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Some Results on the Identity d(x) = λx + ζ(x)

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Abstract The main purpose of this paper is study and investigate some results concerning a derivation d on a 2-torsion free semiprime ring R with the center Z(R),when R admits d to satisfy some conditions, then there exist $\lambda \in C$ and an additive mapping ζ: R →C such that $d(x) = \lambda x + \zeta(x)$ for all $x \in R$.

Keywords: *semiprime rings, derivations, generalized derivation, biadditive mapping*

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1. Introduction

Some people ask why study derivation? at first we can say derivations on rings help us to understand rings better and also derivations on rings can tell us about the structure of the rings. For instance a ring is commutative if and only if the only inner derivation on the ring is zero. Also derivations can be helpful for relating a ring with the set of matrices with entries in the ring (see, [6]). Derivations play a significant role in determining whether a ring is commutative, see [1,3,4]. Derivations can also be useful in other fields. For example, derivations play a role in the calculation of the eigen values of matrices (see, [2]) which is important in mathematics and other sciences, business and engineering. Derivations also are used in quantum physics (see, [5]). Derivations can be added and subtracted and we still get a derivation, but when we compose a derivation with itself we do not necessarily get a derivation. In 1957 in [7] Posner proved that if d_1 and d_2 are two non-zero derivations on a prime ring whose characteristic is not 2, then d_1d_2 is not a derivation. Thus in a prime ring *R* whose characteristic is not 2 if d_2 is a derivation then d must be zero. In particular when d_2 is the zero derivation then d = 0. This means that the only nilpotent derivation with degree of nilpotency 2 on a prime ring whose characteristic is not 2 is the zero derivation. The usual derivative operator is additive and $(fg)' = f'g + fg'$. This definition has been generalized for every ring as follows. For a ring R, an additive map d: R →R is called a derivation on R if it satisfies the product rule d(ab) = d(a)b + ad(b). Many authors of the have studied centralizing derivations, endomorphisms, and some related additive mappings. Matej Bresar [9] proved, let R be a ring with center Z(R). mapping F of R into itself is called centralizing if $F(x)x - xF(x) \in Z(R)$ for all $x \in R$, then every additive centralizing mapping F on a von Neumann algebra R is of the form $F(x) = cx + \zeta(x)$, $x \in R$, where $c \in Z(R)$ and ζ is an additive mapping from R into Z(R),and also consider α-derivations and some related mappings, which are centralizing on rings and Banach algebras. The history of commuting and centralizing mappings goes back to (1955) when Divinsky [10] proved that a simple Artinian ring is commutative if it has a commuting nontrivial automorphism. Two years later, Posner [7] has proved that the existence of a non-zero centralizing derivation on prime ring forces the ring to be commutative (Posner's second theorem).Muhammad A.C. and Mohammed S.S. [11] proved, let R be a semiprime ring and d:R→ R a mapping satisfy d(x)y=xd(y) for all $x,y \in R$. Then d is a centralizer. Muhammad A.C. and A. B.Thaheem [12] proved, let d and g be a pair of derivations of semiprime ring R satisfying $d(x)x+xg(x) \in Z(R)$,then cd and cg are central for all $c \in Z(R)$. B.Zalar [13] has proved, let R be a 2-torsion free semiprime ring and d:R → R an additive mapping which satisfies $d(x^2) = d(x)x$ for all $x \in R$. Then d is a left centralizer. Recently, Mehsin Jabel [18,19,20] proved some results concerning generalized derivations on prime and semiprime rings.

In this paper we study and investigate some results concerning derivation d on semiprime ring R, we give some results about that.

