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# Some New Integral Inequalities for $n$-times Differentiable $s$- Convex functions in the first sense

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**Abstract** In this work, by using an integral identity together with both the Hölder and the Power-Mean integral inequality we establish several new inequalities for $n$-time differentiable $s$-convex functions in the first sense.

***Keywords:*** *convex function, $s$- convex function in the first sense, hölder integral inequality and power-mean integral inequality*

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## 1. Introduction

In this paper, by using the some classical integral inequalities, Hölder and Power-Mean integral inequality, we establish some new inequalities for functions whose $n$th derivatives in absolute value are $s$-convex functions in the first sense. For some inequalities, generalizations and applications concerning convexity see [1-11]. Recently, in the literature there are so many papers about $n$-times differentiable functions on several kinds of convexities and $s$-convex functions. In references [5,6,7,8], readers can find some results about this issue. Many papers have been written by a number of mathematicians concerning inequalities for different classes of convex and $s$-convex functions in the first sense see for instance the recent papers [12-19] and the references within these papers. There are quite substantial literatures on such problems. Here we mention the results of [1-19] and the corresponding references cited therein.

**Definition 1.1:** A function $f: I \subseteq \mathbb{R} \to \mathbb{R}$ is said to be convex if the inequality

$$f(tx+(1-t)y) \leq tf(x)+(1-t)f(y)$$

is valid for all $x, y \in I$ and $t \in [0,1]$. If this inequality reverses, then $f$ is said to be concave on interval $I \neq \emptyset$. This definition is well known in the literature.

**Definition 1.2:** A function $f: [0,\infty) \to \mathbb{R}$ is said to be $s$-convex in the first sense if

$$f(\alpha x+\beta y) \leq \alpha^s f(x)+\beta^s f(y)$$

holds for all $x, y \in [0,\infty)$, $\alpha, \beta \geq 0$ with $\alpha^s + \beta^s = 1$ and for some fixed $s \in (0,1]$. It can be easily seen that every 1-convex function is convex.

In [21], Dragomir and Fitzpatrick proved a variant of Hadamard's inequality which holds for $s$-convex mapping in the first sense.

**Definition 1.3:** The following double inequality is well-known in the literature as Hadamard's inequality for convex mappings [8,10]: Let $f: I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex mapping defined on the interval $I$ in $\mathbb{R}$ and $a, b \in I$ with $a < b$. Then the following inequality holds:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+sf(b)}{s+1}.$$

Throughout this paper we will use the following notations and conventions. Let $J = [0,\infty) \subset \mathbb{R} = (-\infty,+\infty)$, and $a, b \in J$ with $0 < a < b$ and $f' \in L[a,b]$ and

$$A(a,b) = \frac{a+b}{2}, L_p(a,b) = \left(\frac{b^{p+1}-a^{p+1}}{(p+1)(b-a)}\right)^{\frac{1}{p}},$$

$$a \neq b,\ p \in \mathbb{R},\ p \neq -1, 0$$

be the arithmetic and generalized logarithmic mean for $a, b > 0$ respectively.

We will use the following Lemma [20] for we obtain the main results:

**Lemma 1.1:** Let $f: I \subseteq \mathbb{R} \to \mathbb{R}$ be $n$-times differentiable mapping on $I^\circ$ for $n \in \mathbb{N}$ and $f^{(n)} \in L[a,b]$, where $a, b \in I^\circ$ with $a < b$, we have the identity

$$\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx$$

$$=\frac{(-1)^{n+1}}{n!}\int_a^b x^n f^{(n)}(x)dx$$

where an empty sum is understood to be nil.

