Turkish Journal of Analysis and Number Theory, 2018, Vol. 6, No. 5, 136-147
Available online at http://pubs.sciepub.com/tjant/6/5/3

DOI:10.12691/tjant-6-5-3

Some Hypergeometric Generating Relations Motivated by the Work of Srivastava and Their Generalizations

M.I. Qureshi[1], Sulakshana Bajaj[2,*]

[1]Department of Applied Sciences and Humanities, Faculty of Engineering and Technology, Jamia Millia Islamia (A Central University), New Delhi - 110025, India,
[2]Department of Applied Sciences and Humanities, Modi Institute of Technology, Rawatbhata Road, Kota, Rajasthan 324010, India
*Corresponding author: sulakshana80@ymail.com

Received June 19, 2018; Revised August 22, 2018; Accepted October 08, 2018

Abstract In the present paper, we have obtained hypergeometric generating relations associated with two hypergeometric polynomials of one variable $H_n^{(\alpha,\beta)}(x;m)$ and $\mathscr{B}_n^{(\alpha,\beta)}(x;m,\lambda,\mu)$ with their independent demonstrations via Gould's identity. As applications, some well known and new generating relations are deduced. Using bounded sequences, further generalizations of two main hypergeometric generating relations have also been given for two generalized polynomials $S_n^{(\alpha,\beta)}(x;m)$ and $T_n^{(\alpha,\beta)}(x;m,\lambda,\mu)$.

Keywords: *Jacobi Polynomials, generalized Laguerre polynomial, generalized Rice polynomial of Khandekar, Gould's identity.*

Cite This Article: M.I. Qureshi, and Sulakshana Bajaj, “Some Hypergeometric Generating Relations Motivated by the Work of Srivastava and Their Generalizations.” *Turkish Journal of Analysis and Number Theory*, vol. 6, no. 5 (2018): 136-147. doi: 10.12691/tjant-6-5-3.

1. Introduction and Preliminaries

Throughout in the present paper, we use the following standard notations:

$$\mathbb{N} := \{1,2,3,\ldots\},\ \mathbb{N}_0 := \{0,1,2,3,\ldots\} = \mathbb{N}\cup\{0\}$$

$$\mathbb{Z}_0^- := \{0,-1,-2,-3,\ldots\},$$

$$\mathbb{Z}^- := \{-1,-2,-3,\ldots\} = \mathbb{Z}_0^- \setminus \{0\}$$

and $\mathbb{Z} = (\mathbb{Z}_0^- \cup \mathbb{N})$.

Here, as usual, $\mathbb{Z}$ denotes the set of integers, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^+$ denotes the set of positive real numbers and $\mathbb{C}$ denotes the set of complex numbers.

The Pochhammer symbol (or the shifted factorial) $(\lambda)_\nu$ $(\lambda,\nu\in\mathbb{C})$ is defined, in terms of the familiar Gamma function, by

$$(\lambda)_\nu := \frac{\Gamma(\lambda+\nu)}{\Gamma(\lambda)} = \begin{cases} 1 & (\nu=0;\lambda\in\mathbb{C}\setminus\{0\}) \\ \lambda(\lambda+1)\ldots(\lambda+n-1) & (\nu=n\in\mathbb{N};\lambda\in\mathbb{C}) \end{cases} \tag{1.1}$$

it is being understood *conventionally* that $(0)_0 = 1$ and assumed tacitly that the Gamma quotient exists.

Some useful consequences of Lagrange's expansion [[1], p.133; see also [2], p.146, problem 207] include the following generalization [[2], p. 349, problem 216] of the familiar binomial expansion:

$$\sum_{n=0}^{\infty}\binom{\theta+(\beta+1)n}{n}t^n = \frac{(1+\zeta)^{\theta+1}}{(1-\beta\zeta)} \tag{1.2}$$

where $\binom{\theta+(\beta+1)n}{n}$ is a binomial coefficient and θ, β are complex numbers independent of n and ζ is a function of 't' defined implicitly by

$$\zeta = t(1+\zeta)^{1+\beta} \tag{1.3}$$

subject to the condition

$$\zeta(0) = 0 \tag{1.4}$$

Another generalization [[2], p. 348, problem 212] related with the equation (1.2), is given as:

$$\sum_{n=0}^{\infty}\frac{\theta}{\{\theta+(\beta+1)n\}}\binom{\theta+(\beta+1)n}{n}t^n = 1+\theta\sum_{n=1}^{\infty}\binom{\theta+(\beta+1)n-1}{n-1}\frac{t^n}{n} = (1+\zeta)^\theta \tag{1.5}$$

where ζ is defined by the equations (1.3) and (1.4).

When $\beta=-1$, both results (1.2) and (1.5) reduce immediately to the binomial expansion.

Gould [[3], p.90; see also [4], p.169] gave the following identity:

$$\sum_{n=0}^{\infty}\frac{\theta(\sigma+\mu n)}{\{\theta+(\beta+1)n\}}\binom{\theta+(\beta+1)n}{n}t^n = (1+\zeta)^{\theta}\left(\sigma+\frac{\mu\theta\zeta}{(1-\beta\zeta)}\right) \tag{1.6}$$

where $\theta,\ \beta,\ \sigma,\ \mu$ are complex parameters independent of n and ζ is given by the equations (1.3) and (1.4).

If we put $\theta=\{\alpha+(\beta+1)mr\}$ and $\sigma = \{\lambda+\mu mr\}$ in Gould's identity (1.6), we get the first modified form of Gould's identity:

$$\sum_{n=0}^{\infty}\frac{\left[\begin{array}{l}\{\alpha+(\beta+1)mr\}\\ \times(\lambda+\mu n+\mu mr)\end{array}\right]}{\{\alpha+(\beta+1)mr+(\beta+1)n\}}\binom{\alpha+(\beta+1)mr+(\beta+1)n}{n}t^n = (1+\zeta)^{\{\alpha+(\beta+1)mr\}}\left(\lambda+\mu mr+\frac{\mu\{\alpha+(\beta+1)mr\}\zeta}{(1-\beta\zeta)}\right) \tag{1.7}$$

with

$$\zeta=t(1+\zeta)^{\beta+1};\zeta(0)=0.$$

If we put $\theta=\{\alpha+(\beta+1)mr\}$ and $\sigma = \{\lambda+\mu(\beta+1)r\}$ in Gould's identity (1.6), we get the second modified form of Gould's identity

$$\sum_{n=0}^{\infty}\left[\begin{array}{l}\dfrac{\{\alpha+(\beta+1)mr\}\{\lambda+\mu n+\mu(\beta+1)r\}}{\{\alpha+(\beta+1)mr+(\beta+1)n\}}\\ \times\dbinom{\alpha+(\beta+1)mr+(\beta+1)n}{n}t^n\end{array}\right] = (1+\zeta)^{\{\alpha+(\beta+1)mr\}}\left(\begin{array}{l}\lambda+\mu(\beta+1)r\\ +\dfrac{\mu\{\alpha+(\beta+1)mr\}\zeta}{(1-\beta\zeta)}\end{array}\right) \tag{1.8}$$

with

$$\zeta=t(1+\zeta)^{\beta+1};\zeta(0)=0.$$

Gauss's Multiplication Theorem

For every positive integer m, we have

$$(b)_{mr}=m^{mr}\prod_{j=1}^{m}\left(\frac{b+j-1}{m}\right)_r;\ r=0,1,2,\ldots \tag{1.9}$$

Summation identity [[5], *p*, 101, *Lemma* (3), (2.1.6)]

$$\sum_{n=0}^{\infty}\sum_{r=0}^{[\frac{n}{m}]}B(r,n)=\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}B(r,n+mr) \tag{1.10}$$

$\left([x]\text{ denotes the greatest integer in } x;\ m\in\mathbb{N}\right)$,

provided that series involved are absolutely convergent.

The generalized Laguerre polynomials $L_n^{(\alpha)}(x)$ are defined by

$$L_n^{(\alpha)}(x)=\frac{(1+\alpha)_n}{n!}\,{}_1F_1\left[-n;1+\alpha;x\right];n\in N_0. \tag{1.11}$$

Replacing α by $\alpha+n\beta$ in equation (1.11), we get

$${}_1F_1\left[\begin{array}{ll}-n & ;\\ 1+\alpha+n\beta; & \end{array}x\right]=\frac{n!}{(1+\alpha+n\beta)_n}L_n^{(\alpha+n\beta)}(x). \tag{1.12}$$

The Jacobi Polynomials of first kind $P_n^{(\alpha,\beta)}(x)$ [[6], *p*. 254 (132.1), *p*. 255 (132.7)] are defined by the following equations:

$$P_n^{(\alpha,\beta)}(x)=\frac{(1+\alpha)_n}{n!}\,{}_2F_1\left[\begin{array}{ll}-n,1+\alpha+\beta+n; & \frac{1-x}{2}\\ 1+\alpha & ;\ \ \end{array}\right] \tag{1.13}$$

$$P_n^{(\alpha,\beta)}(x) = \frac{(1+\alpha+\beta)_{2n}}{n!(1+\alpha+\beta)_n}\left(\frac{x-1}{2}\right)^n{}_2F_1\left[\begin{array}{ll}-n,-\alpha-n\ ; & \frac{2}{1-x}\\ -\alpha-\beta-2n; & \end{array}\right] \tag{1.14}$$

where n is a non-negative integer.

