

Some Reduction Formulae Associated with Gauss and Fox-Wright Hypergeometric Functions

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Abstract In this paper, we describe some reduction formulae for Gauss' hypergeometric function and Fox-Wright hypergeometric function associated with suitable convergence conditions using series rearrangement technique.

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1. Introduction and Basic Notations

In the present paper, we shall use the following standard notations:

$$\mathbb{N} := \{1, 2, 3, \dots\}, \mathbb{N}_0 := \{0, 1, 2, 3, \dots\} = \mathbb{N} \cup \{0\}$$

$$\mathbb{Z}_0^- := \{0, -1, -2, -3, \dots\},$$

$$\mathbb{Z}^- := \{-1, -2, -3, \dots\} = \mathbb{Z}_0^- \setminus \{0\} \text{ and } \mathbb{Z} = (\mathbb{Z}_0^- \cup \mathbb{Z}^-).$$

Here, as usual, \mathbb{Z} denotes the set of integers, \mathbb{R} denotes the set of real numbers, \mathbb{R}^+ denotes the set of positive real numbers and \mathbb{C} denotes the set of complex numbers.

The Pochhammer symbol (or the shifted factorial) $(\lambda)_\nu$, $(\lambda, \nu \in \mathbb{C})$ is defined in terms of the familiar Gamma function by

$$(\lambda)_\nu := \frac{\Gamma(\lambda + \nu)}{\Gamma(\lambda)}$$

$$= \begin{cases} 1; & (\nu = 0; \lambda \in \mathbb{C} \setminus \{0\}) \\ \lambda(\lambda + 1) \dots (\lambda + n - 1); & (\nu = n \in \mathbb{N}; \lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-) \\ \frac{(-1)^n k!}{(k - n)!}; & (\lambda = -k; \nu = n; n, k \in \mathbb{N}_0; 0 \leq n \leq k) \\ 0; & (\lambda = -k; \nu = n; n, k \in \mathbb{N}_0; n > k) \\ \frac{(-1)^n}{(1 - \lambda)_n}; & (\nu = -n; n \in \mathbb{N}; \lambda \neq 0, \pm 1, \pm 2, \dots). \end{cases} \quad (1.1)$$

it being understood *conventionally* that $(0)_0 = 1$, and assumed tacitly that the Gamma quotient exists.

In the Gaussian hypergeometric series ${}_2F_1(a, b; c; z)$, there are two numerator parameters a, b and one denominator parameter c . A natural generalization of this series is accomplished by introducing any arbitrary number of numerator and denominator parameters. The non-terminating hypergeometric series [[5], pp.42-43]

$${}_pF_q \left[\begin{matrix} \alpha_1, \alpha_2, \dots, \alpha_p; \\ \beta_1, \beta_2, \dots, \beta_q; \end{matrix} z \right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n (\alpha_2)_n \dots (\alpha_p)_n z^n}{(\beta_1)_n (\beta_2)_n \dots (\beta_q)_n n!}, \quad (1.2)$$

is known as the generalized Gauss and Kummer series, or simply, *generalized hypergeometric series*. Here p and q are positive integers or zero (interpreting an empty product as unity), and we assume that the variable z , the numerator parameters $\alpha_1, \alpha_2, \dots, \alpha_p$ and the denominator parameters $\beta_1, \beta_2, \dots, \beta_q$ take on complex values, provided that

$$\beta_j \neq 0, -1, -2, \dots; \quad j = 1, 2, \dots, q. \quad (1.3)$$

Convergence conditions [[5], p.43]:

Suppose that none of the numerator parameters is zero or negative integer (otherwise the question of convergence will not arise), and with the usual restriction (1.3), the ${}_pF_q$ series in the definition (1.2)

(i) converges for $|z| < \infty$, if $p \leq q$,

(ii) converges for $|z| < 1$, if $p = q + 1$.

