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Common Fixed Point Theorems for Occasionally Weakly Compatible Mappings in G-metric Spaces

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Abstract In this paper, we prove some common fixed point results for two pairs of mappings using the concept of occasionally weakly compatible, (E.A)/ common property (E.A) and an inequality involving quadratic terms in the settings of G-metric spaces.

Keywords: *common fixed points, occasionally weakly compatible mappings, G-metric spaces, coincidence points*

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1. Introduction

In 2006, Mustafa and Sims [1] introduced the notion of G-metric spaces as a generalization of the metric spaces. After that, many authors studied fixed and common fixed point in generalized metric spaces, see [2-10]. These results provide the basis for carrying out analysis in G-metric spaces, in particular for the development of G-metric fixed point theory for mappings satisfying a variety of contractive type conditions. In G-metric spaces, fixed point theory is indispensable due to its wide application. The classical result of Banach continues to be the source of inspiration for many researchers working in the area of G-metric fixed point theory. A G-metrical common fixed point theorem generally involves conditions on commutativity, continuity, and completeness and suitable containment of ranges of the involved mappings besides an appropriate contraction conditions and researcher in this domain are aimed at weakening one or more of these conditions.

Now, we give some preliminaries and basic definitions which we use throughout the paper.

Definition 1.1. (G-metric spaces, see [1]).

In 2006, Mustafa and Sims introduced the concept of G-metric spaces as follows:

Let X be a nonempty set and $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following:

(G1) $G(x,y,z)=0 \; if \; x=y=z,$

(G2) $0<G(x,x,y) \; for \; all \; x,y \; in \; X \; with \; x\neq y,$

(G3) $G(x,x,y)\leq G(x,y,z) \; for \; all \; x,y,z \; in \; X \; with \; z\neq y,$

(G4) $G(x,z,y)=G(x,y,z)=G(y,z,x)=\ldots$ (symmetry in all three variables),

(G5) $G(x,y,z)\leq G(x,a,a)+G(a,y,z)$ for all $x, y, z, a \; in \; X$ (rectangle inequality).

Then the function G is called a generalized metric or, more specifically, a G-metric on X and the pair (X,G) is called a G-metric spaces.

Definition 1.2. [1] A $G-$ metric space $(X,G)$is called a symmetric $G-$ metric if

$$G(x,y,y)=G(y,x,x)\,\forall x,y\in X.$$

Definition 1.3. [11] Let $f and g$ be two self mappings on G- metric spaces (X,G). The mappings $f and g$ are said to be compatible if

$$\lim_{n\to\infty} G\left(fgx_{n,},gfx_n,gfx_n\right)=0,$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$$

for some $z \in X$.

Definition 1.4. [12] Two maps $f \; and \; g$ are said to be weakly compatible if they commute at coincidence points.

Definition 1.5. [11] Let f and g be two self mappings of a G- metric space (X,G) . Then the pair (f,g) is said to be satisfy the property E.A. if there exists a sequence (x_n) in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

for some $t \in X$.

Definition 1.6. [12] A pair of self-mappings (f,g) of a G-metric space (X,G) is said to be non compatible if there exists a sequence (x_n) in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t,$$

for some $t \in X$. But $\lim_{n\to\infty} G(fgx_n, gfx_n, gfx_n)$ is either non zero or non – existent.

Definition 1.7. [10] A pair of self-mappings (f, g) of a G-metric space (X, G) is said to be occasionally weakly compatible if $fgx = gfx$ $for\ some\ x \in C(f, g)$.

Definition 1.8. [10] *Let f and g* be self mappings of a G-metric space. If $= gx = w,\ w \in X,\ for\ some\ x \in X$. Then x is called a coincidence point of f *and* g and the set of coincidence point is denoted by $C(f, g)$ and w is called a point of coincidence of f *and* g.

2. Main Results

In this section, we prove some common fixed point theorems for four self-maps in the setting of G-metric spaces.

