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Existence and Stability of Mixed Stochastic Integro-differential Inclusion Equations via Cosine Dynamical System

Salah H Abid, Sameer Q Hasan*, Zainab A Khudhu

Department of Mathematics, College of Education Almustansryah University
*Corresponding author: dr.sameer_kasim @yahoo.com

Abstract In this paper we presented the existence and stability for classes of Mixed stochastic integro-differential inclusion problem via cosine dynamical semi group with illustrative example.

Keywords: *integro-differential inclusions equations, cosine dynamical system, mixed-stochastic mild solution, fractional partial differential equations, Asymptotic Stability*

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1. Introduction

In this paper, gives a nonlocal and sufficient condition of the existence of mild solutions for the following neutral stochastic functional integro-differential inclusions with nonlocal conditions:

$$d[x'(t) - g(t, x(t))]$$

$$\in A[x(t) + F(t, x(t), \int_0^t a(t, x(t))ds)dw] + \sigma(t)\, dw^H$$

$$x(0) = x^0,\ x'(0) = x',\ t \in [0, b] = J$$

Where A is the infinitesimal generator of a compact, analytic resolvent operator $S(t), t \geq 0$ in the Hilbert space H. Suppose $\{w(t): t \geq 0\}$ is a given K-valued Brownian motion or Wiener process with a finite trace nuclear covariance operator $Q \geq 0$ *and* $L(K; H)$ denotes the space of all bounded linear operators from K in to H. Let $h_i: J \to J, i = 1,2,3$ and $f(t), t\epsilon J$, be a bounded linear operator. The random variable $x_0 \in H$ satisfies $E\|x_0\|^2 < \infty$, and g, a, F are given functions specified later.

The theory of integro-differential equations or inclusions has become an active area of investigation due to their applications in the fields such as mechanics, electrical engineering, medicine biology, ecology and so on can (see [1,4,5] and references therein). Several authors have established the existence results of mild solutions for these equations ([3,14,18,23] and references therein). In addition, the nonlinear integro-differential equations with resolvent operators serve as an abstract formulation of partial integro-differential equations that arise in many physical phenomena. One can see [19] and references therein. The deterministic models often fluctuate due to noise, which is random or at least appears to be so. Therefore, we must move from deterministic problems to stochastic problems. As the generalization of classic impulsive integro-differential equations or inclusions, impulsive neutral stochastic functional integro-differential equations or inclusions have attracted the researchers great interest. And some works have done on the existence results of mild solutions for these equations ([15,20] and references therein). To the best of our knowledge, there is no work reported on the existence of mild solutions for the impulsive neutral stochastic functional Integro-differential inclusions with nonlocal initial conditions and resolvent operators, and the aim of this paper is to close the gap. In this paper, motivated by the previously mentioned papers, we will study this interesting problem. Sufficient conditions for the existence are given by means of the fixed point theorem for multi-valued mapping due to Dhage [7] and the fractional power of operators. Especially, the known results appeared in [6] is generalized to the stochastic settings. An example is provided to illustrate the theory.

2. Preliminaries

For more details on this section, We refer the reader to Da prato and Zabczyk [24].throughout the paper $(H, \|.\|_H)$ *and* $(K, \|.\|_K)$denote two real separable Hilbert spaces. In case without confusion, we just use $\langle .,. \rangle$ for the inner product and $\|.\|$ for the norm.

Let $(\Omega, \mathcal{F}, p; F)$ $(F = \{\mathcal{F}(t)\}_{t\geq 0})$ be complete filtered probability space satisfying that $\mathcal{F}_0$ contains all P-null sets of $\boldsymbol{\mathcal{F}}$. An H-valued random variable is an $\mathcal{F}$-measurable function $x(t): \Omega \to H$ and the collection of random variables $S = \{x(t, w): \Omega \to H \setminus t \in J\}$ is called a stochastic process. Generally, we just write *x(t)* instead of

$x(t,w)$ and $x(t): J \to H$ in the space of S. *Let* $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K. Suppose that $\{w(t): t \geq 0\}$ is a cylindrical K −valued wiener process with a finite trace nuclear covariance operator $Q \geq 0$, denote $Tr(Q) = \sum_{i=1}^{\infty} \lambda_i = \lambda < \infty$, which satisfies that $Qe_i = \lambda_i e_i$. *So*, actually, $w(t) = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\, w_i(t) e_i$, where$\{w_i\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard wiener processes. We assume that $\mathcal{F}_t = \sigma\{w(s): 0 \leq s \leq t\}$ *is the* σ − algebra generated by w and $\mathcal{F}_T = \mathcal{F}$. Let $\Psi \in L(K,H)$ and define $\|\Psi\|_Q^2 = Tr(\Psi Q \Psi^*) = \sum_{n=1}^{\infty} \left\|\sqrt{\lambda_n}\Psi e_n\right\|^2$.

If $\|\Psi\|_Q < \infty$, then Ψ is called a Q-Hilbert-Schmidt operator. Let $L_Q(K,H)$denote the space of all Q-Hilbert-Schmidt operators $\Psi: K \to H$. *The* completion $L_Q(K,H)$ *of* *L(K,H)* with respect to topology induced by the norm $\|.\|_Q$ where $\|\Psi\|_Q^2 = \langle \Psi, \Psi \rangle$ is a Hilbert space with the above norm topology. Let $A: D(A) \to H$ be infinitesimal generator of a compact, analytic resolvent operator $S(t), t \geq 0$. Let $L_2(\Omega, \mathcal{F}_{t,H})$ denote the Hilbert space of all $\mathcal{F}_t$ − measurable square integrable random variables with values in H. Let $L_2^{\mathcal{F}}([0,b], H)$ be the Hilbert space of all square integrable and $\mathcal{F}_t$ − measurable processes with values in H. $\boldsymbol{\beta}$([0,b])=$\{x: [0,b] \to H, x_K \in C(J_K, H)\}$ let $L_2^0([,\Omega,], H)$ denote the family of all $\mathcal{F}_0$ −measurable, β –valued random variables $x(0)$. We use the notations $p_{cl}(H)$ for the family of all subsets of H and denote