## 2. Main Results

**Theorem 2.1.** For $\forall n \in \mathbb{N}$; let $f: I \subset [0,\infty) \to \mathbb{R}$ be $n$-times differentiable function on $I^\circ$ and $a,b \in I^\circ$ with $a<b$. If $\left|f^{(n)}\right|^q$ for $q>1$ is $s$-convex function in the first sense on $[a,b]$, then the following inequality holds:

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\le \frac{1}{n!}(b-a)^{\frac{1}{q}} L_{np}^n(a,b)\left[\frac{\left|f^{(n)}(b)\right|^q+s\left|f^{(n)}(a)\right|^q}{s+1}\right]^{\frac{1}{q}},$$

where $1/p+1/q=1$.

**Proof.** If $\left|f^{(n)}\right|^q$ for $q>1$ is $s$-convex function in the first sense on $[a,b]$, using Lemma1.1, the Hölder integral inequality and

$$\left|f^{(n)}(x)\right|^q=\left|f^{(n)}\left(\frac{x-a}{b-a}b+\frac{b-x}{b-a}a\right)\right|^q$$
$$\le\left(\frac{x-a}{b-a}\right)^s\left|f^{(n)}(b)\right|^q+\left[1-\left(\frac{x-a}{b-a}\right)^s\right]\left|f^{(n)}(a)\right|^q,$$

we have

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\le\frac{1}{n!}\int_a^b x^n\left|f^{(n)}(x)\right|dx$$
$$\le\frac{1}{n!}\left(\int_a^b x^{np}dx\right)^{\frac{1}{p}}\left(\int_a^b\left|f^{(n)}(x)\right|^q dx\right)^{\frac{1}{q}}$$
$$\le\frac{1}{n!}\left(\int_a^b x^{np}dx\right)^{\frac{1}{p}}$$
$$\times\left(\int_a^b\left[\begin{array}{l}\left(\frac{x-a}{b-a}\right)^s\left|f^{(n)}(b)\right|^q\\+\left[1-\left(\frac{x-a}{b-a}\right)^s\right]\left|f^{(n)}(a)\right|^q\end{array}\right]dx\right)^{\frac{1}{q}}$$
$$=\frac{1}{n!}\left(\int_a^b x^{np}dx\right)^{\frac{1}{p}}\left(\frac{\left|f^{(n)}(b)\right|^q}{(b-a)^s}\int_a^b(x-a)^s dx\right.$$
$$\left.+\frac{\left|f^{(n)}(a)\right|^q}{(b-a)^s}\int_a^b\left[(b-a)^s-(x-a)^s\right]dx\right)^{\frac{1}{q}}$$
$$=\frac{1}{n!}\left(\left.\frac{x^{np+1}}{np+1}\right|_a^b\right)^{\frac{1}{p}}\left(\frac{\left|f^{(n)}(b)\right|^q}{(b-a)^s}\left.\frac{(x-a)^{s+1}}{s+1}\right|_a^b\right.$$
$$\left.+\frac{\left|f^{(n)}(a)\right|^q}{(b-a)^s}\left[(b-a)^s x-\frac{(x-a)^{s+1}}{s+1}\right]\Bigg|_a^b\right)^{\frac{1}{q}}$$
$$=\frac{1}{n!}\left(\frac{b^{np+1}-a^{np+1}}{np+1}\right)^{\frac{1}{p}}\left\{\frac{\left|f^{(n)}(b)\right|^q}{(b-a)^s}\frac{(b-a)^{s+1}}{s+1}\right.$$
$$\left.+\frac{\left|f^{(n)}(a)\right|^q}{(b-a)^s}\left[(b-a)^s b-\frac{(b-a)^{s+1}}{s+1}-(b-a)^s a\right]\right\}^{\frac{1}{q}}$$
$$=\frac{1}{n!}(b-a)^{\frac{1}{p}}\left[\frac{b^{np+1}-a^{np+1}}{(np+1)(b-a)}\right]^{\frac{1}{p}}$$
$$\times\left[(b-a)\frac{\left|f^{(n)}(b)\right|^q+s\left|f^{(n)}(a)\right|^q}{s+1}\right]^{\frac{1}{q}}$$
$$=\frac{1}{n!}(b-a)^{\frac{1}{q}}L_{np}^n(a,b)\left[\frac{\left|f^{(n)}(b)\right|^q+s\left|f^{(n)}(a)\right|^q}{s+1}\right]^{\frac{1}{q}}.$$

This completes the proof of theorem.

