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New Approach of *F*-Contraction Involving Fixed Point on a Closed Ball

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Abstract The article is written with a view to introducing the new idea of *F*-contraction on a closed ball and have new theorems in a complete metric space. That is why this outcome becomes useful for the contraction of the mapping on a closed ball instead of the whole space. At the same time, some comparative examples are constructed which establish the superiority of our results. It can be stated that the results that have come into being give proof of extension as well as substantial generalizations and improvements of several well known results in the existing comparable literature.

Keywords: *metric space, fixed point, F contraction, closed ball*

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1. Introduction

Shoaib et al. [40] proved significant results concerning the existence of fixed points of the dominated self mappings satisfying some contractive conditions on a closed ball in a 0-complete quasi-partial metric space. Other results on closed ball can be seen in [5,6,7,8,25]. Over the years, Fixed Point Theory has been generalized in different ways by several mathematicians (see [2,3,4,9-15,17-21,23-27]).

For $x \in X$ and $\varepsilon > 0$, $\overline{B(x,\varepsilon)} = \{y \in X : d(x,y) \le \varepsilon\}$ is a closed ball in (X,d).

Definition 1 [18] *Let (X,d) be a metric space. Let T be self mappings and $\alpha,\eta : X \times X \to [0,+\infty)$ be two functions. T is called $\alpha-\eta$ -continuous if for given $x \in X$, and sequence $\{x_n\}$ with*

$$x_n \to x \text{ as } n \to \infty, \alpha(x_n, x_{n+1}) \ge \eta(x_n, x_{n+1})$$
$$\text{for all } n \in \mathbb{N} \Rightarrow Tx_n \to Tx.$$

In 2012, Wardowski [42] introduce a concept of *F*-contraction as follows:

Definition 2 [33] *Let (X,d) be a metric space. A self mapping T is said to be an F contraction if there exists $\tau > 0$ such that*

$$\forall x, y, d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \le F(d(x,y)), \tag{1.1}$$

where $F : \mathbb{R}_+ \to \mathbb{R}$ is a mapping satisfying the following conditions:

(F1) F is strictly increasing, i.e. for all $x, y \in \mathbb{R}_+$ such that $x < y$, $F(x) < F(y)$;

(F2) For each sequence $\{\alpha_n\}_{n=1}^{\infty}$ of positive numbers, $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$;

(F3) There exists $\kappa \in (0,1)$ such that $\lim \alpha \to 0^+ \alpha^k F(\alpha) = 0$.

We denote by Δ_F, the set of all functions satisfying the conditions (F1)-(F3).

Furthermore, it was done by different investigators see [1,3,15,16,23,27,28,29,31,32,33,34,37,38,39].

Hussain et al. [18] introduced the following family of new functions.

Let Δ_G denotes the set of all functions $G : \mathbb{R}^{+4} \to \mathbb{R}^+$ satisfying:

(G) for all $t_1, t_2, t_3, t_4 \in \mathbb{R}^+$ with $t_1t_2t_3t_4 = 0$ there exists $\tau > 0$ such that $G(t_1,t_2,t_3,t_4) = \tau$.

2. Banach Fixed Point Theorem for *F*-Contraction on a Closed Ball

In this section, we introduce Banach fixed point theorem for modified *F*-contraction on a closed ball in complete metric spaces.

Now we state our main result.

Theorem 3 *Let T be a continuous self mapping in a complete metric space (X,d) and x_0 be an arbitrary point in $X, r > 0$. Assume that $\tau > 0$ and $F \in \Delta_F$ for all $x, y \in \overline{B(x_0,r)} \subseteq X$ with $d(Tx,Ty) > 0$ such that*

$$\tau + F\left(d\left(Tx, Ty\right)\right) \le F\left(kd\left(x, y\right)\right), \text{ where } 0 \le k < 1. \quad (2.1)$$

Morever

$$d\left(x_0, Tx_0\right) \le (1-k)r. \quad (2.2)$$

Then there exist a point x^* *in* $\overline{B(x_0, r)}$ *such that* $Tx^* = x^*$.

