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A Class of Irrational Linear Multistep Block Method for the Direct Numerical Solution of Third Order Ordinary Differential Equations

Bamikole Gbenga Ogunware, Ezekiel Olaoluwa Omole*

Department of Mathematics and Statistics, Joseph Ayo Babalola University,
Ikeji-Arakeji, Osun State, Nigeria
*Corresponding author: omolez247@gmail.com

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Abstract This work considers the direct solution of general third order ordinary differential equation by three-step irrational linear multistep method. This method is derived using collocation and interpolation techniques. An irrational three-step method is developed. Taylor series and block methods are used to generate the independent solution at selected points. The properties of the method were also determined. The developed method was applied on general third order ordinary differential equations. And the performance of the numerical results of the method compared favourably with the results of existing authors in the recent literature to test its accuracy and stability.

Keywords: *Irrational Linear Multistep Method, interpolation, collocation, third order, block and Taylor series*

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1. Introduction

The numerical solution of general third order initial value problems of the form:

$$y''' = f\left(x, y, y', y''\right), y\left(x_0\right) = y_0, \quad y'\left(x_0\right) = y', y''\left(x_0\right) = y'' \qquad (1)$$

is considered. Where f is continuous and satisfies Lipchitz's condition that guarantees the uniqueness and existence of a solution.

Problems in the form (1) have wide application in engineering, social science, physical science, etc. Very often, these problems do not have an analytical solution, and this has necessitated the use of numerical method to approximate their solutions. Previously, equation (1) is solved by reducing it to its equivalent system of first order ordinary differential equations and thereafter appropriate numerical method for first order would be applied to solve the systems. However, it is shown in [1,2], that this reduction approach has serious problems which include consumption of human effort, computational burden and non-economization of computer time.

Linear multistep methods for solving equation (1) directly have been proposed by some researchers such as [3,4,5,6]. A P-stable linear multistep method for direct solution of equation (1) was developed by [7] which was implemented in predictor-corrector mode. In order to cater for the setbacks encountered in the reduction approach and also bring about improvement on numerical method accuracy [8,9,10] developed block methods for solving higher order ordinary differential equations directly in which the accuracy is better than when it is reduced to system of first order ordinary differential equations.

Also, various authors such as [11,12,13,14,15,16] developed hybrid methods. This method, while retaining certain characteristics of the continuous linear multistep method, share with Runge-Kutta Methods the property of utilizing data at other points, other than the step point. Block method was found to be efficient in handling equation (1). It is also cost effective.

The use of irrational linear multistep method is uncommon in numerical analysis literatures; hence this article discusses the derivation and application of a three-step block method with one irrational point for the solution of equation (1) directly.

2. Derivation of the Method (Scheme)

In developing this method, power series of the form

$$y(x) = \sum_{j=0}^{k+4} a_j x^j \qquad (2)$$

is considered as the approximate solution to equation(1), where $k = 3$, equation (3) is derived by differentiating equation(2) three times to give.

$$y'''(x) = \sum_{j=0}^{k+4} j(j-1)(j-2)a_j x^{j-3} = f(x, y, y', y'') \quad (3)$$

Interpolating (2) at $x_{n+j}, j = 1, \sqrt{3}, 2$ and collocating equation (3) at $x_{n+j}, j = 0, 1, \sqrt{3}, 2, 3$. These equations are then combined to give a nonlinear system of equations of the form

$$\begin{bmatrix} 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 \\ 1 & x_{n+\sqrt{3}} & x_{n+\sqrt{3}}^2 & x_{n+\sqrt{3}}^3 & x_{n+\sqrt{3}}^4 & x_{n+\sqrt{3}}^5 & x_{n+\sqrt{3}}^6 & x_{n+\sqrt{3}}^7 \\ 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & x_{n+2}^7 \\ 0 & 0 & 0 & 6 & 24x_n & 60x_n^2 & 120x_n^3 & 210x_n^4 \\ 0 & 0 & 0 & 6 & 24x_{n+1} & 60x_{n+1}^2 & 120x_{n+1}^3 & 210x_{n+1}^4 \\ 0 & 0 & 0 & 6 & 24x_{n+\sqrt{3}} & 60x_{n+\sqrt{3}}^2 & 120x_{n+\sqrt{3}}^3 & 210x_{n+\sqrt{3}}^4 \\ 0 & 0 & 0 & 6 & 24x_{n+2} & 60x_{n+2}^2 & 120x_{n+2}^3 & 210x_{n+2}^4 \\ 0 & 0 & 0 & 6 & 24x_{n+3} & 60x_{n+3}^2 & 120x_{n+3}^3 & 210x_{n+3}^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} = \begin{bmatrix} y_{n+1} \\ y_{n+\sqrt{3}} \\ y_{n+2} \\ f_n \\ f_{n+1} \\ f_{n+\sqrt{3}} \\ f_{n+2} \\ f_{n+3} \end{bmatrix} \quad (4)$$