**Corollary 2.1.** Under the conditions of Theorem 2.1 for $s=1$, we obtain the following inequality

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\le\frac{1}{n!}(b-a)^{\frac{1}{q}}L_{np}^n(a,b)A^{\frac{1}{q}}\left(\left|f^{(n)}(a)\right|^q,\left|f^{(n)}(b)\right|^q\right)$$

which coincide with the Theorem 2.1 in [20].

**Proposition 2.1.** Under the conditions of Corollary 2.1 for $n=1$, we obtain the following:

$$\left|\frac{f(b)b-f(a)a}{b-a}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\le(b-a)^{\frac{1}{q}-1}L_p(a,b)A^{\frac{1}{q}}\left(\left|f^{(n)}(a)\right|^q,\left|f^{(n)}(b)\right|^q\right).$$

**Proposition 2.2.** For $q=1$, we obtain the following inequality:

$$\left|\frac{f(b)b-f(a)a}{b-a}-\frac{1}{b-a}\int_a^b f(x)dx\right| \le L_p(a,b)A\left(\left|f'(a)\right|,\left|f'(b)\right|\right).$$

**Theorem 2.2.** For $\forall n\in\mathbb{N}$; let $f:I\subset[0,\infty)\to\mathbb{R}$ be $n$ -times differentiable function on $I^\circ$ and $a,b\in I^\circ$ with $a<b$ . If $\left|f^{(n)}\right|^q\in L[a,b]$ and $\left|f^{(n)}\right|^q$ for $q\ge 1$ is $s$-convex function in the first sense on $[a,b]$, then the following inequality holds:

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\le\frac{1}{n!}(b-a)L_n^{n\frac{q-1}{q}}(a,b)$$
$$\times\left\{\left[\frac{\left|f^{(n)}(b)\right|^q-\left|f^{(n)}(a)\right|^q}{(b-a)^{s+1}}\right]K(s,n,x)+\left|f^{(n)}(a)\right|^q L_n^n(a,b)\right\}^{\frac{1}{q}}.$$

Where $p=1-\frac{1}{q}$ and $p>1$ , $K(s,n,x)=\int_a^b(x-a)^s x^n\,dx$.

**Proof.** From Lemma1.1 and Power-mean integral inequality, we obtain

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\le\frac{1}{n!}\int_a^b x^n\left|f^{(n)}(x)\right|dx$$
$$\le\frac{1}{n!}\left(\int_a^b x^n dx\right)^{1-\frac{1}{q}}$$
$$\times\left(\int_a^b x^n\left[\left(\frac{x-a}{b-a}\right)^s\left|f^{(n)}(b)\right|^q+\left[1-\left(\frac{x-a}{b-a}\right)^s\right]\left|f^{(n)}(a)\right|^q\right]dx\right)^{\frac{1}{q}}$$
$$=\frac{1}{n!}\left(\int_a^b x^n dx\right)^{1-\frac{1}{q}}\left\{\frac{\left|f^{(n)}(b)\right|^q}{(b-a)^s}\int_a^b(x-a)^s x^n dx\right.$$
$$\left.+\frac{\left|f^{(n)}(a)\right|^q}{(b-a)^s}\int_a^b\left[(b-a)^s-(x-a)^s\right]x^n dx\right\}^{\frac{1}{q}}$$

$$=\frac{1}{n!}\left(\int_a^b x^n dx\right)^{1-\frac{1}{q}}\times\left\{\left[\frac{\left|f^{(n)}(b)\right|^q}{(b-a)^s}-\frac{\left|f^{(n)}(a)\right|^q}{(b-a)^s}\right]\int_a^b(x-a)^s x^n dx+\left|f^{(n)}(a)\right|^q\int_a^b x^n\right\}^{\frac{1}{q}}$$

