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On Generalized c*-continuous Functions and Generalized c*-irresolute Functions in Topological Spaces

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Abstract The aim of this paper is to introduce the notion of generalized c*-continuous functions and generalized c*-irresolute functions in topological spaces and study their basic properties.

Keywords: *gc*-continuous functions and gc*-irresolute functions*

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1. Introduction

In 1963, Norman Levine introduced semi-open sets in topological spaces. Also in 1970, he introduced the concept of generalized closed sets. Palaniappan and Rao introduced regular generalized closed (briefly, rg-closed) sets in 1993. In the year 1996, Andrijevic introduced and studied b-open sets. Gnanambal introduced generalized preregular closed (briefly gpr-closed) sets in 1997. N. Levine introduced the concept of semi-continuous function in 1963. In 1980, Jain introduced totally continuous functions. In 1995, T. M. Nour introduced the concept of totally semi-continuous functions as a generalization of totally continuous functions. In 2011, S.S. Benchalli and Umadevi I Neeli introduced the concept of semi-totally continuous functions in topological spaces. In this paper we introduce generalized c*-continuous functions and generalized c*-irresolute functions in topological spaces and study their basic properties.

Section 2 deals with the preliminary concepts. In section 3, generalized c*-continuous functions are introduced and study their basic properties. The generalized c*-irresolute functions in topological spaces are introduced in section 4.

2. Preliminaries

Throughout this paper X denotes a topological space on which no separation axiom is assumed. For any subset A of X, cl(A) denotes the closure of A, int(A) denotes the interior of A, pcl(A) denotes the pre-closure of A and bcl(A) denotes the b-closure of A. Further X\A denotes the complement of A in X. The following definitions are very useful in the subsequent sections.

Definition: 2.1 A subset A of a topological space X is called

i. a semi-open set [1] if $A\subseteq cl(int(A))$ and a semi-closed set if $int(cl(A))\subseteq A$.
ii. a pre-open set [2] if $A\subseteq int(cl(A))$ and a pre-closed set if $cl(int(A))\subseteq A$.
iii. a regular-open set [3] if $A=int(cl(A))$ and a regular-closed set if $A=cl(int(A))$.
iv. a γ-open set [4] (b-open set [5]) if $A\subseteq cl(int(A)) \cup int(cl(A))$ and a γ-closed set (b-closed set) if $int(cl(A))\cap cl(int(A))\subseteq A$.

Definition: 2.2 A subset A of a topological space X is said to be

i. a clopen set if A is both open and closed in X.
ii. a semi-clopen set if A is both semi-open and semi-closed in X.

Definition: 2.3 [6] A subset A of a topological space X is said to be a c*-open set if $int(cl(A))\subseteq A\subseteq cl(int(A))$.

Definition: 2.4 A subset A of a topological space X is called

i. a generalized closed set (briefly, g-closed) [7] if $cl(A)\subseteq H$ whenever $A\subseteq H$ and H is open in X.
ii. a regular-generalized closed set (briefly, rg-closed) [8] if $cl(A)\subseteq H$ whenever $A\subseteq H$ and H is regular-open in X.
iii. a generalized pre-regular closed set (briefly, gpr-closed) [9] if $pcl(A)\subseteq H$ whenever $A\subseteq H$ and H is regular-open in X.
iv. a regular generalized b-closed set (briefly, rgb-closed) [10] if $bcl(A)\subseteq H$ whenever $A\subseteq H$ and H is regular-open in X.
v. a regular weakly generalized closed set (briefly, rwg-closed) [11] if $cl(int(A))\subseteq H$ whenever $A\subseteq H$ and H is regular-open in X.

vi. a semi-generalized b-closed set (briefly, sgb-closed) [12] if bcl(A)⊆H whenever A⊆H and H is semi-open in X.
vii. a weakly closed set (briefly, w-closed) [13] (equivalently, ĝ-closed [14]) if cl(A)⊆H whenever A⊆H and H is semi-open in X.

The complements of the above mentioned closed sets are their respectively open sets.

Definition: 2.5 [6] A subset A of a topological space X is said to be a generalized c*-closed set (briefly, gc*-closed set) if cl(A)⊆H whenever A⊆H and H is c*-open. The complement of the gc*-closed set is gc*-open [15].