2. Preliminaries

Throughout this paper will represent an associative ring with identity with the center Z(R).We recall that R is semiprime if xRx = (o) implies x=o and it is prime if xRy=(o) implies x=o or y=o. A prime ring is semiprime but the converse is not true in general. A ring R is 2-torsion free in case 2x = o implies that x = o for any $x \in R$. An additive mapping d: R → R is called a derivation if d(xy)=d(x)y+xd(y) holds for all $x,y \in R$.A mapping d is called centralizing if $[d(x),x] \in Z(R)$ for all $x \in R$, in particular, if [d(x),x] = o for all $x \in R$, then it is called commuting, and is called central if $d(x) \in Z(R)$ for all $x \in R$. Every central mapping is obviously commuting but not conversely in general. In [14] Q.Deng and H.E.Bell extended the notion of commutativity to one of n-commutativity, where n is an arbitrary positive integer, by defining a mapping d to be n-commuting on U if $[x^n,d(x)]=0$ for all x∈U, where U be a non empty subset of R. A biadditive mapping B: R× R→ R is called a biderivation if for every u∈R the mappings x →B(x, u) and x →B (u,x) are derivations of R. For any semiprime ring R one can construct the ring of quotients Q of R [15], As R can be embedded isomorphically in Q, we consider R as a subring of Q. If the element q∈Q commutes with every element in R then q belongs to C, the center of Q. C contains the centroid of R, and it is called the extended centroid of R. In general, C is a von Neumann regular ring, and C is a field if and only if R is prime [[15], Theorem 12]. As usual, we write [x,y] for xy –yx and make use of the commutator identities [xy,z]=x[y,z]+[x,z]y and [x,yz]=y[x,z]+[x,y]z, and the symbol (xoy) stands for the anti-commutator xy+yx.

The following lemmas are necessary for the paper.

Lemma 1 [[17]: Lemma 1.8]

Let R be a semiprime ring, and suppose that a∈R centralizes all commutators [x,y], x,y ∈ R. Then a∈Z(R).

Lemma 2 [[13]: Lemma1.3]

Let R be a semiprime ring and let a R be a fixed element. If a[x,y]=0 holds for all pairs x,y R, then there exists an ideal U of R such that a∈U ⊂Z(R),where Z(R) denoted to the center of R.

Lemma 3 [[8]: Problem 14, Pag 9]

N has no non-zero nilpotent elements iff $a^2=0$ implies a=0 for all a∈N, where N is a near –ring is a triple (N,+,.) satisfying the condition:

(i) (N,+)is a group which may not be a belian.

(ii) (N,.) is a semigroup.

(iii) For all x,y,x∈ N, x(y+z) = xy + xz. All rings is near –rings.

Lemma 4 [[16], Theorem 4.1]

Let R be a semiprime ring, and let B: R×R→ R be a biderivation. Then there exist an idempotent ε∈C and an element μ∈C such that the algebra (1 - ε)R is commutative and εB(x,y) = με[x,y] for all x, y∈R.

Lemma 5 [[21], Main Theorem]

Let R be a semiprime ring, d a non-zero derivation of R, and U a non-zero left ideal of R. If for some positive integers $t_0,t_1,\ldots,t_n$ and all x∈U, the identity $[\ldots[[d(x^{t0}),x^{t1},x^{t2}],\ldots],x^{tm}]=o$ holds, then either d(U)= o or else d(U) and d(R)U are contained in non-zero central ideal of R. In particular when R is a prime ring, R is commutative**.**

3. The main results

Theorem 3.1.

Let R be a 2-torsion free semiprime ring and d: R → R be a derivation on R such that $d^n(xoy)\pm(xoy)\in Z(R)$ for all x,y∈R, then there exist C and an additive mapping ζ: R→C such that d(x)=λx+ζ(x) for all x∈R, where n is a fixed positive integer.