Proof. Choose a point x_1 in X such that $x_1 = Tx_0$. continuing in this way, so we get $x_{n+1} = Tx_n$, for all $n \ge 0$ and this implies that (x_n) is a nonincreasing sequence. First we show that $x_n \in \overline{B(x_0, r)}$ for all $n \in N$ by using mathematical induction. Since from (2.2), we have

$$d\left(x_0, x_1\right) = d\left(x_0, Tx_0\right) \le (1-k)r < r. \quad (2.3)$$

thus, $x_1 \in \overline{B(x_0, r)}$. Suppose $x_2, ..., x_j \in \overline{B(x_0, r)}$ for some $j \in N$. Thus from (2.1), we obtain

$$F\left(d\left(x_j, x_{j+1}\right)\right) = F\left(d\left(Tx_{j-1}, Tx_j\right)\right) \le F\left(kd\left(x_{j-1}, x_j\right)\right) - \tau.$$

As F is strictly increasing, we have

$$d\left(x_j, x_{j+1}\right) < kd\left(x_{j-1}, x_j\right). \quad (2.4)$$

Now,

$$\begin{aligned} d\left(x_0, x_{j+1}\right) &\le d\left(x_0, x_1\right) + ... + d\left(x_j, x_{j+1}\right) \\ &< d\left(x_0, x_1\right)\left[1 + k + ... + k^j\right] \\ &\le (1-k)r\frac{\left(1-k^{j+1}\right)}{1-k} < r. \end{aligned}$$

Thus $x_{j+1} \in \overline{B(x_0, r)}$. Hence $x_n \in \overline{B(x_0, r)}$ for all $n \in N$. Continuing this process, we get

$$\begin{aligned} F\left(d\left(x_n, x_{n+1}\right)\right) &\le F\left(kd\left(x_{n-1}, x_n\right)\right) - \tau \\ &\le F\left(d\left(Tx_{n-2}, Tx_{n-1}\right)\right) - \tau \\ &\le F\left(kd\left(x_{n-2}, x_{n-1}\right)\right) - 2\tau \\ &\le F\left(d\left(Tx_{n-3}, Tx_{n-2}\right)\right) - 2\tau \\ &\le F\left(d\left(x_{n-3}, x_{n-2}\right)\right) - 3\tau \\ &\vdots \\ &\le F\left(d\left(x_0, x_1\right)\right) - n\tau. \end{aligned}$$

This implies that

$$F\left(d\left(x_n, x_{n+1}\right)\right) \le F\left(d\left(x_0, x_1\right)\right) - n\tau. \quad (2.5)$$

From (2.5), we obtain $\lim_{n\to\infty} F\left(d\left(x_n, x_{n+1}\right)\right) = -\infty$. Since $F \in \Delta_F$, we have

$$\lim_{n\to\infty} d\left(x_n, x_{n+1}\right) = 0. \quad (2.6)$$

From $(F3)$, there exists $\kappa \in (0,1)$ such that

$$\lim_{n\to\infty}\left(\left(d\left(x_n, x_{n+1}\right)\right)^\kappa F\left(d\left(x_n, x_{n+1}\right)\right)\right) = 0. \quad (2.7)$$

From (2.5), for all $n \in \mathbb{N}$, we obtain

$$\begin{aligned} &\left(d\left(x_n, x_{n+1}\right)\right)^\kappa \left(F\left(d\left(x_n, x_{n+1}\right)\right) - F\left(d\left(x_0, x_1\right)\right)\right) \\ &\le -\left(d\left(x_n, x_{n+1}\right)\right)^\kappa n\tau \le 0. \end{aligned} \quad (2.8)$$

By using (2.6), (2.7) and letting $n \to \infty$, in (2.8), we have

$$\lim_{n\to\infty}\left(n\left(d\left(x_n, x_{n+1}\right)\right)^\kappa\right) = 0. \quad (2.9)$$

We observe that from (2.9), then there exists $n_1 \in \mathbb{N}$, such that $n\left(d\left(x_n, x_{n+1}\right)\right)^\kappa \le 1$ for all $n \ge n_1$, we get

$$d\left(x_n, x_{n+1}\right) \le \frac{1}{n^{\frac{1}{\kappa}}} \text{ for all } n \ge n_1. \quad (2.10)$$