$$=\frac{1}{n!}\left(\left.\frac{x^{n+1}}{n+1}\right|_a^b\right)^{1-\frac{1}{q}}\times\left\{\left[\frac{\left|f^{(n)}(b)\right|^q}{(b-a)^s}-\frac{\left|f^{(n)}(a)\right|^q}{(b-a)^s}\right]K(s,n,x)+\left|f^{(n)}(a)\right|^q\left(\left.\frac{x^{n+1}}{n+1}\right|_a^b\right)\right\}^{\frac{1}{q}}$$

$$=\frac{1}{n!}\left(\frac{b^{n+1}-a^{n+1}}{n+1}\right)^{1-\frac{1}{q}}$$
$$\times\left\{\left[\frac{\left|f^{(n)}(b)\right|^q}{(b-a)^s}-\frac{\left|f^{(n)}(a)\right|^q}{(b-a)^s}\right]K(s,n,x)+\left|f^{(n)}(a)\right|^q\left(\frac{b^{n+1}-a^{n+1}}{n+1}\right)\right\}^{\frac{1}{q}}$$

$$=\frac{1}{n!}(b-a)\left[\frac{b^{n+1}-a^{n+1}}{(n+1)(b-a)}\right]^{1-\frac{1}{q}}$$
$$\times\left\{\left[\frac{\left|f^{(n)}(b)\right|^q}{(b-a)^{s+1}}-\frac{\left|f^{(n)}(a)\right|^q}{(b-a)^{s+1}}\right]K(s,n,x)+\left|f^{(n)}(a)\right|^q\left[\frac{b^{n+1}-a^{n+1}}{(n+1)(b-a)}\right]\right\}^{\frac{1}{q}}$$

$$=\frac{1}{n!}(b-a)L_n^{n\frac{q-1}{q}}(a,b)$$
$$\times\left\{\left[\frac{\left|f^{(n)}(b)\right|^q-\left|f^{(n)}(a)\right|^q}{(b-a)^{s+1}}\right]K(s,n,x)+\left|f^{(n)}(a)\right|^q L_n^n(a,b)\right\}^{\frac{1}{q}}$$

This completes the proof of theorem.

**Corollary 2.2**. Under the conditions of Theorem 2.2 for $s=1$, we obtain the inequality

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$

$$\leq\frac{1}{n!}(b-a)L_n^{n\frac{q-1}{q}}(a,b)$$

$$\times\left[\begin{array}{l}\left(\frac{\left|f^{(n)}(b)\right|^q-\left|f^{(n)}(a)\right|^q}{b-a}\right)\left(L_{n+1}^{n+1}(a,b)-aL_n^n(a,b)\right)\\+\left|f^{(n)}(a)\right|^q L_n^n(a,b)\end{array}\right]^{\frac{1}{q}}.$$

**Proposition 2.3**. Under the conditions of Corollary 2.2 for $n=1$, we obtain the inequality

$$\left|\frac{f(b)b-f(a)a}{b-a}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$

$$\leq\left(\frac{1}{6}\right)^{\frac{1}{q}}A^{1-\frac{1}{q}}(a,b)\left[\begin{array}{l}(2b+a)\left|f'(b)\right|^q\\+(2a+b)\left|f'(a)\right|^q\end{array}\right]^{\frac{1}{q}}.$$

**Proposition 2.4**. For $q=1$, we obtain the following inequality

$$\left|\frac{f(b)b-f(a)a}{b-a}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$

$$\leq(2b+a)\left|f'(b)\right|+(2a+b)\left|f'(a)\right|.$$

**Theorem 2.3.** For $\forall n\in\mathbb{N}$; let $f:I\subset[0,\infty)\to\mathbb{R}$ be $n$-times differentiable function on $I^\circ$ and $a,b\in I^\circ$ with $a<b$. If $\left|f^{(n)}\right|^q\in L[a,b]$ and $\left|f^{(n)}\right|^q$ for $q>1$ is $s$-convex function in the first sense on $[a,b]$, then the following inequality holds:

