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First Zagreb Index, F-index and F-coindex of the Line Subdivision Graphs

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Abstract In this paper we investigate first Zagreb index, F-index and F-coindex of the line graph of some chemical graphs using the subdivision concept.

Keywords: *chemical graphs, Zagreb index, F-index, F-coindex*

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1. Introduction

The line graph of a simple graph G, denoted by $L(G)$, is the graph whose vertices correspond to the edges of G such that two vertices of $L(G)$ being adjacent if and only if the corresponding edges of G share a common vertex [see [2,12]]. The *subdivision graph* $S(G)$ of a graph G is obtained from G by deleting every edge uv of G and replacing it by a vertex w of degree 2 that is joined to u and v [see p.151 of [3]]. If $S(G)$ is the *subdivision graph* of a graph G, then the line subdivision of G is $L(S(G))$. Following [16], we can construct the *Line Subdivision* of *a* graph G, as follows:

(i) Replace each vertex $u \in V(G)$ by $K(u)$, the complete graph on $d_G(u)$ vertices;

(ii) There is an edge joining a vertex of $K(u_1)$ and a vertex of $K(u_2)$ in $L(S(G))$ if and only if there is an edge joining u_1 and u_2 in G;

(iii) For each vertex v of $K(u)$, $d_{L(S(G))}(v) = d_G(u)$.

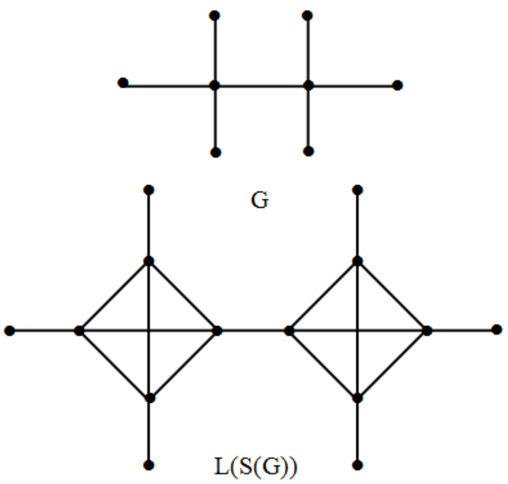

Figure 1. Hydrocarbon graph G and its Line subdivision

A Hydrocarbon graph G and its line subdivision is shown in Figure 1.

Topological indices are numbers associated with molecular graphs for the purpose of allowing quantitative structure-activity/property relationships. Topological indices correlate certain Physico-Chemical properties like boiling point, stability, strain energy etc of chemical compounds. One of the oldest most popular and extremely studied topological indices are well–known Zagreb indices first introduced in 1972 by *Gutman* and *Trinajestic* [8].

Let G be a simple graph and let $V(G)$ and $E(G)$ be its vertex and edge sets, respectively. The edge connecting the vertices u and v will be denoted by uv. The complement $\bar{G}$ of the graph G is the graph with vertex set $V(G)$, in which two vertices in $\bar{G}$ are adjacent if and only if they are not adjacent in G. The degree of the vertex v, denoted by $d_G(v)$, is the number of first neighbors of v in the underlying graph G. Then the first and second Zagreb index are defined as

$$M_1(G) = \sum_{v \in V(G)} d_G^2(v) \tag{1.1}$$

and

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v)$$

respectively.

There is another expression for the first Zagreb index namely

$$M_1(G) = \sum_{uv \in E(G)} \left[d_G(u) + d_G(v)\right]. \tag{1.2}$$

In 2008, bearing in mind Eq. (1.2), Doslic in [6] put forward the first Zagreb coindex, defined as

$$\bar{M}_1(G) = \sum_{uv \notin E(G)} \left[d_G(u) + d_G(v)\right].$$

Recently, *Furtula* and *Gutman* [7] introduced a new topological index and named this index as forgotten topological index. They showed that the predictive ability of this index is almost similar to that of first Zagreb index. Throughout the present paper we name this index as F-index and denote it by $F(G)$, so

$$F(G)=\sum_{v\in V(G)} d_G^3(v). \tag{1.3}$$

There is another expression for the F-index namely

$$F(G)=\sum_{uv\in E(G)} \left[d_G^2(u)+d_G^2(v)\right].$$

Similar to the first Zagreb coindex, the F-coindex of a graph G is defined as

$$\bar{F}(G)=\sum_{uv\notin E(G)} \left[d_G^2(u)+d_G^2(v)\right].$$

For more details on the topological indices and coindices we refer to the articles [1,4,9,10,11,13,18].

