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On Massive MIMO System Convergence: Analysis Using Geometry-Based Stochastic Model

Patrick Danuor[1,*], Reynah Akwafo[2], Stephen Nuagah[3], Emmanuel Nyaho-Tamakloe[4]

[1]Department of Electrical and Electronic Engineering, Ho Technical University, Ho, Ghana
[2]Department of Electrical and Electronic Engineering, Bolgatanga Technical University, Bolgatanga, Ghana
[3]Department of Electrical and Electronic Engineering, Tamale Technical University, Tamale, Ghana
[4]Department of Telecommunications Engineering, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana
*Corresponding author: pdanuor@gmail.com

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Abstract In this paper, three-dimensional massive multi-input multi-output (MIMO) system convergence is analyzed using a geometry-based stochastic channel model. The concept of Maximum Distributive Power is introduced to examine the performance of massive MIMO regarding the Gaussian and the Arbitrary Q-power Cosine distributions of arrival. Verification is accomplished with the help of computer simulation where excellent agreement is found between theoretical and simulation results. We present results to show that higher Q-power values of the Arbitrary Q-power Cosine distribution creates mismatch between the simulated and theoretical correlation coefficients unlike the Gaussian distribution. The practical relevance is that, careful selection of higher Q-power values for 3D massive MIMO channel planning is vital to enhance massive MIMO Convergence. This is demonstrated in the convergence outcomes where higher Q-power values deteriorates the convergence to massive MIMO favorable propagation as the transmit antenna increases. Finally, experimental outcomes illustrate that it requires antenna separation of more than half wavelength to enhance convergence rates at higher Q-power values.

Keywords: *channel model, convergence, favorable propagation, massive MIMO, spatial correlation*

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1. Introduction

In order to reap the full benefits of massive MIMO, the number of Base Station (BS) antennas must be significantly large to converge to favourable propagation, the state where the effects of noise and fast fading vanishes. Also, simple linear processing techniques can be used to maximize the sum capacity at favourable propagation condition [1]. However, increasing the BS antennas inadvertently leads to spatial correlation (SC), a detrimental effect that affects the convergence of massive MIMO [1,2].

A number of works have investigated the effect of SC on convergence using correlated-based stochastic channel models (CBSCMs) [1,2,3]. Unlike the geometry-based stochastic channel model (GBSCMs) that reflect realistic practical channel properties and is more suitable for evaluating massive MIMO, the CBSCM is limited to the theoretical analysis [4]. In view of this, authors in [5] analysed the performance of spatially correlated large arrays using the Saleh-Venzuela channel model which uses millimetre-Wave frequencies and follows GBSM.

So far, the study of 3D massive MIMO convergence and performance using the WINNER+ and 3GPP standard which follows GBSM has received little attention Another challenge is that, the SC is a function of the distribution of arrival [8,10,12]. Therefore, there is the need to lessen the requirements of deducing distinctive SC expressions for massive MIMO antenna arrays regarding different distributions of arrival as in [3,5,6,7,10,12].

We believe that with the innovation in 5G, a precise and direct technique of characterizing the SC of antenna arrays for massive MIMO performance is vital to study the effect of SC on massive MIMO performance.

In this paper, we present the Maximum Distributive Power (MDP) concept to analyse the convergence of 3D massive MIMO. We considered the uniform rectangular array (URA) because of its wide adoption for 3D massive MIMO technology [4]. For the 3D channel model, we considered WINNER+ and 3GPP channel model standard in [8] that follows GBSM and present a new channel realization when the transmitter is modelled as URA.

We summarized the contributions of this paper as follows:

1. We analyse the WINNER+ and 3GPP standard which follows a GBSM approach proposed in [8] and present a new channel realization when the transmitter antenna is represented as URA.
2. We derive analytical expressions for the SC of URA regarding Gaussian, Von Misses and

Arbitrary Q-Power Cosine distributions using the MDP concept to investigate the convergence and uplink capacity of 3D massive MIMO.

3. We demonstrate that even though higher Q-power values of the Cosine distribution enhances massive MIMO performance in GBSM, careful selection of Q-power values above ten (10) is vital to minimize errors, as it causes mismatch between theoretical and simulation correlation coefficients. This deteriorates the convergence to favourable propagation when the number of transmit antenna increases.