Gaussian elimination technique is used in finding the values of a_j's in equation (4) which are then substituted into equation (2) to produce a continuous implicit scheme of the form

$$y(x) = \sum_{j=0}^{k} \alpha_j(x) y_{n+j} + h^3 \left(\sum_{j=0}^{k+1} \beta_j(x) f_{n+j} + \beta_q(x) f_{n+q} \right) \quad (5)$$

Where $y(x)$ is the numerical solution of the initial value problem and $q = \sqrt{3}$.
α_j and β_j are both constant.

$$f_{n+j} = \left(x_{n+j}, y_{n+j}, y'_{n+j}, y''_{n+j}, y'''_{n+j} \right). \quad (6)$$

Using the transformation $= \frac{x - x_{n+2}}{h}, \frac{dt}{dx} = \frac{1}{h}$, we obtain the continuous scheme below.

$$\alpha_1(t) = -\frac{t}{2}(1+\sqrt{3})(\sqrt{3}-t-2)$$

$$\alpha_{\sqrt{3}}(t) = -\frac{t}{2}(5+3\sqrt{3})(t+1)$$

$$\alpha_2(t) = -(2+\sqrt{3})(t+1)(\sqrt{3}-t-2)$$

$$\beta_0(t) = \frac{h^3}{15120}\sqrt{3}\begin{pmatrix} 3t^3\sqrt{3} - 4t^4 - 15\sqrt{3}t^2 \\ -2t^3 + 39\sqrt{3}t + 37t^2 \\ -123\sqrt{3} - 73t + 217 \end{pmatrix} \times t(\sqrt{3}-t-2)(t+1) \quad (7)$$

$$\beta_1(t) = -\frac{h^3}{3360}(1+\sqrt{3})t(\sqrt{3}-t-2)(t+1) \times \begin{pmatrix} 3t^3\sqrt{3} - 4t^4 - 8\sqrt{3}t^2 - 9t^3 \\ -17\sqrt{3}t + 44t^2 + 122\sqrt{3} + 11t - 196 \end{pmatrix}$$

$$\beta_{\sqrt{3}}(t) = -\frac{h^3}{2520}(7+4\sqrt{3})\begin{pmatrix} 2t^3\sqrt{3} + 2t^4 - 3\sqrt{3}t^2 \\ +t^3 - 2\sqrt{3}t \\ -8t^2 - 12\sqrt{3} - 16t + 49 \end{pmatrix} \times t(\sqrt{3}-t-2)(t+1)$$

$$\beta_2(t) = -\frac{h^3}{1680}(2+\sqrt{3})t\begin{pmatrix} 3t^3\sqrt{3} - 4t^4 - \sqrt{3}t^2 \\ -16t^3 - 31\sqrt{3}t + 23t^2 \\ -53\sqrt{3} + 109t + 91 \end{pmatrix} \times (\sqrt{3}-t-2)(t+1)$$

$$\beta_3(t) = \frac{h^3}{30240}(\sqrt{3}+3)\begin{pmatrix} 3t^3\sqrt{3} - 4t^4 + 6\sqrt{3}t^2 \\ -23t^3 - 3\sqrt{3}t - 26t^2 \\ -18\sqrt{3} + 11t + 28 \end{pmatrix} \times t(\sqrt{3}-t-2)(t+1).$$