Theorem 2.1. Let A, B, S and T be four self-mappings of a symmetric G-metric space (X, G) such that

$$G(Ax, By, Bz)^2 \le c_1 \max\left\{\begin{array}{l} G(Sx, Ax, Ax)^2, \\ G(Ty, By, Bz)^2, G(Sx, Ty, Tz)^2 \end{array}\right\} + c_2 \max\left\{\begin{array}{l} G(Sx, Ax, Ax)G(Sx, By, Bz), \\ G(Ty, By, Bz)G(Ty, Ax, Ax) \end{array}\right\} + c_3\{G(Sx, By, Bz)G(Ty, Ax, Ax)\} \tag{2.1}$$

for all

$$x, y \in X, c_1, c_2, c_3 \ge 0 \text{ and } c_1 + c_3 < 1, \tag{2.2}$$

$BX \subseteq SX$, the pair (B, T) satisfies property (E.A) and TX is a closed subspace of X,

OR

$AX \subseteq TX$, the pair (A, S) satisfies property (E.A) and SX is a closed subspace of X.

Then $C(A, S) \ne \phi$ *and* $C(B, T) \ne \phi$.

Moreover, if both the pairs (A, S) *and* (B, T) are occasionally weak compatible mapping in X, then the mappings A, B, S *and* T have a unique common fixed point.

Proof: Since the pair (B, T) satisfy the property E.A., there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z, \text{ for some } z \in X. \tag{2.3}$$

Since $(BX \subseteq SX)$, then there exists a sequence $\{y_n\}$ in X such that $(Bx_n = Sy_n)$ and hence

$$\lim_{n\to\infty} Sy_n = z, \text{ for some } z \in X. \tag{2.4}$$

Now, we prove that $\lim_{n\to\infty} Ay_n = z$. To prove it, we use the inequality (2.1)

$$G(Ay_n, Bx_n, Bx_n)^2 \le c_1 \max\left\{\begin{array}{l} G(Sy_n, Ay_n, Ay_n)^2, \\ G(Tx_n, Bx_n, Bx_n)^2, \\ G(Sy_n, Tx_n, Tx_n)^2 \end{array}\right\} + c_2 \max\left\{\begin{array}{l} G(Sy_n, Ay_n, Ay_n)G(Sy_n, Bx_n, Bx_n), \\ G(Tx_n, Bx_n, Bx_n)G(Tx_n, Ay_n, Ay_n) \end{array}\right\} + c_3\{G(Sy_n, Bx_n, Bx_n)G(Tx_n, Ay_n, Ay_n)\}.$$

Using equations (2.3) and (2.4) we have

$$G(Ay_n, Bx_n, Bx_n)^2 \le c_1 \max\left\{\begin{array}{l} G(Bx_n, Ay_n, Ay_n)^2, \\ G(Tx_n, Bx_n, Bx_n)^2, \\ G(Bx_n, Tx_n, Tx_n)^2 \end{array}\right\} + c_2 \max\left\{\begin{array}{l} G(Bx_n, Ay_n, Ay_n)G(Bx_n, Bx_n, Bx_n), \\ G(Tx_n, Bx_n, Bx_n)G(T, x_n Ay_n, Ay_n) \end{array}\right\} + c_3\{G(Bx_n, Bx_n, Bx_n)G(Tx_n, Ay_n, Ay_n)\};$$

$$G(Ay_n, Bx_n, Bx_n)^2 \le c_1 \max\left\{G(Bx_n, Ay_n, Ay_n)^2, 0, 0\right\} + c_2 \max\{G(Bx_n, Ay_n, Ay_n)0, 0G(T, x_n Ay_n, Ay_n)\} + c_3\{0G(Tx_n, Ay_n, Ay_n)\}$$

$$G(Ay_n, Bx_n, Bx_n)^2 \le c_1\{G(Bx_n, Ay_n, Ay_n)^2\}.$$

Since (X, G) *is a* symmetric G-metric space, we have

$$G(Ay_n, Bx_n, Bx_n)^2 \le c_1\left\{G(Ay_n, Bx_n, Bx_n)^2\right\},$$

which is a contradiction.

