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Theoretical and Numerical Analysis of a Jib Crane Vibration

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Abstract A Comparative approach is used that adopted analytical and numerical methods in analysing the vibration of a jib crane. The analytical method employed the use of planar serial-frame structures analysis which utilizes transfer matrix solution to the governing equation of motion. The jib and the mast are treated as different segment and analysed by the transverse and longitudinal motions of each segment while considering the compatibility requirements across the mast from the jib and the boundary condition of the entire jib crane system. Finite element analysis using ANSYS software was employed to analyse the jib crane system as the numerical method. The jib crane analytical approach derived theoretically was modelled in ANSYS and solved in other to evaluate and compare the vibration parameters of the system.

Keywords: *jib crane, mast, jib, ANSYS*

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1. Introduction

Jib cranes are part of material handling equipment that help in loading and offloading docks, machining and assembly operations, industrial and construction sites. They come in different configurations as application requires ranging from free standing jib cranes to wall bracket and wall cantilever cranes. Several researches were conducted in different aspects of cranes to study, analyses and solve variety of problems associated with them.

An active vibration control based on stability was presented in [1]. The study modelled and developed a nonlinear feedback control scheme and simulated the data using an actual jib crane. The result from this study shows the usefulness of derived model in suppressing the payload vibration. Some works presented dealt with FEM modelling and analysis of such cranes in preventing failures. The study presented in [2] studied the advantages and disadvantages of various FE models for crane structures to avoid service failure while the study presented in [3] modelled a tower crane using FEM and derived the governing equations of motion for the dynamic problem. A comparative analysis of several concept solutions and selected solutions of jib cranes was presented in [4]. The study focused on using the KOMIPS system and the static-dynamic analysis of complex configuration.

Studies related to cranes analysis have been mostly based on FEM approach [5,6,7] without the theoretical background analysis of such systems analysed as a means of comparison and validation. Theoretical approach of a cantilever beam was derived in [8]. The study looked at a cantilever beam with a damped dynamic vibration absorber attached to the beam at desired locations. This gives an inspiration for the development of analytic solution of the jib crane and comparison of such systems.

In this paper, we present a comparative approach of both theoretical and FEM methods of analysing the vibration of a jib crane. The theoretical method explores frame structures approach presented in [9] to model the jib crane as a continuous structure to derive the governing equations of motion and ANSYS [10] will be used to solve same system as the numerical method which will serve as a yardstick for comparison.

2. Methodology

2.1. Theoretical Modelling a Jib Crane as a Frame Structure

The jig crane system is represented in Figure 1 with k frame and angle θ_1. The system is sectioned into k and $k+1$ components inclined at an angle in other to be treated as a sub structural frame. The Jib and Mast having length l_1 and l_2 respectively with the position of the frame angle located at B. The Jib and Mast are analysed independently by their transverse and longitudinal motion.

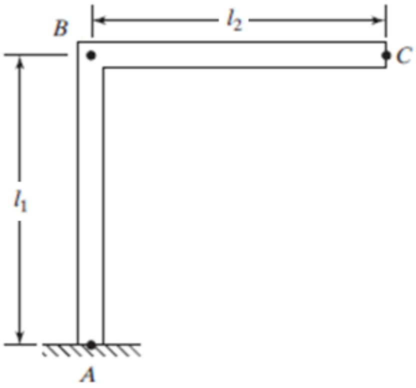

Figure 1. The Jib Crane System

The X axis represent the longitudinal displacement while the Y axis represent the transverse displacement of the Jib as shown in Figure 2. Likewise, for the mast is represented by the Y and X axes respectively. Hence, vibration theories for Euler-Bernoulli beam and axial vibration of a rod are employed to analyse the system.

The amplitude of vibration of the transverse displacement of the jib is denoted by Y_i (X, T) and longitudinal displacements is denoted by $U_i(X, T)$. The total length of the system is $L = (L_1 + L_2)$. The equation of motion for each of the Jib with uniform cross-section, is:

Transverse motion:

$$EA\frac{\partial^4 Y_i(x,t)}{\partial x^4}+\rho A\frac{\partial^2 Y_i(x,t)}{\partial t^2}=0, \qquad (1)$$
$$X_{i-1} < X < X_i, i=1,\ 2\ (k+1)$$

Longitudinal motion:

$$E\frac{\partial^2 U_i(x,t)}{\partial x^2}+\rho\frac{\partial^2 U_i(x,t)}{\partial t^2}=0, \qquad (2)$$
$$X_{i-1} < X < X_i, i=1,\ 2\ (k+1)$$

The boundary conditions at point A is W(0)=0, W’(0)=0, U(0)=0 and U’(0), At point C is W”(l)=0 and W”’(l)=0 and at point B is $Y_2(x) = U_1(x)$, $U_2(x) = -Y_1(x)$, $Y_2'(x) = Y_1'(x)$, $Y_2''(x) = Y_1'''(x)$, $Y_2'''(x) = -U_1'''(x)$, $U_2^p(x) = Y_1^p(x)$.

