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On an Integral Involving Bessel Polynomials and $\overline{H}$-Function of Two Variables and Its Application

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Abstract This paper deals with the evaluation of an integral involving product of Bessel polynomials and $\overline{H}$-function of two variables. By making use of this integral the solution of the time-domain synthesis problem is investigated.

Keywords: *$\overline{H}$-function of two variables, Bessel polynomials, Mellin-Barnes type integral, Time-domain synthesis problem, H-function of two variables*

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1. Introduction

The object of this paper is to evaluate an integral involving Bessel polynomials and the $\overline{H}$-function of two variables due to Singh and Mandia [8], and to apply it in obtaining a particular solution of the classical problem known as the 'time-domain synthesis problem', occurring in the electric network theory. On specializing the parameters, the $\overline{H}$-function of two variables may be reduced to almost all elementary functions and special functions appearing in applied Mathematics Erdelyi, A. et. al. ([2], p.215-222). The special solution derived in the paper is of general character and hence may encompass several cases of interest.

The $\overline{H}$-function of two variables will be defined and represented by Singh and Mandia [8] in the following manner:

$$\overline{H}[x,y]=\overline{H}\left[\begin{smallmatrix}x\\y\end{smallmatrix}\right]$$

$$=\overline{H}^{o,n_1:\ m_2,n_2:m_3,n_2}_{p_1,q_1:p_2,q_2;p_2,q_2}\left[\begin{matrix}x\\y\end{matrix}\left|\begin{matrix}\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},\\(c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3},\\(e_j,E_j)_{n_3+1,p_3}\end{matrix}\right]\\\left[\begin{matrix}(b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2},\\(d_j,\delta_j;L_j)_{m_2+1,q_2},\\(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3}\end{matrix}\right]\end{matrix}\right.\right] \tag{1.1}$$

$$=-\frac{1}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\phi_2(\xi)\phi_3(\eta)x^{\xi}y^{\eta}\,d\xi\,d\eta$$

Where

$$\phi_1(\xi,\eta)=\frac{\prod_{j=1}^{n_1}\Gamma\left(1-a_j+\alpha_j\xi+A_j\eta\right)}{\left[\prod_{j=n_1+1}^{p_1}\Gamma\left(a_j-\alpha_j\xi-A_j\eta\right)\prod_{j=1}^{q_1}\Gamma\left(1-b_j+\beta_j\xi+B_j\eta\right)\right]} \tag{1.2}$$

$$\phi_2(\xi)=\frac{\prod_{j=1}^{n_2}\left\{\Gamma\left(1-c_j+\gamma_j\xi\right)\right\}^{K_j}\prod_{j=1}^{m_2}\Gamma\left(d_j-\delta_j\xi\right)}{\prod_{j=n_2+1}^{p_2}\Gamma\left(c_j-\gamma_j\xi\right)\prod_{j=m_2+1}^{q_2}\left\{\Gamma\left(1-d_j+\delta_j\xi\right)\right\}^{L_j}} \tag{1.3}$$

$$\phi_3(\eta)=\frac{\prod_{j=1}^{n_3}\left\{\Gamma\left(1-e_j+E_j\eta\right)\right\}^{R_j}\prod_{j=1}^{m_3}\Gamma\left(f_j-F_j\eta\right)}{\prod_{j=n_3+1}^{p_3}\Gamma\left(e_j-E_j\eta\right)\prod_{j=m_3+1}^{q_3}\left\{\Gamma\left(1-f_j+F_j\eta\right)\right\}^{S_j}} \tag{1.4}$$

Where x and y are not equal to zero (real or complex), and an empty product is interpreted as unity p_i,q_i,n_i,m_j are non-negative integers such that $0\le n_i\le p_i, o\le m_j\le q_j (i=1,2,3; j=2,3)$. All the $a_j(j=1,2,...,p_1), b_j(j=1,2,...,q_1), c_j(j=1,2,...,p_2)$, $d_j(j=1,2,...,q_2)$, $e_j(j=1,2,...,p_3), f_j(j=1,2,...,q_3)$ are complex parameters. $\gamma_j\ge 0(j=1,2,...,p_2), \delta_j\ge 0(j=1,2,...,q_2)$ (not all zero simultaneously), similarly $E_j\ge 0(j=1,2,...,p_3)$,

$F_j \geq 0 (j=1,2,...,q_3)$ (not all zero simultaneously). The exponents $K_j(j=1,2,...,n_3), L_j(j=m_2+1,...,q_2)$, $R_j(j=1,2,...,n_3), S_j(j=m_3+1,...,q_3)$ can take on non-negative values.