$$p_{cl}(H) = \{Y \in p(H): Y \text{ is closed}\},$$

$$p_{cv}(H) = \{Y \in p(H): Y \text{ is convex}\},$$

$$p_{bd}(H) = \{Y \in p(H): Y \text{ is bounded}\},$$

$$p_{cp}(H) = \{Y \in p(H): Y \text{ is compact}\}.$$

In what follows, we briefly introduce some facts on multi-valued analysis. For details, [5]. A multi-valued map $\Gamma: H \to p(H)$ is convex (closed) valued, if $\Gamma(x)$ is convex (closed) for all $x \in H$. $\Gamma(x)$ *is* bounded on bounded sets if $\Gamma(B) = \cup_{x\in B} \Gamma(x)$ is bounded in H, for any bounded set B *of* H, that is, $sup_{x\in B}\, sup\{\|y\| \in \Gamma(x)\} < \infty$. Γ is called upper semi continuous (u .s .c. for short) on H, if for any $x \in H$, the set Γ(x) is a nonempty, closed subset of H, and if for each open set B of H containing Γ(x), there exists an open neighborhood N of x such that Γ(N) ⊆ B. Γ is said to be completely continuous if Γ(B) is relatively compact, for every bounded subset B ⊆ H. If the multi –valued map Γ is completely continuous with nonempty compact values, then Γ is u .s .c. if and only if Γ has a closed graph, i.e., $x_n \to x, y_n \to y, y_n \in \Gamma(x_n)$ *imply* $y \in \Gamma(x)$. Γ has a fixed point if there is $x \in H$ such that $x \in \Gamma(x)$. A multi-valued map $\Gamma: J \to p_{cl}$ is said to be measurable if for each $x \in H$, the mean –square distance between x and $\Gamma(t)$ is measurable.

Definition (2-1) [11]

The multi-valued map $F: J \times H \to p_{bd,cl,cv}(H)$ is said to be L^2-Caratheodory if

i) $t \mapsto F(t,v)$is measurable for each $v \in H$;
ii) $v \mapsto F(t,v)$ is u.s.c. for almost all $t \in J$;
iii) for each $q > 0$, there exists $h_1 \in L^1(J, R_+)$ such that $\|F(t,v)\|^2 = sup_{f\in F(t,v)}$ $E\|f\|^2 \leq h_q(t)$, for all $\|v\|_\beta^2 \leq q$ and for a. e $t \in J$.

***Lemma (2-1)* [17]**

Let I be a compact interval and a Hilbert space. Let F be an L^2 −Caratheodory multi-valued map with $N_{F,x} \neq \emptyset$ and let Γ be a linear continuous mapping from $L^2(I,H) to C(I,H)$. Then, the operator $\Gamma_\circ N_F : C(I,H) \to p_{cp,cv}(H), x \mapsto (\Gamma_\circ N_F)_{(x)} = \Gamma(N_{F,x})$, is a closed graph operator in $C(I,H) \times C(I,H)$, where $N_{F,x}$ is known as the selectors set from F, is given by $\sigma \in N_{F,x} = \{\sigma \in L^2(L(K,H)): \sigma(t) \in F(t,x) \text{ for } a.e.\, t \in J\}$.

***Lemma (2-2)* [10]**

Let $\{\Phi(t)\}_{t\epsilon[0,T]}$ be a family of deterministic functions with values in L_2^0 *(Y, X)* The stochastic integral of Φ with respect to W^H is defined by

$$\int_0^t \Phi(s) dW_{(s)}^H = \sum_{n=1}^{\infty} \int_0^t \sqrt{\lambda_n}\Phi(s) e_n dB_{n(s)}^H = \sum_{n=1}^{\infty} \int_0^t \sqrt{\lambda_n}\left(K_H^*(\Phi e_n)(s)\right) dB_{n(s)}^H. \quad (2.1)$$

***Lemma (2-3)* [10]**

If φ*: [0, b]* $\to L_2^0(Y,X)$ satisfies $\int_0^T \|\varphi(s)\|_{L_2^0}^2 ds < \infty$ then the above sum in (2.2) is well defined as an X-valued random variable and we have

$$E\left\|\int_0^t \varphi(s) dW_{(S)}^H\right\|^2 \leq 2Ht^{2H-1}\int_0^t \|\varphi(s)\|_{L_2^0}^2 ds. \quad (2.3)$$

Definition (2-2) [2]

A semigroup *T(t),* $0 \leq t < \infty$ of bounded linear operators on a Banach space X is a C_0 −semigroup of bounded linear operators if: $\lim_{t\downarrow 0} T(t)x = x$, for every $x \in X$.

Example (2-1), [12]

Let A ∈ L(X), where X is a Banach space, and set: $T(t) = e^{tA} = \sum_{n=0}^{\infty} \frac{t^n A^n}{n!}$. Then

The family $T(t), t \geq 0$ is strongly continuous semigroup (C_0 −semigroup). The following are briefly the most important facts on semigroup theory of bounded linear operators that needed later on.

***Theorem (2-1)* [8,18]**

A linear (unbounded) operator A is the generator of a C_0 −semigroup of contractions $\{T(t)\}_{t\geq 0}$ if and only if:

(i) A is closed and $\overline{D(A)} = X$.
(ii) The resolvent set $\rho(A)$ of A contains R^+ and for every $\lambda > 0$, $\|(\lambda I - A)^{-1}\|_{L(X)} \leq \frac{1}{\lambda}$.

***Remark (1-1)*, [2]**

Let *T*(*t*) be a C_0 −semigroup then there are constants $w \geq 0$ and $M \geq 1$, such that $\|T(t)\|_{L(X)} \leq Me^{wt}$ for $t \geq 0$. If $w \geq 0$, $T(t)$ is called uniformly bounded Moreover if $M = 1$, it is called a C_0 −semigroup of contractions. The following are briefly the most import.

***Definition (2-3)*, [21]**

A one-parameter family $C(t), t \in R$ of bounded linear operators in the Banach space X is called a strongly continuous cosine family if and only if

i. $C(s+t) + C(s-t) = 2C(s)\,C(t)$, for all $s, t \in R$.
ii. $C(0) = I$

iii. $C(t)x$ is continuous in t on R for each fixed $x \in X$.

Definition (2-4), [9]

If $\{C(t): t \in R\}$ is a strongly continuous cosine family in X,

i. $\{S(t): t \in R\}$, associated to the given strongly continuous cosine family, is defined by $S(t)x = \int_0^t C(s)x\,ds\ x \in X, t \in R$.

ii. The infinitesimal generator $A: X \to X$ of a cosine family $\{C(t): t \in R\}$ is defined by

$$Ax = \frac{d^2}{dt^2} C(t) x_{t=0}, x \in D(A),$$

Where $D(A) = \{x \in X : C(t)\, x \in C^2(R, X)\}$.