Proof: At first we suppose that d in non-zero derivation in our main relation $d^n(xoy)+(xoy)\in Z(R)$ for all x,y∈R, then we have $d^{n-1}(d(xoy))+(xoy)\in Z(R)$ for all x,y∈R, replacing y by (yoz),we get $d^{n-1}(d(xo(yoz)))+(xo(yoz))\in Z(R)$ for all x,y,z ∈ R. Then $d^n(x)(yoz)+(yoz)d^n(x)+2x(d^n(yoz)+(yoz))\in Z(R)$ for all x,y,z∈R.

$$\left[d^n(x)o(yoz),r\right]+2x\left[d^n(yoz)+(yoz),r\right] +2[x,r]\left(d^n(yoz)+(yoz)\right)=0 \text{ for all } x,y,z\in R. \quad (1)$$

According to our main relation $d^n(xoy)+(xoy)\in Z(R)$, the equation (1), reduces to

$$\left[d^n(x)o(yoz),r\right]+2[x,r]\left(d^n(yoz)+(yoz)\right)=0 \text{ for all } x,y,z\in R. \quad (2)$$

Replacing x by (yoz) in (2),we get

$$\left[d^n((yoz))\,o(yoz),r\right] +2[(yoz),\ r]\left(d^n(yoz)+(yoz)\right)=0 \text{ for all } y,z\in R. \quad (3)$$

Form the main relation $d^n(xoy)+(xoy)\in Z(R)$ for all x,y∈ R, we obtain $[d^n(xoy),r]+[xoy,r]=0$ for all x,y,r∈R, the replacing r by (xoy),we obtain

$$\left[d^n(xoy),(xoy)\right]=0 \text{ for all } x,y,r\in R. \quad (4)$$

Then substituting (4) in (3), we get $2[d^n(yoz)(yoz),r]+[yoz,r](d^n(yoz)+(yoz))=0$ for all z,y,r ∈R.

Then $2d^n(yoz)(yoz)r-2rd^n(yoz)(yoz)+(yoz)rd^n(yoz)-r(yoz)rd^n(yoz)+[yoz,r](yoz)=0$ for all z,y,r∈R. Then by using (4) in above equation, we get

$$2d^n(yoz)(yoz)r-rd^n(yoz)(yoz)+(yoz)rd^n(yoz) +[yoz,r]\ (yoz)=0 \text{ for all } z,y,r\hat{I}R. \quad (5)$$

Replacing r by (yoz) with using (4) in (5), we arrive at $2d^n(yoz)(yoz)^2=0$ for all y,z∈R. Since R is 2-torsion free semiprime, we get $d^n(yoz)(yoz)^2=0$ for all y,z∈R.

Rirht-multiplying by [x,r], we obtain $d^n(yoz)(yoz)^2[x,r]=0$ for all x,y,z,r∈R.

According to Lemma 2, the there exists an ideal U of R such that $d^n(yoz)(yoz)^2\in U\subset Z(R)$ for all y,z∈R. This fact lead to have a central ideal U of R, which implies that [x,y]=0 for all x∈U, y∈R.

Apply d to both sided, we obtain [d(x),y]+[x,d(y)]=0 for all x∈U, y∈R. Since U is central ideal, we get [d(x),y]=0 for all x∈U, y∈R. Thus, we get that d is commuting of R, which implies [z,d(z)]=0 for all z∈R. Linearizing [d(z), z] = 0, we obtain [d(z),y] = [z, d(y)]. Hence, we see that the mapping (z, y) → [d(z), y] is a biderivation. By Lemma 4, there exist an idempotent ε ∈C and an element μ∈C such that the algebra (1 - ε)R is commutative (hence, (1 - ε)R ⊆ C), and ε[d(z), y] = εμ[z,y] holds for all z, y∈R. Thus, εd(z) - με z commutes with every element in R, so that εd(z) - με z∈C. Now, let εd(z) =με and define a mapping ζ by ζ(z) = (εd(z) - λz) + (1 - ε)d(z). Note that ζ maps in C and that d(z) = λz + ζ(z) holds for every z∈R, by this we complete our proof.

When d=0, we have (xoy)∈Z(R) for all x,y∈R, replacing y by d(x),we obtain $d(x^2)\in Z(R)$ for all x∈R, then $[x,d(x^2)]=0$ for all x∈R. According to Lemma 2, then there exists a central ideal U of R, so by same technique, we completes the proof of the theorem.

Corollary 3.2.