Furthermore, if we denote

$$\omega = \sum_{j=1}^q \beta_j - \sum_{i=1}^p \alpha_i,$$

it is known that the ${}_pF_q$ series, with $p = q + 1$, is

- (a) absolutely convergent for $|z|=1$, if $\Re(\omega) > 0$,
- (b) conditionally convergent for $|z|=1, z \neq 1$, if $-1 < \Re(\omega) \leq 0$.

Fox-Wright generalized hypergeometric function of one variable:

The Fox-Wright Ψ function of one variable ([3], p.389); see also [4,6,7]) is given by

$$\begin{aligned} & {}_p\Psi_q \left[\begin{matrix} (\alpha_1, A_1), \dots, (\alpha_p, A_p); \\ (\beta_1, B_1), \dots, (\beta_q, B_q); \end{matrix} z \right] \\ &= \sum_{n=0}^{\infty} \frac{\Gamma(\alpha_1 + A_1 n) \dots \Gamma(\alpha_p + A_p n)}{\Gamma(\beta_1 + B_1 n) \dots \Gamma(\beta_q + B_q n)} \frac{z^n}{n!} \\ &= \frac{\Gamma(\alpha_1) \dots \Gamma(\alpha_p)}{\Gamma(\beta_1) \dots \Gamma(\beta_q)} \sum_{n=0}^{\infty} \frac{(\alpha_1)_{nA_1} \dots (\alpha_p)_{nA_p}}{(\beta_1)_{nB_1} \dots (\beta_q)_{nB_q}} \frac{z^n}{n!}, \end{aligned} \quad (1.4)$$

$$= \frac{\Gamma(\alpha_1) \dots \Gamma(\alpha_p)}{\Gamma(\beta_1) \dots \Gamma(\beta_q)} {}_p\Psi_q^* \left[\begin{matrix} (\alpha_1, A_1), \dots, (\alpha_p, A_p); \\ (\beta_1, B_1), \dots, (\beta_q, B_q); \end{matrix} z \right], \quad (1.5)$$

$$= \frac{1}{2\pi\rho} \int_L \frac{\Gamma(\zeta) \prod_{i=1}^p \Gamma(\alpha_i - A_i \zeta)}{q \prod_{j=1}^q \Gamma(\beta_j - B_j \zeta)} (-z)^{-\zeta} d\zeta, \quad (1.6)$$

where $\rho^2 = -1, z \in \mathbb{C}$; parameters $\alpha_i, \beta_j \in \mathbb{C}$; coefficients $A_i, B_j \in \mathbb{R} = (-\infty, +\infty)$ in case of series (1.4) (or $A_i, B_j \in \mathbb{R}^+(0, +\infty)$ in case of contour integral (1.6)), $A_i \neq 0 (i = 1, 2, \dots, p), B_j \neq 0 (j = 1, 2, \dots, q)$. In equation (1.4), the parameters α_i, β_j and coefficients A_i, B_j are adjusted in such a way that the product of Gamma functions in numerator and denominator should be well defined [1,2].

$$\Delta^* = \left(\sum_{j=1}^q B_j - \sum_{i=1}^p A_i \right), \quad (1.7)$$

$$\delta^* = \left(\prod_{i=1}^p |A_i|^{-A_i} \right) \left(\prod_{j=1}^q |B_j|^{B_j} \right), \quad (1.8)$$

$$\mu^* = \sum_{j=1}^q \beta_j - \sum_{i=1}^p \alpha_i + \left(\frac{p-q}{2} \right), \quad (1.9)$$

$$\sigma^* = (1 + A_1 + \dots + A_p) - (B_1 + \dots + B_q) = 1 - \Delta^*. \quad (1.10)$$

Case(I): When contour (L) is a left loop beginning and ending at $-\infty$, then ${}_p\Psi_q$ given by (1.4) or (1.6) holds the

following convergence conditions

- i) when $\Delta^* > -1, 0 < |z| < \infty, z \neq 0$,
- ii) when $\Delta^* = -1, 0 < |z| < \delta^*$,
- iii) when $\Delta^* = -1, |z| = \delta^*$, and $\Re(\mu^*) > \frac{1}{2}$.