Hence $\{G(Ay_n, Bx_n, Bx_n)^2\} = 0$

So, we have.

$$\lim_{n\to\infty} Ay_n = z. \tag{2.5}$$

It is assumed that TX is a closed subspace of X, by equation (2.3), we have.

$$Tv = z \text{ for some } v \in X. \tag{2.6}$$

Now we claim that $Bv = z$. If $Bv \ne z$. Then we have.

$$G(Ay_n, Bv, Bv)^2 \le c_1 \max\left\{\begin{array}{l} G(Sy_n, Ay_n, Ay_n)^2, \\ G(Tv, Bv, Bv)^2, \\ G(Sy_n, Tv, Tv)^2 \end{array}\right\} + c_2 \max\left\{\begin{array}{l} G(Sy_n, Ay_n, Ay_n)G(Sy_n, Bv, Bv), \\ G(Tv, Bv, Bv)G(Tv, Ay_n, Ay_n) \end{array}\right\} + c_3\{G(Sy_n, Bv, Bv)G(Tv, Ay_n, Ay_n)\}.$$

Now taking $n \to \infty$ and using equations (2.3), (2.4) and (2.5), we have.

$G(z, Bv, Bv)^2 \le c_1\{G(z, Bv, Bv)^2\}$, a contradiction.

Hence

$$Bv = z. \tag{2.7}$$

Hence $Bv = Tv = z$. So we have

$$C(B,T) \neq \phi. \tag{2.8}$$

We know that $(BX \subseteq SX)$ *and* $z \in BX$, Then there exists a u in X such that

$$z = Su. \tag{2.9}$$

Now, we claim that $z = Au$. If this does not happen, that is, $z \neq Au$. Then using (2.8) and (2.9), we have

$$G(Au,z,z)^2 = G(\mathrm{Au},\mathrm{Bv},\mathrm{Bv})^2$$
$$\leq c_1 \max\begin{Bmatrix} G(Su,Au,Au)^2, \\ G(T\mathrm{v},Bv,Bv)^2, \\ G(Su,Tv,Tv)^2 \end{Bmatrix}$$
$$+c_2 \max\begin{Bmatrix} G(Su,Au,Au)G(Su,Bv,Bv), \\ G(Tv,Bv,Bv)G(Tv,Au,Au) \end{Bmatrix}$$
$$+c_3 \{G(Su,Bv,Bv)G(Tv,Au,Au)\}$$

$$G(Au,z,z)^2 = G(\mathrm{Au},\mathrm{Bv},\mathrm{Bv})^2$$
$$\leq c_1 \max\left\{G(z,Au,Au)^2, G(z,z,z)^2, G(z,z,z)^2\right\}$$
$$+c_2 \max\begin{Bmatrix} G(z,Au,Au)G(z,z,z), \\ G(z,z,z)G(z,Au,Au) \end{Bmatrix}$$
$$+c_3 \{G(z,z,z)G(z,Au,Au)\}$$

$$G(Au,z,z)^2 = G(\mathrm{Au},\mathrm{Bv},\mathrm{Bv})^2$$
$$\leq c_1 \max\left\{G(z,Au,Au)^2, 0, 0\right\}$$
$$+c_2 \max\{G(z,Au,Au)0, 0G(z,Au,Au)\}$$
$$+c_3 \{0G(z,Au,Au)\}$$

$$G(Au,z,z)^2 = G(\mathrm{Au},\mathrm{Bv},\mathrm{Bv})^2 \leq c_1\left\{G(z,Au,Au)^2\right\},$$

a contradiction. Hence

$$z = Au. \tag{2.10}$$

Hence, we have from (2.9) and (2.10), $z = Au = Su$.
This implies that

$$C(A,S) \neq \phi \tag{2.11}$$

So proof of the theorem holds well when we follow the condition (2.3).