Definition: 2.6 A function f : X → Y is called

i. semi-continuous [1] if the inverse image of each open subset of Y is semi-open in X.
ii. totally-continuous [16] if the inverse image of every open subset of Y is clopen in X.
iii. strongly-continuous [3] if the inverse image of every subset of Y is clopen subset of X.
iv. totally semi-continuous [17] if the inverse image of every open subset of Y is semi- clopen in X .
v. strongly semi-continuous [17] if the inverse image of every subset of Y is semi- clopen in X .
vi. semi-totally continuous [18] if the inverse image of every semi- open subset of Y is clopen in X.
vii. semi-totally semi-continuous [19] if the inverse image of every semi-open subset of Y is semi-clopen in X.
viii. rg-continuous [9] if inverse image of every closed subset of Y is rg-closed in X.
viii. gpr-continuous [9] if inverse image of every closed subset of Y is gpr-closed in X.
ix. w-continuous [20] (ĝ-continuous [14]) if inverse image of every closed subset of Y is w-closed in X.

Definition: 2.7 [21] A function f: X → Y is called an irresolute function if the inverse image of every semi-open subset of Y is semi-open in X.

3. Generalized c*-continuous Functions

In this section, we introduce generalized c*-continuous functions and study its basic properties. Now, we begin with the definition of generalized c*-continuous function.

Definition: 3.1 Let X and Y be two topological spaces. A function f: X → Y is called a generalized c*-continuous (briefly, gc*-continuous) function if $f^{-1}(V)$ is gc*-closed in X for every closed set V of Y .

Example: 3.2 Let X={a,b,c,d} with topology τ={ϕ, {a}, {b}, {a,b}, {b,c}, {a,b,c}, {a,b,d}, X} and Y={1,2,3} with topology σ={ϕ,{1},Y}. Define f : X → Y by f(a)=f(d)=2, f(b)=3, f(c)=1. Then the inverse image of every closed set in Y is gc*-closed in X. Hence f : X → Y is gc*-continuous.

Proposition: 3.3 Let X,Y be two topological spaces. Then f : X → Y is gc*-continuous if and only if $f^{-1}(U)$ is gc*-open in X for every open set U of Y.

Proof: Suppose f : X → Y is gc*-continuous. Let U be an open set in Y. Then Y\U is a closed set in Y. This implies, $f^{-1}(Y\setminus U)$ is a gc*-closed set in X. Since $f^{-1}(Y\setminus U)=X\setminus f^{-1}(U)$, we have $X\setminus f^{-1}(U)$ is a gc*-closed set in X. This implies, $f^{-1}(U)$ is a gc*-open set in X. Conversely, assume that $f^{-1}(U)$ is gc*-open in X for every open set U in Y. Let V be a closed set in Y. Then Y\V is open in Y. Therefore, $f^{-1}(Y\setminus V)$ is gc*-open in X. That is, $X\setminus f^{-1}(V)$ is gc*-open in X. This implies, $f^{-1}(V)$ is gc*-closed in X. Therefore, f is gc*-continuous.

Proposition: 3.4 Let X, Y be two topological spaces. Then every continuous function is gc*-continuous.

Proof: Let f: X → Y be a continuous function. Let V be a closed set in Y. Then $f^{-1}(V)$ is a closed set in X. By Proposition 4.3 [6], $f^{-1}(V)$ is gc*-closed in X. Therefore, f is gc*-continuous.

The converse of the Proposition 3.4 need not be true as seen from the following example.

Example: 3.5 Let X={1,2,3,4} and Y={a,b,c,d,e}. Then, clearly τ={ϕ, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {3,4}, {1,2,3}, {2,3,4}, {1,3,4}, X} is a topology on X and σ={ϕ,{a},{d},{e},{a,d},{a,e}, {d,e},{a,d,e},Y} is a topology on Y. Define f : X → Y by f(1)=b, f(2)=f(3)=d, f(4)=e. Then f is gc*-continuous. Consider the closed set {a,b,c,d} in Y. Then $f^{-1}(\{a,b,c,d\})=\{1,2,3\}$, which is not a closed set in X. Therefore, f is not continuous.

Proposition: 3.6 Let X, Y be two topological spaces. Then every strongly-continuous function is gc*-continuous.