Definition (2-5) [8]

Let X be a Banach space, a one-parameter family $T(t),\ 0 \le t < \infty$ of bounded linear operators from X into X is a semigroup of bounded linear operators on X if:

1. $T\ (0) = I$, where I is the identity operator on X.
2. $T(t+s) = T(t)\, T(s)$, for every $t, s \ge 0$.

Lemma(2-4), [22]

Let $C(t), (resp, S(t)), t \in R$, be a strongly continuous cosine $(resp, \sin)$ family on X, then there exist constants $M \ge 1$ and $w \ge 0$ such that $\|C(t)\| \le Me^{|t|}$, for all $t \in R$,

$$\|S(t_1) - S(t_2)\| \le M \left| \int_{t_1}^{t_2} e^{w|s|} ds \right|, \text{ for all } t_1, t_2 \in R.$$

Theorem (2-2) [7]

Let $B(0, r)$ and $B[0, r]$ denote respectively the open and closed balls in a Hilbert space H centered at the origin and of radius r and let $\Phi_1: H \to P_{bd,cl,cv}(H)$ and $\Phi_2: B[0, r] \to P_{bd,cl,cv}(H)$, two multi-valued operators satisfying

(i) Φ_1 is a contraction, and

(ii) Φ_2 is u.s.c. and completely continuous.

Then, either

(1) the operator inclusion $x \in \Phi_1 x + \Phi_2 x$ has a solution, or

(2) there exists an $x \in H$ with $\|x\| = r$ such that $\lambda x \in f\Phi_1 x + \Phi_2 x\ for\ some\ \lambda > 1$.

3. Main Result of the Existence and Stability

The following lemma and definition are begging to explain the main results.

Lemma (2-5)

Let $\{C(t)\}_{t \ge 0}$ be a cosine semigroup and the H-valued function

$$v(s) = C(t-s)x(s) + S(t-s)\left[x'(s) - g(s, x(s))\right]$$

Then (2.1) has a mixed-stochastic mild solution with $x(0), x'(0) \in L_2^0(\Omega, B)$,

$$x(t) = S(t)[x'(0) - g(0, x(0))]$$
$$+ \int_0^t AS(t-s) F\left(s, x(s)\right), \int_0^t g(s, x(s)) ds dw$$
$$+ \int_0^t S(t-s)\sigma(s) dw^H + \int_0^t C(t-s) g(s, x(s)) ds$$

Proof:

Take the H-valued function

$$v(s) = C(t-s)x(s) + S(t-s)\left[x'(s) - g(s, x(s))\right].$$

Then, different both sides for s and use properties in definition(2.4), we get

$$dv(s)/ds = C(t-s)x'(s) - AS(t-s)x(s)$$
$$+ S(t-s)\frac{d}{ds}\left[x'(s) - g(s, x(s))\right]$$
$$- C(t-s)[x'(s) - g(s, x(s))]$$
$$= C(t-s)x'(s) - AS(t-s)x(s)$$
$$+ S(t-s)A[x(s) + F(s, x(s), \int_0^t g(s, x(s) dw)]$$
$$+ \sigma(s) dw^H - C(t-s)x'(s)$$
$$+ C(t-s) g(s, x(s))$$

Integrate both sides, we get

$$= -AS(t-s)x(s) + AS(t-s)x(s)$$
$$+ S(t-s)AF\left(s, x(s), \int_0^t g(s, x(s)) dw\right)$$
$$+ S(t-s)\sigma(s) dw^H + C(t-s) g(s, x(s))$$
$$v(t) - v(0)$$
$$= \int_0^t AS(t-s)F\left(s, x(s), \int_0^t g(s, x(s)) dw\right)$$
$$+ S(t-s)\sigma(s) dw^H$$
$$+ \int_0^t C(t-s) g(s, x(s)) ds$$
$$x(t) - C(t)x(0) - S(t)\left[x'(0) - g(0, x(0))\right]$$
$$= \int_0^t AS(t-s)F(s, x(s), \int_0^t a(s, x(s)) ds) dw$$
$$+ \int_0^t S(t-s)\sigma(s) dw^H + \int_0^t C(t-s) g(s, x(s)) ds$$

Definition (2-6)

A bounded function $x(t): R \to X$ is called mixed-stochastic mild solution of the inclusion system (2.1) if for any $t \in J, x(0)\ , x'(0) \in L_2^0(\Omega, B)$,

$$x(t) = C(t)x(0) + S(t)\left[x'(0) - g(0, x(0))\right]$$
$$+ \int_0^t AS(t-s)F(s, x(s), \int_0^t a(s, x(s)) ds) dw$$
$$+ \int_0^t S(t-s)\sigma(s) dw^H + \int_0^t C(t-s) g(s, x(s)) ds.$$

Hypotheses

To investigate the existence of the mixed-stochastic mild solution to the system (2.1), and for the operators A we make the following assumption:

1. A is the infinitesimal generator of a compact, analytic resolvent operator $S(t)$, $C(t)$, $t \ge 0$ in the Hilbert space H and there exist constants $N^{\wedge}, M^{\wedge}$ and M_1 such that $\|S(t)\|^2 \le N^{\wedge}, \|C(t)\|^2 \le M^{\wedge}, t \in J$ on $I = [0, T]$, $\|f(t)\|^2 \le M_1$.

2. There exist constant M_2 such that $g : J\, x\, H \to H$, satisfies the following Lipchitz condition, that is, for any $s,t \in J$, $x,y \in H$ such that $\|g(s,x)-g(t,y)\|^2 \le M_2[|s-t| + \|x-y\|^2]$ the multi-valued map $F: J \times H \times H \to P_{bd,cl,cv}(L(K,H))$ is an L^2 − Caratheodory function satisfies the following condition:-

i. for each $t \in J$, the function $F(t,.,.): J \times H \times H \to P_{bd,cl,cv}(L(k,H))$ is u. s. c , and for each $x\, y, \in H$, the function $F(.,x,y)$ is measurable and for each fixed x , $y \in B$, the set

$$N_{AF,x,y} = \left\{ \begin{array}{l} \sigma \in L^2(L(k,H)) : \sigma(t) \in AF(t,x,y) \\ for\ t \in J \end{array} \right\}$$

is nonempty.

ii. for some positive numbers $L_2, L_4 > 0$,

$$\left\| \int_0^t a(s,x) - a(s,y) ds \right\| \le L_2 \|x-y\|$$

and $L_{4=} \sup_{t\in J} \|a(t,0)\|$. Where L_2, L_4 are positive constants.