Now as our supposition the pair (A,S) is occasionally weak compatible, then there exists $u' \in C(A,S)$, so we have

$$Au' = Su' = z' \tag{2.12}$$

and

$$ASu' = SAu' \tag{2.13}$$

From equation (2.12) and (2.13) we have.

$$Az' = Sz' = z''\ (say) \tag{2.14}$$

Again as our supposition the pair(B,T) is occasionally weak compatible, then there exists $v' \in C(B,T)$, so we have

$$Bv' = Tv' = w\ (say) \tag{2.15}$$

and

$$BTv' = TBv' \tag{2.16}$$

From equations (2.15) and (2.16), we have

$$Bw = Tw = w'\ (say) \tag{2.17}$$

Now, we claim that $w' = z''$. If this does not happen that is $w' \neq z''$. Then using equations (2.14) and (2.17), we have

$$G(z'',w',w')^2 = G(\mathrm{Az}',\mathrm{Bw},\mathrm{Bw})^2$$
$$\leq c_1 \max\begin{Bmatrix} G(Sz',Az',Az')^2, \\ G(T\mathrm{w},Bw,Bw)^2, \\ G(Sz',Tw,Tw)^2 \end{Bmatrix}$$
$$+c_2 \max\begin{Bmatrix} G(Sz',Az',Az')G(Sz',Bw,Bw), \\ G(Tw,Bw,Bw)G(Tw,Az',Az') \end{Bmatrix}$$
$$+c_3 \{G(Sz',Bw,Bw)G(Tw,Az',Az')\},$$

$$G(z'',w',w')^2 = G(\mathrm{Az}',\mathrm{Bw},\mathrm{Bw})^2$$
$$\leq (\mathrm{c}_1 + \mathrm{c}_3)G(z'',w',w')^2$$

a contradiction. So we have

$$w' = z'' \tag{2.18}$$

So from equations (2.14) and (2.18), we have

$$Az' = Sz' = w'. \tag{2.19}$$

Now we claim that $w' = z'$. If this does not happen that is $w' \neq z'$. Then from equations (2.12) and (2.19) we have

$$G(z',w',w')^2 = G(\mathrm{Au}',\mathrm{Bw},\mathrm{Bw})^2$$
$$\leq c_1 \max\begin{Bmatrix} G(Su',Au',Au')^2, \\ G(T\mathrm{w},Bw,Bw)^2, \\ G(Su',Tw,Tw)^2 \end{Bmatrix}$$
$$+c_2 \max\begin{Bmatrix} G(Su',Au',Au')G(Su',Bw,Bw), \\ G(Tw,Bw,Bw)G(Tw,Au',Au') \end{Bmatrix}$$
$$+c_3 \{G(Su',Bw,Bw)G(Tw,Au',Au')\}$$
$$= (c_1 + c_3)G(z',w',w')^2,$$

which is a contradiction. So we can conclude that

$$w' = z'. \tag{2.20}$$

Now from equations (2.17), (2.19) and (2.20), we have

$$Az' = Sz' = z' \tag{2.21}$$

and

$$Bw = Tw = w \tag{2.22}$$

Now, we claim that $w = z'$. If this does not happen, then from equations (2.15) and (2.22), we have

$$G(z',w,w)^2 = G(\mathrm{A}z',\mathrm{B}v',\mathrm{B}v')^2$$
$$\leq c_1 \max\left\{\begin{array}{l} G(Sz',Az',Az')^2, \\ G(Tv',Bv'Bv')^2, \\ G(Sz',Tv',Tv')^2 \end{array}\right\}$$
$$+c_2 \max\left\{\begin{array}{l} G(Sz',Az',Az')G(Sz',Bv',Bv'), \\ G(Tv'Bv',Bv')G(Tv',Az',Az') \end{array}\right\}$$
$$+c_3\{G(Sz',Bv',Bv')G(Tv',Az',Az')\}$$
$$=(c_1+c_3)G(z',w,w)^2,$$

which is a contradiction. So we have

$$w = z'. \tag{2.23}$$

So from equations (2.22) and (2.23), we have

$$Bz' = Tz' = z' \tag{2.24}$$

So from equations (2.21) and (2.24), we have. $Az' = Bz' = Sz' = Tz' = z'$.

