. [36] A mapping $F: [0, T] \rightarrow E^n$ is measurable if for all $\alpha \in [0, 1]$ the set-valued map $F_\alpha: [0, T] \rightarrow \text{conv}(R^n)$ defined by $F_\alpha(t) = [F(t)]^\alpha$ is measurable.

Definition 2. [36] A mapping $F: [0, T] \rightarrow E^n$ is said to be integrably bounded if there is an integrable function $h(t)$ such that $\|x(t)\| \leq h(t)$ for every $x(t) \in F_0(t)$.

Definition 3. [36] The integral of a fuzzy mapping $F: [0, T] \rightarrow E^n$ is defined levelwise by

$$\left[\int_0^T F(t) dt \right]^\alpha = \int_0^T F_\alpha(t) dt = \left\{ \int_0^T f(t) dt : f: [0, T] \rightarrow R^n \text{ is a measurable selection of } F_\alpha: [0, T] \rightarrow \text{conv}(R^n) \right\} \text{ for all } \alpha \in [0, 1].$$

Definition 4. [36] A measurable and integrably bounded mapping $F: [0, T] \rightarrow E^n$ is said to be integrable over $[0, T]$ if $\int_0^T F(t) dt \in E^n$.

Note that if $F: [0, T] \rightarrow E^n$ is measurable and integrably bounded, then F is integrable. Further if $F: [0, T] \rightarrow E^n$ is continuous, then it is integrable.

Now we consider following control differential equations with the fuzzy parameter

$$\dot{x} = f(t, x, w, v), \quad x(0) = x_0, \quad (1)$$

where $\dot{x}$ means $\frac{dx}{dt}$; $t \in R_+$; $x \in R^n$; $w \in R^m$ is the control; $v \in V \in E^k$ is the fuzzy parameter; $f: R_+ \times R^n \times R^m \times R^k \rightarrow R^n$.

Let $W: R_+ \rightarrow \text{conv}(R^m)$ be the measurable set-valued map.

Definition 5. The set LW of all measurable single-valued branches of the set-valued map $W(\cdot)$ is the set of the admissible controls.

Further we consider following control fuzzy differential inclusions

$$\dot{x} \in F(t, x, w), \quad x(0) = x_0, \quad (2)$$

where $F: R_+ \times R^n \times R^m \rightarrow E^n$ is the fuzzy map such that $F(t, x, w) \equiv f(t, x, w, V)$.

Obviously, the control fuzzy differential inclusion (2) turns into the ordinary fuzzy differential inclusion [29-32]

$$\dot{x} \in \Phi(t, x), \quad x(0) = x_0, \quad (3)$$

if the control $\tilde{w}(\cdot) \in LW$ is fixed and $\Phi(t, x) \equiv F(t, x, \tilde{w}(t))$.

If right-hand side of the fuzzy differential inclusion (3) satisfies some conditions (for example look [32]) then the fuzzy differential inclusions (3) has the fuzzy R-solution [32].

Let $X(t)$ denote the fuzzy R-solution of the differential inclusion (3), then $X(t, w)$ denotes the fuzzy R-solution of the control differential inclusion (2) for the fixed $w(\cdot) \in LW$.

Definition 6. The set $Y(T) = \{X(T, w): w(\cdot) \in LW\}$ is called the attainable set of the fuzzy system (2).

3. The Some Properties of The Fuzzy R-Solution

Further consider the following control linear fuzzy integro-differential inclusions

$$\begin{aligned} \dot{x} &\in A(t)x + \int_0^t K(t, s)x(s)ds \\ &+ B(t)w + C(t)V, \quad x(0) = 0, \end{aligned} \quad (4)$$

where $A(t), K(t, s)$ are $(n \times n)$ -dimensional matrix-valued functions; $B(t)$ is $(n \times m)$ - dimensional matrix-valued function; $C(t)$ is $(n \times k)$ - dimensional matrix-valued function; $V \in E^k$ is fuzzy set.

In this section, we consider some properties of the fuzzy R-solution of the control fuzzy integro-differential inclusion (4).

Let the following condition be true.

Condition A:

A1. $A(\cdot), B(\cdot), C(\cdot)$ are measurable on $[0, T]$;

A2. There exist $a, b, c > 0$ such that

$$\|A(t)\| \leq a, \|B(t)\| \leq b, \|C(t)\| \leq c$$

almost everywhere on $[0, T]$;

A3. $K(t, s)$ is measurable on $[0, T] \times [0, T]$;

A4. There exists $l > 0$ such that $\|K(t, s)\| \leq l$ almost everywhere on $[0, T] \times [0, T]$;

A5. The set-valued map $W: [0, T] \rightarrow \text{conv}(R^m)$ is measurable on $[0, T]$;

A6. There exists such $v(\cdot) \in L_2[0, T]$ that $h(W(t), 0) \leq v(t)$ almost everywhere on $[0, T]$.

Theorem 1. [37] Let condition A be true.