Proof: Let f: X → Y be a strongly-continuous function and let V be a closed set in Y. Then $f^{-1}(V)$ is a clopen set in X. This implies, $f^{-1}(V)$ is closed in X. By Proposition 4.3 [6], $f^{-1}(V)$ is gc*-closed. Therefore, f : X → Y is gc*-continuous.

The converse of the Proposition 3.6 need not be true as seen from the following example.

Example: 3.7 Let X={1,2,3,4,5} and Y={a,b,c,d}. Then, clearly τ={φ {1}, {4}, {5}, {1,4}, {1,5}, {4,5}, {1,4,5}, X} is a topology on X and σ={ϕ, {a}, {c}, {d}, {a,c}, {a,d}, {c,d}, {a,c,d}, Y} is a topology on Y. Define f : X → Y by f(1)=f(4)=f(5)=b, f(2)=f(3)=a. Then f is gc*-continuous. Consider the subset {a} in Y. Then $f^{-1}(\{a\})=\{2,3\}$, which is not a clopen set in X. Therefore, f is not strongly-continuous.

Proposition: 3.8 Let X,Y be two topological spaces. Then every semi-totally continuous function is gc*-continuous.

Proof: Let f: X → Y be a semi-totally continuous function and let V be an open set in Y. Since every open set is semi-open, we have V is semi-open in Y. Therefore, by our assumption, $f^{-1}(V)$ is clopen in X. Then by Proposition 3.7 [15], $f^{-1}(V)$ is a gc*-open set in X. Therefore, f : X → Y is gc*-continuous.

The converse of the Proposition 3.8 need not be true as seen from the following example.

Example: 3.9 In Example 3.5, define f : X → Y by f(1)=b, f(2)=f(3)=d, f(4)=e. Then f is gc*-continuous. Consider the semi-open set {d} in Y. Then $f^{-1}(\{d\})=\{2,3\}$, which is not a clopen set in X. Therefore, f is not semi-totally continuous.

Proposition: 3.10 Let X, Y be two topological spaces. Then every totally-continuous function is gc*-continuous.

Proof: Let f: X → Y be a totally-continuous function and let V be an open set in Y. Then $f^{-1}(V)$ is clopen in X. Therefore, by Proposition 3.7 [15], $f^{-1}(V)$ is gc*-open in X. Therefore, f is gc*-continuous.

The converse of the Proposition 3.10 need not be true as seen from the following example.

Example: 3.11 In Example 3.5, define f : X → Y by f(1)=b, f(2)=f(3)=d, f(4)=e. Then f is gc*-continuous. Consider the open set {d} in Y. Then $f^{-1}(\{d\})=\{2,3\}$, which is not a clopen set in X. Therefore, f is not a totally-continuous function.

Proposition: 3.12 Let X,Y be two topological space. Then every w-continuous (ĝ-continuous) function is gc*-continuous.

Proof: Let f: X → Y be a w-continuous function. Let V be a closed set in Y. Then $f^{-1}(V)$ is w-closed in X. By Proposition 4.5 [6], we have $f^{-1}(V)$ is gc*-closed in X. Therefore, f is gc*-cotinuous.

The converse of the Proposition 3.12 need not be true as seen from the following example.

Example: 3.13 Let X={1,2,3} and Y={a,b,c}. Then, clearly τ={φ{2},{1,2},X} is a topology on X and σ={φ{a},Y} is a topology on Y. Define f :X → Y by f(1)=a, f(2)=c, f(3)=b. Then f is gc*- continuous. Consider the closed set {b,c} in Y. Then f^{-1}({b,c})={2,3} which is not a w-closed set in X. Therefore, f is not a w-continuous function.

Proposition: 3.14 Let X, Y be two topological spaces. Then every gc*-continuous function is rg-continuous.

Proof: Letf: X → Y be a gc*-continuous function. Let V be a closed set in Y. Then $f^{-1}(V)$ is a gc*-closed set in X. By Proposition 4.7 [6], we have $f^{-1}(V)$ is rg-closed (gpr-closed) in X. Therefore, f is rg-continuous.

Proposition: 3.15 Let X,Y be two topological spaces. Then every gc*-continuous function is gpr-continuous.

Proof: Let f: X → Y be a gc*-continuous function. Let V be a closed set in Y. Then $f^{-1}(V)$ is a gc*-closed set in X. By Proposition 4.9 [6], we have $f^{-1}(V)$ is gpr-closed in X. Therefore, f is gpr-continuous.