Replacing α by $(\alpha+bn)$ and β by $\{\beta-(b+1)n\}$ in equation (1.13), we get

$${}_2F_1\left[\begin{array}{ll}-n,1+\alpha+\beta; & \frac{1-x}{2}\\ 1+\alpha+bn; & \end{array}\right] = \frac{n!\Gamma(\alpha+bn+1)}{\Gamma\{\alpha+(b+1)n+1\}}P_n^{(\alpha+bn,\beta-(b+1)n)}(x) \tag{1.15}$$

Replacing α by $(\alpha-n)$ and β by $\{\beta-(b+1)n\}$ in equation (1.14), we get the following result

$${}_2F_1\left[\begin{array}{ll}-n,\quad -\alpha\ ; & \frac{2}{1-x}\\ -\alpha-\beta+bn; & \end{array}\right] = \frac{n!\Gamma(1+\alpha+\beta-bn-n)}{\Gamma(1+\alpha+\beta-bn)}\left(\frac{2}{x-1}\right)^n P_n^{(\alpha-n,\beta-bn-n)}(x) \tag{1.16}$$

The generalized Rice Polynomials $H_n^{(\alpha,\beta)}[\nu,\sigma,x]$ of Khandekar [[7], *p*. 158, *eq*. (2.3)] are defined by

$$H_n^{(\alpha,\beta)}[\nu,\sigma,x] = \binom{\alpha+n}{n}{}_3F_2\left[\begin{array}{ll}-n,\alpha+\beta+n+1,\nu; & \\ \alpha+1,\sigma\quad ; & \end{array}x\right] \tag{1.17}$$

$$H_n[\nu,\sigma,x]=H_n^{(0,0)}[\nu,\sigma,x] \tag{1.18}$$

$$P_n^{(\alpha,\beta)}(x)=H_n^{(\alpha,\beta)}\left[\nu,\nu,\frac{1-x}{2}\right]. \tag{1.19}$$

Replacing α by $(\alpha+bn)$ and β by $\{\beta-(b+1)n\}$ in eq.(1.17), we get

$${}_3F_2\left[\begin{array}{ll}-n,\nu,1+\alpha+\beta; & \\ 1+\alpha+bn,\sigma\ \ ; & \end{array}x\right] = \frac{n!}{(1+\alpha+bn)_n}H_n^{(\alpha+bn,\beta-(b+1)n)}\left[\nu,\sigma,x\right] \tag{1.20}$$

Some useful Pochhammer's relations

$$\frac{(\lambda+\mu n)\left(\frac{(\lambda+\mu n)}{\mu(\beta+1-m)}+1\right)_r}{\left(\frac{(\lambda+\mu n)}{\mu(\beta+1-m)}\right)_r} = \lambda+\mu n+\mu(\beta+1)r-\mu mr \tag{1.21}$$

$$\frac{\lambda+\mu(\beta+1)r}{\alpha+m(\beta+1)r}=\frac{\lambda}{\alpha}\frac{\left(\frac{\lambda}{\mu(\beta+1)}+1\right)_r\left(\frac{\alpha}{m(\beta+1)}\right)_r}{\left(\frac{\lambda}{\mu(\beta+1)}\right)_r\left(\frac{\alpha}{m(\beta+1)}+1\right)_r} \quad (1.22)$$

$$\alpha+m(\beta+1)r=\alpha\frac{\left(\frac{\alpha}{m(\beta+1)}+1\right)_r}{\left(\frac{\alpha}{m(\beta+1)}\right)_r} \quad (1.23)$$

$$\frac{\begin{Bmatrix}\mu\zeta\alpha+\mu\zeta\beta mr\\+\mu\zeta mr\end{Bmatrix}}{(1-\beta\zeta)}=\frac{\mu\zeta\alpha}{(1-\beta\zeta)}\frac{\left(\frac{\alpha}{m(\beta+1)}+1\right)_r}{\left(\frac{\alpha}{m(\beta+1)}\right)_r} \quad (1.23a)$$

where $r=0,1,2,3,\ldots$

Now we shall discuss some special cases of the implicit functions defined by equation (1.3) subject to the condition (1.4). Using Mathematica 9.0, we can find the roots of resulting cubic equation in ζ for different values of β in equation (1.3).

Case I:- When $\beta=0$ in (1.3), then particular value of ζ (satisfying the condition (1.4)) is denoted by

$$\Theta=\frac{t}{1-t} \quad (1.24)$$

Case II:- When $\beta=1$ in (1.3),we get

$$t\zeta^2+(2t-1)\zeta+t=0$$

then one of the values of ζ (satisfying the condition (1.4)) is given by

$$\Lambda=\frac{1-2t-\sqrt{(1-4t)}}{2t}. \quad (1.25)$$

Case III:- When $\beta=-2$ in (1.3), we get

$$\zeta^2+\zeta-t=0$$

then the particular value of ζ (satisfying the condition (1.4)) is given by

$$\Xi=\frac{-1+\sqrt{(1+4t)}}{2}. \quad (1.26)$$

Case IV:- When $\beta=-3$ in (1.3), we get

$$\zeta^3+2\zeta^2+\zeta-t=0$$

then one of the roots (satisfying the condition (1.4)) of above equation is given by

$$\Upsilon=\frac{1}{3}\left[-2+\frac{2^{\frac{1}{3}}}{\left\{2+27t+3\sqrt{3}\sqrt{(4t+27t^2)}\right\}^{\frac{1}{3}}}+\frac{\left\{2+27t+3\sqrt{3}\sqrt{(4t+27t^2)}\right\}^{\frac{1}{3}}}{2^{\frac{1}{3}}}\right] \quad (1.27)$$

Case V:- When $\beta=-\frac{1}{2}$ in (1.3), we get

$$\zeta^2-t^2\zeta-t^2=0$$

then one of the roots (satisfying the condition (1.4)) of above equation is given by

$$U=\frac{t}{2}\{t+\sqrt{(t^2+4)}\} \quad (1.28)$$

Case VI:- When $\beta=\frac{-3}{2}$ in (1.3), we get

$$\zeta^3+\zeta^2-t^2=0$$

then one of the roots (satisfying the condition (1.4)) of above equation is given by

$$\Psi=-\frac{1}{3}\left[1+\frac{(1+\iota\sqrt{3})}{2^{\frac{2}{3}}\begin{Bmatrix}-2+27t^2\\+3\sqrt{3}\sqrt{(-4t^2+27t^4)}\end{Bmatrix}^{\frac{1}{3}}}+\frac{(1-\iota\sqrt{3})\begin{Bmatrix}-2+27t^2\\+3\sqrt{3}\sqrt{(-4t^2+27t^4)}\end{Bmatrix}^{\frac{1}{3}}}{2^{\frac{4}{3}}}\right] \quad (1.29)$$

where $\iota=\sqrt{(-1)}$.

Case VII:- When $\beta=-\frac{1}{3}$ in (1.3),we obtain

$$\zeta^3-t^3\zeta^2-2t^3\zeta-t^3=0$$

then one of the values of ζ (satisfying the condition (1.4)) is denoted by

$$\Pi=\frac{t^3}{3}-\frac{2^{\frac{1}{3}}(-6t^3-t^6)}{3\{27t^3+18t^6+2t^9+3\sqrt{3}\sqrt{(27t^6+4t^9)}\}^{\frac{1}{3}}}+\frac{\{27t^3+18t^6+2t^9+3\sqrt{3}\sqrt{(27t^6+4t^9)}\}^{\frac{1}{3}}}{3.2^{\frac{1}{3}}} \quad (1.30)$$

Case VIII:-When $\beta=-\frac{2}{3}$ in (1.3), we obtain

$$\zeta^3-t^3\zeta-t^3=0,$$

then one of the values of ζ (satisfying the condition (1.4)) is denoted by

$$\Phi=\frac{\left(\frac{2}{3}\right)^{\frac{1}{3}}t^3}{\left\{9t^3+\sqrt{3}\sqrt{(27t^6-4t^9)}\right\}^{\frac{1}{3}}}+\frac{\left\{9t^3+\sqrt{3}\sqrt{(27t^6-4t^9)}\right\}^{\frac{1}{3}}}{2^{\frac{1}{3}}3^{\frac{2}{3}}}. \quad (1.31)$$

2. Main Generating Relations

First Generating Relation:

If any values of variables and parameters leading to the results which do not make sense, are tacitly excluded, then

$$\sum_{n=0}^{\infty}\frac{(\lambda+\mu n)}{\{\alpha+(\beta+1)n\}}H_n^{(\alpha,\beta)}(x;m)t^n$$
$$=(1+\zeta)^{\alpha}\left\{\begin{array}{l}\frac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{c}\frac{\alpha}{m(\beta+1)},1+\frac{\lambda}{m\mu},(a_p);\\ 1+\frac{\alpha}{m(\beta+1)},\frac{\lambda}{m\mu},(b_q);\end{array} x(-\zeta)^m\right]\\ +\frac{\mu\zeta}{(1-\beta\zeta)}\,{}_pF_q\left[\begin{array}{c}(a_p);\\(b_q);\end{array}x(-\zeta)^m\right]\end{array}\right\}, \quad (2.1)$$

where,

$$\zeta=t(1+\zeta)^{1+\beta};\ \zeta(0)=0,$$

provided that involved series on both sides are absolutely convergent.