Let R be a 2-torsion free semiprime ring and d: R → R be a non-zero derivation on R such that $d^n(xoy)\pm(xoy)$

Z(R) for all x,y∈R, then $[d^n(xoy),(xoy)^n]=0$ for all x,y ∈R, where n is a fixed positive integer.

Proof: According to Theorem 3.1, we have the relation

$$\left[d^n\left(xoy\right),\left(xoy\right)\right]=0 \text{ for all } x,y\in R. \quad (6)$$

And

$$d^n\left(xoy\right)\left(xoy\right)^2=0 \text{ for all } x,y\in R. \quad (7)$$

According to (6), we obtain

$$\left(xoy\right)d^n\left(xoy\right)\left(xoy\right)=\left(xoy\right)^2 d^n\left(xoy\right)=0 \text{ for all } x,y\in R. \quad (8)$$

Subtracting (7) and (8),we arrive at

$$\left[d^n\left(xoy\right),\left(xoy\right)^2\right]=0 \text{ for all } x,y\in R. \quad (9)$$

Again right-multiplying of relation (7), by (xoy) gives $d^n(xoy)(xoy)^3=0$ for all x,y∈R. Then according to (9) and (6),we arrive at $[d^n(xoy),(xoy)^3]=0$ for all x,y∈R.

So, if we continue by same technique, arrive to the relation $[d^n(xoy),(xoy)^n]=0$ for all x,y∈R, which completes the proof of the corollary.

Corollary 3.3.

Let R be a 2-torsion free semiprime ring and d: R→ R be a non-zero derivation on R such that $d^n(xoy)\pm(xoy)\in Z(R)$ for all x, y ∈R, then R contains a central ideal, where n is a fixed positive integer.

Theorem 3.4.

Let R be a 2-torsion free semiprime ring and d: R→ R be a derivation on R such that $d^n([x,y])\pm[x,y]\in Z(R)$ for all x,y∈R, then there exist C and an additive mapping ζ: R → C such that d(x)=λx+ζ(x) for all x∈R, where n is a fixed positive integer.

Proof: We have the main relation $d^n([x,y])+[x,y]\in Z(R)$ for all x,y∈R, now we suppose that d is non-zero derivation on R, then $d^n([x,y])+[x,y]\in Z(R)$ for all x,y∈R, then

$$\left[d^n(\left[x,y\right]),r\right]+\left[\left[x,y\right],r\right]=0 \text{ for all } x,y,r\in R. \quad (10)$$

Replacing r by [x,y],gives $[d^n([x,y]),[x,y]]=0$ for all x,y∈R.

Applying Lemma 3, we obtain $d^n([x,y])\in Z(R)$ for all x,y∈R.

The substitution this fact in the relation (10), gives

$$\left[\left[x,y\right],r\right]=0 \text{ for all } x,y,r\in R. \quad (11)$$

Replacing r by d(z) in (11) with apply Lemma 1, we arrive to d(z)∈Z(R) for all z∈R, which leads to [d(z),z]=0 for all z∈R. Linearizing [d(x), x] = 0 we obtain [d(x), y] = [x, d(y)]. Hence, we see that the mapping (x, y)→[d(x), y] is a biderivation. By Lemma 4, there exist an idempotent ε∈C and an element μ∈C such that the algebra (1 - ε)R is commutative (hence, (1 - ε)R ⊆ C), and ε[d(x), y] = εμ[x,y] holds for all x, y∈R. Thus, εd(x) - με x commutes with every element in R, so that εd(x) - με x∈C. Now, let εd(x) =με and define a mapping ζ by ζ(x) = (εd(x) - λx) + (1 - ε)d(x). Note that ζ maps in C and that d(x) = λx + ζ(x) holds for every x ∈R, by this we complete our proof.

When d=0, we have [x,y]∈Z(R) for all x,y∈R, then by same technique in the first part of proof, we completes the proof.

Theorem 3.5.