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$

$$\leq\frac{1}{n!}(b-a)\left[\begin{array}{l}\frac{\left|f^{(n)}(b)\right|^q-\left|f^{(n)}(a)\right|^q}{(b-a)^{s+1}}S(n,s,q,x)\\+\left|f^{(n)}(a)\right|^q L_{nq}^n\end{array}\right]^{\frac{1}{q}}$$

Where $S(n,s,q,x)=\int_a^b x^{nq}(x-a)^s dx$.

**Proof:** If $\left|f^{(n)}\right|^q$ for $q>1$ is $s$-convex function the first sense on $[a,b]$, using Lemma1.1 and the Hölder integral inequality, we have the following inequality:

$$\left|\frac{(-1)^{n+1}}{n!}\int_a^b x^n f^{(n)}(x)dx\right|$$

$$\leq\frac{1}{n!}\left(\int_a^b 1^p dx\right)^{\frac{1}{p}}\left(\int_a^b x^{nq}\left|f^{(n)}(x)\right|^q dx\right)^{\frac{1}{q}}$$

$$\leq\frac{1}{n!}\left(\int_a^b 1.dx\right)^{\frac{1}{p}}\left(\int_a^b x^{nq}\left\{\begin{array}{l}\left(\frac{x-a}{b-a}\right)^s\left|f^{(n)}(b)\right|^q\\+\left[1-\left(\frac{x-a}{b-a}\right)^s\right]\left|f^{(n)}(a)\right|^q\end{array}\right\}dx\right)^{\frac{1}{q}}$$

$$=\frac{1}{n!}\left(\int_a^b 1.dx\right)^{\frac{1}{p}}\left(\frac{\left|f^{(n)}(b)\right|^q-\left|f^{(n)}(a)\right|^q}{(b-a)^s}\int_a^b x^{nq}(x-a)^s dx\right.$$

$$\left.+\left|f^{(n)}(a)\right|^q\int_a^b x^{nq}dx\right)^{\frac{1}{q}}$$

$$=\frac{1}{n!}(b-a)^{\frac{1}{p}}\left[\begin{array}{l}\frac{\left|f^{(n)}(b)\right|^q-\left|f^{(n)}(a)\right|^q}{(b-a)^s}S(n,s,q,x)\\+\left|f^{(n)}(a)\right|^q\left(\frac{b^{nq+1}-a^{nq+1}}{nq+1}\right)\end{array}\right]^{\frac{1}{q}}$$

$$=\frac{1}{n!}(b-a)^{\frac{1}{p}}\left[\frac{\left|f^{(n)}(b)\right|^q-\left|f^{(n)}(a)\right|^q}{(b-a)^s}S(n,s,q,x)\right.$$

$$\left.+(b-a)\left|f^{(n)}(a)\right|^q\left[\frac{b^{nq+1}-a^{nq+1}}{(nq+1)(b-a)}\right]\right]^{\frac{1}{q}}$$

$$=\frac{1}{n!}(b-a)\left[\begin{array}{l}\frac{\left|f^{(n)}(b)\right|^q-\left|f^{(n)}(a)\right|^q}{(b-a)^{s+1}}S(n,s,q,x)\\+\left|f^{(n)}(a)\right|^q L_{nq}^n\end{array}\right]^{\frac{1}{q}}$$

**Corollary 2.3**. Under the conditions of Theorem 2.3 for $s=1$ we obtain the inequality

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$

$$\leq\frac{(b-a)}{n!}\left[\begin{array}{l}\frac{\left|f^{(n)}(b)\right|^q-\left|f^{(n)}(a)\right|^q}{b-a}\\\times\left[L_{nq+1}^{nq+1}(a,b)-aL_{nq}^{nq}(a,b)\right]+\left|f^{(n)}(a)\right|^q L_{nq}^{nq}\end{array}\right]^{\frac{1}{q}}.$$