Here Srivastava's generalized hypergeometric polynomials $H_n^{(\alpha,\beta)}(x;m)$ [[5], p. 360, eq. (7.3.3.); see also [8], pp. 331-332] are given by

$$H_n^{(\alpha,\beta)}(x;m)=\left\{\begin{array}{l}\binom{\alpha+(\beta+1)n}{n}\\ \times\,{}_{p+m}F_{q+m}\left[\begin{array}{cc}\Delta(m;-n),(a_p) & ;\\ \Delta(m;1+\alpha+\beta n),(b_q); & x\end{array}\right]\end{array}\right\}, \quad (2.2)$$

where α and β are complex parameters independent of 'n' and $\Delta(m;\lambda)$ abbreviates the array of m number of parameters given by

$$\frac{\lambda}{m},\frac{\lambda+1}{m},\ldots,\frac{\lambda+m-1}{m};\ m\in\mathbb{N}.$$

Independent Demonstration:

Using the definition (2.2) of $H_n^{(\alpha,\beta)}(x;m)$ and then the power series form of ${}_{p+m}F_{q+m}[x]$ in left hand side of equation (2.1), we get

$$\Omega=\sum_{n=0}^{\infty}\frac{(\lambda+\mu n)}{\{\alpha+(\beta+1)n\}}H_n^{(\alpha,\beta)}(x;m)t^n$$
$$=\sum_{n=0}^{\infty}\frac{(\lambda+\mu n)\Gamma\{\alpha+(\beta+1)n+1\}}{\{\alpha+(\beta+1)n\}\Gamma(n+1)\Gamma\{\alpha+\beta n+1\}}$$
$$\times\sum_{r=0}^{[\frac{n}{m}]}\frac{\prod_{j=1}^{m}\left(\frac{-n+j-1}{m}\right)_r[(a_p)]_r x^r t^n}{\prod_{j=1}^{m}\left(\frac{1+\alpha+n\beta+j-1}{m}\right)_r[(b_q)]_r r!}.$$

Using Gauss's multiplication theorem (1.9) in above equation, we get

$$\Omega=\sum_{n=0}^{\infty}\sum_{r=0}^{[\frac{n}{m}]}\frac{(\lambda+\mu n)(-n)_{mr}(\alpha)_{n(\beta+1)}\left[(\alpha_p)\right]_r x^r t^n}{\alpha(\alpha+1)_{n\beta}(1+\alpha+n\beta)_{mr}\left[(b_q)\right]_r r!n!}. \quad (2.3)$$

Now applying summation identity (1.10) and then simplifying further, we get

$$\Omega=\sum_{r=0}^{\infty}\frac{(\alpha)_{m(\beta+1)r}\left[(a_q)\right]_r \mathrm{x}^r(-t)^{mr}}{\alpha(\alpha+1)_{m(\beta+1)r}\left[(b_q)\right]_r \mathrm{r}!}$$
$$\times\sum_{n=0}^{\infty}\left\{\begin{array}{l}\frac{\{\alpha+(\beta+1)mr\}(\lambda+\mu n+\mu mr)}{\{\alpha+(\beta+1)mr+(\beta+1)n\}}\\ \times\binom{\alpha+(\beta+1)mr+(\beta+1)n}{n}t^n\end{array}\right\}. \quad (2.4)$$

Now using first modified Gould's identity (1.7) and then (1.23*a*), we get

$$\Omega=(1+\zeta)^{\alpha}\sum_{r=0}^{\infty}\left(\begin{array}{l}\frac{(\alpha)_{m(\beta+1)r}\left[(a_p)\right]_r \mathrm{x}^r(-t)^{mr}}{\alpha(\alpha+1)_{m(\beta+1)r}\left[(b_q)\right]_r \mathrm{r}!}\\ \times(1+\zeta)^{m(\beta+1)r}\left[\begin{array}{l}(\lambda+\mu mr)\\ +\frac{\mu\zeta\{\alpha+(\beta+1)mr\}}{(1-\beta\zeta)}\end{array}\right]\end{array}\right). \quad (2.5)$$

Simplifying it further, we get

$$\Omega=(1+\zeta)^{\alpha}\sum_{r=0}^{\infty}\frac{(\frac{\alpha}{m(\beta+1)})_r\left[(a_p)\right]_r x^r(-\zeta)^{mr}}{\alpha(\frac{\alpha}{m(\beta+1)}+1)_r\left[(b_q)\right]_r \mathrm{r}!}$$
$$\times\left[\lambda\frac{\left(\frac{\lambda}{\mu m}+1\right)_r}{\left(\frac{\lambda}{\mu m}\right)_r}+\frac{\mu\zeta\alpha}{(1-\beta\zeta)}\frac{\left(\frac{\alpha}{m(\beta+1)}+1\right)_r}{\left(\frac{\alpha}{m(\beta+1)}\right)_r}\right]. \quad (2.6)$$

After solving it further, we get the result (2.1) in the form of sum of two generalized hypergeometric functions of one variable.

Second Generating Relation:

If any values of variables and parameters leading to the results which do not make sense, are tacitly excluded, then

$$\sum_{n=0}^{\infty}\frac{(\lambda+\mu n)}{\{\alpha+(\beta+1)n\}}\mathscr{B}_n^{(\alpha,\beta)}(x;m,\lambda,\mu)\mathrm{t}^n$$
$$=(1+\zeta)^{\alpha}\left\{\begin{array}{l}\frac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{c}\frac{\alpha}{m(\beta+1)},1+\frac{\lambda}{\mu(\beta+1)},(a_p);\\ 1+\frac{\alpha}{m(\beta+1)},\frac{\lambda}{\mu(\beta+1)},(b_q);\end{array} x(-\zeta)^m\right]\\ +\frac{\mu\zeta}{(1-\beta\zeta)}\,{}_pF_q\left[\begin{array}{c}(a_p);\\(b_q);\end{array}x(-\zeta)^m\right]\end{array}\right\}, \quad (2.7)$$

where

$$\zeta=t(1+\zeta)^{1+\beta};\ \zeta(0)=0,$$

provided that involved series on both sides are absolutely convergent.

Here we define new generalized hypergeometric polynomials $\mathscr{B}_n^{(\alpha,\beta)}(x;m,\lambda,\mu)$ known as "Pathan's generalized hypergeometric polynomials of one variable", given by

$$\mathscr{B}_n^{(\alpha,\beta)}(x;m,\lambda,\mu) = \left\{ \begin{array}{l} \binom{\alpha+(\beta+1)n}{n} \times \\ {}_{p+m+1}F_{q+m+1}\left[\begin{array}{l} \Delta(m;-n), 1+\frac{(\lambda+\mu n)}{\mu(\beta+1-m)}, (a_p); \\ \Delta(m;1+\alpha+\beta n), \frac{(\lambda+\mu n)}{\mu(\beta+1-m)}, (b_q); \end{array} x\right] \end{array} \right\}, \quad (2.8)$$

where α and β are complex parameters independent of 'n' and $\Delta(m;\lambda)$ abbreviates the array of m number of parameters given by

$$\frac{\lambda}{m}, \frac{\lambda+1}{m}, \ldots, \frac{\lambda+m-1}{m}; m \in \mathbb{N}.$$

Independent Demonstration:

Using the definition (2.8) of $\mathscr{B}_n^{(\alpha,\beta)}(x;m,\lambda,\mu)$ and then the power series form of ${}_{p+m+1}F_{q+m+1}[x]$ in left hand side of equation (2.7), using Gauss's multiplication theorem (1.9) and result (1.21), we get

$$\Omega^* = \sum_{n=0}^{\infty} \frac{(\lambda+\mu n)}{\{\alpha+(\beta+1)n\}} \mathscr{B}_n^{(\alpha,\beta)}(x;m,\lambda,\mu) = \sum_{n=0}^{\infty} \sum_{r=0}^{[\frac{n}{m}]} \frac{\left\{ \begin{array}{l} [\lambda+\mu n+\mu(\beta+1)r-\mu m r] \\ \times(-n)_{mr}\Gamma\{\alpha+n(\beta+1)\}\left[\left(a_p\right)\right]_r x^r t^n \end{array}\right\}}{\Gamma\{\alpha+n\beta+1+mr\}\left[\left(b_q\right)\right]_r r!n!}. \quad (2.9)$$

Now applying summation identity (1.10) in above equation then simplifying further, we get

$$\Omega^* = \left[\begin{array}{l} \sum_{r=0}^{\infty} \frac{[(a_p)]_r x^r (-t)^{mr}}{[\alpha+m(\beta+1)r]\left[\left(b_q\right)\right]_r r!} \\ \times \sum_{n=0}^{\infty} \left(\begin{array}{l} \frac{\{\alpha+(\beta+1)mr\}(\lambda+\mu n+\mu(\beta+1)r)}{\{\alpha+(\beta+1)mr+(\beta+1)n\}} \\ \times \binom{\alpha+(\beta+1)mr+(\beta+1)n}{n} t^n \end{array}\right) \end{array}\right] \quad (2.10)$$

.

Now using second modified Gould's identity (1.8), we get

$$\Omega^* = (1+\zeta)^{\alpha} \sum_{r=0}^{\infty} \left\{ \begin{array}{l} \frac{[(a_p)]_r x^r (-t)^{mr} (1+\zeta)^{m(\beta+1)r}}{[(b_q)]_r r!} \\ \cdot \left[\frac{\lambda+\mu(\beta+1)r}{\alpha+m(\beta+1)r} + \frac{\mu\zeta}{(1-\beta\zeta)} \right] \end{array} \right\}. \quad (2.11)$$

Now using equation (1.22) in above equation and summing it up into hypergeometric form further, we get the desired result (2.7).

3. Known Applications of Generating Relation (2.1)

(i). Putting $\lambda = 1, \mu = \frac{\beta+1}{\alpha}$ in equation (2.1) and after simplifying, we get

$$\sum_{n=0}^{\infty} H_n^{(\alpha,\beta)}(x;m)t^n = (1+\zeta)^{\alpha} {}_pF_q\left[\begin{array}{l}(a_p); \\ (b_p);\end{array} x(-\zeta)^m\right]\left\{\frac{\zeta(1+\beta)}{(1-\beta\zeta)}+1\right\} = \frac{(1+\zeta)^{\alpha+1}}{(1-\beta\zeta)} {}_pF_q\left[\begin{array}{l}(a_p); \\ (b_q);\end{array} x(-\zeta)^m\right], \quad (3.1)$$

which is the result of Srivastava [[9], p. 975; [10], p. 233, eq. (12)]. Here ζ being given by equations (1.3), (1.4) and $H_n^{(\alpha,\beta)}(x;m)$ is given by equation (2.2).

(ii). Putting $\lambda = 1, \mu = \frac{\beta+1}{\alpha}$ and $m = 1$ in equation (2.1) and using the definition of $H_n^{(\alpha,\beta)}(x;1)$ and after simplification, we get

$$\sum_{n=0}^{\infty} \binom{\alpha+(\beta+1)n}{n} {}_{p+1}F_{q+1}\left[\begin{array}{l} -n, (a_p) \quad ; \\ 1+\alpha+n\beta, (b_q); \end{array} x\right] t^n = \frac{(1+\zeta)^{\alpha+1}}{(1-\beta\zeta)} {}_pF_q\left[\begin{array}{l}(a_p); \\ (b_q);\end{array} x(-\zeta)\right], \quad (3.2)$$

which is the result of Srivastava [[11], p. 591, eq. (9); see also 7, p. 1186]. ζ is given by equations (1.3) and (1.4).