Evaluating the above continuous method at the end point, gives the discrete scheme

$$y_{n+3} = (2\sqrt{3}+6)y_{n+2} + \sqrt{3}y_{n+1} + (-5-3\sqrt{3})y_{n+\sqrt{3}} + h^3\begin{bmatrix} \left(\frac{463}{2520} - \frac{271}{2520}\sqrt{3}\right) f_n + \left(\frac{5}{14} - \frac{11}{60}\sqrt{3}\right) f_{n+1} \\ + \left(\frac{3}{140} + \frac{1}{252}\sqrt{3}\right) f_{n+\sqrt{3}} + \left(\frac{121}{280} - \frac{43}{840}\sqrt{3}\right) f_{n+2} \\ + \left(\frac{1}{120}\sqrt{3} + \frac{1}{180}\right) f_{n+3} \end{bmatrix}. \quad (8)$$

2.1. Derivation of the Block

Normalizing the combination of the discrete scheme with the evaluation of the first and second derivatives of the continuous method at all the points yield the block below.

$$
\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+1}\\y_{n+\sqrt{3}}\\y_{n+2}\\y_{n+3}\end{bmatrix}=\begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n-1}\\y_{n-\sqrt{3}}\\y_{n-2}\\y_{n}\end{bmatrix}+h\begin{bmatrix}0&0&0&1\\0&0&0&\sqrt{3}\\0&0&0&2\\0&0&0&3\end{bmatrix}\begin{bmatrix}y'_{n-1}\\y'_{n-\sqrt{3}}\\y'_{n-2}\\y'_{n}\end{bmatrix}+h^2\begin{bmatrix}0&0&0&\frac{1}{2}\\0&0&0&3/2\\0&0&0&2\\0&0&0&9/2\end{bmatrix}\begin{bmatrix}y''_{n-1}\\y''_{n-\sqrt{3}}\\y''_{n-2}\\y''_{n}\end{bmatrix}
$$

$$
+h^3\begin{bmatrix}\left(\frac{47}{1680}\sqrt{3}+\frac{97}{840}\right)&\left(\frac{-47}{360}-\frac{47}{630}\sqrt{3}\right)&\left(\frac{47}{840}\sqrt{3}+\frac{11}{140}\right)&\left(\frac{-47}{15120}\sqrt{3}-\frac{1}{420}\right)\\ \left(\frac{45}{56}+\frac{9}{112}\sqrt{3}\right)&\left(\frac{-13}{40}\sqrt{3}-\frac{9}{14}\right)&\left(\frac{9}{28}\sqrt{3}+\frac{9}{35}\right)&\left(\frac{-3}{112}\sqrt{3}+\frac{3}{280}\right)\\ \left(\frac{39}{35}+\frac{19}{105}\sqrt{3}\right)&\left(\frac{-152}{315}\sqrt{3}-\frac{38}{45}\right)&\left(\frac{38}{105}\sqrt{3}+\frac{11}{21}\right)&\left(\frac{-19}{945}\sqrt{3}-\frac{1}{63}\right)\\ \left(\frac{243}{70}+\frac{243}{560}\sqrt{3}\right)&\left(\frac{-81}{70}\sqrt{3}-\frac{81}{40}\right)&\left(\frac{243}{280}\sqrt{3}+\frac{243}{140}\right)&\left(\frac{-27}{560}\sqrt{3}-\frac{9}{280}\right)\end{bmatrix}\begin{bmatrix}f_{n+1}\\f_{n+\sqrt{3}}\\f_{n+2}\\f_{n+3}\end{bmatrix}
$$

$$
+h^3\begin{bmatrix}0&0&0&\left(\frac{19}{180}-\frac{47}{7560}\sqrt{3}\right)\\0&0&0&\left(\frac{9}{20}\sqrt{3}-\frac{3}{7}\right)\\0&0&0&\left(\frac{-38}{945}\sqrt{3}+\frac{5}{9}\right)\\0&0&0&\left(\frac{-27}{280}\sqrt{3}+\frac{27}{20}\right)\end{bmatrix}\begin{bmatrix}f_{n-1}\\f_{n-\sqrt{3}}\\f_{n-2}\\f_{n}\end{bmatrix} \tag{9}
$$