2. Three-Dimension Massive MIMO Channel Model

In this section, we examine the WINNER+ and 3GPP standard which follows GBSM approach in [8] and present a new channel realization when the BS antenna is modelled as URA. This standard is considered because the propagation paths in the azimuth only does not enhance performance [9]. The WINNER+ and 3GPP standard under consideration present the elevation angle of the antenna boresight θ_{tilt} into the channel and permits dynamic adaptation of the antenna downtilt angles. Further, it opens up several benefits for 3D beamforming which can lead to substantial system performance [9].

According to [8], the effective 3GPP channel realization between the BS antenna port, s^{th} and the MS antenna port u^{th} is expressed as

$$[H_{s,u}] = \sum_{n=1}^{N} \alpha_n \sqrt{g_t(\varphi_n^{AoD}, \vartheta_n^{AoD}, \theta_{tilt})} \times\sqrt{g_r(\phi_n^{AoA}, \theta_n^{AoA})} \times [a_t(\varphi_n^{AoD}, \vartheta_n^{AoD})]_s \times [a_r(\phi_n^{AoA}, \theta_n^{AoA})]_u \tag{1}$$

where $u = 1,\dots,N_{MS}$ and $s = 1,\dots,N_{BS}$. α_n is defined as the complex random amplitude of the nth path. $(\varphi_n^{AoD}, \vartheta_n^{AoD})$ represents the azimuth and elevation angles of departure (AoDs), respectively. $(\phi_n^{AoA}, \theta_n^{AoA})$ denote the azimuth and elevation angles of arrival (AoAs), respectively of the n^{th} path. $g_t(\varphi_n^{AoD}, \vartheta_n^{AoD}, \theta_{tilt}) \approx g_{t,H}(\varphi_n^{AoD} g_t, V(\vartheta_n^{AoD}, \theta_{tilt}))$ is the gain of antenna array at the BS given in [8].

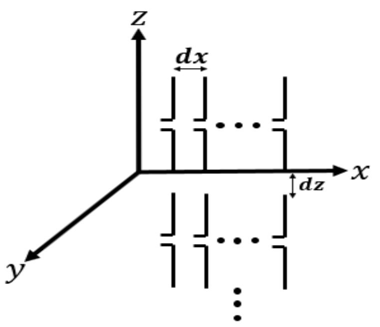

Figure 1. Uniform Rectangular Antenna

To demonstrate our idea, the vertical and horizontal antenna patterns at the BS is approximated in dB as in [10]. At the MS, $g_r(\phi^{AoA}, \theta^{AoA})$ is taken as 0dB as the MS generally does not favour any direction [7]. The antenna array response at the transmitter is given in eqn. (2) as

$$[a_t(\varphi_n^{AoD}, \vartheta_n^{AoD})]_s = exp(ik(s-1)d_t sin\vartheta^{AoD}) \tag{2}$$

and the antenna array response at the receiver represented in eqn. (3) as

$$[a_r(\phi_n^{AoA}, \theta_n^{AoA})]_u = exp(ik(s-1)d_r sin\phi sin\theta^{AoA}) \tag{3}$$

From Figure 1, on the scalars x-and z-axes [10], $\Psi = \sqrt{\kappa_x^2 + \kappa_z^2}$, where $\kappa_x = 2\pi dx(n-m)/\lambda$, $\kappa_z = 2\pi dz(p-q)$ and $\beta = cos^{-1}(\kappa_x/\kappa_z)$ [10]. d_x and d_z represent the distances between the elements. The position of an element at the URA transmitter given by $\Psi\angle\beta$ and d_r represents the separation between the receiving antenna ports. The location vector of s^{th} transmit element for URA can be expressed as $v_t.x_s = cos(\varphi_n - \beta_s) sin\vartheta_n^{AoD}$. Therefore, the BS port of URA based on Eq. (1) can be expressed as

$$[a_t(\varphi_n^{AoD}, \vartheta_n^{AoD})]_s = exp(ik\Psi cos(\varphi_n^{AoD} - \beta_s) sin\vartheta_n^{AoD}) \tag{4}$$