The contour L_1 is in ξ-plane and runs from $-i\infty$ to $+i\infty$. The poles of $\Gamma(d_j-\delta_j\xi)(j=1,2,...,m_2)$ lie to the right and the poles of $\Gamma\left\{(1-c_j+\gamma_j\xi)\right\}^{K_j}$ $(j=1,2,...,n_2)$, $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)(j=1,2,...,n_1)$ to the left of the contour. For $K_j(j=1,2,...,n_2)$ not an integer, the poles of gamma functions of the numerator in (1.3) are converted to the branch points.

The contour L_2 is in η-plane and runs from $-i\infty$ to $+i\infty$. The poles of $\Gamma(f_j-F_j\eta)(j=1,2,...,m_3)$ lie to the right and the poles of $\Gamma\left\{(1-e_j+E_j\eta)\right\}^{R_j}$ $(j=1,2,...,n_3)$, $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)(j=1,2,...,n_1)$ to the left of the contour. For $R_j(j=1,2,...,n_3)$ not an integer, the poles of gamma functions of the numerator in (1.4) are converted to the branch points.

The functions defined in (1.1) is an analytic function of x and y, if

$$U=\sum_{j=1}^{p_1}\alpha_j+\sum_{j=1}^{p_2}\gamma_j-\sum_{j=1}^{q_1}\beta_j-\sum_{j=1}^{q_2}\delta_j<0 \quad (1.5)$$

$$V=\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_3}F_j<0 \quad (1.6)$$

The integral in (1.1) converges under the following set of conditions:

$$\begin{aligned}\Omega=&\sum_{j=1}^{n_1}\alpha_j-\sum_{j=n_1+1}^{p_1}\alpha_j+\sum_{j=1}^{m_2}\delta_j-\sum_{j=m_2+1}^{q_2}\delta_jL_j\\&+\sum_{j=1}^{n_2}\gamma_jK_j-\sum_{j=n_2+1}^{p_2}\gamma_j-\sum_{j=1}^{q_1}\beta_j>0\end{aligned} \quad (1.7)$$

$$\begin{aligned}\Lambda=&\sum_{j=1}^{n_1}A_j-\sum_{j=n_1+1}^{p_1}A_j+\sum_{j=1}^{m_2}F_j-\sum_{j=m_2+1}^{q_2}F_jS_j\\&+\sum_{j=1}^{n_3}E_jR_j-\sum_{j=n_2+1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j>0\end{aligned} \quad (1.8)$$

$$|\arg x|<\frac{1}{2}\Omega\pi,\ |\arg y|<\frac{1}{2}\Lambda\pi \quad (1.9)$$

The behavior of the $\overline{H}$-function of two variables for small values of $|z|$ follows as:

$$\overline{H}[x,y]=0(|x|^{\alpha}|y|^{\beta}),\max\{|x|,|y|\}\to 0 \quad (1.10)$$

Where

$$\alpha=\min_{1\le j\le m_2}\left[\mathrm{Re}\left(\frac{d_j}{\delta_j}\right)\right],\beta=\min_{1\le j\le m_2}\left[\mathrm{Re}\left(\frac{f_j}{F_j}\right)\right] \quad (1.11)$$

For large value of $|z|$,

$$\overline{H}[x,y]=0\left\{|x|^{\alpha'},|y|^{\beta'}\right\},\min\{|x|,|y|\}\to 0 \quad (1.12)$$

Where

$$\begin{aligned}\alpha'&=\max_{1\le j\le n_2}\mathrm{Re}\left(K_j\frac{c_j-1}{\gamma_j}\right),\\ \beta'&=\max_{1\le j\le n_3}\mathrm{Re}\left(R_j\frac{e_j-1}{E_j}\right)\end{aligned} \quad (1.13)$$

Provided that $U<0$ and $V<0$.

If we take $K_j=1(j=1,2,...,n_2), L_j=1(j=m_2+1,...,q_2)$, $R_j=1(j=1,2,...,n_3), S_j=1(j=m_3+1,...,q_3)$ in (1.1), the $\overline{H}$-function of two variables reduces to H-function of two variables due to [7].

The following results are needed in the analysis that follows:

Bessel polynomials are defined as

$$\begin{aligned}y_n(x;a,b)&=\sum_{r=0}^{n}\frac{(-n)_r(a+n-1)_r}{r!}\left(-\frac{x}{b}\right)^r\\&={}_2F_0\left[-n,a+n-1;-\frac{x}{b}\right]\end{aligned} \quad (1.14)$$

Orthogonality property of Bessel polynomials is derived by Exton ([4], p.215, (14)):

$$\begin{aligned}&\int_0^\infty x^{a-2}e^{-\frac{1}{x}}y_m(x;a,1)\,y_n(x;a,1)dx\\&=\frac{(-1)^m n!(n+a-1)\pi}{\Gamma(a+n)(2n+a-1)\sin\pi a}\delta_{m,n}\end{aligned} \quad (1.15)$$

Where $\mathrm{Re}(a)<1-m-n$.