Hence z' is a common fixed point of the mappings A, B, S and T.

Uniqueness: Let m be another common fixed point of the mappings A, B, S and T other than z'. So, we have, $Am = Bm = Sm = Tm = m$.

Then from condition (2.1), we have

$$G(z',m,m)^2 = G\left(\mathrm{A}z',Am,A\,m\right)^2$$
$$\leq c_1 \max\left\{\begin{array}{l} G(Sz',Az',Az')^2, \\ G(\mathrm{T}m,Bm,Bm)^2, \\ G(Sz',Tm,Tm)^2 \end{array}\right\}$$
$$+c_2 \max\left\{\begin{array}{l} G(Sz',Az',Az')G(Sz',Bm,Bm), \\ G(Tm'Bm,Bm)G(Tm,Az',Az') \end{array}\right\}$$
$$+c_3\{G(Sz',Bm,Bm)G(Tm,Az',Az')\}$$

$$G(z',m,m)^2 = G\left(\mathrm{A}z',Am,A\,m\right)^2$$
$$\leq c_1 \max\left\{G(z',z',z')^2, G(m,,m,m)^2, G(z',m,m)^2\right\}$$
$$+c_2 \max\left\{\begin{array}{l} G(z',z',z')G(z',m,m), \\ G(m',m,m)G(m,z',z') \end{array}\right\}$$
$$+c_3\{G(z',m,m)G(m,z',z')\}$$

$$G(z',m,m)^2 = G(\mathrm{A}z',Am,A\,m)^2$$
$$\leq c_1 \max\left\{0,0,G(z',m,m)^2\right\}$$
$$+c_2 \max\{0G(z',m,m),0G(m,z',z')\}$$
$$+c_3\{G(z',m,m)G(m,z',z')\}$$

Since (X,G) is a symmetric G-metric space, so $G(z',m,m) = G(m,z',z')$.

$$G(z',m,m)^2 = G(\mathrm{A}z',Am,Am)^2$$
$$\leq c_1\left\{G(z',m,m)^2\right\} + c_3\{G(z',m,m)G(z',m,m)\}$$

$$G(z',m,m)^2 \leq (c_1+c_3)G(z',m,m)^2,$$

which is a contradiction. Hence $z' = m$. This implies that z' is a unique common fixed point of the mappings A, B, S and T.

Theorem 2.2. Let A, B, S and T be four self-mappings of a symmetric G-metric space (X,G) such that

$$G(Ax,By,Bz)^2$$
$$\leq c_1 \max\left\{\begin{array}{l} G(Sx,Ax,Ax)^2, \\ G(Ty,By,Bz)^2, \\ G(Sx,Ty,Tz)^2 \end{array}\right\} \tag{2.25}$$
$$+c_2 \max\left\{\begin{array}{l} G(Sx,Ax,Ax)G(Sx,By,Bz), \\ G(Ty,By,Bz)G(Ty,Ax,Ax) \end{array}\right\}$$
$$+c_3\{G(Sx,By,Bz)G(Ty,Ax,Ax)\}$$

for all

$$x,y,z \in X,\ c_1,c_2,c_3 \geq 0 \ and \ c_1+c_3 < 1, \tag{2.26}$$

The pair (B,T) satisfies property (E.A) and TX is a closed subspace of X,

OR

The pair (A,S) satisfies property (E.A) and SX is a closed subspace of X.