Hence, the proposed 3D channel realization between the s^{th} URA transmit antenna port and the u^{th} receiving antenna port regarding the WINNER+ and 3GPP standard model is proposed as

$$[H_{s,u}]_{URA} = \sum_{n=1}^{N} \alpha_n \sqrt{g_t(\varphi_n^{AoD}, \vartheta_n^{AoD}, \theta_{tilt})} \sqrt{g_r(\phi_n^{AoA}, \theta_n^{AoA})} \times exp(ik\Psi cos(\varphi_n^{AoD} - \beta_s) sin\vartheta_n^{AoD}) \times exp(ik(u-1)d_r sin\phi^{AoA} \sin\theta^{AoA}) \tag{5}$$

3. Concept of Maximum Distributive Power

In this section, we present the MDP concept to derive the SC of the URA regarding arbitrary Q-power Cosine and Gaussian distributions.

From [11], the phase gradient of an incident wave, $\bar{E} = e^{jkzsin\varpi} = e^{j\xi}$, impinging on an antenna array is given as $\partial\xi/\partial z = ksin\varpi$ where ϖ is the angle of arrival. z represents the distance along the array axis and k denotes the wave number. The phase gradient, $\partial\xi/\partial$ is proportional to $x = sin\varpi$ which has a known Probability Density Function (PDF) that follows closely that of Cosine Q-power and Gaussian distributions [11,12].

Definition of Maximum Distributive Power: Let $f(\varpi)$ be the distribution of arrival and $\partial\xi/\partial z$ be the phase gradient of incident wave as a function of $x = sin\varpi$, where $\xi = kzsin\varpi$ and ϖ is the angle of arrival of the incident wave. The *MDP* varies relatively to the distribution and is expressed as

$$MDP = \int_x f(\varpi)dx \text{ [11]}. \quad (6)$$

4. Proposed 3D SC of URA

In this section, we derive the SC of URA based on the MDP regarding Arbitrary Q-power Cosine and Gaussian distributions.

From [7], the steering vector (SV) for the $N \times P$ URA is given in eqn. (5) as

$$a(\theta,\phi)_{URA} = [1, e^{j\upsilon},..., e^{j(P-1)\tau}, e^{j\upsilon},..., e^{j(\upsilon+\tau)}, \\ ..., e^{j[\upsilon+(P-1)\tau]},..., e^{j(N-1)\upsilon},..., e^{[(N-1)\upsilon+(P-1)\tau]}]^T \quad (7)$$

where υ and τ are defined as $2\pi d_x cos\phi sin\theta / \lambda$ and $2\pi d_z cos\phi sin\theta / \lambda$, respectively. λ represents the wavelength. The azimuth and elevation angles are bounded between $0 \le \phi \le 2\pi$ and $0 \le \theta \le \pi$. Using the SV of URA and the 3D SC expression in [13], the SC between the antennas at positions (a,b) and (c,d) can expressed in eqn. (6) as

$$\rho_{[(a,b),(c,d)]} = \int_{\phi^{AoA}} \int_{\theta^{AoA}} e^{j\Psi sin(\theta^{AoA} sin(\beta+\phi^{AoA}))} \\ \times P(\phi^{AoA}, \theta^{AoA}) sin(\theta^{AoA}) d\theta d\phi \quad (8)$$

where $P(\phi^{AoA}, \theta^{AoA})$ is the joint pdf of the azimuth and elevation angles of arrival. It is clear from section III that, the incident wave, $\varpi = \theta^{AoA} sin(\beta + \phi^{AoA})$. Motivated by the MDP definition, the joint pdf in equation (6) is represented by the MDP concept and expressed

$$\rho_{[(a,b),(c,d)]} \\ = MDP(\varpi) \int_{\phi^{AoA}} \int_{\theta^{AoA}} e^{j\Psi sin(\theta^{AoA} sin(\beta+\phi^{AoA}))} \\ \times sin(\theta^{AoA}) d\theta d\phi \quad (9)$$

where $MDP(\varpi)$ is obtained by evaluating the integral of $P(\phi^{AoA}, \theta^{AoA})$. Using the Bessel substitutions for two periodic functions in [14] as

$$J_0(v) = \frac{1}{2} \int_{-\pi}^{\pi} e^{-j(n\omega - vsin(\omega))} d\omega \quad (10)$$

$$= \int_0^{\pi} J_0(v) sin(\theta) d\theta = \sqrt{(2\pi / v)} J_{1/2}(v) \quad (11)$$

and

$$J_{1/2}(v) = \sqrt{(2/\pi v)} sin(v) \text{ [14]}, \quad (12)$$

the analytical SC expression of large-scale URA based on MDP becomes

$$\rho_{[(a,b),(c,d)]} = \frac{8\pi}{\Psi sin\theta^{AoA}} MPA(\varpi) sin(\Psi sin\theta^{AoA}) \quad (13)$$