(iii). Putting $\lambda = 1, \mu = \frac{1}{2\alpha}, \beta = \frac{-1}{2}$ and $m = 1$ in equation (2.1), using the definition (2.2) of $H_n^{(\alpha,-\frac{1}{2})}(x;1) = f_n^{(\alpha)}(x)$ and replacing ζ by U, we get

$$\sum_{n=0}^{\infty} f_n^{(\alpha)}(x)t^n = \sum_{n=0}^{\infty} \binom{\alpha+\frac{n}{2}}{n} {}_{p+1}F_{q+1}\left[\begin{array}{l} -n, (a_p); \\ 1+\alpha-\frac{n}{2}, (b_q); \end{array} x\right] t^n = \frac{[1+U]^{\alpha+1}}{(1+\frac{1}{2}U)} {}_pF_q\left[\begin{array}{l}(a_p); \\ (b_q);\end{array} -xU\right], \quad (3.3)$$

where

$$U = \frac{t}{2}\{t+\sqrt{(t^2+4)}\},$$

which is the known result of Brown [[12], p. 264, eq. (7)] and $f_n^{(\alpha)}(x)$ is Brown's generalized hypergeometric polynomial [[5], p. 358, eq. (7.2.4)].

(iv). Putting $\alpha = \nu - 1,\ \beta = 0,\ \zeta = \frac{t}{1-t}$ and replacing q by $(q-m)$ in equation (3.1) and after simplifying, we get

$$\sum_{n=0}^{\infty}\binom{\nu+n-1}{n}{}_{p+m}F_q\left[\begin{matrix}\Delta(m;-n),(a_p);\\ \Delta(m;\nu),(b_{q-m});\end{matrix}\ x\right]t^n = (1-t)^{-\nu}\ {}_pF_{q-m}\left[\begin{matrix}(a_p);\\ (b_{q-m});\end{matrix}\ x\left(\frac{t}{t-1}\right)^m\right]. \tag{3.4}$$

Now replacing p to $p+m$ in equation (3.4), we get

$$\sum_{n=0}^{\infty}\binom{\nu+n-1}{n}{}_{p+2m}F_q\left[\begin{matrix}\left(\begin{matrix}\Delta(m;-n),a_1,\ldots,a_p,\\ a_{p+1},\ldots,a_{p+m}\end{matrix}\right);\\ \Delta(m;\nu),\ b_1,\ldots,b_{q-m}\ ;\end{matrix}\ x\right]t^n = (1-t)^{-\nu}\ {}_{p+m}F_{q-m}\left[\begin{matrix}\left(\begin{matrix}a_1,\ldots,a_p,\\ a_{p+1},\ldots,a_{p+m}\end{matrix}\right);\\ b_1,\ldots,b_{q-m}\ ;\end{matrix}\ x\left(\frac{t}{t-1}\right)^m\right]. \tag{3.5}$$

Further taking

$$a_{p+1} = \frac{\nu}{m},\ a_{p+2} = \frac{\nu+1}{m},\ldots,\ a_{p+m} = \frac{\nu+m-1}{m},$$

in equation (3.5), we get

$$\sum_{n=0}^{\infty}\binom{\nu+n-1}{n}{}_{p+m}F_{q-m}\left[\begin{matrix}\Delta(m;-n),a_1,\ldots,a_p;\\ b_1,\ldots,b_{q-m}\ ;\end{matrix}\ x\right]t^n = (1-t)^{-\nu}\ {}_{p+m}F_{q-m}\left[\begin{matrix}\Delta(m;\nu),a_1,\ldots,a_p;\\ b_1,\ldots,b_{q-m};\end{matrix}\ x\left(\frac{t}{t-1}\right)^m\right]. \tag{3.6}$$

Replacing q by $q+m$ and x by $f(x)$ in equation (3.6), we get

$$\sum_{n=0}^{\infty}\binom{\nu+n-1}{n}{}_{p+m}F_q\left[\begin{matrix}\Delta(m;-n),a_1,\ldots,a_p;\\ b_1,\ldots,b_q;\end{matrix}\ f(x)\right]t^n = (1-t)^{-\nu}\ {}_{p+m}F_q\left[\begin{matrix}\Delta(m;\nu),a_1,\ldots,a_p;\\ b_1,\ldots,b_q;\end{matrix}\ f(x)\left(\frac{t}{t-1}\right)^m\right], \tag{3.7}$$

which is the known result of Srivastava [[10]; p. 233, equation (13)].

4. Some Special Cases of Generating Relation (3.2)

(i). Taking $p = 0 = q$ in equation (3.2), we get

$$\sum_{n=0}^{\infty}\binom{\alpha+(\beta+1)n}{n}{}_1F_1\left[\begin{matrix}-n\ ;\\ 1+\alpha+n\beta;\end{matrix}\ x\right]t^n = \frac{(1+\zeta)^{\alpha+1}}{(1-\beta\zeta)}\ {}_0F_0\left[\begin{matrix}—;\\ —;\end{matrix}\ x(-\zeta)\right] = \frac{(1+\zeta)^{\alpha+1}}{(1-\beta\zeta)}exp(-x\zeta), \tag{4.1}$$

which is the known result of Srivastava [[11], p. 591, eq.(7)] subject to the conditions (1.3) and (1.4).

Now using the definition of generalized Laguerre polynomials (1.12) and solving, we get

$$\sum_{n=0}^{\infty}L_n^{(\alpha+n\beta)}(x)t^n = \frac{(1+\zeta)^{\alpha+1}}{(1-\beta\zeta)}e^{-x\zeta}, \tag{4.2}$$

where ζ is given by equations (1.3) and (1.4). It is the known result of Brown [[13], p. 822] and Carlitz [[14], p. 826; see also [11], p. 590, eq. (4)]. The generating relation (4.2) is a unification and generalization of the following two generating relations (4.3) and (4.4):
Taking $\beta = 0$ in equations (1.3), (1.4) and (4.2), we have

$$\sum_{n=0}^{\infty}L_n^{(\alpha)}(x)t^n = (1-t)^{-\alpha-1}\ exp\left\{-\frac{xt}{(1-t)}\right\}, \tag{4.3}$$

and taking $\beta = -1$ in equations (1.3), (1.4) and (4.2), we have

$$\sum_{n=0}^{\infty}L_n^{(\alpha-n)}(x)t^n = (1+t)^{\alpha}\ exp\left(-xt\right). \tag{4.4}$$

(ii). In equations (1.3), (1.4) and (3.2), putting $p=1, q=0$, replacing x by $\frac{1-x}{2}$, β by b and ζ by v, we get

$$\sum_{n=0}^{\infty}\binom{\alpha+(b+1)n}{n}{}_2F_1\left[\begin{matrix}-n,a_1\ ;\\ 1+\alpha+nb;\end{matrix}\ \frac{(1-x)}{2}\right]t^n = \frac{(1+v)^{\alpha+1}}{(1-bv)}\ {}_1F_0\left[\begin{matrix}a_1;\\ -;\end{matrix}\ \frac{(1-x)}{2}(-v)\right] = \frac{(1+v)^{\alpha+1}}{(1-bv)}\left\{1+\frac{(1-x)}{2}v\right\}^{-a_1}, \tag{4.5}$$

where v is a function of 't', defined implicitly by

$$v = t(1+v)^{(b+1)};\ v(0) = 0. \tag{4.6}$$

It is the known Generating relation of Srivastava [[11], p. 591, eq. (8)]. Putting $a_1 = 1+\alpha+\beta$ in equation (4.5) and using the definition (1.15) of Jacobi polynomials, we get

$$\sum_{n=0}^{\infty}P_n^{(\alpha+bn,\ \beta-(b+1)n)}(x)t^n = \frac{(1+v)^{\alpha+1}}{(1-bv)}\left\{1+\frac{(1-x)}{2}v\right\}^{-(1+\alpha+\beta)}, \tag{4.7}$$

where v is given by equation (4.6). This is the known result of Srivastava [[11], p. 594, eq. (22); [15], p. 748; see also [16], p. A654].

Taking $b = 0$ in equations (4.6) and (4.7), we get the following generating relation of E. Feldheim, recorded in the monograph of Srivastava-Manocha [[5], p. 90, Q. 15 (second equation); see also [11], p. 594, eq. (23)].

$$\sum_{n=0}^{\infty}P_n^{(\alpha,\ \beta-n)}(x)t^n = (1-t)^{\beta}\left\{1-\frac{(1+x)t}{2}\right\}^{-(1+\alpha+\beta)}. \tag{4.8}$$

(iii). Putting p=1, q =0, $a_1 = \lambda$, $\alpha = a$, $\beta = b$ and replacing ζ by v in equations (1.3), (1.4) and (3.2), we get

$$\sum_{n=0}^{\infty}\binom{a+(b+1)n}{n}{}_2F_1\left[\begin{matrix}-n,\ \lambda;\\1+a+nb;\end{matrix}x\right]t^n = \frac{(1+v)^{a+1}}{(1-bv)}(1+xv)^{-\lambda}, \tag{4.9}$$

where

$$v=t(1+v)^{(b+1)};\ v(0)=0.$$

Replacing x by $\frac{2}{1-x}$ and λ by $-\alpha$ and then a by $-\alpha-\beta-1$ and using the definition (1.16) of Jacobi Polynomials, we get the following result

$$\sum_{n=0}^{\infty}\left(\frac{2}{1-x}\right)^n P_n^{(\alpha-n,\,\beta-bn-n)}(x)t^n = \frac{(1+v)^{-\alpha-\beta}}{(1-bv)}\left(1+\frac{2}{1-x}v\right)^{\alpha}, \tag{4.10}$$

where

$$v=t(1+v)^{(b+1)};\ v(0)=0.$$

Replacing t by $t\frac{(1-x)}{2}$; v by w, we get

$$\sum_{n=0}^{\infty}P_n^{(\alpha-n,\,\beta-bn-n)}(x)t^n = \frac{(1+w)^{-\alpha-\beta}}{(1-bw)}\left(1+\frac{2w}{1-x}\right)^{\alpha}, \tag{4.11}$$

where

$$w(x,t)=t\frac{(1-x)}{2}(1+w)^{(b+1)};w(x,0)=0, \tag{4.12}$$

which is known result of Srivastava [[11], p. 593, eq. (16); see also [15], p. 748].