3. the map $AS(t-s)F\left(s, x(s), \int_0^t a(s,x(s)ds)\right)$: $J \times H \times H \to H$ and there exist positive constants L_1, L_2, L_3, such that

$$\|AS(t_1-s)F(s,x_1,y_1) - AS(t_2-s)F(s,x_2,y_2)\|$$
$$\le L_1(|t_1-t_2| + \|x_1-x_2\| + \|y_1-y_2\|),$$
$$L_3 = \sup_{t\in J} \|AF(t,0,0)\|.$$

4. r > $max\{6N^{\wedge 2} 2Ht^{2H-1} C_1 / 1 - (18N^{\wedge 2} b +$ 6M^2+12N^2Ex^02+L3)+M2L4(1+12M^2 , M^∅1−ψ1+ N^∅2−ψ2+L4∅1− ψ1+M^bL5 ∅2−ψ1/(L1N^+bL2)Tr(Q)μI .

4. Existence of the Inclusion Nonlinear Stochastic Differential System

In this section , the existence of the mixed-stochastic mild solution to the inclusion Problem formulation (2.1) has been discussed.

Theorem (2-3)

Assume the Hypotheses (1-5) are hold . Then for initial value $x(0) = x^0$, $x'(0) = x'$ $\in L_2^0(\Omega, B)$, and $\sup_{t\in J} E\|x'(t)\|^2 \le L\, sup_{t\in J} E\|x(t)\|^2$. Then the initial value mixed-stochastic inclusion system (2.1) has mixed - stochastic mild solution $x \in B$.

Proof:

Let the operator $\emptyset: B \to P(B)$ defined by

$$\emptyset(x) = x \in B, x(t)$$
$$= C(t)x(0) + S(t)[x'(0) - g(0,x(0))]$$
$$+\int_0^t AS(t-s)F\left(s, x(s)), \int_0^t a(s,x(s))ds\right)dw$$
$$+\int_0^t S(t-s)\sigma(s)dw^H + \int_0^t C(t-s)g(s,x(s))ds.$$

a fixed point of $\emptyset$ are stochastic- mild solutions of the equation (2.1). Let

$$\emptyset_1(x) = x \in B, x(t)$$
$$= C(t)x(0) + S(t)[x'(0) - g(0,x(0))]$$
$$+\int_0^t C(t-s)g(s,x(s))ds$$
$$\emptyset_2(x) = x \in B, x(t)$$
$$= \int_0^t AS(t-s)F\left(s,x(s)), \int_0^t g(s,x(s))dw\right)ds$$
$$+\int_0^t S(t-s)\sigma(s)dw^H$$

We prove that the operators $\emptyset_1$ and $\emptyset_2$ are satisfy all the condition for theorem(2-2).

Let $B_l = \{x \in B, E\|x\|^2 \le l\}$.

Step(1):

Now to prove that $\emptyset_1$is contraction.

Let $x_1,\ x_2 \in B_1$, from assuming that

$$\emptyset_1(x) = \{C(t)x(0) + S(t)[x'(0) - g(0,x(0))]\}$$
$$+\int_0^t C(t-s)g(s,x(s))ds,$$

we have that

$$E\|\emptyset_1(x_1)(t) - \emptyset_1(x_2)(t)\|^2$$
$$\le 4\|C(t)\|^2 E\|x_1(0) - x_2(0)\|^2$$
$$+4\|S(t)\|^2 E\|x_1'(0) - x_2'(0)\|^2$$
$$+4\|S(t)\|^2 E\|g(0,x_1(0)) - g(0,x_2(0))\|^2$$
$$+4\|C(t-s)\|^2 E\left\|\int_0^t g(s,x_1(s)) - g(s,x_2(s))ds\right\|^2$$
$$\le 4M^{\wedge 2} E\|x_1(0) - x_2(0)\|^2$$
$$+4N^{\wedge 2} E\left[\begin{array}{l} \|x_1'(0) - x_2'(0)\|^2 \\ +4N^{\wedge 2} M_2 \|x_1(0) - x_2(0)\|^2 \end{array}\right]$$
$$+4M^{\wedge 2} bM_2 E\|x_1(t) - x_2(t)\|^2$$
$$\le \left(4M^{\wedge 2} + 4N^{\wedge 2} M_2\right) E\|x_1(0) - x_2(0)\|^2$$
$$+4N^{\wedge 2} E\|x_1'(0) - x_2'(0)\|^2$$
$$+4M^{\wedge 2} bM_2 Sup_{t\in J} E\|x_1(t) - x_2(t)\|^2$$
$$\le (4M^{\wedge 2} + 4N^{\wedge 2} M_2) \sup_{t\in J} E\|x_1(t) - x_2(t)\|^2$$
$$+4N^{\wedge 2} \sup_{t\in J} E\|x_1'(t) - x_2'(t)\|^2$$
$$\le \left(\left(4M^{\wedge 2} + 4N^{\wedge 2} M_2 + 4M^{\wedge 2} bM_2\right) + 4N^{\wedge 2} L\right)$$
$$\sup_{t\in J} E\|x_{111}(t) - x_2(t)\|^2$$
$$= L_0 Sup_{t\in J} E\|x_1(t) - x_2(t)\|^2,$$

Where $L_0 = \left(\left(4M^{\wedge 2} + 4N^{\wedge 2}M_2 + 4M^{\wedge 2}bM_2\right) + 4N^{\wedge 2}L\right)$, $E\|\emptyset_1(x_1)(t) - \emptyset_1(x_2)(t)\|^2 \le L_0 E\|x_1 - x_2\|^2$

Step (2):

Now to prove that $\emptyset_2(x)$ is convex for each $x \in B$, let $u_{1,1}, u_{1,2} \in \emptyset_2(x)$, then, there exists $\sigma_{1,1}, \sigma_{1,2} \in F_{N,x}$ such that

$$u_{1,i}(t) = \int_0^t AS(t-s)\sigma_{1,i}(s)dw + \int_0^t S(t-s)\sigma_{2,i}(s)dw^H.$$