4.1. Maximum Distributive Power of Arbitrary Q-Power Cosine and Gaussian Distributions

Next is to evaluate the MDP of arbitrary Q-power cosine distribution and Gaussian distributions. The arbitrary Q-power cosine distributions is expressed in [12] as

$$F_{cos}(\varpi) = \frac{k_c}{\pi} cos^Q(\varpi - \varpi_m), \quad (14)$$

where Q represents the shape of the distribution and k_c is used to modify the distribution area to unity value. We assume Q to be even as in [12]. The MDP of arbitrary Q-power cosine distribution can therefore be derived as

$$MDP_{cos}(\varpi) = \frac{k_c}{\pi} \int cos^Q(\varpi - \varpi_m) d\varpi. \quad (15)$$

This integral is evaluated as [12]

$$MDP_{cos}(\varpi) = \frac{k_c}{\pi}((\frac{1}{2^n})\binom{2n}{n}(\varpi - \varpi_m) \\ +(\frac{1}{2^{n-1}}) \sum_{k=0}^{n-1} \binom{2n}{n} sin(2m-2k)(\varpi - \varpi_m)/2n-2k) \quad (16)$$

Likewise, the Gaussian distribution is expressed in [8] as

$$F_{gau}(\varpi) = \frac{\kappa}{\sigma\sqrt{2\pi}} e^{-(\varpi-\varpi_m)^2/2\sigma^2}, \quad (17)$$

where σ and ϖ_m are defined as the standard deviation and mean direction of arrival of the distribution respectively.

The MDP of Gaussian distribution can be derived as

$$MDP_{gau}(\varpi) = \frac{k}{\sigma\sqrt{2\pi}} \int exp - (-\varpi - \varpi_m)^2 / 2\sigma^2 d\varpi \quad (18)$$

which can be further expressed as

$$MDP_{gau}(\varpi) = \frac{\kappa\sigma}{\sqrt{2\pi(\varpi - \varpi_m)}} e^{-(\varpi-\varpi_m)^2/2\sigma^2} \quad (19)$$

4.2. SC of URA regarding Arbitrary Q-Power Cosine and Gaussian Distributions

Now, we substitute the MDP expressions in eqns. (16) and (19) into (9) to obtain the Bessel SC expression of arbitrary Q-power cosine and Gaussian distributions in eqns. (20) and (21), respectively.

$$\rho^{cos}_{[(a,b),(c,d)]} = \frac{16_{kc}}{Zsin\theta^{AoA}}((\frac{1}{2^n})\binom{2n}{n}(\varpi - \varpi_m) \\ +(\frac{1}{2^{n-1}}) \times \sum_{k=0}^{n-1} \binom{2n}{n} sin(2n-2k) \frac{\varpi - \varpi_m}{2n-2k} \\ \times \sum_{k=0}^{\infty} J_{2k+1}(\Psi) sin((2k+1)\theta^{AoA}) \quad (20)$$

$$\rho^{gau}_{[(a,b),(c,d)]} = \frac{16\pi\kappa\sigma}{\sqrt{2\pi}\Psi sin\theta^{AoA}(\theta^{AoA}sin(\beta+\phi^{AoA})-\varpi_m)} \times exp(\frac{-(\theta sin(\beta+\phi^{AoA})-\alpha_a)^2}{2\sigma^2}) \times \sum_{k=0}^{\infty} J_{2k+1}(\Psi)sin((2k+1)\theta^{AoA}) \quad (21)$$