Taking $b=-1$ in equations (4.6) and (4.7); (4.11) and (4.12), we obtain another result of E. Feldheim, recorded in the monograph of Srivastava-Manocha [[5], p. 90, Q. 15 (first equation); see also [11], p. 593, eq. (19)].

$$\sum_{n=0}^{\infty}P_n^{(\alpha-n,\,\beta)}(x)t^n=(1+t)^{\alpha}\left(1-\frac{t(x-1)}{2}\right)^{-\alpha-\beta-1}. \tag{4.13}$$

And taking $b=0$ in equations (4.11) and (4.12), we obtain

$$\sum_{n=0}^{\infty}P_n^{(\alpha-n,\,\beta-n)}(x)t^n = \left(1+\frac{t(x+1)}{2}\right)^{\alpha}\left(1+\frac{t(x-1)}{2}\right)^{\beta}, \tag{4.14}$$

which is the result of Milch [17] and also recorded in the monograph of Srivastava-Manocha [[5], p. 82, (1.11.2)].

(iv). Putting $p=2$, $q=1$, $\beta=b$, $a_1=1+\alpha+\gamma$, $a_2=\mu, b_1=\sigma$ and replacing ζ by v in equations (1.3), (1.4) and (3.2), we get

$$\sum_{n=0}^{\infty}\binom{\alpha+(b+1)n}{n}{}_3F_2\left[\begin{matrix}-n,1+\alpha+\gamma,\ \mu;\\1+\alpha+nb,\ \sigma;\end{matrix}x\right]t^n = \frac{(1+v)^{\alpha+1}}{(1-bv)}{}_2F_1\left[\begin{matrix}1+\alpha+\gamma,\ \mu;\\\sigma;\end{matrix}x(-v)\right] \tag{4.15}$$

$$v=t(1+v)^{(b+1)};\ v(0)=0.$$

Now using the definition (1.20) of Generalized Rice polynomials of Khandekar in above equation (4.15), we get a well known result of Joshi and Prajapat [[18], p. 272]:

$$\sum_{n=0}^{\infty}H_n^{(\alpha+bn,\ \gamma-bn-n)}[\mu,\ \sigma,\ x]t^n = \frac{(1+v)^{\alpha+1}}{(1-bv)}{}_2F_1\left[\begin{matrix}1+\alpha+\gamma,\ \mu;\\\sigma;\end{matrix}x(-v)\right], \tag{4.16}$$

where

$$v=t(1+v)^{(b+1)};\ v(0)=0.$$

(v). Putting $b=\frac{-1}{2}$ and replacing v by U in equations (4.6) and (4.7), we get

$$\sum_{n=0}^{\infty}P_n^{(\alpha-\frac{n}{2},\ \beta-\frac{n}{2})}(x)t^n = \frac{(1+U)^{\alpha+1}}{(1+\frac{U}{2})}\left\{1+\frac{(1-x)}{2}U\right\}^{-(1+\alpha+\beta)}, \tag{4.17}$$

$$U=\frac{t\{t+\sqrt{(t^2+4)}\}}{2},$$

which is the known result of Brown [13].

5. New Applications of Generating Relation (2.1)

The results from (5.1) to (5.8) are believed to be new in author's knowledge and are not found in the literature of generating relations.

(i). Putting $\beta=0$ and $\zeta=\Theta=\frac{t}{1-t}$ from equation (1.24) in equation (2.1), we get

$$\sum_{n=0}^{\infty}\frac{(\lambda+\mu n)}{(\alpha+n)}\binom{\alpha+n}{n}{}_{p+m}F_{q+m}\left[\begin{matrix}\Delta(m;-n),(a_p);\\\Delta(m;1+\alpha),(b_q);\end{matrix}x\right]t^n = (1-t)^{-\alpha}\left\{\begin{matrix}\frac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{matrix}\frac{\alpha}{m},1+\frac{\lambda}{m\mu},(a_p);\\1+\frac{\alpha}{m},\frac{\lambda}{m\mu},(b_q);\end{matrix}x\left(\frac{-t}{1-t}\right)^m\right]\\+\frac{\mu t}{(1-t)}\,{}_pF_q\left[\begin{matrix}(a_p);\\(b_q);\end{matrix}x\left(\frac{-t}{1-t}\right)^m\right]\end{matrix}\right\}. \tag{5.1}$$

(ii). Putting $\beta=1$ and $\zeta=\Lambda=\dfrac{1-2t-\sqrt{(1-4t)}}{2t}$ from equation (1.25) in equation (2.1), we get

$$\sum_{n=0}^{\infty}\left\{\begin{matrix}\dfrac{(\lambda+\mu n)}{(\alpha+2n)}\dbinom{\alpha+2n}{n}\\ \times\,{}_{p+m}F_{q+m}\left[\begin{matrix}\Delta(m;-n),(a_p);\\ \Delta(m;1+\alpha+n),(b_q);\end{matrix}\,x\right]t^n\end{matrix}\right\}$$
$$=(1+\Lambda)^{\alpha}\left\{\begin{matrix}\dfrac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{matrix}\dfrac{\alpha}{2m},1+\dfrac{\lambda}{m\mu},(a_p);\\ 1+\dfrac{\alpha}{2m},\dfrac{\lambda}{m\mu},(b_q);\end{matrix}\,x(-\Lambda)^m\right]\\ +\dfrac{\mu\Lambda}{(1-\Lambda)}\,{}_pF_q\left[\begin{matrix}(a_p);\\ (b_q);\end{matrix}\,x(-\Lambda)^m\right]\end{matrix}\right\}. \tag{5.2}$$

(iii). Putting $\beta=-2$ and $\zeta=\Xi=\dfrac{-1+\sqrt{(1+4t)}}{2}$ from equation (1.26) in equation (2.1), we get

$$\sum_{n=0}^{\infty}\left\{\begin{matrix}\dfrac{(\lambda+\mu n)}{(\alpha-n)}\dbinom{\alpha-n}{n}\\ \times\,{}_{p+m}F_{q+m}\left[\begin{matrix}\Delta(m;-n),(a_p);\\ \Delta(m;1+\alpha-2n),(b_q);\end{matrix}\,x\right]t^n\end{matrix}\right\}$$
$$=(1+\Xi)^{\alpha} \tag{5.3}$$
$$\times\left\{\begin{matrix}\dfrac{\lambda}{\alpha}\cdot{}_{p+2}F_{q+2}\left[\begin{matrix}\dfrac{-\alpha}{m},1+\dfrac{\lambda}{m\mu},(a_p);\\ 1-\dfrac{\alpha}{m},\dfrac{\lambda}{m\mu},(b_q);\end{matrix}\,x(-\Xi)^m\right]\\ +\dfrac{\mu\Xi}{(1+2\Xi)}\,{}_pF_q\left[\begin{matrix}(a_p);\\ (b_q);\end{matrix}\,x(-\Xi)^m\right]\end{matrix}\right\}.$$

(iv). Putting $\beta=-3$ and $\zeta=\Upsilon$ from equation (1.27) in equation (2.1), we get

$$\sum_{n=0}^{\infty}\left\{\begin{matrix}\dfrac{(\lambda+\mu n)}{(\alpha-2n)}\dbinom{\alpha-2n}{n}\\ \times\,{}_{p+m}F_{q+m}\left[\begin{matrix}\Delta(m;-n),(a_p);\\ \Delta(m;1+\alpha-3n),(b_q);\end{matrix}\,x\right]t^n\end{matrix}\right\}$$
$$=(1+\Upsilon)^{\alpha}\left\{\begin{matrix}\dfrac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{matrix}\dfrac{-\alpha}{2m},1+\dfrac{\lambda}{m\mu},(a_p);\\ 1-\dfrac{\alpha}{2m},\dfrac{\lambda}{m\mu},(b_q);\end{matrix}\,x(-\Upsilon)^m\right]\\ +\dfrac{\mu\Upsilon}{(1+3\Upsilon)}\,{}_pF_q\left[\begin{matrix}(a_p);\\ (b_q);\end{matrix}\,x(-\Upsilon)^m\right]\end{matrix}\right\}. \tag{5.4}$$

(v). Putting $\beta=\dfrac{-1}{2}$ and $\zeta=U$ from equation (1.28) in equation (2.1), we get

$$\sum_{n=0}^{\infty}\left\{\begin{matrix}\dfrac{(\lambda+\mu n)}{(\alpha+\frac{1}{2}n)}\dbinom{\alpha+\frac{1}{2}n}{n}\\ \times\,{}_{p+m}F_{q+m}\left[\begin{matrix}\Delta(m;-n),(a_p);\\ \Delta(m;1+\alpha-\frac{1}{2}n),(b_q);\end{matrix}\,x\right]t^n\end{matrix}\right\}$$
$$=(1+U)^{\alpha}\left\{\begin{matrix}\dfrac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{matrix}\dfrac{2\alpha}{m},1+\dfrac{\lambda}{m\mu},(a_p);\\ 1+\dfrac{2\alpha}{m},\dfrac{\lambda}{m\mu},(b_q);\end{matrix}\,x(-U)^m\right]\\ +\dfrac{\mu U}{(1+\frac{1}{2}U)}\,{}_pF_q\left[\begin{matrix}(a_p);\\ (b_q);\end{matrix}\,x(-U)^m\right]\end{matrix}\right\}. \tag{5.5}$$