Writing out the block explicitly gives the following

$$
y_{n+1}=\frac{1}{2}y''_n h^2+y'_n h+y_n+h^3\left[\begin{matrix}\left(\frac{19}{180}-\frac{47}{7560}\sqrt{3}\right)f_n+\left(\frac{47}{1680}\sqrt{3}+\frac{97}{840}\right)f_{n+1}+\left(-\frac{47}{360}-\frac{47}{630}\sqrt{3}\right)\\ f_{n+\sqrt{3}}+\left(\frac{47}{840}\sqrt{3}+\frac{11}{140}\right)f_{n+2}+\left(-\frac{47}{15120}\sqrt{3}-\frac{1}{420}\right)f_{n+3}\end{matrix}\right] \tag{10}
$$

$$
y_{n+\sqrt{3}}=\frac{3}{2}y''_n h^2+\sqrt{3}y'_n h+y_n+h^3\left[\begin{matrix}\left(\frac{9}{20}\sqrt{3}-\frac{3}{7}\right)f_n+\left(\frac{45}{56}+\frac{9}{112}\sqrt{3}\right)f_{n+1}+\left(-\frac{13}{40}\sqrt{3}-\frac{9}{14}\right)\\ f_{n+\sqrt{3}}+\left(\frac{9}{28}\sqrt{3}+\frac{9}{35}\right)f_{n+2}+\left(-\frac{3}{112}\sqrt{3}+\frac{3}{280}\right)f_{n+3}\end{matrix}\right] \tag{11}
$$

$$
y_{n+2}=2y''_n h^2+2y'_n h+y_n+h^3\left[\begin{matrix}\left(-\frac{38}{945}\sqrt{3}+\frac{5}{9}\right)f_n+\left(\frac{39}{35}+\frac{19}{112}\sqrt{3}\right)f_{n+1}+\left(-\frac{152}{315}\sqrt{3}-\frac{38}{45}\right)\\ f_{n+\sqrt{3}}+\left(\frac{38}{105}\sqrt{3}+\frac{11}{21}\right)f_{n+2}+\left(-\frac{19}{945}\sqrt{3}-\frac{1}{63}\right)f_{n+3}\end{matrix}\right] \tag{12}
$$

$$
y_{n+3}=\frac{9}{2}y''_n h^2+3y'_n h+y_n+h^3\left[\begin{matrix}\left(-\frac{27}{280}\sqrt{3}+\frac{27}{20}\right)f_n+\left(\frac{243}{70}+\frac{243}{560}\sqrt{3}\right)f_{n+1}+\left(-\frac{81}{70}\sqrt{3}-\frac{81}{40}\right)\\ f_{n+\sqrt{3}}+\left(\frac{243}{280}\sqrt{3}+\frac{243}{140}\right)f_{n+2}+\left(-\frac{27}{560}\sqrt{3}-\frac{9}{280}\right)f_{n+3}\end{matrix}\right]. \tag{13}
$$

3. Analysis of the Basic Properties of the Block

3.1. Order and Error Constant of the Block

A block linear multistep method is said to be of order p if $\bar{c}_0=\bar{c}_1=\bar{c}_2=...=\bar{c}_p=0, \quad \bar{c}_{p+3}\neq 0$

Thus C_{P+3} is the error constant. See [7]

For our three step irrational method, expanding the block in Taylor series expansion gives