5. System Model

In this analysis, an uplink 3D massive MIMO system employing several BS antennas M, serving simultaneously K single-antenna users is considered. The K terminals receive a $K\times 1$ vector on the downlink where Time Division Duplex is assumed as

$$\mathbf{x}_f = \sqrt{\rho_f}\mathbf{H}^T\mathbf{s}_f + \mathbf{w}_f, \mathbf{H} = R_t^{1/2}H_{s,u} \quad (22)$$

where $\mathbf{x}_f$ is the received signal at the UE, ρ_f denotes the transmit SNR, $[.]^T$ represents matrix transpose and $\mathbf{s}_f$ defines an $M\times 1$ precoded vector of data symbols. $\mathbf{w}_f$ represent a $K\times 1$ noise vector with an independent (iid) $\mathcal{CN}(0,1)$ noise vector. $\mathbf{H}$ denotes the correlated channel matrix, $H_{s,u}$ represents the $M\times K$ channel matrix and R_t is the $N_{TX}\times N_{TX}$ transmit SC matrix at the transmitter.

5.1. Convergence of Massive MIMO User Channel

Convergence demonstrates favourable propagation, the state where the effect of noise and fast fading vanishes in massive MIMO channel [1]. According to [1],

$$\frac{1}{M}\mathbf{H}^T\mathbf{H} \to D_\beta, \text{ for } M\to\infty, \quad (23)$$

where D_β is the $K\times K$ diagonal matrix whose diagonal elements comprise the vector $k = 1,\dots,K$. The convergence is measured by employing the Diagonal Dominance (Θ), metric defined in eqn. (24) as [2,3]

$$\Theta = \frac{\sum_{i=1}^{K}\mathbf{H}_{iid}}{\sum_{i=1}^{K}\sum_{j=1, j\neg i}^{K}|\mathbf{H}_{ij}|} \quad (24)$$

6. Numerical Results

For numerical evaluation, we consider a 10×10 URA antenna and set $\theta_{tilt} = 90^\circ$, $\theta_{3dB} = 15^\circ$, and $\phi_{3dB} = 70^\circ$ as in [8] at the BS for the 3-D channel modelling. The AoD and AoA are drawn from Laplacian distribution. The BS and MS height is considered as 25m and 1.5m, respectively. The mean azimuth and elevation angle of arrival is taken at 90°.

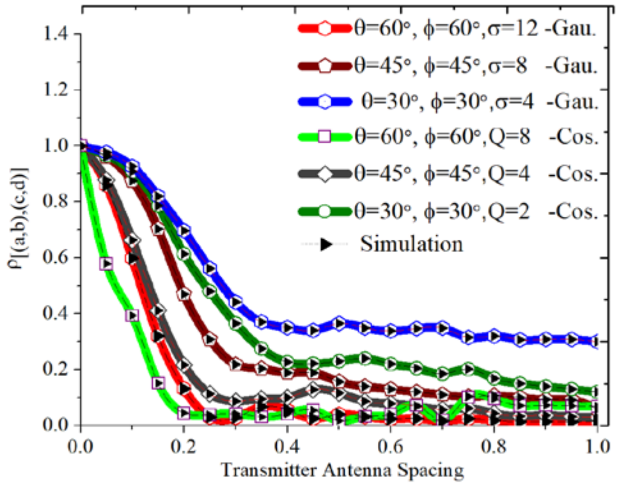

Figure 2. SC for Gaussian and arbitrary Q-power Cosine PDF for varying θ, ϕ, Q and σ

In the SC computation, the analytical results in eqns. (20) and (21) are computed over one hundred (100) channel realizations for different values of σ and Q. We determine $|\rho_{[(a,b),(c,d)]}|$ as illustrated in Figure 2 and Figure 3. Our outcomes demonstrate perfect agreement between theoretical and simulation correlation coefficients for the Gaussian distribution at different key performance values and arbitrary Q Power values below ten (10) as shown in Figure 2. However, Q values of Cosine distribution above ten (10), recorded a mismatch between simulation and theoretical correlation coefficients as illustrated in Figure 3. Table 1 presents the mean percentage error registered at different key performance values.