Let R be a 2-torsion free semiprime ring and d:R → R be a derivation on R such that $d^n(xoy)\pm[x,y]\in Z(R)$ for all x,y∈R, then there exist C and an additive mapping ζ: R → C such that d(x)=λx+ζ(x) for all x∈R, where n is a fixed positive integer.

Proof: We have the relation $d^n(xoy)+[x,y]\in Z(R)$ for all x,y∈R.

Now, when d is non-zero derivation on R, we have the relation $d^n(xoy)+[x,y]\in Z(R)$ for all x,y∈R. Then

$$\left[d^n\left(xoy\right),r\right]+\left[\left[x,y\right],r\right]=o \text{ for all } x,y,r\in R. \quad (12)$$

Replacing r by [x,y], we obtain $[d^n(xoy),[x,y]]=o$ for all x,y∈R. Applying Lemma 3, we obtain $d^n(xoy)\in Z(R)$ for all x,y∈R, thus by using this fact in (12), we arrive to [[x,y],r]=o for all x,y,r∈R.

Replaicing r by d(z) with applying Lemma 1, we obtain [d(z),z]=0 for all z∈R. Then Linearizing [d(z), z] = 0 we obtain [d(x), y] = [x, d(y)]. Hence, we see that the mapping (x, y)→[d(x), y] is a biderivation. By Lemma 4, there exist an idempotent ε∈C and an element μ∈C such that the algebra (1 - ε)R is commutative (hence, (1 - ε)R ⊆ C), and ε[d(x), y] = εμ[x,y] holds for all x, y∈R. Thus, εd(x) - με x commutes with every element in R, so that εd(x) - με x∈C. Now, let εd(x) =με and define a mapping ζ by ζ(x) = (εd(x) - λx) + (1 - ε)d(x). Note that ζ maps in C and that d(x) = λx + ζ(x) holds for every x ∈ R, by this we complete our proof.

When d=0, we have [x,y]∈Z(R) for all x,y∈R, then by same technique in the first part of proof of Theorem 3.4, we completes the proof.

Theorem 3.6.

Let R be a 2-torsion free semiprime ring and U is a non-zero ideal of R, d: R → R be a derivation on R. If R admits d to satisfy $d^n([x,y])\pm(xoy)\in Z(R)$ for all x,y∈U, then there exist C and an additive mapping ζ: R→ C such that d(x)=λx+ζ(x) for all x∈R, where n is a fixed positive integer.

Proof: In our theorem, we have the main relation $d^n([x,y])+(xoy)\in Z(R)$ for all x,y∈U. Now, we assume that d is non-zero derivation of R, then $d^n([x,y])+(xoy)\in Z(R)$ for all x,y∈U, which gives $[d^n([x,y]),r]+[xoy,r]=0$ for all x,y∈U.

Replacing x by y and r by d(y),we obtain $2[d(y),y^2]=0$ for all y∈U. Since R is 2-torsion free semiprime with apply

Lemma 5, we have R contains a non-zero central ideal U, then for all x∈U, there exists y∈R, such that [x,y]=0 for all x∈U, y∈R.

Apply d to both sided, we obtain [d(x),y]+[x,d(y)]=0 for all x∈U,y∈R. Since U is central ideal, we get [d(x),y]=0 for all x∈U, y∈R. Thus, we get that d is commuting of R, which implies [z,d(z)]=0 for all z∈R.

Linearizing [d(z), z] = 0, we obtain [d(z),y] = [z, d(y)]. Hence, we see that the mapping (z, y)→[d(z), y] is a biderivation. By Lemma 4, there exist an idempotent ε ∈C and an element μ ∈C such that the algebra (1 - ε)R is commutative (hence, (1 - ε)R ⊆ C), and ε[d(z), y] = εμ[z,y] holds for all z, y ∈ R. Thus, εd(z) - με z commutes with every element in R, so that εd(z) - με z∈C. Now, let εd(z) =με and define a mapping ζ by ζ(z) = (εd(z) - λz) + (1 - ε)d(z). Note that ζ maps in C and that d(z) = λz + ζ(z) holds for every z∈R, by this we complete our proof. When d=0, we have (xoy)∈Z(R) for all x,y∈R, the we completes the proof of the theorem by same method in part second of Theorem 3.1.