Now, $m, n \in \mathbb{N}$ such that $m > n \ge n_1$. Then, by the triangle inequality and from (2.10) we have

$$\begin{aligned} &d\left(x_n, x_m\right) \\ &\le d\left(x_n, x_{n+1}\right) + d\left(x_{n+1}, x_{n+2}\right) + ... + d\left(x_{m-1}, x_m\right) \\ &= \sum_{i=n}^{m-1} d\left(x_i, x_{i+1}\right) \le \sum_{i=n}^{\infty} d\left(x_i, x_{i+1}\right) \le \sum_{i=n}^{\infty} \frac{1}{i^{\frac{1}{k}}}. \end{aligned} \quad (2.11)$$

The series $\sum_{i=n}^{\infty} \frac{1}{i^{\frac{1}{\kappa}}}$ is convergent. By taking the limit as $n \to \infty$, in (2.11), we have $\lim_{n,m\to\infty} d\left(x_n, x_m\right) = 0$. Hence $\{x_n\}$ is a Cauchy sequence. Since X is a complete metric space there exists $x^* \in \overline{B(x_0, r)}$ such that $x_n \to x^*$ as $n \to \infty$. T is a continuous then $x_{n+1} = Tx_n \to Tx^*$ as $n \to \infty$. That is, $x^* = Tx^*$. Hence x^* is a fixed point of T. To prove uniqueness, let $x, y \in \overline{B_p(x_0, r)}$ and $x \ne y$ be any two fixed point of T, then from (2.1), we have

$$\tau + F\left(d\left(Tx, Ty\right)\right) \le F\left(kd\left(x, y\right)\right)$$

we obtain

$$\tau + F\left(d\left(x, y\right)\right) \le F\left(d\left(x, y\right)\right)$$

which is a contradiction. Hence, $x = y$. Therefore, T has a unique fixed point in $\overline{B(x_0, r)}$.

Example 4 *Let* $X = \mathbb{R}^+$ *and* $d(x, y) = |x - y|$. *Then* (X, d) *is a complete metric space. Define the mapping* $T : X \to X$ *by,*

$$T(x) = \begin{cases} \dfrac{x}{4} & \text{if } x \in [0,1] \\ x - \dfrac{1}{2} & \text{if } x \in (1, \infty). \end{cases}$$

$x_0 = 1$, $r = 2$, $\overline{B(x_0, r)} = [0,1]$. *If* $F(\alpha) = \ln \alpha$, $\alpha > 0$ *and* $\tau > 0$, *then*

$$d(1, T1) = \left|1 - \frac{1}{4}\right| = \frac{3}{4} < r.$$

If $x, y \in \overline{B(x_0, r)}$, *then*

$$\frac{1}{4}|x-y| < \frac{1}{2}|x-y|$$

$$\frac{x}{4} - \frac{y}{4} < k|x-y|$$

$$d(Tx, Ty) < kd(x, y).$$

This implies that

$$\tau + F(d(Tx,Ty)) = \tau + \ln d(Tx,Ty) \\ \le \ln kd(x,y) = F(kd(x,y)).$$

If $x, y \in (1, \infty)$, *then*

$$\left|x - \frac{1}{2} - y + \frac{1}{2}\right| = |x-y|$$

$$\tau + |Tx - Ty| > |x-y|$$

$$\tau + F(d(Tx,Ty)) > F(d(x,y)).$$

Hence the contraction does not satisfy on X.