The converse of Proposition 3.14 and Proposition 3.15 need not be true as seen from the following example.

Example: 3.16 Let X={a, b, c, d, e} and Y={1,2,3,4}. Then, clearly τ={φ{a},{d},{e},{a,d},{a,e}, {d,e}, {a,d,e}, X} is a topology on X and σ={φ {1}, {3}, {4}, {1,3}, {1,4}, {3,4}, {1,3,4}, Y} is a topology on Y. Define f : X → Y by f(a)=1, f(b)=3, f(c)=2, f(d)=f(e)=4. Then f is rg-continuous and gpr-continuous. Consider the closed set {2,4} in Y. Then f^{-1}({2,4})={c,d,e}, which is not a gc*-closed set in X. Therefore f is not gc*-continuous.

The semi-continuous functions and gc*-continuous functions are independent. For example, In Example 3.7, define f : X → Y by f(1)=f(2)=a, f(3)=b, f(4)=f(5)=d. Then f is semi-continuous. Consider the closed set {b,c,d} in Y. Then f^{-1}({b,c,d})={3,4,5}, which is not a gc*-closed set in X. Therefore, f is not a gc*-continuous. Now, define g: X → Y by g(1)=g(4)= g(5)=b, g(2)=g(3)=a. Then g is gc*-continuous. Consider the closed set {b,d} in Y. Then g^{-1}({b,d})={1,4,5}, which is not a semi-closed set in X. Therefore, g is not a semi-continuous function.

The totally-semi continuous functions and gc*-continuous functions are independent. For example, let X={1,2,3,4} and Y={a,b,c,d,e}. Then, clearly τ={φ {1}, {2}, {1,2}, {1,3}, {1,2,3}, X} is a topology on X and σ={φ {a}, {d}, {e}, {a,d}, {a,e}, {d,e}, {a,d,e}, Y} is a topology on Y. Define f : X → Y by f(1)=a, f(2)=b, f(3)=f(4)=c. Then f is gc*-continuous. Consider the open set {a} in Y. Then f^{-1}({a})={1}, which is not a semi-clopen set in X. Therefore, f is not a totally-semi continuous function. Now, define g: X → Y by g(1)=g(3)=a, g(2)=g(4)=d. Then g is totally-semi continuous. Consider the closed set {a,b,c} in Y. Then g^{-1}({a,b,c})={1,3}, which is not a gc*-closed set in X. Therefore, g is not a gc*-continuous function.

The strongly semi-continuous functions and gc*-continuous functions are independent. For example, let X={1,2,3,4,5} and Y={a,b,c,d,e}. Then, clearly τ={φ {1}, {4}, {5}, {1,4}, {1,5}, {4,5}, {1,4,5}, X} is a topology on X and σ={φ {a,b}, {c,d}, {a,b,c,d}, Y} is a topology on Y. Define f : X → Y by f(1)=f(2)=a, f(3)=f(5)=c, f(4)=d. Then f is strongly-semi continuous. Consider the closed set {a,b,e} in Y. Then f^{-1}({a,b,e})={1,2}, which is not a gc*-closed set in X. Therefore, f is not gc*-continuous. Now, define g : X → Y by g(1)=g(3)=g(4)=g(5)=e, g(2)=c. Then g is gc*-continuous. Consider the subset {c} in Y. Then g^{-1}({c})={2}, which is not a semi-clopen set in X. Therefore, g is not a strongly-semi continuous function.

The semi-totally semi-continuous functions and gc*-continuous functions are independent. For example, let X={1,2,3,4} and Y={a,b,c,d,e}. Then, clearly τ={φ {1}, {2}, {1,2}, {1,3}, {1,2,3}, X} is a topology on X and σ={φ{a},{d},{e},{a,d},{a,e},{d,e},{a,d,e},Y} is a topology on Y. Define f: X → Y by f(1)=a, f(2)=b, f(3)=f(4)=c. Then f is gc*-continuous. Consider the semi-open set {a,b} in Y. Then f^{-1}({a,b})={1,2}, which is not a semi-clopen set in X. Therefore, f is not a semi-totally semi-continuous function. Now, define g:X → Y by g(1)=g(3)=a, g(2)=g(4)=d. Then g is semi-totally-semi-continuous. Consider the closed set {a,b,c} in Y. Then g^{-1}({a,b,c})={1,3}, which is not a gc*-closed set in X. Therefore, g is not a gc*-continuous function.