$$\begin{bmatrix} \sum_{q}^{\infty}\left(\frac{h^{q}}{q!}y^{q}\right)-\left(y_{n+1}-y_{n}-hy_{n}'-\frac{h^{2}}{2}y''-\left(\frac{19}{180}-\frac{47}{7560}\sqrt{3}\right)h^{3}y'''\right)- \\ -\sum_{q}^{\infty}\left(\frac{h^{q+3}}{q!}y^{q+3}\right)\left(\left(\frac{47}{1680}\sqrt{3}+\frac{97}{840}\right)(1)^{q}+\left(-\frac{47}{360}-\frac{47}{630}\sqrt{3}\right)(\sqrt{3})^{q}+\left(\frac{47}{840}\sqrt{3}+\frac{11}{140}\right)(2)^{q}+\left(-\frac{47}{15120}\sqrt{3}-\frac{1}{420}\right)(3)^{q}\right) \\ \sum_{q}^{\infty}\left(\frac{(\sqrt{3}h)^{q}}{q!}y^{q}\right)-\left(y_{n+\sqrt{3}}-y_{n}-\sqrt{3}hy_{n}'-\frac{3}{2}h^{2}y_{n}''-\left(\frac{9}{20}\sqrt{3}-\frac{3}{7}\right)h^{3}y_{n}'''\right) \\ -\sum_{q}^{\infty}\left(\frac{h^{q+3}}{q!}y^{q+3}\right)\left(+\left(\frac{45}{56}+\frac{9}{112}\sqrt{3}\right)(1)^{q}+\left(-\frac{13}{40}\sqrt{3}-\frac{9}{14}\right)(\sqrt{3})^{q}+\left(\frac{9}{28}\sqrt{3}+\frac{9}{35}\right)(2)^{q}+\left(-\frac{3}{112}\sqrt{3}+\frac{3}{280}\right)(3)^{q}\right) \\ \sum_{q}^{\infty}\left(\frac{(2h)^{q}}{q!}y^{q}\right)-\left(y_{n+2}-y_{n}-2hy_{n}'-2h^{2}y_{n}''-\left(-\frac{38}{945}\sqrt{3}+\frac{5}{9}\right)y_{n}'''\right) \\ -\sum_{q}^{\infty}\left(\frac{h^{q+3}}{q!}y^{q}\right)\left(\left(\frac{39}{35}+\frac{19}{112}\sqrt{3}\right)(1)^{q}+\left(-\frac{152}{315}\sqrt{3}-\frac{38}{45}\right)(\sqrt{3})^{q}+\left(\frac{38}{105}\sqrt{3}+\frac{11}{21}\right)(2)^{q}+\left(-\frac{19}{945}\sqrt{3}-\frac{1}{63}\right)(3)^{q}\right) \\ \sum_{q}^{\infty}\left(\frac{(3h)^{q}}{q!}y^{q}\right)-\left(y_{n+3}-y_{n}-3hy_{n}'-\frac{9h^{2}}{2}y''_{n}-\left(-\frac{27}{280}\sqrt{3}+\frac{27}{20}\right)y_{n}'''\right) \\ -\sum_{q}^{\infty}\left(\frac{h^{q+3}}{q!}y^{q}\right)\left(\left(\frac{243}{70}+\frac{243}{560}\sqrt{3}\right)(1)^{q}+\left(-\frac{81}{70}\sqrt{3}-\frac{81}{40}\right)(\sqrt{3})^{q}+\left(\frac{243}{280}\sqrt{3}+\frac{243}{140}\right)(2)^{q}+\left(-\frac{27}{560}\sqrt{3}-\frac{9}{280}\right)(3)^{q}\right) \end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix} \quad (14)$$

Hence, the irrational block is of order 5, with error constant

$$[0.00133246803, 0.0060061738, 0.0085425287, 0.0214368064]^{T}.$$

3.2. Zero Stability of the Block Method

Given the general form of block method

$$A^{(0)}Y_{m}=A^{(i)}Y_{m-1}+h^{\mu}[B^{(i)}F_{m}+B^{(0)}F_{m-1}]$$

A block method is said to be zero stable, if the roots

$$\det\left[\lambda A^{(0)}-A^{(i)}\right]=0$$

of the first characteristic polynomial satisfy $|\lambda|\le 1$ and for the roots with $|\lambda|\le 1$, the multiplicity must not exceed the order of the differential equation. See [7]

$$A=\left[z\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix}-\begin{pmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{pmatrix}\right]=0$$

$$A=z^{4}-z^{3}=0, z=0,0,0,1$$

Hence the block is zero stable.

3.3. Region of Absolute Stability

The Stability nature of the method is found in the Spirit of [7] and [4] as shown below