(vi). Putting $\beta=\dfrac{-3}{2}$ and $\zeta=\Psi$ from equation (1.29) in equation (2.1), we get

$$\sum_{n=0}^{\infty}\left[\begin{matrix}\dfrac{(\lambda+\mu n)}{(\alpha-\frac{1}{2}n)}\dbinom{\alpha-\frac{1}{2}n}{n}\\ \times\,{}_{p+m}F_{q+m}\left[\begin{matrix}\Delta(m;-n),(a_p);\\ \Delta(m;1+\alpha-\frac{3}{2}n),(b_q);\end{matrix}\,x\right]t^n\end{matrix}\right]$$
$$=(1+\Psi)^{\alpha}\left\{\begin{matrix}\dfrac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{matrix}\dfrac{-2\alpha}{m},1+\dfrac{\lambda}{m\mu},(a_p);\\ 1-\dfrac{2\alpha}{m},\dfrac{\lambda}{m\mu},(b_q);\end{matrix}\,x(-\Psi)^m\right]\\ +\dfrac{\mu\Psi}{(1+\frac{3}{2}\Psi)}\,{}_pF_q\left[\begin{matrix}(a_p);\\ (b_q);\end{matrix}\,x(-\Psi)^m\right]\end{matrix}\right\}. \tag{5.6}$$

(vii). Putting $\beta=\dfrac{-1}{3}$ and $\zeta=\Pi$ from equation (1.30) in equation (2.1), we get

$$\sum_{n=0}^{\infty}\left\{\begin{matrix}\dfrac{(\lambda+\mu n)}{(\alpha+\frac{2}{3}n)}\dbinom{\alpha+\frac{2}{3}n}{n}\\ \times\,{}_{p+m}F_{q+m}\left[\begin{matrix}\Delta(m;-n),(a_p);\\ \Delta(m;1+\alpha-\frac{1}{3}n),(b_q);\end{matrix}\,x\right]t^n\end{matrix}\right\}$$
$$=(1+\Pi)^{\alpha}\left\{\begin{matrix}\dfrac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{matrix}\dfrac{3\alpha}{2m},1+\dfrac{\lambda}{m\mu},(a_p);\\ 1+\dfrac{3\alpha}{2m},\dfrac{\lambda}{m\mu},(b_q);\end{matrix}\,x(-\Pi)^m\right]\\ +\dfrac{\mu\Pi}{(1+\frac{1}{3}\Pi)}\,{}_pF_q\left[\begin{matrix}(a_p);\\ (b_q);\end{matrix}\,x(-\Pi)^m\right]\end{matrix}\right\}. \tag{5.7}$$

(viii). Putting $\beta=\frac{-2}{3}$ and $\zeta=\Phi$ from equation (1.31) in equation (2.1), we get

$$\sum_{n=0}^{\infty}\left\{\begin{array}{l}\dfrac{(\lambda+\mu n)}{(\alpha+\frac{1}{3}n)}\dbinom{\alpha+\frac{1}{3}n}{n}\\ \times\,{}_{p+m}F_{q+m}\left[\begin{array}{l}\Delta(m;-n),(a_p);\\ \Delta(m;1+\alpha-\frac{2}{3}n),(b_q);\end{array}x\right]t^n\end{array}\right\}$$

$$=(1+\Phi)^{\alpha}\left\{\begin{array}{l}\dfrac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{l}\frac{3\alpha}{m},1+\frac{\lambda}{m\mu},(a_p);\\ 1+\frac{3\alpha}{m},\frac{\lambda}{m\mu},(b_q);\end{array}x(-\Phi)^m\right]\\ +\dfrac{\mu\Phi}{(1+\frac{2}{3}\Phi)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\ (b_q);\end{array}x(-\Phi)^m\right]\end{array}\right\}. \tag{5.8}$$

6. New Applications of Generating Relation (2.7)

The following Generating relations of this section are new in the author′s knowledge and are not available in the literature of Generating relations.

(i). Putting $\lambda=1,\ \mu=\frac{\beta+1}{\alpha}$ in equation (2.7) and after simplifying, we get

$$\sum_{n=0}^{\infty}\left\{\begin{array}{l}\dbinom{\alpha+(\beta+1)n}{n}\times\\ {}_{p+m+1}F_{q+m+1}\left[\begin{array}{l}\Delta(m;-n),1+\frac{\{\alpha+(\beta+1)n\}}{(\beta+1)(\beta+1-m)},\ (a_p);\\ \Delta(m;1+\alpha+\beta n),\frac{\{\alpha+(\beta+1)n\}}{(\beta+1)(\beta+1-m)},(b_q);\end{array}x\right]t^n\end{array}\right\}$$

$$=(1+\zeta)^{\alpha}\left\{\begin{array}{l}{}_{p+2}F_{q+2}\left[\begin{array}{l}\frac{\alpha}{m(\beta+1)},1+\frac{\alpha}{(\beta+1)^2},(a_p);\\ 1+\frac{\alpha}{m(\beta+1)},\frac{\alpha}{(\beta+1)^2},(b_q);\end{array}x(-\zeta)^m\right]\\ +\dfrac{(\beta+1)\zeta}{(1-\beta\zeta)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\ (b_q);\end{array}x(-\zeta)^m\right]\end{array}\right\}, \tag{6.1}$$

where ζ is given by equations (1.3) and (1.4).

(ii). Putting $\lambda=1,\ \mu=\frac{\beta+1}{\alpha},m=1$ in equation (2.7) and using the definition of $\mathscr{B}_n^{(\alpha,\beta)}\left(x;1,1,\frac{\beta+1}{\alpha}\right)$, we get

$$\sum_{n=0}^{\infty}\left\{\begin{array}{l}\dbinom{\alpha+(\beta+1)n}{n}\\ {}_{p+2}F_{q+2}\left[\begin{array}{l}-n,1+\frac{\alpha+(\beta+1)n}{(\beta+1)\beta},(a_p);\\ 1+\alpha+\beta n,\frac{\alpha+(\beta+1)n}{(\beta+1)\beta},(b_q);\end{array}x\right]\mathrm{t}^n\end{array}\right\}$$

$$=(1+\zeta)^{\alpha}\left\{\begin{array}{l}{}_{p+2}F_{q+2}\left[\begin{array}{l}\frac{\alpha}{(\beta+1)},1+\frac{\alpha}{(\beta+1)^2},(a_p);\\ 1+\frac{\alpha}{(\beta+1)},\frac{\alpha}{(\beta+1)^2},(b_q);\end{array}x(-\zeta)\right]\\ +\dfrac{(\beta+1)\zeta}{(1-\beta\zeta)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\ (b_q);\end{array}x(-\zeta)\right]\end{array}\right\}, \tag{6.2}$$

where ζ is given by equations (1.3) and (1.4).

(iii). Putting $\lambda=1,\ \mu=\frac{1}{2\alpha},\ \beta=\frac{-1}{2},\ m=1$ in equation (2.7), using the definition of $\mathscr{B}_n^{(\alpha,-\frac{1}{2})}\left(x;1,1,\frac{1}{2\alpha}\right)$ and replacing ζ by U from equation (1.28), we get

$$\sum_{n=0}^{\infty}\dbinom{\alpha+\frac{n}{2}}{n}{}_{p+2}F_{q+2}\left[\begin{array}{l}-n,1-4\alpha-2n,(a_p);\\ 1+\alpha-\frac{n}{2},-4\alpha-2n,(b_q);\end{array}x\right]t^n$$

$$=(1+U)^{\alpha}\left\{\begin{array}{l}{}_{p+2}F_{q+2}\left[\begin{array}{l}2\alpha,1+4\alpha,(a_p);\\ 1+2\alpha,4\alpha,(b_q);\end{array}x(-U)\right]\\ +\dfrac{U}{(2+U)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\ (b_q);\end{array}x(-U)\right]\end{array}\right\}. \tag{6.3}$$

(iv). Putting $\beta=0$ and $\zeta=\Theta=\frac{t}{1-t}$ from equation (1.24) in equation (2.7), we get

$$\sum_{n=0}^{\infty}\left\{\begin{array}{l}\dfrac{(\lambda+\mu n)}{(\alpha+n)}\dbinom{\alpha+n}{n}\times\\ {}_{p+m+1}F_{q+m+1}\left[\begin{array}{l}\Delta(m;-n),1+\frac{(\lambda+\mu n)}{\mu(1-m)},(a_p);\\ \Delta(m;1+\alpha),\frac{(\lambda+\mu n)}{\mu(1-m)},(b_q);\end{array}x\right]\mathrm{t}^n\end{array}\right\}$$

$$=(1-t)^{-\alpha}\left\{\begin{array}{l}\dfrac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{l}\frac{\alpha}{m},1+\frac{\lambda}{\mu},(a_p);\\ 1+\frac{\alpha}{m},\frac{\lambda}{\mu},(b_q);\end{array}x\left(\frac{t}{t-1}\right)^m\right]\\ +\mu\left(\dfrac{t}{1-t}\right){}_pF_q\left[\begin{array}{l}(a_p);\\ (b_q);\end{array}x\left(\frac{t}{t-1}\right)^m\right]\end{array}\right\}. \tag{6.4}$$

(v). Putting $\beta=1$ and $\zeta=\Lambda=\frac{1-2t-\sqrt{(1-4t)}}{2t}$ from equation (1.25) in equation (2.7), we get

$$\sum_{n=0}^{\infty}\left[\begin{array}{l}\dfrac{(\lambda+\mu n)}{(\alpha+2n)}\dbinom{\alpha+2n}{n}\\ {}_{p+m+1}F_{q+m+1}\left[\begin{array}{l}\Delta(m;-n),1+\frac{\lambda+\mu n}{\mu(2-m)},(a_p);\\ \Delta(m;1+\alpha+n),\frac{\lambda+\mu n}{\mu(2-m)},(b_q);\end{array}x\right]\mathrm{t}^n\end{array}\right]$$

$$=(1+\Lambda)^{\alpha}\left\{\begin{array}{l}\frac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{l}\frac{\alpha}{2m},1+\frac{\lambda}{2\mu},(a_p);\\ 1+\frac{\alpha}{2m},\frac{\lambda}{2\mu},(b_q);\end{array}x(-\Lambda)^m\right]\\ +\frac{\mu\Lambda}{(1-\Lambda)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\(b_q);\end{array}x(-\Lambda)^m\right]\end{array}\right\}. \quad (6.5)$$