3. Fixed Point Theoreminvolving *GF*-Contraction on a Closed Ball

In this section, we define a new contraction called $\alpha - \eta$ -*GF*-contraction on a closed ball and obtained Banach fixed point theorems for such contraction in the setting of complete metric spaces. We define $\alpha - \eta$ -*GF*-contraction on a closed ball as follows:

Definition 5 *Let* T *be a self mapping in a metric space* (X, d) *and* x_0 *an arbitrary point in* X *with* $r > 0$. *Also suppose that* $\alpha : X \times X \to \{-\infty\} \cup (0, +\infty)$, $\eta : X \times X \to \mathbb{R}^+$ *two functions. We say that* T *is called* $\alpha - \eta$ *-GF-contraction on a closed ball if for all* $x, y \in \overline{B(x_0, r)} \subseteq X$, *with* $\eta(x, Tx) \le \alpha(x, y)$ *and* $d(Tx, Ty) > 0$, *we have*

$$G(d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)) \\ + F(d(Tx,Ty)) \le F(kd(x,y)), \tag{3.1}$$

and

$$d(x_0, Tx_0) \le (1-k)r, \tag{3.2}$$

where $0 \le k < 1$, $G \in \Delta_G$ *and* $F \in \Delta_F$.

Definition 6 ([36]). *Let* $T : X \to X$ *and* $\alpha, \eta : X \times X \to [0, +\infty)$ *be two functions. We say that* T *is* α *-admissible mapping with respect to* η *if* $x, y \in X$, $\alpha(x, y) \ge \eta(x, y)$ *implies that* $\alpha(Tx, Ty) \ge \eta(Tx, Ty)$.

Theorem 7 *Let* (X, d) *be a complete metric space. Let* $T : X \to X$ *be* $\alpha - \eta -$ *GF-contraction mapping on a closed ball satisfying the following assertions:*

(i) T is an α -admissible mapping with respect to η;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \ge \eta(x_0, Tx_0)$;

(iii) T is an $\alpha - \eta -$ continuous.

Then there exist a point x^* in $\overline{B(x_0, r)}$ such that $Tx^* = x^*$.

Proof. Let x_0 in X such that $\alpha(x_0, Tx_0) \ge \eta(x_0, Tx_0)$. For $x_0 \in X$, we construct a sequence $\{x_n\}_{n=1}^{\infty}$ such that $x_1 = Tx_0, x_2 = Tx_1 = T^2 x_0$. Continuing this way, $x_{n+1} = Tx_n = T^{n+1} x_0$, for all $n \in \mathbb{N}$. Now since, T is an α -admissible mapping with respect to η then $\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \ge \eta(x_0, Tx_0) = \eta(x_0, x_1)$. By continuing in this process we have,

$$\eta(x_{n-1}, Tx_{n-1}) = \eta(x_{n-1}, x_n) \\ \le \alpha(x_{n-1}, x_n), \text{ for all } n \in \mathbb{N}. \tag{3.3}$$

If there exists $n \in \mathbb{N}$ such that $d(x_n, Tx_n) = 0$, there is nothing to prove. So, we assume that $x_n \ne x_{n+1}$ with

$$d(Tx_{n-1}, Tx_n) = d(Tx_{n-1}, Tx_n) > 0, \forall n \in \mathbb{N}.$$

First we show that $x_n \in \overline{B(x_0, r)}$ for all $n \in N$. Since T be a $\alpha - \eta - GF$-contraction on closed ball, we have

$$d(x_0, x_1) = d(x_0, Tx_0) \le (1-k)r < r. \tag{3.4}$$

thus, $x_1 \in \overline{B(x_0, r)}$. Suppose $x_2, ..., x_j \in \overline{B(x_0, r)}$ for some $j \in N$, such that

$$G(d(x_{j-1}, Tx_{j-1}), d(x_j, Tx_j), d(x_{j-1}, Tx_j), d(x_j, Tx_{j-1})) \\ + F(d(Tx_{j-1}, Tx_j)) \le F(kd(x_{j-1}, x_j)),$$

which implies

$$G(d(x_{j-1}, x_j), d(x_j, x_{j+1}), d(x_{j-1}, x_{j+1}), 0) \\ + F(d(Tx_{j-1}, Tx_j)) \le F(kd(x_{j-1}, x_j)). \tag{3.5}$$