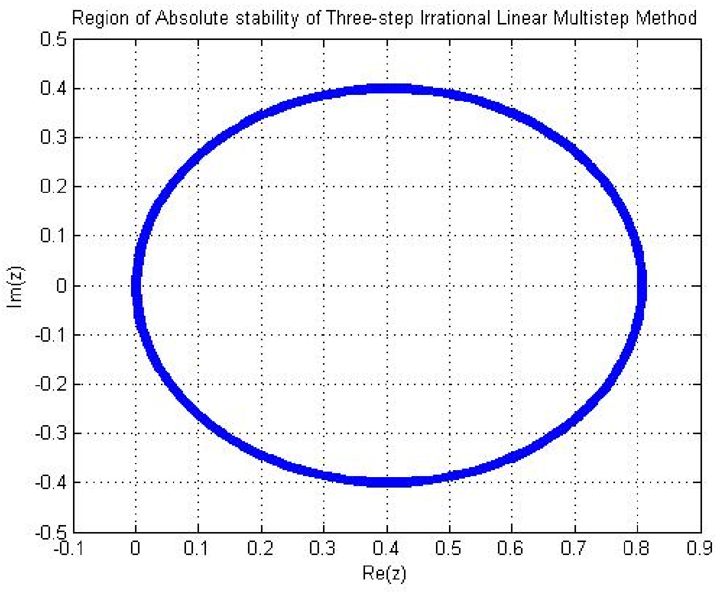

Figure 1. Showing the region of absolute stability of the irrational method

3.4. Convergence

Theorem 1: Convergence [17]

The necessary and sufficient condition for a linear multistep method to be convergent is for it to be consistent and zero stable. From the theorem above, the irrational block method is convergent.

4. Numerical Experiments

Here, the performance of the new irrational method is examined on some test examples. The results obtained from the test examples are shown in tabular and graphical form. We used MATLAB codes for the computational purposes.

Example 1:

$y''' = 3\sin x,\ y(0) = 1,\ y'(0) = 0, y''(0) = -2,\ 0 \le x \le 1, h = 0.1$

Exact solution: $y(x) = 3\cos x + (x^2 / 2) - 2$

Source: [18] and [10]

Table 1. Comparison of the result of problem 1 with [18] and [10]

x	Exact solution	Computed solution	Error in irrational method K=3, p=5	Error in [10] K=4, p=6	Error in[18] K=3, p=5
0.1	0.990012495834	0.990012495794	3.9587E-11	1.017524E-10	1.6592250E-10
0.2	0.960199733524	0.960199733270	2.5377E-10	6.558604E-10	4.7627491E-10
0.3	0.911009467377	0.911009466740	6.3689E-10	1.601112E-09	6.2318195E-10
0.4	0.843182982009	0.843182980542	1.4670E-09	2.998699E-09	2.9134462E-10
0.5	0.757747685671	0.757747683083	2.5885E-09	5.670582E-09	8.7111829E-10
0.6	0.656006844729	0.656006840423	4.3061E-09	9.092473E-09	3.9290352E-09
0.7	0.539526561853	0.539526555377	6.4768E-09	1.347564E-08	9.5534655E-09
0.8	0.410120128041	0.410120118674	9.3677E-09	1.950064E-08	1.8041497E-08
0.9	0.269829904812	0.269829891958	1.2854E-08	2.673002E-08	3.0311993E-08
1.0	0.120906917604	0.120906900449	1.7155E-08	3.533854E-08	4.7304419E-08
1.10	-0.034211635723	-0.034211657892	2.2169E-08	4.580460E-08	7.0036732E-08
1.20	-0.192926736570	-0.192926764627	2.8057E-08	5.781661E-08	9.9630021E-08

Example 2:

$y''' = x - 4y', y(0) = 0, y'(0) = 0, y''(0) = 1, h = 0.01$

Exact solution: $y(x) = \frac{-3}{16}\cos(2x) + \frac{3}{16} + \frac{x^2}{8}$

Source: [19] and [20]

Table 2. Comparison of the result of problem 2 with [19] and [20]

X	Exact solution	Computed solution	Error in irrational method K=3, p=6	Error in [19] k=4, p=7	Error in [20] k=4, p=7
0.1	1.050041729278	1.050041729273	5.4554E-12	1.1189E-11	1.1889E-11
0.2	1.100335347731	1.100335347500	2.3062E-10	3.0422E-09	3.0422E-09
0.3	1.151140435936	1.151140434017	1.9194E-09	7.7779E-08	7.7956E-08
0.4	1.202732554054	1.202732545402	8.6524E-09	1.5559E-07	7.7467E-07
0.5	1.255412811883	1.255412783644	2.8239E-08	3.0544E-07	4.5990E-06
0.6	1.309519604203	1.309519528495	7.5708E-08	4.6102E-07	6.4783E-06
0.7	1.365443754271	1.371153015061	1.7829E-07	7.0374E-07	5.7839E-06
0.8	1.423648930194	1.423648546102	3.8409E-07	1.0177E-06	2.3547E-06
0.9	1.484700278594	1.484699500907	7.7769E-07	1.6528E-06	3.7665E-06
1.0	1.549306144334	1.549304634965	1.5094E-06	3.0768E-06	1.2331E-05