Then for every $w(\cdot) \in LW$ there exists the fuzzy R-solution $X(\cdot, w)$ such that

1) the fuzzy map $X(\cdot, w)$ is equal to

$$X(t, w) = \int_0^t [\Phi(t, s) + \int_s^t \Phi(t, \tau) R(\tau, s) d\tau] B(s) w(s) ds + \int_0^t [\Phi(t, s) + \int_s^t \Phi(t, \tau) R(\tau, s) d\tau] C(s) V ds$$

where $t \in [0, T]$; $\Phi(t, s)$ is Cauchy matrix of the differential equation $\dot{x} = A(t)x$; $R(t, s)$ is solution of the system

$$R(t, s) - Q(t, s) = \int_s^t Q(t, \tau) R(\tau, s) d\tau, \\ Q(t, \tau) = \int_\tau^t K(t, \xi) \Phi(\xi, \tau) d\xi;$$

2) $X(t, w) \in E^n$ for every $t \in [0, T]$;

3) the fuzzy map $X(\cdot, w)$ is the absolutely continuous fuzzy map on $[0, T]$.

Theorem 2. [37] Let condition A be true.

Then the attainable set $Y(T)$ is compact and convex.

We obtained the basic properties of the fuzzy R-solution of systems (4). Now, we consider some fuzzy control problems.

4. Time-Optimal Problems

Consider the following time-optimal problem [10,26]: it is necessary to find the minimal time T and the control $w^*(\cdot) \in LW$ such that the fuzzy R-solution of system (4) satisfies one of the conditions:

$$X(T, w^*) \cap S_k \neq \emptyset \quad (5)$$

$$X(T, w^*) \subset S_k \quad (6)$$

$$X(T, w^*) \supset S_k \quad (7)$$

where $S_k \in E^n$ is the fuzzy terminal set.

It is obvious that optimum time and optimum controls for these problems will be different.

Theorem 3. (necessary optimal condition for time-optimal problem (4),(5)). Let condition A be true and the pair $(T, w^*(\cdot))$ be optimal in control problem (4),(5).

Then there exists the vector-function $\psi(\cdot)$, that is the solution of the system

$$\dot{\psi} = -A^T(t)\psi - \int_t^T K^T(s, t)\psi(s)ds, \quad \psi(T) \in S_1(0)$$

such that

$$1) (B(t)w^*(t), \psi(t)) = C(B(t)W(t), \psi(t))$$

almost everywhere on $[0, T]$;

$$2) C\left(\left[X(T, w^*)\right]^1, \psi(T)\right) = -C\left([S_k]^1, -\psi(T)\right),$$

where $C(P, \psi) = \max_{p \in P} (p_1\psi_1 + \dots + p_n\psi_n)$, $\psi \in R^n$,

$$P \in \text{conv}(R^n).$$

Proof. Let $w^*(\cdot)$ be the optimal control and $X(\cdot, w^*)$ be the optimal fuzzy R-solution of problem (4),(5), i.e.

$$1) X(T, w^*) \in Y(T);$$

$$2) X(T, w^*) \cap S_k \neq \emptyset.$$

From 1) and 2) we have

$$\max_{X \in [Y(T)]^1} C(X, \psi) \geq -C([S_k]^1, -\psi) \quad (8)$$

for all $\psi \in S_1(0)$.

Consequently

$$p = \max_{X \in [Y(T)]^1} \min_{\psi \in S_1(0)} C(X, \psi) + C([S_k]^1, -\psi) \geq 0.$$

From $[X(T, w^*)]^1 \cap [S_k]^1 \neq \emptyset$ we have

$$q(T, \psi) = C\left(\left[X(T, w^*)\right]^1, \psi\right) + C([S_k]^1, -\psi) \geq 0$$

for all $\psi \in S_1(0)$.

From Theorem 1 we have that the function $q(T, \psi)$ is continuous on $R_+ \times S_1(0)$.

If $q(T, \psi) > 0$ for all $\psi \in S_1(0)$ then we have $q^0(T) = \min_{\psi \in S_1(0)} q(T, \psi) \geq \gamma > 0$. Hence there exists

$\tau < T$ such that $q^0(\tau) \geq 0$. Consequently we have

$$C\left(\left[X(\tau, w^*)\right]^1, \psi\right) + C([S_k]^1, -\psi) \geq 0$$

for all $\psi \in S_1(0)$, i.e. $[X(\tau, w^*)]^1 \cap [S_k]^1 \neq \emptyset$.

It contradicts that T is optimal time.

If $p > 0$,

$$\max_{X \in [Y(T)]^1} \min_{\psi \in S_1(0)} C(X, \psi) + C([S_k]^1, -\psi) = \\ = C(\tilde{X}, \tilde{\psi}) + C([S_k]^1, -\tilde{\psi})$$

and $[X(T, w^*)]^1 \neq \tilde{X}$, then we have a contradiction.