Let $\lambda \in [0,1]$

$$\lambda u_{1,1}(t) = \int_0^t AS(t-s)\sigma_{1,1}(s)dw + \int_0^t S(t-s)\sigma_{2,1}(s)dw^H$$

and

$$(1-\lambda)u_{1,2}(t) = \int_0^t AS(t-s)\sigma_{1,1}(s)dw + \int_0^t S(t-s)\sigma_{2,2}(s)dw^H.$$

For each $t \in J$, we have

$$\left(\lambda u_{1,1}(t) + (1-)u_{1,2}(t)\right) = A\int_0^t S(t-s)\left(\sigma_{1,1}(s) + (1-)\sigma_{1,2}(s)\right)dw + \int_0^t S(t-s)\left(\sigma_{2,1}(s) + (1-)\sigma_{2,2}(s)\right)dw^H.$$

From the condition (3-i) and since

$$F_{N,x}(s, x(s), \int_0^t a(s, x(s))ds)$$

is convex then we have that $\lambda u_{1,1}(t) + (1-\lambda)u_{1,2}(t) \in \emptyset_2(x)$.

Step (3):

Now to prove $\emptyset_2$maps bounded set into bounded set in *B*. Indeed, it is enough to show that there exists appositive constant $\wedge$ such that for $u \in \emptyset_2(x), x \in B$, we have $E\|u(t)\|^2 \le \wedge$ if $u \in \emptyset_2(x)$ then there exists $\sigma_1 \in AF_{N,x}\left(s, x(s), \int_0^t a(s, x(s))ds\right)$ for each $t \in J$ such that

$$u(t) = \int_0^t AS(t-s)F(s, x(s), \int_0^t a(s, x(s))ds)dw + \int_0^t S(t-s)\sigma(s)dw^H \varnothing_2(x).$$

We have that

$$E\|u(t)\|^2 \le 2\int_0^t S(s-t)^2\sigma_1(s)^2 dw + 2E\|S(s-t)\|^2 \int_0^t \sigma(s)dw^H$$
$$\le 2N^{\wedge 2}bTr(Q) + 2N^{\wedge 2}\left(2Ht^{2H-1}\int_0^t \|\sigma(s)\|^2 ds\right)$$
$$\le 2N^{\wedge 2}bTr(Q) + 4N^{\wedge 2}Hb^{2H-1}C_1 = \Lambda$$

for the condition the function $\sigma: [0,T] \to L_2^0(Y;X)$ satisfies from $\int_0^t \|\sigma(s)\|_{L_2}^2 . ds < \infty$ and there exists $C_1 > 0$ such that $\mathrm{Sup}\|\sigma(s)(t)\|_{L_2}^2 \le C_1$.

Then $E\|u(t)\|^2 \le \wedge$.

Step (4):

$\emptyset_2$ maps bounded set into equicontinuous sets of B. Let $0 < \tau_1 < \tau_2 \le b$. Then we have for each $x \in B_l$. and $u \in \emptyset_2(x)$, there exists $\sigma \in N_{AF,x}(s, x(s) \int_0^t a(s, x(s)ds)$ such that for each $t \in J$, we have.

$$u(t) = \int_0^t AS(t-s)F(s, x(s), \int_0^t a(s, x(s))ds)dw + \int_0^t S(t-s)\sigma(s)dw^H$$

$$E\|u(\tau_2) - u(\tau_1)\|^2$$

$$\le 6E\left\|\int_0^{\tau_1-\varepsilon}\left\{\begin{matrix}[S(\tau_2-s) - S(\tau_1-s)] \\ AF(s, x(s), \int_0^t a(s, x(s)ds)\end{matrix}\right\}dw\right\|^2 + 6E\left\|\int_{\tau_1-\varepsilon}^{\tau_1}\left\{\begin{matrix}[S(\tau_2-s) - S(\tau_1-s)] \\ AF(s, x(s)\int_0^t a(s, x(s)ds)\end{matrix}\right\}dw\right\|^2$$
$$+6E\left\|\int_{\tau_1}^{\tau_2}[S(\tau_2-s)]AF(s, x(s)\int_0^t a(s, x(s))dw\right\|^2 + 6E\left\|\int_0^{\tau_1-\varepsilon}[S(\tau_2-s) - S(\tau_1-s)]\sigma(s)dw^H\right\|^2$$
$$+6E\left\|\int_{\tau_1-\varepsilon}^{\tau_1}[S(\tau_2-s) - S(\tau_1-s)]\sigma(s)dw^H\right\|^2 + 6E\left\|\int_{\tau_1}^{\tau_2}S(\tau_2-s)\sigma(s)dw^H\right\|^2$$

$$\le 6E\|[S(\tau_2-s) - S(\tau_1-s)]\|^2 \left\|\int_0^{\tau_1-\varepsilon}\begin{pmatrix}AF(s, x(t), h(x(s)) \\ -AF(s,0,0) + AF(s,0,0)\end{pmatrix}dw\right\|^2$$
$$+6E\|[S(\tau_2-s) - S(\tau_1-s)]\|^2 \left\|\int_{\tau_1-\varepsilon}^{\tau_1}\begin{bmatrix}(AF(s, x(t), h(x(s))) \\ -AF(s,0,0) + AF(s,0,0))\end{bmatrix}dw\right\|^2$$
$$+6E\|S(\tau_2-s)\|^2 \left\|\int_{\tau_1}^{\tau_2}\begin{pmatrix}AF(s, x(s), h(x(s)) \\ -AF(s,0,0) + AF(s,0,0)\end{pmatrix}dw\right\|^2$$
$$+6E\|[S(\tau_2-s) - S(\tau_1-s)]\|^2 \left\|\int_0^{\tau_1-\varepsilon}\sigma(s)dw^H\right\|^2$$
$$+6E(S(\tau_2-s) - S(\tau_1-s)]]^2 \left\|\int_{\tau_1-\varepsilon}^{\tau_1}\sigma(s)dw^H\right\|^2$$
$$+E\|S(\tau_2-s)]\|^2 \left\|\int_{\tau_1}^{\tau_2}\sigma(s)dw^H\right\|^2$$

$$\le 6E\|[S(\tau_2-s) - S(\tau_1-s)]\|^2 \left\|\int_0^{\tau_1-\varepsilon}L_1(x(s) + bL_2(x(t) + L_4) + L_3)dw\right\|^2$$
$$+6E\|[S(\tau_2-s) - S(\tau_1-s)]\|^2 \left\|\int_{\tau_1-\varepsilon}^{\tau_1}L_1(x(s) + bL_2(x(t) + L_4) + L_3)dw\right\|^2$$
$$+6E\|S(\tau_2-s)\|^2 \left\|\int_{\tau_1}^{\tau_2}(L_1(x(s) + bL_2(x(t) + L_4) + L_3)dw\right\|^2$$

$$+6E\left\|\left[S(\tau_2-s)-S(\tau_1-s)\right]\right\|^2\left\|\int_0^{\tau_1-\varepsilon}\sigma(s)dw^H\right\|^2$$

$$+6E\left\|S(\tau_2-s)-S(\tau_1-s)\right]\right\|^2\left\|\int_{\tau_1-\varepsilon}^{\tau_1}\sigma(s)dw^H\right\|^2$$

$$+E\left\|S(\tau_2-s)\right]\right\|^2\left\|\int_{\tau_1}^{\tau_2}\sigma(s)dw^H\right\|^2.$$