Case(II): When contour (L) is a right loop beginning and ending at $+\infty$, then ${}_p\Psi_q$ given by (1.4) or (1.6) holds the following convergence conditions

- i) when $\Delta^* < -1, 0 < |z| < \infty, z \neq 0$,
- ii) when $\Delta^* = -1, |z| > \delta^*$,
- iii) when $\Delta^* = -1, |z| = \delta^*$, and $\Re(\mu^*) > \frac{1}{2}$.

Case (III): When contour (L) is starting from $\gamma - i\infty$ and ending at $\gamma + i\infty$, where $\gamma \in \mathbb{R}$, then ${}_p\Psi_q$ is also convergent under the following conditions

- i) when $\sigma^* > 0, |\arg(-z)| < \frac{\pi}{2} \sigma^*, 0 < |z| < \infty, z \neq 0$,
- ii) when $\sigma^* = 0, \arg(-z) = 0, 0 < |z| < \infty, z \neq 0$ such that $-\gamma \Delta^* + \Re(\mu^*) > \frac{1}{2} + \gamma$,
- iii) when $\gamma = 0, \sigma^* = 0, \arg(-z) = 0, 0 < |z| < \infty, z \neq 0$ such that, $\Re(\mu^*) > \frac{1}{2}$.

Next we collect some results that we will need in the sequel.

Decomposition identity:

The idea of separation of a power series into its even and odd terms exhibited by the elementary identity

$$\sum_{r=0}^{\infty} \Phi(r) = \sum_{r=0}^{\infty} \Phi(2r) + \sum_{r=0}^{\infty} \Phi(2r+1), \quad (1.11)$$

is at least as old as the series themselves and concerned power series is absolutely convergent.

Pfaff-Kummer's linear transformation [5], p.33(eq.19):

$${}_2F_1 \left[\begin{matrix} a, b; \\ d; \end{matrix} z \right] = (1-z)^{-a} {}_2F_1 \left[\begin{matrix} a, d-b; \\ d; \end{matrix} \frac{-z}{1-z} \right], \quad (1.12)$$

($a, b \in \mathbb{C}; d \in \mathbb{C} \setminus \mathbb{Z}_0^-$ and $|\arg(1-z)| < \pi$).

Binomial theorem:

$$(1-z)^{-b} = {}_1F_0 \left[\begin{matrix} b; \\ -; \end{matrix} z \right] = \sum_{r=0}^{\infty} \frac{(b)_r z^r}{r!}, \quad (1.13)$$

($|z| < 1; b \in \mathbb{C} \setminus \mathbb{Z}_0$),

$$(1-z)^m = {}_1F_0 \left[\begin{matrix} -m; \\ -; \end{matrix} z \right] = \sum_{r=0}^m \frac{(-m)_r z^r}{r!}, \quad m \in \mathbb{N}_0.$$

In sections 2 and 3, we obtain some reduction formulae for Gauss' hypergeometric function and Fox-Wright hypergeometric function associated with suitable convergence conditions by using binomial theorem, decomposition identity and Pfaff-Kummer's linear transformation.

2. Some Reduction Formulae for Gauss' Hypergeometric Function

The following reduction formulae associated with suitable convergence conditions hold true

$$\begin{aligned}
 & 2\sqrt{1+x} {}_2F_1 \left[\begin{matrix} b, 1-b; \\ 1/2; \end{matrix} -x \right] \\
 &= \left(\sqrt{1+x} + \sqrt{x} \right)^{2b-1} + \left(\sqrt{1+x} - \sqrt{x} \right)^{2b-1}, \quad (2.1) \\
 & \left(\left| \frac{x}{1+x} \right| < 1; 1-2b \in \mathbf{C} \setminus \mathbf{Z}_0^- \right).
 \end{aligned}$$