In 2011, Ranjini et al. calculated the explicit expressions for the Shultz indices of the subdivision graphs of the Tadpole, Wheel, Helm and Ladder graphs [15]. They also studied the Zagreb indices of the line graphs of the Tadpole, Wheel and Ladder graphs with subdivision in [14]. In 2015, Su and Xu calculated the general Sum-connectivity indices and coindices of the line graphs of the Tadpole, Wheel and Ladder graphs with subdivision in [17]. In [12], Nadeem et al. computed ABC_4 and GA_5 indices of the line graphs of the Tadpole, Wheel and Ladder graphs by using the notion of subdivision.

In this paper we compute first Zagreb index, F-index and F-coindex of Dandelion graph $D(n,l)$, Comet graph $C(n,l)$, Fence graph $P_n[P_2]$, Closed fence graph $C_n[P_2]$, Friendship graph F_m, t-fold bristled of C_n and t-fold bristled of P_n, Tadpole graph $T_{n,k}$, Wheel W_{n+1} and Ladder graph L_n.

2. Main Results

We begin with a lemma used in the proof of our results.

Lemma 2. 1. ([5]) *Let G be a simple graph with n vertices and m edges. Then*

$$\bar{F}(G)=(n-1)M_1(G)-F(G).$$

Let $D(n,l)$ be a Dandelion graph with n vertices consisted of a copy of the star S_{n-1} and a copy of the path P_l with vertices $P_0,P_1,\ldots,P_{l-1}$, where P_0 is identified with a star center. (See Figure 2)

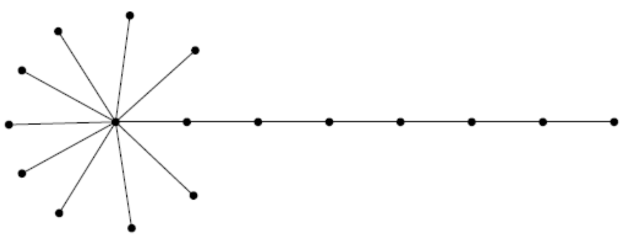

Figure 2. Dandelion graph $D(17,8)$

Theorem 2.2. *Let G be the line graph of the subdivision graph of a Dandelion graph. Then*

$$M_1(G)=(n-l+1)^3+(n-l+1)+8(l-2)$$

$$F(G)=(n-l+1)^4+(n-l+1)+16(l-2)$$

$$\bar{F}(G)=(2n-2)\left[(n-l+1)^3+(n-l+1)+8(l-2)\right]$$
$$-[(n-l+1)^4+(n-l+1)+16(l-2)].$$

Proof. The number of vertices in G are $2n-2$ among which $n-l+1$ vertices are of degree $n-l+1$, $n-l+1$ vertices are of degree 1 and the remaining $2l-4$ vertices are of degree 2. Using Eqs.(1.1) and (1.3) and lemma 2.1 we have,

$$M_1(G)=(n-l+1)(n-l+1)^2+(n-l+1)(1)^2+(2l-4)(2)^2$$
$$=(n-l+1)^3+(n-l+1)+8(l-2)$$

$$F(G)=(n-l+1)(n-l+1)^3+(n-l+1)(1)^3+(2l-4)(2)^3$$
$$=(n-l+1)^4+(n-l+1)+16(l-2).$$

$$\bar{F}(G)=(2n-2)\left[(n-l+1)^3+(n-l+1)+8(l-2)\right]$$
$$-\left[(n-l+1)^4+(n-l+1)+16(l-2)\right].$$

For a positive integer $l\le n$ let $C(n,l)$ be a Comet graph with n vertices consisted of a copy of the Complete graph K_{n-l+1} and a copy of the Path P_l with vertices $P_0,P_1,\ldots,P_{l-1}$, where P_0 is identified with a vertex from K_{n-l+1}. (See Figure 3)