Proposition: 3.17 Let X, Y be two topological spaces. Then for any bijective function f: X → Y, the following statements are equivalent.

i. f : X → Y is gc*-continuous.
ii. f^{-1} : Y → X is gc*-open.

Proof: (i) ⇒ (ii) Assume that f : X → Y is gc*-continuous. Let U be an open set in Y. Since f is gc*-continuous, we have $f^{-1}(U)$ is gc*-open in X. Therefore, f^{-1} is a gc*-open map. **(ii) ⇒ (i)** Assume that f^{-1} : Y → X is a gc*-open map. Let V be an open subset of Y. By our assumption, $f^{-1}(V)$ is gc*-open in X. Therefore, f is gc*-continuous.

The composition of two gc*-continuous functions need not be gc*-continuous. For example, let X={a,b,c}, Y={1,2,3}, Z={p,q,r}. Then, clearly τ={φ {b}, {c}, {b,c}, X} is a topology on X, σ={φ{1},Y} is a topology on Y and η={φ{p},{p,q},Z} is a topology on Z. Define f: X → Y by f(a)=1, f(b)=3, f(c)=2 and g : Y → Z by g(1)=q, g(2)=p, g(3)=r. Then f and g are gc*-continuous. Consider the closed set {r} in Z. Then $(g∘f)^{-1}$({r})= f^{-1}(g^{-1}({r}))= f^{-1}({3})= {b}, which is not a gc*-closed set in X. Therefore, g∘f is not a gc*-continuous function.

Proposition: 3.18 Let X,Y and Z be topological spaces. If f : X → Y is gc*-continuous and g: Y → Z is continuous, then g∘f : X → Z is gc*-continuous.

Proof: Let V be a closed set in Z. Since g is continuous, we have $g^{-1}(V)$ is closed in Y. Also, since f is gc*-continuous, we have $f^{-1}(g^{-1}(V))$ is gc*-closed in X. But $f^{-1}(g^{-1}(V))=(g∘f)^{-1}(V)$. Therefore, $(g∘f)^{-1}(V)$ is gc*-closed in X. Hence g∘f is gc*-continuous.

Proposition: 3.19 Let X,Y and Z be topological spaces. If f : X → Y and g : Y → Z are continuous. Then g∘f : X → Z is gc*-continuous.

Proof: Let V be a closed set in Z. Since g is continuous, we have $g^{-1}(V)$ is closed in Y. Also, since f is continuous, we have $f^{-1}(g^{-1}(V))$ is closed in Y. That is, $(g∘f)^{-1}(V)$ is closed in Y. By Proposition 4.3 [6], $(g∘f)^{-1}(V)$ is gc*-closed in Y. Therefore, g∘f is a gc*-continuous map.

4. Generalized c*-irresolute Functions

In this section, we introduce generalized c*-irresolute functions in topological spaces. Also, we discuss about some of their basic properties.

Definition: 4.1 Let X and Y be two topological spaces. A function f : X → Y is said to be a generalized c*-irresolute (briefly, gc*-irresolute) function if $f^{-1}(V)$ is gc*-closed in X for every gc*-closed set V in Y.

Example: 4.2 Let X={a,b,c,d,e} and Y={1,2,3,4}. Then, clearly τ={ϕ,{a},{d},{e},{a,d},{a,e}, {d,e},{a,d,e},X} is a topology on X and σ={ϕ, {1}, {3}, {4}, {1,3}, {1,4}, {3,4}, {1,3,4}, Y} is a topology on Y. Define f: X → Y by f(a)=f(d)=3, f(b)=f(c)=2, f(e)=4. Then the inverse image of every gc*-closed set in Y is gc*-closed in X. Hence f is gc*-irresolute.

Proposition: 4.3 Let X,Y be two topological spaces. Then f : X → Y is gc*-irresolute if and only if $f^{-1}(U)$ is gc*-open in X for every gc*-open set U of Y.