The integral defined by Bajpai et.al. [1] is also required:

$$\int_0^\infty x^{\sigma-1}e^{-\frac{1}{x}}y_n(x;a,1)dx=\frac{\Gamma(-\sigma-n)\Gamma(a-\sigma-1+n)}{\Gamma(a-\sigma-1)} \quad (1.16)$$

Where $\mathrm{Re}(\sigma+n)<0, \mathrm{Re}(a-\sigma-1+n)>0, \sigma\neq-1,-2,...$.

2. Integral

The integral to be evaluated is

$$\int_0^\infty\left\{\begin{array}{l}x^{\sigma-1}e^{-\frac{1}{x}}y_n(x;a,1)\overline{H}^{o,n_1:\ m_2,n_2:m_3,n_2}_{p_1,q_1:p_2,q_2;p_2,q_2}\\ \left[\begin{array}{c|c} \begin{matrix}u\,x^{\lambda}\\ v\end{matrix} & \begin{array}{l}\left[\begin{array}{l}(a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},\\(c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3},\\(e_j,E_j)_{n_3+1,p_3}\end{array}\right]\\ \left[\begin{array}{l}(b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2},\\(d_j,\delta_j;L_j)_{m_2+1,q_2},\\(f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3}\end{array}\right]\end{array}\end{array}\right]\end{array}\right\}dx$$

$$= \overline{H}^{0,n_1:\;\; m_2,n_2:m_3,n_2}_{p_1+1,q_1+2:p_2,q_2;p_2,q_2} \left[\begin{matrix} x \\ y \end{matrix} \left| \begin{matrix} \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p_1},(a-\sigma-1;\lambda), \\ (c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2}, \\ (e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \end{matrix}\right] \\ \left[\begin{matrix} (b_j,\beta_j;B_j)_{1,q_1}, \\ (-\sigma-n;\lambda),(a-\sigma+1+n;\lambda),(d_j,\delta_j)_{1,m_2}, \\ (d_j,\delta_j;L_j)_{m_2+1,q_2},(f_j,F_j)_{1,m_3}, \\ (f_j,F_j;S_j)_{m_3+1,q_3} \end{matrix}\right] \end{matrix} \right. \right] \tag{2.1}$$

Where

$$R\left[\sigma+\lambda\frac{a_j-1}{\alpha_j}+n\right]<0, R\left[\sigma-a-n+1+\lambda\frac{a_j-1}{\alpha_j}\right]<0$$

For $j=1,2,...,n_1;\sigma\neq-1,-2,...$, and conditions (1.7), (1.8) and (1.9) are also satisfied.

Proof: To establish (2.1), express the $\overline{H}$ -function of two variables in its integrand as a Mellin-Barnes type integral (1.1) and interchange the order of integration which is permissible due to the absolute convergence of the integrals involved in the process, we obtain

$$-\frac{1}{4\pi^2}\int_{L_1}\int_{L_2} \phi_1(\xi,\eta)\phi_2(\xi)\phi_3(\eta)u^{\xi}v^{\eta} \left\{\int_0^\infty x^{a+\lambda(\xi+\eta)-1}e^{-\frac{1}{x}}y_n(x;a,1)dx\right\}d\xi d\eta$$

Now evaluating the inner integral with the help of (1.16), it becomes

$$-\frac{1}{4\pi^2}\int_{L_1}\int_{L_2} \frac{\phi_1(\xi,\eta)\phi_2(\xi)\phi_3(\eta)\times\begin{bmatrix}\Gamma(-\sigma-n-\xi-\eta)\\ \Gamma(a-\sigma-1+n-\xi-\eta)\end{bmatrix}u^{\xi}v^{\eta}}{\Gamma(a-\sigma-1-\xi-\eta)}d\xi d\eta$$

Which on applying (1.1), yields the desired result (2.1).