Now by definition of G,

$$d(x_{j-1}, x_j).d(x_j, x_{j+1}).d(x_{j-1}, x_{j+1}).0 = 0,$$

so there exists $\tau > 0$ such that,

$$G(d(x_{j-1}, x_j).d(x_j, x_{j+1}).d(x_{j-1}, x_{j+1}).0) = \tau.$$

Therefore

$$F(d(x_j, x_{j+1})) = F(d(Tx_{j-1}, Tx_j)) \\ \le F(kd(x_{j-1}, x_j)) - \tau. \tag{3.6}$$

Rest of the proof follows the similar lines of Theorem 3. Since X is a complete metric space there exists

$x^* \in \overline{B(x_0,r)}$ such that $x_n \to x^*$ as $n \to \infty$. T is an $\alpha-\eta-$continuous and $\eta(x_{n-1},x_n) \le \alpha(x_{n-1},x_n)$, for all $n \in \mathbb{N}$ then $x_{n+1} = Tx_n \to Tx^*$ as $n \to \infty$. That is, $x^* = Tx^*$. Hence x^* is a fixed point of T.

Example 8 *Let $X = \mathbb{R}^+$ and d be the usual metric on X. define $T: X \to X$, $\alpha: X \times X \to \{-\infty\} \cup (0,+\infty)$, $\eta: X \times X \to \mathbb{R}^+$, $G: (\mathbb{R}^+)^4 \to \mathbb{R}^+$ and $F: \mathbb{R}^+ \to \mathbb{R}$ by*

$$Tx = \begin{cases} \sqrt{x} & \text{if } x \in [0,1], \\ 2x & \text{if } x \in (1,\infty) \end{cases} \quad \alpha(x,y) = \begin{cases} e^{x+y} & \text{if } x \in [0,1], \\ \frac{1}{3} & \text{otherwise} \end{cases}$$

$\eta(x,y) = \frac{1}{2}$ for all $x, y \in X$, $G(t_1,t_2,t_3,t_4) = \tau > 0$ and $F(t) = \ln t$ with $t > 0$. $x_0 = \frac{1}{2}, r = 1, \overline{B(x_0,r)} = [0,1]$, *then*

$$d\left(\frac{1}{2}, T\frac{1}{2}\right) = \left|\frac{1}{2} - \frac{1}{\sqrt{2}}\right| = 0.20710 < r.$$

If $x, y \in \overline{B(x_0,r)}$, then $\alpha(x,y) = e^{x+y} \ge \frac{1}{2} = \eta(x,y)$. On the other hand, $Tx \in [0,1]$ for all $x \in [0,1]$. Then $\alpha(Tx,Ty) \ge \eta(x,Tx)$ with $d(Tx,Ty) = \left|\sqrt{x} - \sqrt{y}\right| > 0$, clearly $\alpha(0,T0) \ge \eta(0,T0)$. Hence we have

$$d(Tx,Ty) = \left|\frac{\sqrt{x} - \sqrt{y} \times \sqrt{x} + \sqrt{y}}{\sqrt{x} + \sqrt{y}}\right|$$

$$= \left|\frac{x-y}{\sqrt{x} + \sqrt{y}}\right| < k|x-y|.$$

Consequently,

$$\tau + F(d(Tx,Ty)) = \tau + \ln d(Tx,Ty)$$
$$\le \ln kd(x,y) = F(kd(x,y)).$$

If $x \notin \overline{B(x_0,r)}$ or $y \notin \overline{B(x_0,r)}$ then $\alpha(x,y) = \frac{1}{3} \not\ge \frac{1}{2}$ $= \eta(x,y)$, either

$$2|x-y| > |x-y|$$

$$|2x-2y| > |x-y|$$

$$|Tx-Ty| > |x-y|$$

$$\tau + F(d(Tx,Ty)) \ge F(d(x,y)).$$

On the other hand the contraction does not satisfy.

4. Conclusion

This research focus on introducing new idea of F-contraction on a closed ball which is different from F-contraction given in [18,33,42]. Therefore a generalization of results is very useful so far as it requires the F-contraction mapping only on a closed ball rather than the whole space. This new idea however guides the researcher towards further investigations and applications. At the same time, it will be interesting to apply these concepts in a various spaces.

Conflict of Interests

The author declare that he has no competing interests.

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