Proof: Suppose f: X → Y is gc*-irresolute. Let U be an gc*-open set in Y. Then Y\U is a gc*-closed set in Y. This implies, $f^{-1}(Y\backslash U)$ is a gc*-closed set in X. Since $f^{-1}(Y\backslash U)=X\backslash f^{-1}(U)$, we have $X\backslash f^{-1}(U)$ is a gc*-closed set in X. This implies, $f^{-1}(U)$ is a gc*-open set in X. Conversely, assume that $f^{-1}(U)$ is gc*-open in X for every gc*-open set U in Y. Let V be a gc*-closed set in Y. Then Y\V is gc*-open in Y. Therefore, $f^{-1}(Y\backslash V)$ is gc*-open in X. That is, $X\backslash f^{-1}(V)$ is gc*-open in X. This implies, $f^{-1}(V)$ is gc*-closed in X. Therefore, f is gc*-continuous.

The irresolute and gc*-irresolute functions are independent. For example, let X={1,2,3, 4,5} and Y={a,b,c,d}. Then, clearly τ={ϕ, {1}, {4}, {5}, {1,4}, {1,5}, {4,5}, {1,4,5}, X} is a topology on X and σ={ϕ, {a}, {c}, {d},{a,c}, {a,d}, {c,d}, {a,c,d}, Y} is a topology on Y. Define f : X → Y by f(1)=f(2)=a, f(3)=f(4)=f(5)=d. Then f is irresolute. Consider the gc*-closed set {a,b} in Y. Then $f^{-1}(\{a,b\})=\{1,2\}$ which is not a gc*-closed set in X. Therefore, f is not gc*-irresolute. Define g : X → Y by g(1)=g(4)=g(5)=b, g(2)=g(3)=a. Then g is gc*-irresolute. Consider the semi-open set {a} in Y. Then $g^{-1}(\{a\})=\{2,3\}$, which is not a semi-open set in X. Therefore, g is not irresolute.

Proposition: 4.4 Let X,Y be two topological spaces. Then every gc*-irresolute function is rg-continuous.

Proof: Let f : X → Y be a gc*-irresolute function. Let V be a closed set in Y. Then by Proposition 4.3 [6], V is gc*-closed set in Y. Since f is gc*-irresolute, $f^{-1}(V)$ is a gc*-closed set in X. Therefore, by Proposition 4.7 [6], $f^{-1}(V)$ is a rg-closed set in X. Hence f is rg-continuous.

Proposition: 4.5 Let X,Y be two topological spaces. Then every gc*-irresolute function is gpr-continuous.

Proof: Let f : X → Y be gc*-irresolute and V be a closed set in Y . Then by Proposition 4.3 [6], V is gc*-closed in Y. Since f is gc*-irresolute, we have $f^{-1}(V)$ is a gc*-closed set in X. Therefore, by Proposition 4.9 [6], $f^{-1}(V)$ is a gpr-closed set in X. Hence f is gpr-continuous.

The converse of Proposition 4.4 and Proposition 4.5 need not be true as seen from the following example.

Example: 4.6 Let X={a,b,c,d,e} with topology τ={ϕ, {a}, {d}, {e}, {a,d}, {a,e}, {d,e}, {a,d,e}, X} and Y={1,2,3,4,5} with topology σ={ϕ,{1,2},{3,4},{1,2,3,4},Y}. Define f: X→ Y by f(a)=2, f(b)=1, f(c)=5, f(d)=3, f(e)=4. Then f is rg-coninuous and gpr-continuous. Consider the gc*-closed set {1,3} in Y. Then $f^{-1}(\{1,3\})=\{b,d\}$, which is not a gc*-closed set in X. Therefore, f is not gc*-irresolute.

The gc*-irresolute and w-continuous functions are independent. For example,

1. let X={1,2,3,4,5} and Y={a,b,c,d}. Then, clearly τ={ϕ,{1,2},{3,4},{1,2,3,4},X} is a topology on X and σ={ϕ,{a},{c},{d},{a,c},{a,d},{c,d},{a,c,d},Y} is a topology on Y. Define f: X → Y by f(1)=f(4)=a, f(2)=f(3)=b, f(5)=c. Then f is gc*-irresolute. Consider the closed set {a,b,d} in Y. Now, $f^{-1}(\{a,b,d\})$ ={1,2,3,4}, which is not a w-closed set in X. Therefore, f is not a w-continuous function.
2. let X={1,2,3} and Y={a,b,c}. Then, clearly τ={ϕ, {2}, {3}, {2,3}, X} is a topology on X and σ={ϕ, {a}, {a,b}, {a,c}, Y} is a topology on Y. Define f: X → Y by f(1)=f(3)=b, f(2)=a. Then f is w-continuous. Consider the gc*-closed set {a} in Y. Then $f^{-1}(\{a\})=\{2\}$ is not a gc*-closed set in X. Therefore f is not a gc*-irresolute function.