$$\begin{aligned}
 & 2(2b-1)\sqrt{x} {}_2F_1 \left[\begin{matrix} b, 1-b; \\ 3/2; \end{matrix} -x \right] \\
 &= \left(\sqrt{1+x} + \sqrt{x} \right)^{2b-1} - \left(\sqrt{1+x} - \sqrt{x} \right)^{2b-1}, \quad (2.2) \\
 & \left(\left| \frac{x}{1+x} \right| < 1; 1-2b \in \mathbf{C} \setminus \mathbf{Z}_0^- \right).
 \end{aligned}$$

$$\begin{aligned}
 & 2 {}_2F_1 \left[\begin{matrix} -b, b; \\ 1/2; \end{matrix} -x \right] \\
 &= \left(\sqrt{1+x} + \sqrt{x} \right)^{2b} + \left(\sqrt{1+x} - \sqrt{x} \right)^{2b}, \quad (2.3) \\
 & \left(\left| \frac{x}{1+x} \right| < 1; -2b \in \mathbf{C} \setminus \mathbf{Z}_0^- \right).
 \end{aligned}$$

$$\begin{aligned}
 & 4(b-1)\sqrt{x+x^2} {}_2F_1 \left[\begin{matrix} b, 2-b; \\ 3/2; \end{matrix} -x \right] \\
 &= \left(\sqrt{1+x} + \sqrt{x} \right)^{2b-2} - \left(\sqrt{1+x} - \sqrt{x} \right)^{2b-2}, \quad (2.4) \\
 & \left(\left| \frac{x}{1+x} \right| < 1; 2-2b \in \mathbf{C} \setminus \mathbf{Z}_0^- \right).
 \end{aligned}$$

Proof of reduction formula (2.1):

Suppose right hand side of equation (2.1) is denoted by Ω . Then by using binomial theorem (1.13), we obtain

$$\begin{aligned}
 \Omega &= \left(\sqrt{1+x} + \sqrt{x} \right)^{2b-1} + \left(\sqrt{1+x} - \sqrt{x} \right)^{2b-1} \\
 &= \left(\sqrt{1+x} \right)^{2b-1} \left\{ {}_1F_0 \left[\begin{matrix} 1-2b; \\ -; \end{matrix} -\sqrt{\frac{x}{1+x}} \right] \right. \\
 & \quad \left. + {}_1F_0 \left[\begin{matrix} 1-2b; \\ -; \end{matrix} \sqrt{\frac{x}{1+x}} \right] \right\} \quad (2.5) \\
 &= \left(\sqrt{1+x} \right)^{2b-1} \\
 & \quad \times \sum_{r=0}^{\infty} \left\{ \frac{(1-2b)_r}{r!} \left(\sqrt{\frac{x}{1+x}} \right)^r \left[(-1)^r + 1 \right] \right\}.
 \end{aligned}$$

Now using decomposition identity (1.11) in equation (2.5), we get

$$\Omega = 2 \left(\sqrt{1+x} \right)^{2b-1} {}_2F_1 \left[\begin{matrix} (1-2b)/2, (2-2b)/2; \\ 1/2; \end{matrix} \frac{x}{1+x} \right]. \quad (2.6)$$

Further using Pfaff-Kummer's linear transformation (1.12) in equation (2.6), and simplifying further, we arrive at left hand side of equation (2.1).

Proof of reduction formula (2.2):

Suppose right hand side of equation (2.2) is denoted by Δ . Then by using binomial theorem (1.13), we get

$$\begin{aligned}
 \Delta &= \left(\sqrt{1+x} + \sqrt{x} \right)^{2b-1} - \left(\sqrt{1+x} - \sqrt{x} \right)^{2b-1} \\
 &= \left(\sqrt{1+x} \right)^{2b-1} \left\{ {}_1F_0 \left[\begin{matrix} 1-2b; \\ -; \end{matrix} -\sqrt{\frac{x}{1+x}} \right] \right. \\
 & \quad \left. - {}_1F_0 \left[\begin{matrix} 1-2b; \\ -; \end{matrix} \sqrt{\frac{x}{1+x}} \right] \right\} \quad (2.7) \\
 &= \left(\sqrt{1+x} \right)^{2b-1} \sum_{r=0}^{\infty} \left\{ \frac{(1-2b)_r}{r!} \left(\sqrt{\frac{x}{1+x}} \right)^r \left[(-1)^r - 1 \right] \right\}.
 \end{aligned}$$