Hence there exists $\tilde{\psi} \in S_1(0)$ such that

$$C\left(\left[X(T, w^*)\right]^1, \tilde{\psi}\right) = \max_{X \in [Y(T)]^1} C(X, \tilde{\psi}),$$

$$C\left(\left[X(T, w^*)\right]^1, \tilde{\psi}\right) = -C\left([S_k]^1, -\tilde{\psi}\right).$$

Consequently

$$\left(\int_0^T [\Phi(t, s) + \int_s^T \Phi(T, \tau) R(\tau, s) d\tau] B(s) w^*(s) ds, \tilde{\psi}\right)$$

$$= \max_{w(\cdot) \in LW} \left(\int_0^T [\Phi(t, s) + \int_s^T \Phi(T, \tau) R(\tau, s) d\tau] B(s) w(s) ds, \tilde{\psi}\right)$$

Then we have

$$\left(\left[\Phi(t, s) + \int_s^T \Phi(T, \tau) R(\tau, s) d\tau\right] B(s) w^*(s), \tilde{\psi}\right)$$

$$= \max_{w \in W(s)} \left(\left[\Phi(t, s) + \int_s^T \Phi(T, \tau) R(\tau, s) d\tau\right] B(s) w, \tilde{\psi}\right)$$

for almost every $s \in [0, T]$. If

$$\psi(t) = \frac{\left[\Phi(t, s) + \int_s^T \Phi(T, \tau) R(\tau, s) d\tau\right]^T \tilde{\psi}}{\left\|\left[\Phi(t, s) + \int_s^T \Phi(T, \tau) R(\tau, s) d\tau\right]^T \tilde{\psi}\right\|}$$

than theorem 3 is proved.

Following theorems are similarly proved.

Theorem 4. (necessary optimal condition for time-optimal problem (4),(6)). Let condition A be true and the pair $(T, w^*(\cdot))$ be optimal in control problem (4),(6).

Then there exists the vector-function $\psi(\cdot)$, that is the solution of the system

$$\dot{\psi} = -A^T(t)\psi - \int_t^T K^T(s, t)\psi(s)ds, \quad \psi(T) \in S_1(0)$$

such that

$$1) (B(t)w^*(t), \psi(t)) = C(B(t)W(t), \psi(t))$$

almost everywhere on $[0, T]$;

$$2) \text{ for all } \alpha \in [0, 1]$$

$$C\left(\left[X(T, w^*)\right]^\alpha, \psi(T)\right) \leq C\left([S_k]^\alpha, \psi(T)\right)$$

and there exists $\beta \in [0, 1]$

$$C\left(\left[X(T, w^*)\right]^\beta, \psi(T)\right) = C\left([S_k]^\beta, \psi(T)\right)$$

This theorem can be proved similar to theorem 5 with little changes in condition (8):

for all $\alpha \in [0, 1]$

$$\max_{X \in [Y(T)]^\alpha} C(X, \psi) - C([S_k]^\alpha, \psi) \leq 0$$

for all $\psi \in S_1(0)$ and there exist $\beta \in [0, 1]$ and $\bar{\psi} \in R^n$ such that

$$\max_{X \in [Y(T)]^\beta} C(X, \bar{\psi}) - C([S_k]^\beta, \bar{\psi}) = 0$$

Theorem 5. (necessary optimal condition for time-optimal problem (4),(7)). Let condition A be true and the pair $(T, w^*(\cdot))$ be optimal in control problem (4),(7).

Then there exists the vector-function $\psi(\cdot)$, that is the solution of the system

$$\dot{\psi} = -A^T(t)\psi - \int_t^T K^T(s, t)\psi(s)ds, \quad \psi(T) \in S_1(0)$$

such that

$$1) (B(t)w^*(t), \psi(t)) = C(B(t)W(t), \psi(t))$$

almost everywhere on $[0, T]$;

$$2) \text{ for all } \alpha \in [0, 1]$$

$$C\left(\left[X(T, w^*)\right]^\alpha, \psi(T)\right) \geq C\left([S_k]^\alpha, \psi(T)\right)$$

and there exists $\beta \in [0, 1]$

$$C\left(\left[X(T, w^*)\right]^\beta, \psi(T)\right) = C\left([S_k]^\beta, \psi(T)\right)$$

Also this theorem can be proved similar to theorem 5 with little changes in condition (8):

for all $\alpha \in [0, 1]$

$$\max_{X \in [Y(T)]^\alpha} C(X, \psi) - C([S_k]^\alpha, \psi) \geq 0$$

for all $\psi \in S_1(0)$ and there exist $\beta \in [0, 1]$ and $\bar{\psi} \in R^n$ such that

$$\max_{X \in [Y(T)]^\beta} C(X, \bar{\psi}) - C([S_k]^\beta, \bar{\psi}) = 0$$

5. Conclusions

In this paper we have considered time-optimal problems for systems when the behavior of object is described by linear control fuzzy integro-differential inclusion. For these problems we have received necessary conditions of optimality. These results generalize results of papers [7,10,14,16,21,24].

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