Where $h(x(s))$ is defined by $\int_0^t a(s,x_1(s))ds$.

The right- hand side of the above the quality tends to zero as $\tau_2\to\tau_1$ with ε sufficiently small, also *S(t)* is a continuous simegroup .

$$u(\tau_2)-u(\tau_1)^2\le\in.$$

Step (5):

Now to prove$(\emptyset_2B_l)(t)$ is relativity compact in H for each $t\in J$.
Where $(\emptyset_2B_l)(t)=\{u(t):u\in\emptyset_2B_l\}$, $t\in J$, the set $(\emptyset_2B_1)(t)$ is relatively compact in B for $t=0$. Let $0<t\le b$ and $0<\in<t$, for $x\in B_l$ and $u\in\emptyset_2(x)$, there exists $\sigma\in N_{F,x}$ such that

$$u(t)=\int_0^{t-\in}AS(t-s)F(s,x(s),\int_0^t a(s,x(s)))dw$$
$$+\int_{t-\in}^{t}AS(t-s)F\left(s,x(s),\int_0^t a(s,x(s))\right)dw$$
$$+\int_0^{t-\in}S(t-s)\sigma(s)dw^H$$
$$+\int_{t-\in}^{t}S(t-s)\sigma(s)dw^H.$$

Now, we define

$$u_\in(t)=\int_0^{t-\in}AS(t-s)F(s,x(s),\int_0^t a(s,x(s))dw$$
$$+\int_0^{t-\in}S(t-s)\sigma(s)dw^H$$

for each $0<\in<t$,

$$E\left\|u(t)-u_\in(t)\right\|^2$$
$$\le 6E\,\|\int_0^{t-\in}S(t-s)AF\left((s,x(s),\int_0^t a(s,x(s))\right)dw$$
$$+\int_{t-\in}^{t}S(t-s)AF\left((s,x(s),\int_0^t a(s,x(s))\right)dw$$
$$+\int_0^{t-\in}S(t-s)\sigma(s)dw^H+\int_{t-\in}^{t}S(t-s)\sigma(s)dw^H$$
$$-\int_0^{t-\in}S((t-\in)-(s-\in))AF\left((s,x(s),\int_0^t a(s,x(s))dw\right)$$
$$-\int_0^{t-\in}S((t-\in)-(s-\in))\sigma(s)dw^H\,\|^2$$

From definition (2-3), and lemma (2-5) for the cosine simegroup continuous we have

$$\le 6E\left\|\int_{t-\in}^{t}S(t-s)AF\left((s,x(s),\int_0^t a(s,x(s))ds\right)dw\right\|^2$$

$$+6E\left\|\int_{t-\in}^{t}S(t-s)\sigma(s)dw^H\right\|^2$$

$$\le 6E\left\|\int_{t-\in}^{t}\begin{bmatrix}S(t-s)(AF(s,x(s),\int_0^t a(s,x(s))ds\\-AF(s,0,0)+AF(s,0,0))\end{bmatrix}dw\right\|^2$$

$$+6E\left\|\int_{t-\in}^{t}S(t-s)\sigma(s)dw^H\right\|^2$$

$$\le 6N^{\wedge2}bTr(Q)\mu(l)L_1(\left\|x(t)\right\|+b\left\|\begin{matrix}(a(t,x(t))\\-a(t,0)+a(t,0)\end{matrix}\right\|)$$

$$+6N^{\wedge2}\left(2Ht^{2H-1}\int_{t-\in}^{t}\sigma(s)^2dS\right)$$

$$\le 6N^{\wedge2}bTr(Q)\mu(l)L_1(r+b(L_2r+L_3)+L_4)$$

$$+12N^{\wedge2}Hb^{2H-1}C_1$$

The relative compact sets arbitrarily close to the set $\{u(t):u\in\emptyset_2B_l\}$ then its relative compact in *B*, thus $\emptyset_2$ is a compact multi-valued closed graph.

Step (6):

Now to show that $\emptyset_2$ has a closed graph.

Let $x_n\to x_*$, $x_n\epsilon B_l$, $u_n\epsilon\emptyset_2(x_n)$ and $u_n\to u_*$, we aim to show that $u_*\epsilon\emptyset_2(x_*)$ indeed, $u_n\epsilon\emptyset_2(x_n)$ means that there exists $\sigma_n\epsilon N_{AF}\left(s,x(s),\int_0^t a(s,x(s))ds\right)$ such that

$$u_n(t)=\int_0^t AS(t-s)F_n(s,x(s),\int_0^t a(s,x(s)ds)dw$$
$$+\int_0^t S(t-s)\sigma_n(s)dw^H.$$

there exists $\sigma_{1,n}\in N_{AF,x}$, thus

$$u_n(t)=\int_0^t\sigma_{1,n}(s)dw+\int_0^t S(t-s)\sigma_n(s)dw^H.$$

We must prove that there exists $\sigma_1^*\in N_{AF,x_*}$ such that

$$u_*(t)=\int_0^t S(t-s)\sigma_1^*(s)dw+\int_0^t S(t-s)\sigma_*(s)dw^H.$$

Suppose the linear continuous operator $\Gamma_{1,2}:L^2(J,H)\to C(J,H)$.