Theorem 2.3. *Let G be the line graph of the subdivision graph of a Comet graph, then*

$$M_1(G)=(n-l+1)^3+(n-l)^4+4l-3$$

$$F(G)=(n-l+1)^4+(n-l)^5+8l-7$$

$$\bar{F}(G)=\left[(n-l)^2+n+l-3\right]$$
$$\times\left[(n-l+1)^3+(n-l)^4+4l-3\right]$$
$$-[(n-l+1)^4+(n-l)^5+8l-7].$$

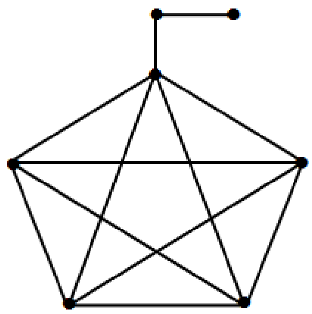

Figure 3. Comet graph $C(7,3)$

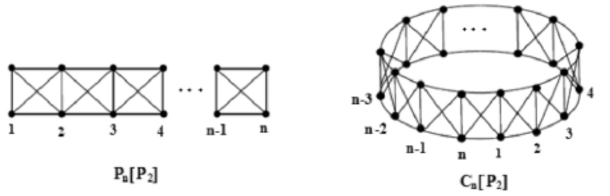

Figure 4. Fence and Closed fence graph

Proof. For $l = 2$ the proof is easy, so we consider the case $l > 2$. The subdivision graph $S(C(n,l))$ contains $(n-l)^2 + n + l - 2$ edges, so G contain $(n-l)^2 + n + l - 2$ vertices among which, $n - l + 1$ vertices are of degree $n - l + 1$, $(n-l)^2$ vertices are of degree $n - l$, $l - 1$ vertices are of degree 2 and one vertex is of degree 1. Now, using Eqs. (1.1) and (1.3) and lemma 2.1 we have the proof.

Theorem 2.4. *Let G be the line graph of the subdivision graph of a Fence graph* $P_n[P_2]$ *(See Figure 4), then*

$$M_1(G) = 250n - 392$$

$$F(G) = 1250n - 2176$$

$$\overline{F}(G) = 2500n^2 - 7420n + 5703.$$

Proof. The number of vertices in G are $12 + 10(n-2)$ among which $10(n-2)$ vertices are of degree 5 and 12 vertices are of degree 3. Thus, using Eqs. (1.1) and (1.3) and lemma 2.1 we have the proof.

A Friendship graph (or Dutch windmill graph) F_m is a graph with *2m+1* vertices and *3m* edges constructed by joining *m* copies of the cycle graph C_3 with a common vertex.(See Figure 5)

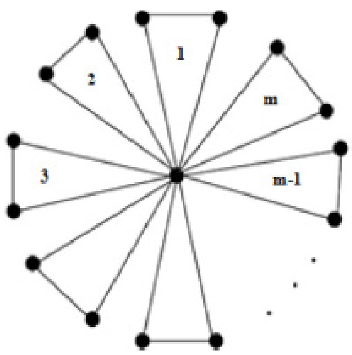

Figure 5. Friendship graph F_m

Theorem 2.5. *Let G be the line graph of the subdivision graph of a Friendship graph* F_m, *then*

$$M_1(G) = 8m(m^2 + 2)$$

$$F(G) = 16m(m^3 + 2)$$

$$\overline{F}(G) = 8m(4m^3 - m^2 + 12m - 6).$$

Proof. The number of vertices in G are $6m$ among which $2m$ vertices are of degrees $2m$, and $4m$ vertices are of degree 2. Hence, using Eqs. (1.1) and (1.3) and lemma 2.1 we can get the proof.