Proposition: 4.7 Let X,Y be two topological spaces. Then every gc*-irresolute function is gc*-continuous.

Proof: Let f: X → Y be a gc*-irresolute function and V be a closed set in Y. Then by Proposition 4.3 [6], V is a gc*-closed set in Y. Since f is gc*-irresolute, we have $f^{-1}(V)$ is a gc*-closed set in X. Therefore, f is gc*-continuous.

The converse of the Proposition 4.7 need not be true as seen from the following example.

Example: 4.8 Let X={1,2,3} and Y={a,b,c}. Then, clearly τ={ϕ, {2}, {3}, {2,3}, X} is a topology on X and σ={ϕ, {a}, Y} is a topology on Y. Define f : X → Y by f(1)=a, f(2)=c, f(3)=b. Then f is gc*-continuous. Consider the gc*-closed set {b} in Y. Then $f^{-1}(\{b\})=\{3\}$, which is not a gc*-closed set in X. Therefore, f is not a gc*-irresolute function.

Proposition: 4.9 Let X, Y and Z be topological spaces. If f : X → Y and g: Y → Z are gc*-irresolute functions, then g∘f : X → Z is gc*-irresolute.

Proof: Let V be a gc*-closed set in Z . Then $g^{-1}(V)$ is gc*-closed. This implies, $f^{-1}(g^{-1}(V))$ is gc*-closed. But $f^{-1}(g^{-1}(V))= (g\circ f)^{-1}(V)$. Therefore $(g\circ f)^{-1}(V)$ is gc*-closed in X. Hence, g∘f is gc*-irresolute.

Proposition: 4.10 Let X,Y and Z be topological spaces. If f: X → Y and g: Y → Z are gc*-irresolute functions, then g∘f: X → Z is gc*-continuous.

Proof: Let f : X → Y and g : Y → Z be two gc*-irresolute functions. Let V be a closed set in Z. By Proposition 4.3 [6], V is a gc*-closed set in Z. Then $g^{-1}(V)$ is gc*-closed. This implies, $f^{-1}(g^{-1}(V))$ is gc*-closed. But $f^{-1}(g^{-1}(V))= (g\circ f)^{-1}(V)$. Therefore $(g\circ f)^{-1}(V)$ is gc*-closed in X. Hence g∘f : X → Z is gc*-continuous.

Proposition: 4.11 Let X, Y and Z be topological spaces. If f: X → Y is gc*-irresolute and g : Y → Z is continuous, then g∘f: X → Z is gc*-continuous.

Proof: Let V be a closed set in Z . Since g is continuous, we have $g^{-1}(V)$ is closed in Y. By Proposition 4.3 [6], $g^{-1}(V)$ is a gc*-closed set in Y. Since f is gc*-irresolute, we have $f^{-1}(g^{-1}(V))$ is gc*-closed in X. But $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$. Therefore, $(g\circ f)^{-1}(V)$ is gc*-closed in X. Hence, g∘f is gc*-continuous.

Proposition: 4.12 Let X,Y and Z be topological spaces. If f: X → Y is gc*-irresolute and g: Y → Z is gc*-continuous, then g∘f : X → Z is gc*-continuous.

Proof: Let V be a closed set in Z. Since g is gc*-continuous, we have $g^{-1}(V)$ is gc*-closed in Y. Since f is gc*-irresolute, we have $f^{-1}(g^{-1}(V))$ is gc*-closed in X. But $fs^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)$. Therefore, $(g\circ f)^{-1}(V)$ is gc*-closed in X. Hence, g∘f is gc*-continuous.

5. Conclusion

In this paper we have introduced gc*-continuous and gc*-irresolute functions in topological spaces and studied some of their basic properties. Also we have studied the relationship between gc*-continuous functions and some of the functions already exist.

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