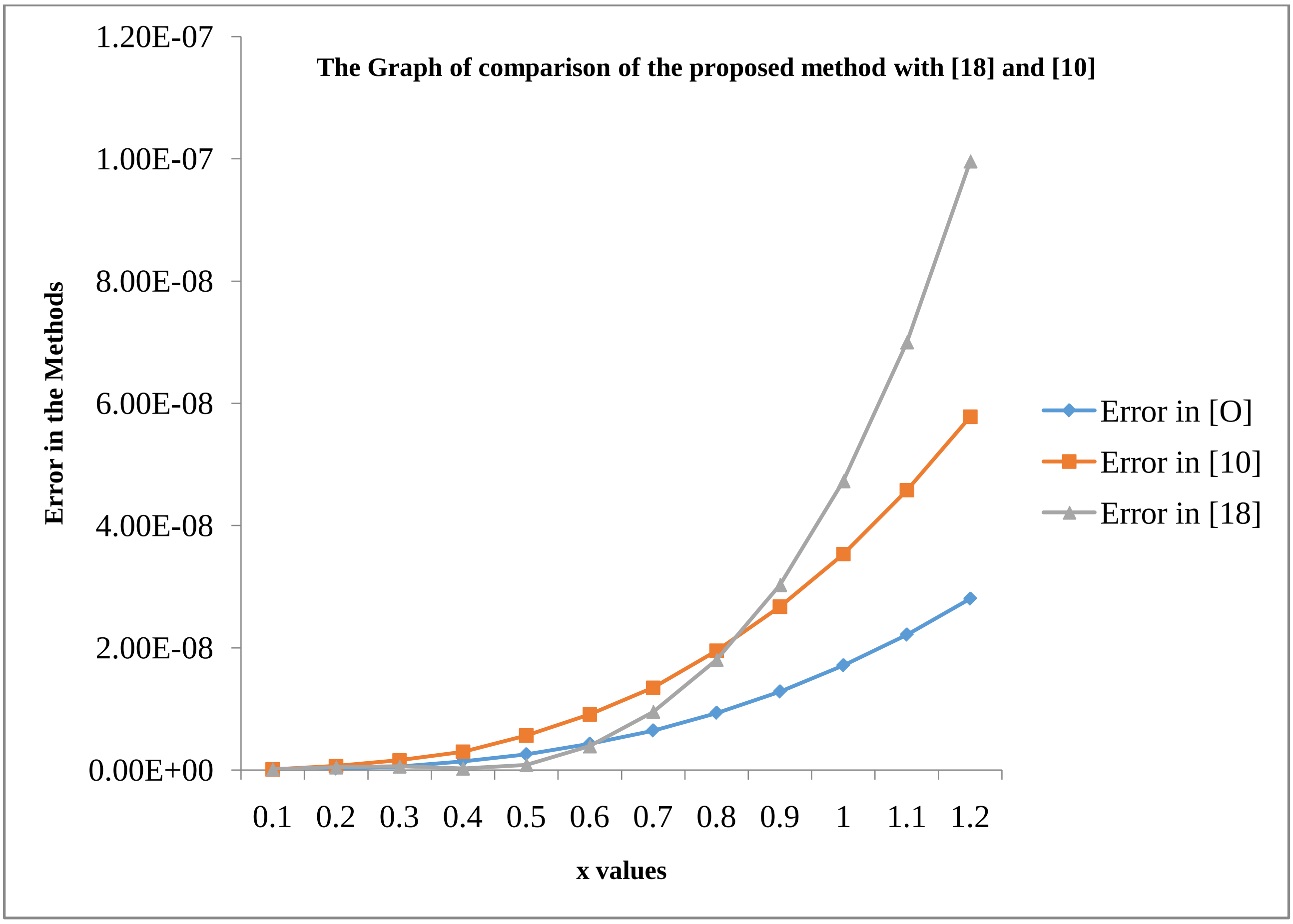

Figure 2. Showing the comparison of the result of problem 1 with [18] and [10]