Now using decomposition identity (1.11) in equation (2.7), we get

$$\Delta = 2\sqrt{x}(2b-1)(1+x)^{b-1} {}_2F_1 \left[\begin{matrix} -b+1, -b+3/2; \\ 3/2; \end{matrix} \frac{x}{1+x} \right]. \quad (2.8)$$

On using Pfaff-Kummer's linear transformation (1.12) in equation (2.8), and simplifying further, we have left hand side of equation (2.2).

Proof of reduction formula (2.3):

Suppose right hand side of equation (2.3) is denoted by Λ . Then by using binomial theorem (1.13), we have

$$\begin{aligned}
 \Lambda &= \left(\sqrt{1+x} + \sqrt{x} \right)^{2b} + \left(\sqrt{1+x} - \sqrt{x} \right)^{2b} \\
 &= \left(\sqrt{1+x} \right)^{2b} \left\{ {}_1F_0 \left[\begin{matrix} -2b; \\ -; \end{matrix} -\sqrt{\frac{x}{1+x}} \right] \right. \\
 & \quad \left. + {}_1F_0 \left[\begin{matrix} -2b; \\ -; \end{matrix} \sqrt{\frac{x}{1+x}} \right] \right\} \quad (2.9) \\
 &= \left(\sqrt{1+x} \right)^{2b} \sum_{r=0}^{\infty} \left\{ \frac{(-2b)_r}{r!} \left(\sqrt{\frac{x}{1+x}} \right)^r \left[(-1)^r + 1 \right] \right\}.
 \end{aligned}$$

Now using decomposition identity (1.11) in equation (2.9), we get

$$\Lambda = 2(1+x)^b {}_2F_1 \left[\begin{matrix} -b, -b+1/2; \\ 1/2; \end{matrix} \frac{x}{1+x} \right]. \quad (2.10)$$

On using Pfaff-Kummer's linear transformation (1.12) in equation (2.10), and simplifying further, we arrive at left hand side of equation (2.3).

Proof of reduction formula (2.4):

Suppose right hand side of equation (2.4) is denoted by Ξ . Then by using binomial theorem (1.13), we obtain

$$\begin{aligned}
 \Xi &= \left(\sqrt{1+x} + \sqrt{x} \right)^{2b-2} - \left(\sqrt{1+x} - \sqrt{x} \right)^{2b-2} \\
 &= \left(\sqrt{1+x} \right)^{2b-2} \left\{ {}_1F_0 \left[\begin{matrix} 2-2b; \\ -; \end{matrix} -\sqrt{\frac{x}{1+x}} \right] \right. \\
 & \quad \left. - {}_1F_0 \left[\begin{matrix} 2-2b; \\ -; \end{matrix} \sqrt{\frac{x}{1+x}} \right] \right\}
 \end{aligned}$$

$$= \left(\sqrt{1+x} \right)^{2b-2} \sum_{r=0}^{\infty} \left\{ \frac{(2-2b)_r \left(\sqrt{\frac{x}{1+x}} \right)^r}{r! \times [(-1)^r - 1]} \right\}. \quad (2.11)$$

Now using decomposition identity (1.11) in equation (2.11), we get

$$\Xi = 4\sqrt{x}(b-1)(1+x)^{b-3/2} \times {}_2F_1 \left[\begin{matrix} -b+2, -b+3/2; \\ 3/2; \end{matrix} \frac{x}{1+x} \right]. \quad (2.12)$$

Applying Pfaff-Kummer's linear transformation (1.12) in equation (2.12), and simplifying further, we get left hand side of equation (2.4).