Corollary 3.7.

Let R be a 2-torsion free semiprime ring and d: R → R be a derivation on R. If R admits d to satisfy $d^n([x,y])\pm(xoy) \in Z(R)$ for all x,y∈ R, then R contains anon-zero central ideal, where n is a fixed positive integer.

Theorem 3.8.

Let R be a 2-torsion free semiprime ring and d: R→ R be a derivation on R. If R admits d to satisfy $d^n(xoy)\pm d^q(xoy)\pm[x,y] \in Z(R)$ for all x,y∈ R, then there exist C and an additive mapping ζ: R→ C such that d(x)=λx+ζ(x) for all x∈R, where n and q be a fixed positive integers.

Proof: We have the relation $d^n(xoy)+d^q(xoy)+[x,y] \in Z(R)$ for all x,y∈R. Suppose that d is non-zero derivation of R, we have $d^n(xoy)+d^q(xoy)+[x,y] \in Z(R)$ for all x,y∈R, then

$$\left[d^n(xoy)+d^q(xoy),r\right]+\left[[x,y],r\right]=0 \quad \text{for all } x,y,r\in R. \tag{13}$$

Replacing r by [x,y], we obtain $[d^n(xoy)+d^q(xoy),[x,y]]=0$ for all x,y∈R. Apply Lemma1, and substituting that result in relation (13), we get [[x,y],r]=0 for all x,y,r∈R. Then replacing r by d(z), we get [[x,y],d(z)]=0 for all x,y,z∈R. Then by apply Lemma1, we arrive to [d(z),z]=0 for all z ∈R. Linearizing [d(x), x] = 0 we obtain [d(x), y] = [x, d(y)]. Hence, we see that the mapping (x, y)→[d(x), y] is a biderivation. By Lemma 4, there exist an idempotent ε ∈C and an element μ ∈C such that the algebra (1 - ε)R is commutative (hence, (1 - ε)R ⊆ C), and ε[d(x), y] = εμ[x,y] holds for all x, y ∈ R. Thus, εd(x) - με x commutes with every element in R, so that εd(x) - με x ∈C. Now, let εd(x) =με and define a mapping ζ by ζ(x) = (εd(x) - λx) + (1 - ε)d(x). Note that ζ maps in C and that d(x) = λx + ζ(x) holds for every x ∈R, by this we complete our proof.

When d=0, we get the relation [x,y]∈Z(R) for all x,y∈R. Therefore, by using same method in the first part of proof of the theorem, we completes our proof.

Proceeding on the same line with necessary variations, we can prove the following theorem.

Theorem 3.9.

Let R be a 2-torsion free semiprime ring and d: R→ R be a derivation on R. If R admits d to satisfy

(i) $d^n(xoy)\pm d^q([x,y])\pm[x,y]\in Z(R)$ for all x,y∈R.

(ii) $d^n([x,y])\pm d^q([x,y])\pm[x,y]\in Z(R)$ for all x,y∈R.

then there exist C and an additive mapping ζ: R → C such that d(x)=λx+ζ(x) for all x∈R, where n and q be a fixed positive integers.

Theorem 3.10.

Let R be a 2-torsion free semiprime ring and d: R→ R be a derivation on R. If R admits d to satisfy $d^n([x,y])\pm d^q([x,y])\pm(xoy) \in Z(R)$ for all x,y∈R, then there exist C and an additive mapping ζ: R →C such that d(x)=λx+ζ(x) for all x ∈R, where n and q be a fixed positive integers.