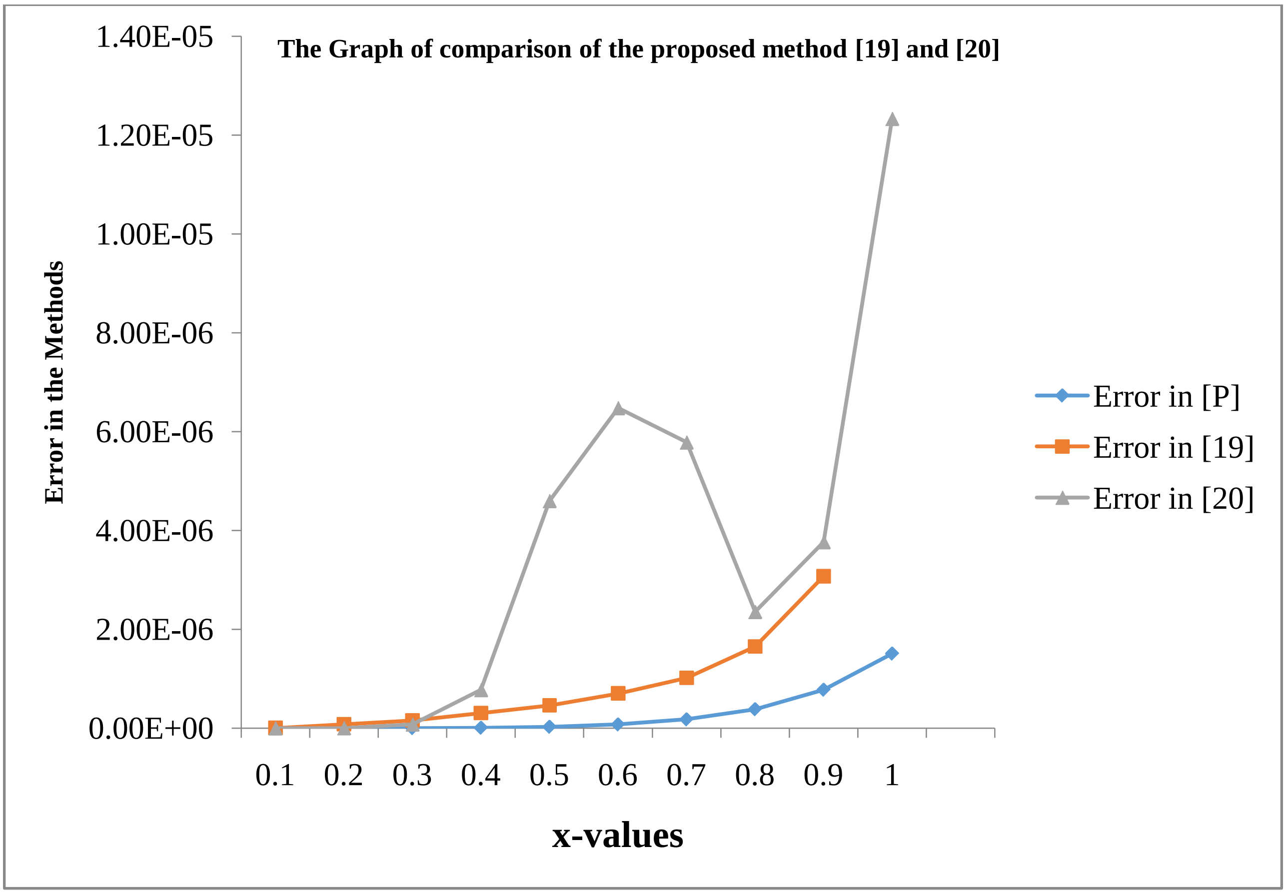

Figure 3. Showing the comparison of the result of problem 2 with [19] and [20]

5. Conclusion

From the two test problems solved by the new irrational method, we can conclude that the new method is effective in handling third order ordinary differential equations initial value problems. This fact is clearly seen from the accuracy of the results presented in the Table 1 and Table 2 and graphs in Figure 2 and Figure 3 above.

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