The compatibility conditions at point B are

$$Y(0,T) = Y(L,T) = 0 \qquad (3a)$$

$$Y'(0,T) = Y'(L,T) = 0 \qquad (3b)$$

$$U(0,T) = U(L,T) = 0. \qquad (3c)$$

The compatibility conditions transfer the transverse and longitudinal displacement, slope, bending moment, shear force and axial force, respectively from the mast to the jib at angle θ_i. This compatibility requirements across the mast from the jib angle θ_i are Y_i and U_i that represent the transverse and longitudinal displacements of the jib at point B as shown in Figure 3(a) while the force compatibility requirements across the jib angle θ_i are V_i and F_i that represent the shear and axial forces of the Jib at point B as shown in Figure 3(b). The mast is the ith segment while the Jib is the $ith+1$ segment.

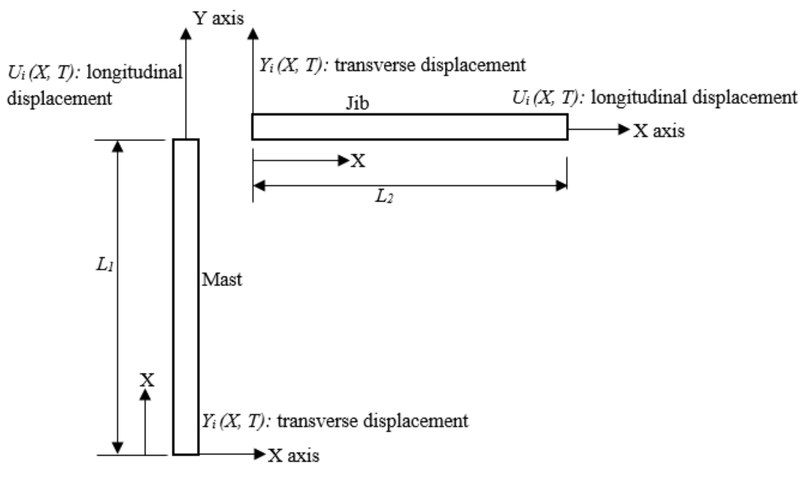

Figure 2. Transverse and longitudinal motion of the system

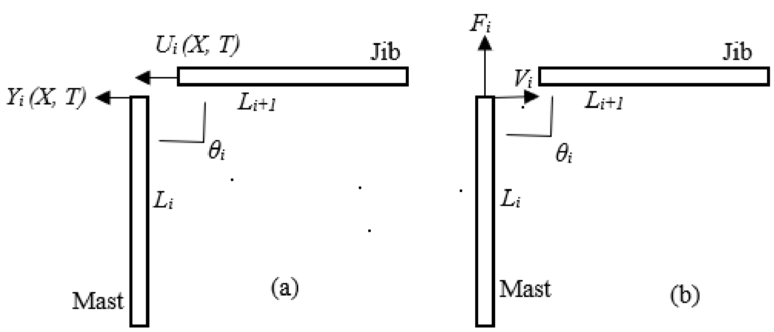

Figure 3. (a) Displacement Compatibility, (b) Force Compatibility

$$Y_{i+1}(X_i^+,T) = -Y_i(X_i^-,T)\cos\theta_i + U_i(X_i^-,T)\sin\theta_i \quad \text{displacement continuity} \tag{4a}$$

$$U_{i+1}(X_i^+,T) = -Y_i(X_i^-,T)\sin\theta_i + U_i(X_i^-,T)\cos\theta_i \quad \text{displacement continuity} \tag{4b}$$

$$Y'_{i+1}(X_i^+,T) = Y'_i(X_i^-,T) \text{ slope continuity} \tag{4c}$$

$$Y''_{i+1}(X_i^+,T) = Y''_i(X_i^-,T) \text{ moment continuity} \tag{4d}$$

$$EIY'''_{i+1}(X_i^+,T) = -EIY'''_i(X_i^-,T)\cos\theta_i - EIU_i(X_i^-,T)\sin\theta_i \quad \text{shear continuity} \tag{4e}$$

$$EIU'_{i+1}(X_i^+,T) = EIY'''_i(X_i^-,T)\sin\theta_i - EIU'_i(X_i^-,T)\cos\theta_i \quad \text{axial force continuity} \tag{4f}$$

The symbols X_i^+ and X_i^- refers to the mast and jib above and below the angle between the two point at B referred to as X_i. The assumptions in the mentioned compatibility conditions are the same as the normal analysis of the transverse vibrations of Euler–Bernoulli beam and the axial vibrations of a rod and the angle between the mast and jib is assumed to be constant during the motion of the system.