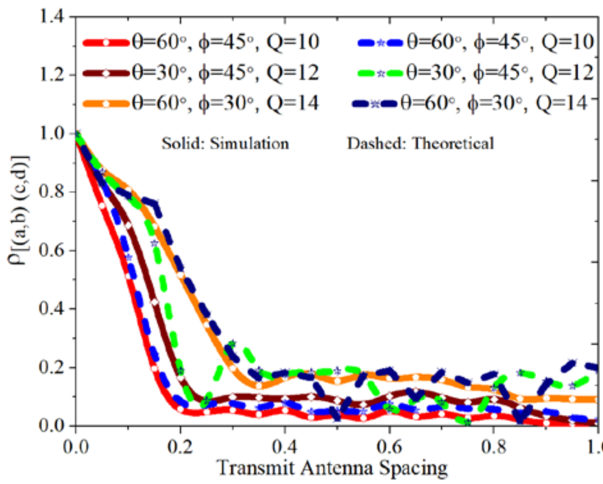

Figure 3. SC for Arbitrary Q-Power Cosine PDF for varying θ, ϕ and Q

For convergence analysis, the diagonal dominance metric defined in section 1V (A) was examined using the proposed channel model in eqn.(5) at higher θ, ϕ and key performance parameters of the distributions.

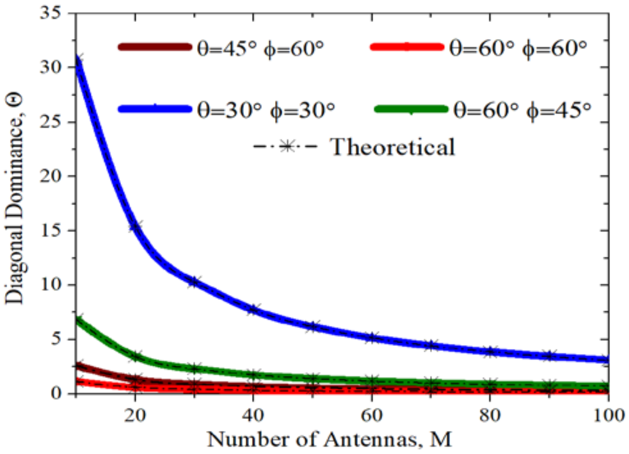

Figure 4. Convergence for Gaussian PDF at varying θ and ϕ

As shown in Figure 4, the convergence rate for Gaussian and Cosine distributions at $Q \leq 8$ distribution is very fast as θ and ϕ increases. However, higher values of $Q \geq 10$ deteriorates the convergence for $M \geq 70$ as shown in Figure 5.

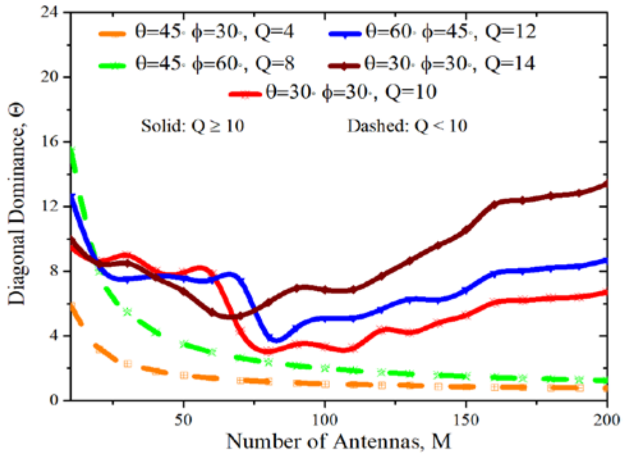

Figure 5. Convergence for Arbitrary Q-Power Cosine PDF at $Q \geq 10$, varying θ and ϕ

Table 1. COMPARISON OF MEAN PERCENTAGE ERROR OF ANGULAR DISTRIBUTIONS

Distributions	Parameter	Value	Mean Square Error
Gaussian	Standard Deviation, σ	4	0.0358
		8	0.0504
		12	0.1104
Arbitrary Q- Power Cosine	Arbitrary Q-power, Q	4	0.0817
		8	0.1985
		10	2.1184
		12	3.8515
		14	7.0124

7. Conclusion

In this paper, we have presented the MDP concept to derive analytical expressions of SC to study the performance of 3D massive MIMO using GBSM. Results indicate that Q-power values above ten (10) of the cosine distribution deteriorates the convergence performance. This is due to the mismatch in the correlation coefficients between theoretical and simulation results. The MDP concept can be extended to the Laplacian and Student's -t distribution for further studies.

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