(vi). Putting $\beta=-2$ and $\zeta=\Xi=\frac{-1+\sqrt{(1+4t)}}{2}$ from equation (1.26) in equation (2.7), we get

$$\sum_{n=0}^{\infty}\left\{\begin{array}{l}\frac{(\lambda+\mu n)}{(\alpha-n)}\binom{\alpha-n}{n}\\ {}_{p+m+1}F_{q+m+1}\left[\begin{array}{l}\Delta(m;-n),1-\frac{(\lambda+\mu n)}{\mu(1+m)},(a_p)?\ ;\\ \Delta(m;1+\alpha-2n),-\frac{(\lambda+\mu n)}{\mu(1+m)},(b_q);\end{array}x\right]t^n\end{array}\right\}$$

$$=(1+\Xi)^{\alpha}\left\{\begin{array}{l}\frac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{l}\frac{-\alpha}{m},1-\frac{\lambda}{\mu},(a_p);\\ 1-\frac{\alpha}{m},\frac{-\lambda}{\mu},(b_q);\end{array}x(-\Xi)^m\right]\\ +\frac{\mu\Xi}{(1+2\Xi)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\(b_q);\end{array}x(-\Xi)^m\right]\end{array}\right\}. \quad (6.6)$$

(vii). Putting $\beta=-3$ and $\zeta=\Upsilon$ from equation (1.27) in equation (2.7), we get

$$\sum_{n=0}^{\infty}\left\{\begin{array}{l}\frac{(\lambda+\mu n)}{(\alpha-2n)}\binom{\alpha-2n}{n}\\ {}_{p+m+1}F_{q+m+1}\left[\begin{array}{l}\Delta(m;-n),1-\frac{(\lambda+\mu n)}{\mu(2+m)},(a_p);\\ \Delta(m;1+\alpha-3n),-\frac{(\lambda+\mu n)}{\mu(2+m)},(b_q);\end{array}x\right]t^n\end{array}\right\}$$

$$=(1+\Upsilon)^{\alpha}\left\{\begin{array}{l}\frac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{l}\frac{-\alpha}{2m},1-\frac{\lambda}{2\mu},(a_p);\\ 1-\frac{\alpha}{2m},\frac{-\lambda}{2\mu},(b_q);\end{array}x(-\Upsilon)^m\right]\\ +\frac{\mu\Upsilon}{(1+3\Upsilon)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\(b_q);\end{array}x(-\Upsilon)^m\right]\end{array}\right\}. \quad (6.7)$$

(viii). Putting $\beta=\frac{-1}{2}$ and $\zeta=U$ from equation (1.28) in equation (2.7), we get

$$\sum_{n=0}^{\infty}\left\{\begin{array}{l}\frac{(\lambda+\mu n)}{(\alpha+\frac{1}{2}n)}\binom{\alpha+\frac{1}{2}n}{n}\\ {}_{p+m+1}F_{q+m+1}\left[\begin{array}{l}\Delta(m;-n),1+\frac{(\lambda+\mu n)}{\mu(\frac{1}{2}-m)},(a_p);\\ \Delta(m;1+\alpha-\frac{1}{2}n),\frac{(\lambda+\mu n)}{\mu(\frac{1}{2}-m)},(b_q);\end{array}x\right]t^n\end{array}\right\}$$

$$=(1+U)^{\alpha}\left\{\begin{array}{l}\frac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{l}\frac{2\alpha}{m},1+\frac{2\lambda}{\mu},(a_p);\\ 1+\frac{2\alpha}{m},\frac{2\lambda}{\mu},(b_q);\end{array}x(-U)^m\right]\\ +\frac{\mu U}{(1+\frac{1}{2}U)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\(b_q);\end{array}x(-U)^m\right]\end{array}\right\}. \quad (6.8)$$

(ix). Putting $\beta=\frac{-3}{2}$ and $\zeta=\Psi$ from equation (1.29) in equation (2.7), we get

$$\sum_{n=0}^{\infty}\left\{\begin{array}{l}\frac{(\lambda+\mu n)}{(\alpha-\frac{1}{2}n)}\binom{\alpha-\frac{1}{2}n}{n}\\ {}_{p+m+1}F_{q+m+1}\left[\begin{array}{l}\Delta(m;-n),1-\frac{(\lambda+\mu n)}{\mu(\frac{1}{2}+m)},(a_p);\\ \Delta(m;1+\alpha-\frac{3}{2}n),-\frac{(\lambda+\mu n)}{\mu(\frac{1}{2}+m)},(b_q);\end{array}x\right]t^n\end{array}\right\}$$

$$=(1+\Psi)^{\alpha}\left\{\begin{array}{l}\frac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{l}\frac{-2\alpha}{m},1-\frac{2\lambda}{\mu},(a_p);\\ 1-\frac{2\alpha}{m},\frac{-2\lambda}{\mu},(b_q);\end{array}x(-\Psi)^m\right]\\ +\frac{\mu\Psi}{(1+\frac{3}{2}\Psi)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\(b_q);\end{array}x(-\Psi)^m\right]?\end{array}\right\}. \quad (6.9)$$

(x). Putting $\beta=\frac{-1}{3}$ and $\zeta=\Pi$ from equation (1.30) in equation (2.7), we get

$$\sum_{n=0}^{\infty}\left\{\begin{array}{l}\frac{(\lambda+\mu n)}{(\alpha+\frac{2}{3}n)}\binom{\alpha+\frac{2}{3}n}{n}\\ {}_{p+m+1}F_{q+m+1}\left[\begin{array}{l}\Delta(m;-n),1+\frac{(\lambda+\mu n)}{\mu(\frac{2}{3}-m)},(a_p);\\ \Delta(m;1+\alpha-\frac{1}{3}n),\frac{(\lambda+\mu n)}{\mu(\frac{2}{3}-m)},(b_q);\end{array}x\right]t^n\end{array}\right\}$$

$$=(1+\Pi)^{\alpha}\left\{\begin{array}{l}\frac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{l}\frac{3\alpha}{2m},1+\frac{3\lambda}{2\mu},(a_p);\\ 1+\frac{3\alpha}{2m},\frac{3\lambda}{2\mu},(b_q);\end{array}x(-\Pi)^m\right]\\ +\frac{\mu\Pi}{(1+\frac{1}{3}\Pi)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\(b_q);\end{array}x(-\Pi)^m\right]\end{array}\right\}. \quad (6.10)$$

(xi). Putting $\beta=\frac{-2}{3}$ and $\zeta=\Phi$ from equation (1.31) in equation (2.7), we get

$$\sum_{n=0}^{\infty}\left\{\begin{array}{l}\dfrac{(\lambda+\mu n)}{(\alpha+\frac{1}{3}n)}\begin{pmatrix}\alpha+\frac{1}{3}n\\ n\end{pmatrix}\\ {}_{p+m+1}F_{q+m+1}\left[\begin{array}{l}\Delta(m;-n),1+\dfrac{(\lambda+\mu n)}{\mu(\frac{1}{3}-m)},(a_p);\\ \Delta(m;1+\alpha-\frac{2}{3}n),\dfrac{(\lambda+\mu n)}{\mu(\frac{1}{3}-m)},(b_q);\end{array} x\right]\end{array}\right\}\mathrm{t}^n$$

$$=(1+\Phi)^{\alpha}\left\{\begin{array}{l}\dfrac{\lambda}{\alpha}\,{}_{p+2}F_{q+2}\left[\begin{array}{l}\dfrac{3\alpha}{m},1+\dfrac{3\lambda}{\mu},(a_p);\\ 1+\dfrac{3\alpha}{m},\dfrac{3\lambda}{\mu},(b_q);\end{array} x(-\Phi)^m\right]\\ +\dfrac{\mu\Phi}{(1+\frac{2}{3}\Phi)}\,{}_pF_q\left[\begin{array}{l}(a_p);\\ (b_q);\end{array} x(-\Phi)^m\right]\end{array}\right\}. \quad (6.11)$$

Making suitable adjustments of parameters and variables in all generating relations of sections 5 and 6, we can also obtain a number of new generating relations involving restricted generalized Laguerre polynomials, restricted Jacobi polynomials, restricted generalized Rice polynomials of Khandekar and other orthogonal polynomials.

7. Further Generalizations of Generating Relations (2.1) and (2.7)

Generalization of (2.1):

Let

$$S_n^{(\alpha,\beta)}(x;m)=\sum_{r=0}^{\left[\frac{n}{m}\right]}\begin{pmatrix}\alpha+(\beta+1)n\\ n-mr\end{pmatrix}\gamma_r x^r, \quad (7.1)$$

where α,β are complex parameters independent of `n'; m is an arbitrary positive integer and $\{\gamma_r\}$ is a bounded sequence of arbitrary real and complex numbers such that $\gamma_r\neq 0$. Then

$$\sum_{n=0}^{\infty}\frac{\lambda+\mu n}{\{\alpha+(\beta+1)n\}}S_n^{(\alpha,\beta)}(x;m)\,\mathrm{t}^n$$
$$=(1+\zeta)^{\alpha}\left\{\begin{array}{l}\displaystyle\sum_{n=0}^{\infty}\frac{(\lambda+\mu mn)}{\{\alpha+(\beta+1)mn\}}\gamma_n x^n\zeta^{mn}\\ +\dfrac{\mu\zeta}{(1-\beta\zeta)}\displaystyle\sum_{n=0}^{\infty}\gamma_n x^n\zeta^{mn}\end{array}\right\}, \quad (7.2)$$

where ζ is given by

$$\zeta=t(1+\zeta)^{(\beta+1)};\ \zeta(0)=0, \quad (7.3)$$

provided that each of the series involved is absolutely convergent.