For a given graph *G*, its t-fold bristled graph *Brst(G)* is obtained by attaching *t* vertices of degree one to each vertex of *G*.(See Figure 6)

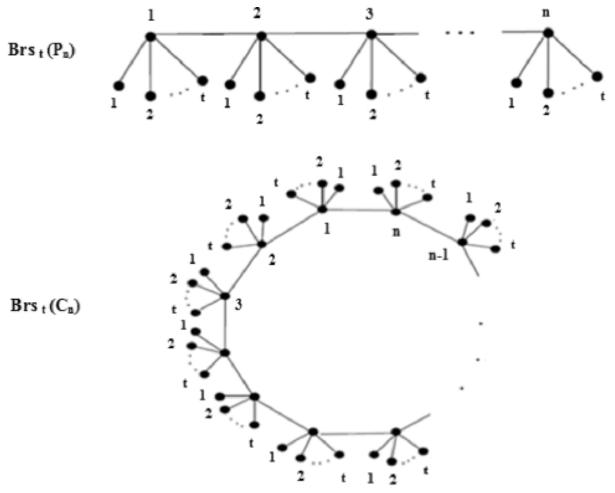

Figure 6. t-fold bristled graph of P_n and C_n

Theorem 2.6. *Let G be the line graph of the subdivision of a t-fold bristled graph of* C_n, *then*

$$M_1(G) = tn + (2n + tn)(t+2)^2$$

$$F(G) = tn + (2n + tn)(t+2)^3$$

$$\overline{F}(G) = (2n + 2nt - 1)\left[tn + (2n + tn)(t+2)^2\right] - [tn + (2n + tn)(t+2)^3].$$

Proof. The number of vertices of G are $2n(1 + t)$, among which tn vertices are of degree one and $2n + tn$ vertices are of degree $t + 2$. Thus, using Eqs.(1.1) and (1.3) and lemma 2.1 we have the proof.

With reference to the above theorems, the proof of next theorems are easy, so we omit the proofs.

Theorem 2.7. *Let G be the line graph of the subdivision of a Closed fence graph* $C_n[P_2]$ *(See Figure 4), then*

$$M_1(G) = 250(n-1)$$

$$F(G) = 1250(n-1)$$

$$\overline{F}(G) = 2500n^2 - 6500n + 400.$$

Theorem 2.8. *Let G be the line graph of the subdivision of a t-fold bristled graph of* P_n *(See Figure 6), then*

$$M_1(G) = t(tn - 4) + n(5t + 4) - 6$$

$$F(G) = nt\left(t^2 + 6t + 13\right) - 6t(t+3) + 8n - 14$$

$$\overline{F}(G) = nt\left(nt^2 - 11t + 6tn - t^2 + 9n - 28\right) + 4n^2 - 18n + 6t^2 + 18t + 20.$$

Theorem 2.9. *Let G be the line graph of the subdivision graph of Wheel* W_{n+1}, *then*

$$M_1(G) = n^3 + 27n$$

$$F(G) = n(n^3 + 81)$$

$$\overline{F}(G) = 2n(3n^3 - n^2 + 108n - 108).$$

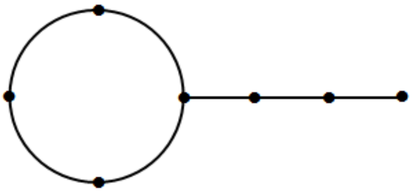

Figure 7. Tadpole graph $T_{4,3}$

A Tadpole graph $T_{n,k}$ is a special type of graph consisting of a Cycle graph with *n* (at least 3) vertices and a Path graph with *k* vertices, connected with a bridge. (See Figure 7)

Theorem 2.10. *Let G be the line graph of the subdivision graph of a Ttadpole graph $T_{n,k}$, then*

$$M_1(G) = 4(2n+2k+3)$$

$$F(G) = 16n+16k+50$$

$$\overline{F}(G) = 4(2n+2k-1)(2n+2k+3)-16(n+k)-50.$$

A Ladder graph L_n is a graph obtained as the Cartesian Product of two path graphs, one of which has only one edge(See Figure 8).

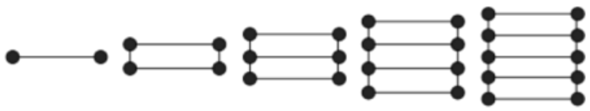

Figure 8. Ladder graphs L_1, L_2, L_3, L_4 and L_5

Theorem 2.11. *Let G be the line graph of the subdivision graph of a Ladder graph L_n, then*

$$M_1(G) = 54n-76$$

$$F(G) = 162n-260$$

$$\overline{F}(G) = 324n^2-888n+640.$$

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