**Proposition 2.5.** Under the conditions of Corollary 2.3 for $n = 1$ we obtain the inequality

$$\left|\frac{f(b)b-f(a)a}{b-a}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$

$$\leq\left[\frac{\left|f'(b)\right|^q-\left|f'(a)\right|^q}{b-a}\begin{pmatrix}L_{q+1}^{q+1}(a,b)\\ -aL_q^q(a,b)\end{pmatrix}+\left|f'(a)\right|^q L_q^q\right]^{\frac{1}{q}}.$$

**Proposition 2.6.** For $q = 1$, we obtain the following inequality

$$\left|\frac{f(b)b-f(a)a}{b-a}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$

$$\leq\frac{1}{6}\left[(2b+a)\left|f'(b)\right|+(2a+b)\left|f'(a)\right|\right].$$

**Theorem 2.4.** For $\forall n \in \mathbb{N}$; let $f: I\subset[0,\infty)\to\mathbb{R}$ be $n$ -times differentiable function on $I^\circ$ and $a,b\in I^\circ$ with $a<b$. If $\left|f^{(n)}\right|^q \in L[a,b]$ and $\left|f^{(n)}\right|^q$ for $q > 1$ is $s$-concave function in the first sense on $[a, b]$, then the following inequality holds:

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$

$$\leq\frac{1}{n!}(b-a)L_{np}^n(a,b)\left|f^{(n)}\left(\frac{a+b}{2}\right)\right|,$$

where $1/p+1/q=1$.

**Proof:** If $\left|f^{(n)}\right|^q$ for $q > 1$ is $s$-concave function the first sense on $[a, b]$, using Lemma1.1, the Hermite-Hadamard inequality and the Hölder integral inequality, we have the following inequality:

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$

$$\leq\frac{1}{n!}\int_a^b x^n\left|f^{(n)}(x)\right|dx$$

$$\leq\frac{1}{n!}\left(\int_a^b x^{np}dx\right)^{\frac{1}{p}}\left(\int_a^b\left|f^{(n)}(x)\right|^q dx\right)^{\frac{1}{q}}$$

$$\leq\frac{1}{n!}\left(\int_a^b x^{np}dx\right)^{\frac{1}{p}}\left((b-a)\left|f^{(n)}\left(\frac{a+b}{2}\right)\right|^q\right)^{\frac{1}{q}}$$

$$=\frac{1}{n!}(b-a)^{\frac{1}{q}}\left(\left.\frac{x^{np+1}}{np+1}\right|_a^b\right)^{\frac{1}{p}}\left|f^{(n)}\left(\frac{a+b}{2}\right)\right|$$

$$=\frac{1}{n!}(b-a)^{\frac{1}{q}}\left(\frac{b^{np+1}-a^{np+1}}{np+1}\right)^{\frac{1}{p}}\left|f^{(n)}\left(\frac{a+b}{2}\right)\right|$$

$$=\frac{1}{n!}(b-a)^{\frac{1}{q}}(b-a)^{\frac{1}{p}}\left[\frac{b^{np+1}-a^{np+1}}{(np+1)(b-a)}\right]^{\frac{1}{p}}\left|f^{(n)}\left(\frac{a+b}{2}\right)\right|$$

$$=\frac{1}{n!}(b-a)L_{np}^n(a,b)\left|f^{(n)}\left(\frac{a+b}{2}\right)\right|.$$

**Corollary 2.4**. Under the conditions of Theorem 2.4 for $n = 1$, we obtain the inequality

$$\left|\frac{f(b)b-f(a)a}{b-a}-\frac{1}{b-a}\int_a^b f(x)dx\right|\leq L_p(a,b)\left|f'\left(\frac{a+b}{2}\right)\right|.$$

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