Then $C(A,S) \neq \phi$ and $C(B,T) \neq \phi$.

Moreover, if both the pairs (A,S) and (B,T) are occasionally weakly compatible mapping in X, then the mappings A, B, S and T have a unique common fixed point.

Proof: Since the pairs (A,S) and (B,T) satisfy the property E.A. Then there exists two sequences (x_n) and (y_n) in X such that.

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z \ \ for\ some\ z \in X. \tag{2.27}$$

As our supposition that SX and TX are a closed subspaces of X, then,

$$z = Su = Tv \ \ for\ some\ u,v \in X. \tag{2.28}$$

Now, we claim that $Bv = z$. If this does not happen then by inequality (2.25), we have

$$G(Ay_n,Bv,Bv)^2$$
$$\leq c_1 \max\left\{\begin{array}{l} G(Sy_n,Ay_n,Ay_n)^2, \\ G(\mathrm{T}v,Bv,Bv)^2, \\ G(Sy_n,Tv,Tv)^2 \end{array}\right\}$$
$$+c_2 \max\left\{\begin{array}{l} G(Sy_n,Ay_n,Ay_n)G(Sy_n,Bv,Bv), \\ G(Tv,Bv,Bv)G(Tv,Ay_n,Ay_n) \end{array}\right\}$$
$$+c_3\{G(Sy_n,Bv,Bv)G(Tv,Ay_n,Ay_n)\}.$$

On taking $n \to \infty$ and using equations (2.27) and (2.28), we have $G(z,Bv,Bv)^2 \leq c_1\{G(z,Bv,Bv)^2\}$, a contradiction.

Hence

$$Bv = z. \quad (2.29)$$

Hence $Bv = Tv = z$. So we have

$$C(B,T) \neq \phi. \quad (2.30)$$

Now we claim that $z = Au$. If this does not happen that is $z \neq Au$. Then using (2.27) and (2.28), we have

$$G(Au,z,z)^2 = G(\mathrm{Au},\mathrm{Bv},\mathrm{Bv})^2$$
$$\leq c_1 \max\begin{Bmatrix} G(Su,Au,Au)^2, \\ G(Tv,Bv,Bv)^2, \\ G(Su,Tv,Tv)^2 \end{Bmatrix}$$
$$+c_2 \max\begin{Bmatrix} G(Su,Au,Au)G(Su,Bv,Bv), \\ G(Tv,Bv,Bv)G(Tv,Au,Au) \end{Bmatrix}$$
$$+c_3\{G(Su,Bv,Bv)G(Tv,Au,Au)\}$$

$$G(Au,z,z)^2 = G(\mathrm{Au},\mathrm{Bv},\mathrm{Bv})^2$$
$$\leq c_1 \max\left\{G(z,Au,Au)^2, G(z,z,z)^2, G(z,z,z)^2\right\}$$
$$+c_2 \max\begin{Bmatrix} G(z,Au,Au)G(z,z,z), \\ G(z,z,z)G(z,Au,Au) \end{Bmatrix}$$
$$+c_3\{G(z,z,z)G(z,Au,Au)\}$$

$$G(Au,z,z)^2 = G(Au,Bv,Bv)^2$$
$$\leq c_1 \max\left\{G(z,Au,Au)^2, 0, 0\right\}$$
$$+c_2 \max\{G(z,Au,Au)0, 0G(z,Au,Au)\}$$
$$+c_3\{0G(z,Au,Au)\}$$

$$G(Au,z,z)^2 = G(\mathrm{Au},\mathrm{Bv},\mathrm{Bv})^2 \leq c_1\left\{G(z,Au,Au)^2\right\},$$

which is a contradiction. Hence $z = Au$.

Hence we have $z = Au = Su$.

This implies that $C(A,S) \neq \phi$.

Now, remaining part of the proof follows as that of Theorem 2.1.

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