Proof: The main relation it is $d^n([x,y])+d^q([x,y])+(xoy) \in Z(R)$ for all x,y∈ R. Suppose that d is non-zero derivation of R, we have $d^n([x,y])+d^q([x,y])+(xoy)\in Z(R)$ for all x,y∈ R, then $[d^n([x,y])+d^q([x,y]),r]+[xoy,r]=0$ for all x,y∈R

Replacing x by y, gives $2[y^2,r]=0$ for all y,r∈R. Since R is 2-torsion free semiprime, we obtain $y^2\in Z(R)$ with apply Lemma 3, we arrive to [y,r]=[y,d(y)]=0 for all y,r ∈R.

When d=0, the our main relation reduces to (xoy)∈Z(R) for all x,y ∈R, replacing y by x, with using R is 2-torsion free semiprime, we get

$[x^2,r]=0$ for all x,r ∈R. This relation gives $x^2\in Z(R)$,with apply Lemma 3, we arrive to

[x,r]=[x,d(x)]=0 for all x,r ∈R. Linearizing [d(x), x] = 0 we obtain [d(x), y] = [x, d(y)]. Hence, we see that the mapping (x, y) → [d(x), y] is a biderivation. By Lemma 4, there exist an idempotent ε∈C and an element μ ∈C such that the algebra (1 - ε)R is commutative (hence, (1 - ε)R ⊆ C), and ε[d(x), y] = εμ[x,y] holds for all x, y ∈ R. Thus, εd(x) - με x commutes with every element in R, so that εd(x) - με x∈C. Now, let εd(x) =με and define a mapping ζ by ζ(x) = (εd(x) - λx) + (1 - ε)d(x). Note that ζ maps in C and that d(x) = λx + ζ(x) holds for every x ∈R, by this we complete our proof.

When d=0, we have (xoy) ∈ Z(R) for all x,y∈ R, the we completes the proof of the theorem by same method in part second of Theorem 3.1.

Theorem 3.11.

Let R be a 2-torsion free semiprime ring and d:R→ R be a derivation on R. If R admits a derivation d to satisfy $d^n(xoy)\pm d^q(xoy)\pm(xoy)\in Z(R)$ for all x,y∈ R, then there exist C and an additive mapping ζ: R → C such that d(x)=λx+ζ(x) for all x∈ R, where n and q be a fixed positive integer.

Proof: We have the relation $d^n(xoy)+d^q(xoy)+(xoy) \in Z(R)$ for all x,y∈ R, which can be written in the form $g^w(xoy)+(xoy)\in Z(R)$ for all x,y∈ R, where g(xoy) stands for $d^n(xoy)+d^q(xoy)$ and w stands for a fixed positive integer n and q.

Then, we have $[g^w(xoy)+(xoy),r]=0$ for all x,y∈ R. Thus by same technique in the proof of the Theorem 3.1, we completes the proof.

Corollary 3.12.

Let R be a 2-torsion free semiprime ring and d: R→ R be a non-zero derivation on R such that $d^n(xoy)\pm d^q(xoy)\pm(xoy)\in Z(R)$ for all x,y ∈R then $[d^n(xoy)+d^q(xoy),(xoy)^n]=0$ for all x,y∈ R, where n and q be a fixed positive integers.

Corollary 3.13.

Let R be a 2-torsion free semiprime ring and d:R→ R be a non-zero derivation on R such that $d^n(xoy)\pm d^q(xoy)\pm(xoy) \in Z(R)$ for all x,y ∈R, then R contains a central ideal, where n and q be a fixed positive integers.

Remark 2.13.

In our theorems we cannot exclude the condition torsion free, the following example explain that.

Example 2.14.

We denote by Z the integer system, let:

$$R = \begin{pmatrix} \frac{Z}{2Z} & \frac{Z}{2Z} \\ \frac{Z}{2Z} & \frac{Z}{2Z} \end{pmatrix}, a = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and d is the inner derivation induced by a (every derivation is inner derivation), that is, d(x) =[a, x], for all x $\in$ R. Then R is a non-commutative prime ring with char R=2, and d([x,y])±[x,y] $\in$ Z(R), for all x, y $\in$ R.

Then by similar approach we can show otherwise.

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