Special Case: If we take $K_j=1(j=1,2,...,n_2)$, $L_j=1(j=m_2+1,...,q_2)$, $R_j=1(j=1,2,...,n_3)$, $S_j=1(j=m_3+1,...,q_3)$ in (1.1), the $\overline{H}$ -function of two variables reduces to H -function of two variables due to [7], and we get

$$\int_0^\infty \left\{ x^{\sigma-1}e^{-\frac{1}{x}}y_n(x;a,1)\overline{H}^{o,n_1:\; m_2,n_2:m_3,n_2}_{p_1,q_1:p_2,q_2;p_2,q_2} \left[\begin{matrix} u\,x^{\lambda} \\ v \end{matrix} \left| \begin{matrix} \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;1)_{1,n_2}, \\ (c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;1)_{1,n_3}, \\ (e_j,E_j)_{n_3+1,p_3} \end{matrix}\right] \\ \left[\begin{matrix} (b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2}, \\ (d_j,\delta_j;1)_{m_2+1,q_2}, \\ (f_j,F_j)_{1,m_3},(f_j,F_j;1)_{m_3+1,q_3} \end{matrix}\right] \end{matrix} \right. \right] \right\} dx$$

$$= H^{0,n_1:\;\; m_2,n_2:m_3,n_2}_{p_1+1,q_1+2:p_2,q_2;p_2,q_2} \left[\begin{matrix} u \\ v \end{matrix} \left| \begin{matrix} \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p_1},(a-\sigma-1;\lambda), \\ (c_j,\gamma_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2}, \\ (e_j,E_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \end{matrix}\right] \\ \left[\begin{matrix} (b_j,\beta_j;B_j)_{1,q_1}, \\ (-\sigma-n;\lambda),(a-\sigma+1+n;\lambda), \\ (d_j,\delta_j)_{1,m_2},(d_j,\delta_j)_{m_2+1,q_2}, \\ (f_j,F_j)_{1,m_3},(f_j,F_j)_{m_3+1,q_3} \end{matrix}\right] \end{matrix} \right. \right] \tag{2.2}$$

Provided all condition are satisfied given in (2.1).

3. Solution of the Time-Domain Synthesis Problem of Signals

The classical time-domain synthesis problem occurring in electric network theory is as follows ([4], p. 139):

Given an electrical signal described by a real valued conventional function $f(t)$ on $0<t<\infty$, construct an electrical network consisting of finite number of components R, C and I which are all fixed, linear and positive, such that output of $f_N(t)$, resulting from a delta-function $\delta(t)$ approximates $f(t)$ on $0<t<\infty$ in some sense.

In order to obtain a solution of this problem, we expand the function $f(t)$ into a convergent series:

$$f(t)=\sum_{n=0}^{\infty}\psi_n(t) \tag{3.1}$$

Or real-valued function $\psi_n(t)$. Let every partial sum

$$f_N(t)=\sum_{n=0}^{N}\psi_n(t);\; N=0,1,2,... \tag{3.2}$$

Possesses the two properties

(i) $f_N(t)=0$, for $-\infty<t<0$

(ii) The Laplace transform $F_N(s)$ 0f $F_N(t)$ is a rational function having a zero as $s=\infty$ and all its poles in the left-hand s -plane, except possibly for a simple pole at the origin.

After choosing N in (3.2) sufficiently large whatever approximation criterion is being used, an orthogonal series expansion may be employed. The Bessel polynomial transformation and (1.15) yields as immediate solution in the following form:

$$f(t)=\sum_{n=0}^{\infty}C_n t^{\frac{a-2}{2}}e^{-\frac{1}{2}t}y_n(t;a,1)$$

Where

$$C_n=(-1)^n\frac{\Gamma(a+n)(2n+a-1)\sin\pi a}{n!(n+a-1)\pi} \times\int_0^\infty f(t)t^{\frac{a-2}{2}}y_n(t;a,1)dt \tag{3.3}$$

Where $R(a) < 1-2n$.

The function $f(t)$ is continuous and of bounded variation in the open interval $(0,\infty)$.

4. Particular Solution of the Problem

The particular solution of the problem is:

$$f(t) = \frac{\sin \pi a}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n \Gamma(a+n)(2n+a-1)}{n!(n+a-1)} t^{\frac{a-2}{2}} e^{-\frac{1}{2}t}$$

$$H^{0,n_1:\ m_2,n_2:m_3,n_2}_{p_1+1,q_1+2:p_2,q_2;p_2,q_2} \left[\begin{matrix} u \\ v \end{matrix} \left| \begin{matrix} \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p_1},(a-\sigma-1;\lambda), \\ (c_j,\gamma_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2}, \\ (e_j,E_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \end{matrix} \right] \\ \left[\begin{matrix} (b_j,\beta_j;B_j)_{1,q_1},(-\sigma-n;\lambda),(a-\sigma+1+n;\lambda), \\ (d_j,\delta_j)_{1,m_2},(d_j,\delta_j)_{m_2+1,q_2}, \\ (f_j,F_j)_{1,m_3},(f_j,F_j)_{m_3+1,q_3} \end{matrix} \right] \end{matrix} \right. \right] y_n(t;a,1) \tag{4.1}$$

Where $\sigma < 0, R(a) < 1-2n, R\left(a-\sigma+\frac{a_j-1}{\alpha_j}\right) < 2,$ $j = 1,2,...,n_1;$ $\sigma \neq -1,-2,....$ and result (1.7), (1.8) and (1.9) are also holds.