From lemma (2-1) it follows that $\Gamma_1N_{AF,x}\left(s,x(s),\int_0^t g(s,x(s))ds\right)$ is closed graph operator and we have

$$\left\|u_n(t)-u_*(t)+\int_0^t S(t-s)(\sigma_n(s)-\sigma_*(s))dw^H\right\|\to 0$$

as $n\to\infty$, thus

$$u_n(t)-\int_0^t S(t-s)\sigma_n(s)dw^H$$
$$\in\Gamma_1N_{AF,x}\left(s,x(s),\int_0^t g(s,x(s))ds\right)$$

Since $u_n \rightarrow u_*$, it follows from lemma (2-1) that

$$u_*(t)-\int_0^t S(t-s)\sigma_*(s)dw^H \in \Gamma_1 N_{AF,x}\left(s,x(s),\int_0^t g(s,x(s))ds\right).$$

That is, there exists $a\ \sigma_1^* \in N_{AF,x}$ such that

$$u_*(t)-\int_0^t S(t-s)\sigma_*(s)dw^H = \int_0^t S(t-s)\sigma_1^*(s)dw,$$

hence $\emptyset_2$ has a closed graph.

As in lemma (2-1) Let Γ be alinear continues mapping from $L^2(I,H)to\ C(I,H)$, Then, the operator Γ o N_F : $C(I\ ,H) \rightarrow P_{cp,cv}\ (H). x \rightarrow (\Gamma\ oN_{AF})(x) = \Gamma\ (N_{AF},x)$.

Is a closed graph operator in $C(J\ ,H) \times\ C(J,H)$.

Step (7):

The operator inclusion $x \in \emptyset_1(x) + \emptyset_2(x)$ has a solution in $B[0,r]$. Define an open ball *B(0, r)* in B, where r satisfies the inequality given in (5), we that $\emptyset_1$ and $\emptyset_2$ satisfy all conditions of theorem (2-2) . Therefore , if we can show that the second condition of theorem (2-2) is not true , then , we show that the system (2.1) has Least one mild solution, for $\lambda\, u \in \emptyset_1 x + \emptyset_2 x$. For some $\lambda > 1$ with $E\|x\|^2 = r$, then, we have

$$x(t)=\lambda^{-1}\big(C(t)x(0)\big)$$
$$+\lambda^{-1}\big(S(t)[x'(0)-g(s,x(0))]\big)$$
$$+\lambda^{-1}\left(\int_0^t AS(t-s)F\left(s,x(s),\int_0^t g(s,x(s))ds\right)dw\right)$$
$$+\lambda^{-1}\left(\int_0^t S(t-s)\sigma(s)dw^H\right)$$
$$+\lambda^{-1}\left(\int_0^t C(t-s)g(s,x(s))ds\right).$$

$$E\|x(t)\|^2 \le 6\|C(t)x(0)\|^2$$
$$+12\|S(t)\|^2\Big[\|x'(0)-g(0,x(0))\|^2\Big]$$
$$+18\left\|\int_0^t AS(t-s)F(s,x(s),\int_0^t g(s,x(s))dw\right\|^2$$
$$+6\left\|\int_0^t S(t-s)\sigma(s)dw^H\right\|^2+12\left\|\int_0^t C(t-s)g(s,x(s))ds\right\|^2$$
$$\le 6\|C(t)x(0)\|^2+12S(t)^2[\|x'(0)-g(0,x(0)\|^2$$
$$+18\|S(t-s)\|^2\left\|\begin{matrix}\int_0^t AF(s,x(s),\int_0^t g(s,x(s))dw) \\ -AF(s,0,o)+AF(s,0,0)\end{matrix}\right\|^2$$
$$+6\|S(t-s)\|^2\|\sigma(s)dW^H\|^2$$
$$+12\|C(t-s)\|^2\left\|\int_0^t g(s,x(s))-g(s,0)+g(s,0)ds\right\|^2$$
$$\le 6M^{\wedge 2}Ex(0)^2+12N^{\wedge 2}\left(Ex'(0)^2+L_3\right)$$
$$+18N^{\wedge 2}b[E\|x\|^2+\left\|\int_0^t g((s,x(s))-g(s,0)+g(s,0))dw\right\|^2$$
$$+6N^{\wedge 2}\left(2Ht^{2H-1}\left\|\int_0^t \sigma(s)\right\|^2 ds\right)$$
$$+12M^{\wedge 2}\left\|\int_0^t g(s,x(s)-g(s,0)+g(s,0)ds\right\|^2$$
$$\le 6M^{\wedge 2}E\|x(0)\|^2+12N^{\wedge 2}\left(E\|x(0)\|^2+L_3\right)$$
$$+18N^{\wedge 2}b\Big[Ex^2+M_2\left(Ex^2+L_4\right)\Big]$$
$$+6N^{\wedge 2}\left(2Ht^{2H-1}\int_0^t \sigma(s)^2 ds\right)+12M^{\wedge 2}M_2(Ex^2+L_4)$$
$$\mathrm{E}\|x(t)\|^2-18N^{\wedge 2}bEx^2-M_2\left(Ex^2+L_4\right)$$
$$-6M^{\wedge 2}E\|x(0)\|^2-12N^{\wedge 2}\left(E\|x'(0)\|^2+L_3\right)$$
$$-12M^{\wedge 2}M_2\left(Ex^2+L_4\right)\le 6N^{\wedge 2}\left(2Ht^{2H-1}C_1\right).$$

$$E\|x(t)\|^2 \le 4N^{\wedge 2}2Ht^{2H-1}C_1/1-\begin{pmatrix}18N^{\wedge 2}b+M_2L_4+6M^{\wedge 2}\\ +12N^{\wedge 2}(Ex'(0)^2+L_3)\\ +12M^{\wedge 2}M_2L_4\end{pmatrix}$$

thus

$$r \le 6N^{\wedge 2}2Ht^{2H-1}C_1/1-(18N^{\wedge 2}b+6M^{\wedge 2}$$
$$+12N^{\wedge 2}(Ex'(0)^2+L_3)+M_2L_4(1+12M^{\wedge 2}))$$

is a contradiction to condition (5), thus, $x \in \emptyset_1(x) + \emptyset_2(\mathrm{x})$ has a solution in B[0,r].

Hence the system(2.1) has at least one mild solution.