3. Applications of Reduction Formulae in Fox-Wright function

As an application of formulae (2.1)-(2.4), we obtain the following two formulae for Fox-Wright hypergeometric function associated with convergence conditions

$${}_1\Psi_1^* \left[\begin{matrix} (b, 1/2); \\ (b, -1/2); \end{matrix} z \right] = \frac{2}{\sqrt{z^2+4}} \left(\frac{z + \sqrt{z^2+4}}{2} \right)^{2b-1} \quad (3.1)$$

and

$${}_1\Psi_1^* \left[\begin{matrix} (b, 1/2); \\ (1+b, -1/2); \end{matrix} z \right] = \left(\frac{z + \sqrt{z^2+4}}{2} \right)^{2b}, \quad (3.2)$$

$$\left(\left| \frac{z^2}{4+z^2} \right| < 1; b \in \mathbb{C} \setminus \{0, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 2, \pm \frac{5}{2}, \pm 3, \dots\} \right).$$

Proof of reduction formula (3.1):

Suppose left hand side of equation (3.1) is denoted by ζ . Then by using the definitions of ${}_p\Psi_q^*$ (1.4) and (1.5), we obtain

$$\zeta = {}_1\Psi_1^* \left[\begin{matrix} (b, 1/2); \\ (b, -1/2); \end{matrix} z \right] = \sum_{n=0}^{\infty} \frac{(b)_{n/2} z^n}{(b)_{-n/2} n!}. \quad (3.3)$$

Now using decomposition identity (1.11) in equation (3.3), we have

$$\zeta = \sum_{n=0}^{\infty} \frac{(b)_n z^{2n}}{(b)_{-n} (2n)!} + \sum_{n=0}^{\infty} \frac{(b)_{(2n+1)/2} z^{2n+1}}{(b)_{-(2n+1)/2} (2n+1)!}.$$

Solving further, we obtain

$$\zeta = {}_2F_1 \left[\begin{matrix} b, 1-b; \\ 1/2; \end{matrix} \frac{-z^2}{4} \right] + \frac{z \Gamma(b+1/2)}{\Gamma(b-1/2)} {}_2F_1 \left[\begin{matrix} b+1/2, -b+3/2; \\ 3/2; \end{matrix} \frac{-z^2}{4} \right]. \quad (3.4)$$

Now applying the reduction formulae (2.1), (2.4), and simplifying further, we get the right hand side of equation (3.1).

Proof of reduction formula (3.2):

Suppose left hand side of equation (3.2) is denoted by Θ . Then by using the definitions of ${}_p\Psi_q^*$ (1.4) and (1.5), we get

$$\Theta = {}_1\Psi_1^* \left[\begin{matrix} (b, 1/2); \\ (1+b, -1/2); \end{matrix} z \right] = \sum_{n=0}^{\infty} \frac{(b)_{n/2} z^n}{(1+b)_{-n/2} n!}. \quad (3.5)$$

On using decomposition identity (1.11) in equation (3.5), we get

$$\Theta = \sum_{n=0}^{\infty} \frac{(b)_n z^{2n}}{(1+b)_{-n} (2n)!} + \sum_{n=0}^{\infty} \frac{(b)_{(2n+1)/2} z^{2n+1}}{(1+b)_{-(2n+1)/2} (2n+1)!}.$$

Solving further, we get

$$\Theta = {}_2F_1 \left[\begin{matrix} b, -b; \\ 1/2; \end{matrix} \frac{-z^2}{4} \right] + \frac{z (b)_{1/2}}{(1+b)_{-1/2}} {}_2F_1 \left[\begin{matrix} b+1/2, -b+1/2; \\ 3/2; \end{matrix} \frac{-z^2}{4} \right] \quad (3.6) \\ = {}_2F_1 \left[\begin{matrix} b, -b; \\ 1/2; \end{matrix} \frac{-z^2}{4} \right] + z b {}_2F_1 \left[\begin{matrix} b+1/2, -b+1/2; \\ 3/2; \end{matrix} \frac{-z^2}{4} \right].$$

Now applying the reduction formulae (2.2), (2.3), and solving further, we obtain the right hand side of equation (3.2).

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