Independent Demonstration:

Using the definition (7.1) of $S_n^{(\alpha,\beta)}(x;m)$ in left hand side of equation (7.2), we get

$$\Omega^{**}=\sum_{n=0}^{\infty}\frac{(\lambda+\mu n)}{\{\alpha+(\beta+1)n\}}S_n^{(\alpha,\beta)}(x;m)\,\mathrm{t}^n$$
$$=\sum_{n=0}^{\infty}\sum_{r=0}^{[\frac{n}{m}]}\frac{(\lambda+\mu n)\Gamma\{\alpha+(\beta+1)\mathrm{n}+1\}}{\left[\begin{array}{l}\{\alpha+(\beta+1)n\}\Gamma(n-mr+1)\\ \times\Gamma\{\alpha+\beta n+mr+1\}\end{array}\right]}\gamma_r x^r t^n. \quad (7.4)$$

Applying summation identity (1.10) and then simplifying further, we get

$$\Omega^{**}=\sum_{r=0}^{\infty}\frac{\gamma_r x^r\,(\mathrm{t})^{mr}}{\{\alpha+m(\beta+1)r\}}\times\sum_{n=0}^{\infty}\left\{\begin{array}{l}\dfrac{\{\alpha+(\beta+1)mr\}(\lambda+\mu n+\mu mr)}{\{\alpha+(\beta+1)mr+(\beta+1)n\}}\\ \times\begin{pmatrix}\alpha+(\beta+1)mr+(\beta+1)n\\ n\end{pmatrix}t^n\end{array}\right\}. \quad (7.5)$$

Now using first modified Gould's identity (1.7) with conditions (7.3), we get

$$\Omega^{**}=(1+\zeta)^{\alpha}\sum_{r=0}^{\infty}\left[\begin{array}{l}\{\lambda+\mu mr\}\\ +\left\{\begin{array}{l}\dfrac{\mu\zeta\alpha}{(1-\beta\zeta)}\\ +\dfrac{\mu\zeta(\beta+1)mr\}}{(1-\beta\zeta)}\end{array}\right\}\end{array}\right]\frac{\gamma_r x^r\,(\zeta)^{mr}}{\{\alpha+m(\beta+1)r\}}. \quad (7.6)$$

Changing the summation index from r to n and after solving it further,we get the general result (7.2) corresponding to our first generating relation (2.1) subject to the conditions (7.3).

Generalization of (2.7):

Let

$$T_n^{(\alpha,\beta)}(x;m,\lambda,\mu)=\frac{1}{(\lambda+\mu n)}\sum_{r=0}^{\left[\frac{n}{m}\right]}\left\{\begin{pmatrix}\alpha+(\beta+1)n\\ n-mr\end{pmatrix}\begin{bmatrix}\lambda+\mu n\\ +\mu(\beta+1-m)r\end{bmatrix}\right\}\gamma_r x^r, \quad (7.7)$$

where α,β,λ,μ are complex parameters independent of `n'; m is an arbitrary positive integer and $\{\gamma_r\}$ is a bounded sequence of arbitrary real and complex numbers such that $\gamma_r\neq 0$. Then

$$\sum_{n=0}^{\infty}\frac{(\lambda+\mu n)}{\{\alpha+(\beta+1)n\}}T_n^{(\alpha,\beta)}(x;m,\lambda,\mu)t^n$$
$$=(1+\zeta)^{\alpha}\left\{\begin{array}{l}\displaystyle\sum_{n=0}^{\infty}\frac{\{\lambda+\mu(\beta+1)n\}}{\{\alpha+(\beta+1)mn\}}\gamma_n x^n\zeta^{mn}\\ +\dfrac{\mu\zeta}{(1-\beta\zeta)}\displaystyle\sum_{n=0}^{\infty}\gamma_n x^n\zeta^{mn}\end{array}\right\}, \quad (7.8)$$

where ζ is given by

$$\zeta=t(1+\zeta)^{(\beta+1)};\ \zeta(0)=0,$$

provided that each of the series involved is absolutely convergent.

Independent Demonstration:

Using the definition (7.7) of $T_n^{(\alpha,\beta)}(x;m,\lambda,\mu)$ in left hand side of equation (7.8), we get

$$\Omega^{***}=\sum_{n=0}^{\infty}\frac{(\lambda+\mu n)}{\{\alpha+(\beta+1)n\}}T_n^{(\alpha,\beta)}(x;m,\lambda,\mu)t^n$$

$$=\sum_{n=0}^{\infty}\sum_{r=0}^{[\frac{n}{m}]}\left\{\binom{\alpha+(\beta+1)n}{n-mr}\frac{\begin{Bmatrix}\lambda+\mu n\\+\mu(\beta+1)r-\mu mr\end{Bmatrix}}{\{\alpha+(\beta+1)n\}}\right\}\gamma_r x^r t^n. \quad (7.9)$$

Applying summation identity (1.10) and then simplifying further, we get

$$\Omega^{***}=\sum_{r=0}^{\infty}\frac{\gamma_r x^r (t)^{mr}}{\{\alpha+m(\beta+1)r\}}\sum_{n=0}^{\infty}\left\{\begin{matrix}\dfrac{\begin{Bmatrix}\{\alpha+(\beta+1)mr\}\\\times\{\lambda+\mu n+\mu(\beta+1)r\}\end{Bmatrix}}{\{\alpha+(\beta+1)mr+(\beta+1)n\}}\\\times\dbinom{\alpha+(\beta+1)mr+(\beta+1)n}{n}t^n\end{matrix}\right\}. \quad (7.10)$$

Now using second modified Gould's identity (1.8) with conditions (7.3), we get

$$\Omega^{***}=(1+\zeta)^{\alpha}\sum_{r=0}^{\infty}\left[\begin{matrix}\{\lambda+\mu(\beta+1)r\}\\+\left\{\dfrac{\mu\zeta\alpha}{(1-\beta\zeta)}+\dfrac{\mu\zeta(\beta+1)mr\}}{(1-\beta\zeta)}\right\}\end{matrix}\right] \quad (7.11)$$
$$\times\frac{\gamma_r x^r (\zeta)^{mr}}{\{\alpha+m(\beta+1)r\}}.$$

Changing the summation index from r to n and after solving it further, we get the general result (7.8) corresponding to our second generating relation (2.7) subject to the conditions (7.3).

In the definitions of generalized polynomials given by $S_n^{(\alpha,\beta)}(x;m)$ and $T_n^{(\alpha,\beta)}(x;m,\lambda,\mu)$, putting

$$\gamma_r=\frac{(-1)^{mr}(a_1)_r\dots(a_p)_r}{(b_1)_r\dots(b_q)_r\,r!},$$

we obtain Srivastava's generalized hypergeometric polynomials of one variable $H_n^{(\alpha,\beta)}(x;m)$ and Pathan's generalized hypergeometric polynomials of one variable $\mathcal{B}_n^{(\alpha,\beta)}(x;m,\lambda,\mu)$ respectively.

References

[1] Whittaker, E. T. and Watson, G. N.; *A Course of Modern Analysis*, Fourth ed., Cambridge Univ. Press, Cambridge, London and New York (1927).

[2] P´olya, G. and Szeg¨o, G; Problems and Theorems in Analysis, Vol. I, (Translated from theGerman by D.Aeppli) Springer-Verlag, New York, Heidelberg and Berlin (1972).

[3] Gould, H. W.; Some generalizations of Vandermonde's Convolution, Amer. Math. Monthly, 63(1956); 84-91.

[4] Riordan, J.; *Combinatorial Identities*, John Wiley & Sons, New York, London and Sydney (1968).

[5] Srivastava, H. M. and Manocha, H. L.; *A Treatise on Generating functions,* Halsted Press (Ellis Horwood Ltd., Chichester, U. K.), John Wiley and Sons, New York, Chichester, Brisbane and Toronto, (1984).

[6] Rainville, E. D.; Special Functions, The Macmillan, New York, (1960); Reprinted by Chelsea Publishing Company, Bronx, New York, (1971).

[7] Khandekar, P. R.; On a generalization of Rice's polynomial, I. Proc. Nat. Acad. Sci. India Sect. A, 34(1964); 157-162.

[8] Srivastava, H. M.; *A Note on Certain Generating functions for the Classical Polynomi-als*, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8)*;* 63(1977), 328-333.

[9] Srivastava, H. M.; *A class of Generating Functions for generalized Hypergeometric Poly-nomials (Abstract)*, Notices Amer. Math. Soc., 16(1969)*;* 975 (Abstract #69*T-B*198).

[10] Srivastava, H. M.; *A class of Generating Functions for generalized Hypergeometric Poly-nomials*, J. Math. Anal. Appl., 35 (1971)*;* 230-235.

[11] Srivastava, H. M.; *Generating Functions for Jacobi and Laguerre Polynomials*, Proc. Amer. Math. Soc., 23(1969)*;* 590-595.

[12] Brown, J. W.; New Generating Functions for Classical Polynomials, Proc. Amer. Math. Soc., 21(1969); 263-268.

[13] Brown, J. W.; On Zero Type Sets of Laguerre Polynomials, Duke Math. J., 35(No.4) (1968); 821-823.

[14] Carlitz, L.; Some Generating Functions for Laguerre Polynomials, Duke Math. J., 35(1968); 825-827.

[15] Srivastava, H. M. and Singhal, J. P.; *New Generating Functions for Jacobi and Related Polynomials*, J. Math. Anal. Appl., 41 (1973)*;* 748-752.

[16] Calvez, L.-C. and G'enin, R. Sur les relations entre les fonctions g´en´eratrices et les for-mules de type Rodrigues, C. R. Acad. Sci. Paris S'er. A 269(1969); 651-654.

[17] Milch, P. R.; A Probabilistic proof of a formula for Jacobi Polynomials by L. Carlitz, Proc. Cambridge Philos. Soc., 64(1968); 695-698.

[18] Joshi, C. M. and Prajapat, M. L.; A Triple integral transformation and its applications to generating functions. Boll. Un. Mat. Ital. (5), 14A (1977); 264-274.

[19] Karande, B. K. and Thakare, N. K.; Note on the generating function for generalized hy-pergeometric polynomials, Indian J. Pure Appl. Math., 6(1975); 1185-1187.