5. Example

In this section we will take the following example an fractional partial differential equations

$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}u(t,x)+G(t,x,u(t,x))\right]$$
$$\in \frac{\partial^2}{\partial x^2}[u(t,x)+k\left(t,x,u(t,x),D^{\propto}(u(t,x)\right)dw^H(t)],0\le t\le 1$$

$$u(t,a) = u(t,b) = 0,\ 0 \le t \le \pi$$
$$u(0,x) = u_0(x),\ a \le x \le b$$
$$u'(t,0) = u_1(x), H = L^2([0,\pi]).$$

Define $A : H \to H$ by $AZ = Z''$ with domain

$$D(A) = \{Z \in H : Z'' \in H, Z(0) = Z(\pi) = 0\}$$

it is well known that A is the infinitesimal generator for a strongly continuous cosine family $\{C(t)\}_{t\in}$ on H, A has a spectrum the eigenvaluses $-n^2, n = 1,2, \ldots$ *and eigenvector* $Zn(\tau) = \left(\frac{2}{\pi}\right)^2 sin(n\tau)$, with the following

a) $\{Z_n : n \in IN\}$is an orthonormal basis of X and

$$A\,Z = -\sum_{n=1}^{\infty} n^2 < Z, Z_n > Z_n, Z \in D(A)$$

b) For $Z \in X, C(t)\,Z = \sum_{n=1}^{\infty} cos\,(nt) < Z, Z_n > Z_n$, $C(t)\,Z = \sum_{n=1}^{\infty} cos\,(nt) < Z, Z_n > Z_n$. and

$$S(t)Z = \sum_{n=1}^{\infty} \frac{\sin(nt)}{n} < Z, Z_n > Z_n.$$

c) $\|C(t)\| = \|S(t)\| = 1$ for all $t \in IR$

d)
$k\left(t,.,u(t,.), \int_0^t a(t,.)ds\right) =$
$k(t,.,u(t,.), D^{\propto}(u(t,.)) =$
$k(t,.,u(t,.), I^{1-\propto}(v(t,.)) = u(t,.) + I^{1-\propto}(v(t,.))$

(e) $v(t,.) = b(t-s)f(s,.)$.

(f) $g(t,.) = G(t,.,u(t,.))$.

Under the appropriate conditions (1-5) of *k*, *b*, *g*, then theorem (2-3) , ensures the existence of mild solution to problem(2.1).

Asymptotically stable for the mild solution of inclution formulation problem (2.1).

Given as follows*:*

6. Asymptotically Stable for the Mild Solution of Inclusion Formulation Problem (2.1)

The assymptoically stable of the problem (2.1) has been given in details with necessary and sufficient conditions.

We need to investigate the definition (2-6) on the inclusion problem (2.1).

Theorem (2-4)

Assume the hypotheses (1-5) are hold then the solution has asymptotically stable behivours

Proof

$$x(t) = C(t)x(0) - S(t)\left[x'(0) - g(0, x(0))\right]$$
$$+\int_0^t AS(t-s)F\left(s, x(s), \int_0^t a(s, x(s))dw\right)$$
$$+\int_0^t S(t-s)\sigma(s)dw^H + \int_0^t C(t-s)g(s, x(s))ds.$$

Let $x(t) = x(t, 0, \emptyset_1)$ and $y(t) = y(t, 0, \psi_1)$ be a two solutions of equation (2.1)

$$x(t) = C(t)\varnothing_1 + S(t)\left[(\varnothing_2 - g(0,\varnothing_1))\right]$$
$$+\int_0^t AS(t-s)F\left(s, x(s), \int_0^t a(s, x(s))dw\right)$$
$$+\int_0^t S(t-s)\sigma(s)dw^H + \int_0^t C(t-s)[g(s,\varnothing_1)]ds.$$

$$y(t) = C(t)\psi_1 + S(t)\left[(\psi_2 - g(0,\psi_1))\right]$$
$$+\int_0^t AS(t-s)F\left(s, x(s), \int_0^t a(s, x(s))dw\right)$$
$$+\int_0^t S(t-s)\sigma(s)dw^H + \int_0^t C(t-s)[g(s,\psi_1)]ds.$$

$$\|x(t) - y(t)\| \le \|C(t)\|\|\varnothing_1 - \psi_1\|$$
$$+\|S(t)\|\left(\|\varnothing_2 - \psi_2\| + \|g(0,\varnothing_1) - g(0,\psi_1)\|\right)$$
$$+\int_0^t \|AS(t-s)\| \left\| \begin{matrix} F\left(s, x(s), \int_0^t a(s, x(s))\right) \\ -F\left(s, y(s), \int_0^t a(s, y(s))\right) \end{matrix} \right\| dw$$
$$+\int_0^t \|C(t-s)\|\|g(0,\varnothing_1) - g(0,\psi_1)\|ds$$
$$\le M^\wedge \|\varnothing_1 - \psi_1\| +, N^\wedge \left(\|\varnothing_2 - \psi_2\| + \|g(0,\varnothing_1) - g(0,\psi_1)\|\right)$$
$$+E\int_0^t, N^\wedge \left\| \begin{matrix} AF\left(s, x(s), \int_0^t a(s, x(s))\right)dw \\ -AF\left(s, y(s), \int_0^t a(s, y(s))\right)dw \end{matrix} \right\|$$
$$\le M^\wedge \|\varnothing_1 - \psi_1\| +, N^\wedge \left(\|\varnothing_2 - \psi_2\| + \|g(0,\varnothing_1) - g(0,\psi_1)\|\right)$$
$$+\int_0^t \left(N^\wedge L_1 \begin{pmatrix} E(x(s) - y(s) \\ +bL_2 E(x(s) - y(s)) \end{pmatrix} \right) dw$$
$$+M^\wedge \int_0^t \|g(0,\varnothing_1) - g(0,\psi_1)\|ds$$
$$\le M^\wedge \|\varnothing_1 - \psi_1\| +, N^\wedge \left(\|\varnothing_2 - \psi_2\| + \|g(0,\varnothing_1) - g(0,\psi_1)\|\right)$$
$$+L_1 N^\wedge (E\|x(s) - y(s)\| + bL_2 E\|x(s) - y(s)\|)Tr(Q)\mu(I)$$
$$+M^\wedge bL_5 \|\varnothing_2 - \psi_1\|$$

Hence,

$$E\|x(s) - y(s)\| \le M^\wedge \|\varnothing_1 - \psi_1\|$$
$$+N^\wedge \left(\|\varnothing_2 - \psi_2\| + L_4 \|\varnothing_1 - \psi_1\|\right)$$
$$+M^\wedge bL_5 / (L_1 N^\wedge + bL_2)Tr(Q)\mu(I)$$

such that $(L_1 N^\wedge + bL_2)Tr(Q)\mu(I) > 0$.

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