Proof: Let us consider

$$f(t) = t^{\sigma-\frac{1}{2}} e^{-\frac{1}{2}t} \overline{H}^{o,n_1:\ m_2,n_2:m_3,n_2}_{p_1,q_1:p_2,q_2;p_2,q_2}$$

$$\times \left[\begin{matrix} u\,x^{\lambda} \\ v \end{matrix} \left| \begin{matrix} \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2}, \\ (c_j,\gamma_j)_{n_2+1,p_2},(e_j,E_j;R_j)_{1,n_3}, \\ (e_j,E_j)_{n_3+1,p_3} \end{matrix} \right] \\ \left[\begin{matrix} (b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2}, \\ (d_j,\delta_j;L_j)_{m_2+1,q_2},(f_j,F_j)_{1,m_3}, \\ (f_j,F_j;S_j)_{m_3+1,q_3} \end{matrix} \right] \end{matrix} \right. \right] \tag{4.2}$$

$$= \sum_{n=0}^{\infty} C_n\, t^{\frac{a-2}{2}} e^{-\frac{1}{2}t} y_n(t;a,1)$$

Equation (4.2) is valid, since $f(t)$ is continuous and of bounded variation in the open interval $(0,\infty)$.

Multiplying both sides of (4.2) by $t^{\frac{a-2}{2}} e^{-\frac{1}{2}t} y_m(t;a,1)$ and integrating with respect to t from 0 to ∞, we get

$$\int_0^{\infty} \left\{ \begin{matrix} x^{\sigma-1} e^{-\frac{1}{t}} y_n(t;a,1) \overline{H}^{o,n_1:\ m_2,n_2:m_3,n_2}_{p_1,q_1:p_2,q_2;p_2,q_2} \\ \left[\begin{matrix} u\,t^{\lambda} \\ v \end{matrix} \left| \begin{matrix} \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p_1},(c_j,\gamma_j;K_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2}, \\ (e_j,E_j;R_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \end{matrix} \right] \\ \left[\begin{matrix} (b_j,\beta_j;B_j)_{1,q_1},(d_j,\delta_j)_{1,m_2},(d_j,\delta_j;L_j)_{m_2+1,q_2}, \\ (f_j,F_j)_{1,m_3},(f_j,F_j;S_j)_{m_3+1,q_3} \end{matrix} \right] \end{matrix} \right. \right] \end{matrix} \right\} dt$$

$$= \sum_{n=0}^{\infty} C_n \int_0^{\infty} t^{\frac{a-2}{2}} e^{-\frac{1}{2}t} y_m(t;a,1) y_n(t;a,1) dt$$

Now using (2.1) and (1.15), we obtain

$$C_m = \frac{(-1)^m \Gamma(a+m)(2m+a-1)}{m!(m+a-1)} \frac{\sin \pi a}{\pi}$$

$$H^{0,n_1:\ m_2,n_2:m_3,n_2}_{p_1+1,q_1+2:p_2,q_2;p_2,q_2} \left[\begin{matrix} u \\ v \end{matrix} \left| \begin{matrix} \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p_1},(a-\sigma-1;\lambda), \\ (c_j,\gamma_j)_{1,n_2},(c_j,\gamma_j)_{n_2+1,p_2}, \\ (e_j,E_j)_{1,n_3},(e_j,E_j)_{n_3+1,p_3} \end{matrix} \right] \\ \left[\begin{matrix} (b_j,\beta_j;B_j)_{1,q_1},(-\sigma-m;\lambda),(a-\sigma+1+m;\lambda), \\ (d_j,\delta_j)_{1,m_2},(d_j,\delta_j)_{m_2+1,q_2}, \\ (f_j,F_j)_{1,m_3},(f_j,F_j)_{m_3+1,q_3} \end{matrix} \right] \end{matrix} \right. \right] \tag{4.3}$$

On account of the most general character of the result (4.2) due to presence of the $\overline{H}$-function of two variables, numerous special cases can be derived but further sake of brevity those are not presented here.

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