The following quantities are introduced:

$$y = \frac{Y}{L}, x = \frac{X}{L}, u = \frac{U}{L}, t = \frac{T}{L}, l_i = \frac{L_i}{L}, x_i = \frac{X_i}{L}. \tag{5}$$

Thus, in each beam, Eqns. (1) and (2) can be expressed in a non-dimensional form as

$$\frac{EI}{L^3}\frac{\partial^4 y_i(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y_i(x,t)}{\partial t^2} = 0, \quad X_{i-1} < X < X_i, i = 1,\ 2\ (k+1) \tag{6}$$

$$\frac{E}{L}\frac{\partial^2 u_i(x,t)}{\partial x^2} + \rho\frac{\partial^2 u_i(x,t)}{\partial t^2} = 0, \quad X_{i-1} < X < X_i, i = 1,\ 2\ (k+1). \tag{7}$$

The non-dimensional ''compatibility conditions'' from the mast to the jib angle are (from Equations (4a) – (4f))

$$y_{i+1}(x_i^+,t) = -y_i(x_i^-,t)\cos\theta_i + u_i(x_i^-,t)\sin\theta_i \tag{8a}$$

$$u_{i+1}(x_i^+,t) = -y_i(x_i^-,t)\sin\theta_i + u_i(x_i^-,t)\cos\theta_i \tag{8b}$$

$$y'_{i+1}(x_i^+,T) = y'_i(x_i^-,t) \tag{8c}$$

$$y''_{i+1}(x_i^+,t) = y''_i(x_i^-,t) \tag{8d}$$

$$y'''_{i+1}(x_i^+,t) = -y'''_i(x_i^-,t)\cos\theta_i - \frac{AL^2}{I}u_i(x_i^-,t)\sin\theta_i \tag{8e}$$

$$u'_{i+1}(x_i^+,y) = \frac{I}{AL^2}y'''_i(x_i^-,t)\sin\theta_i - u'_i(x_i^-,t)\cos\theta_i \tag{8f}$$

where $i = 1,\ 2$ (i+1) with the 1 standing for the mast and 2 for the jib. Similarly, the non-dimensional boundary conditions from Equations (3a) - (3c), for point B having a fixed–fixed ends, is written as

$$y(0,t) = 0, \quad y(l,t) = 0 \tag{9a}$$

$$y'(0,t) = 0, \quad y'(l,t) = 0 \tag{9b}$$

$$u(0,t) = 0, \quad u(l,t) = 0. \tag{9c}$$

2.2. Calculation of the Eigen Solutions

The solutions of the other boundary conditions can then be obtained through same procedure. Using the separable solutions: $y_i(x,t) = w_i(x)e^{i\omega t}$ and $u_i(x,t) = \upsilon_i(x)e^{i\omega t}$ in Equations (6) and (7) will lead to the associated eigenvalue problem,

$$w_i''''(x) - \lambda^4 w_i(x) = 0, x_{i-1} < x < x_i, i = 1,\ 2\ (k+1) \tag{10}$$

$$\upsilon_i''(x) - \gamma^2\upsilon_i(x) = 0, x_{i-1} < x < x_i, i = 1,\ 2\ (k+1) \tag{11}$$

Where

$$\lambda^4 = \frac{\rho A L^4\omega^2}{EI} \text{ and } \gamma^2 = \frac{\rho L\omega^2}{E} \tag{12}$$

From Equation (12), the relationship between λ and γ is expressed as

$$\gamma = \lambda^2\frac{1}{L}\sqrt{\frac{I}{A}} = a\lambda^2 \tag{13}$$

where a is a constant expressed as $a = (1/L)\sqrt{I/A}$. From Equations (8a) – (8f), the corresponding compatibility conditions from the mast to the jib angle gives

$$w_{i+1}(x_i^+) = -w_i(x_i^-)\cos\theta_i + u_i(x_i^-)\sin\theta_i \tag{14a}$$

$$\upsilon_{i+1}(x_i^+) = -\upsilon_i(x_i^-)\sin\theta_i + \upsilon_i(x_i^-)\cos\theta_i \tag{14b}$$

$$w'_{i+1}(x_i^+,T) = w'_i(x_i^-,t) \tag{14c}$$

$$w''_{i+1}(x_i^+) = w''_i(x_i^-) \tag{14d}$$

$$w'''_{i+1}(x_i^+) = -w'''_i(x_i^-)\cos\theta_i - \frac{AL^2}{I}\upsilon_i(x_i^-)\sin\theta_i \tag{14e}$$

$$\upsilon'_{i+1}(x_i^+) = \frac{I}{AL^2}w'''_i(x_i^-)\sin\theta_i - \upsilon'_i(x_i^-)\cos\theta_i. \tag{14f}$$

For $i = 1$ and 2, the boundary conditions, from Equations (9a) – (9c), are

$$w(0) = 0, \tag{15a}$$

$$w(l) = 0 \tag{15b}$$

$$w'(0) = 0, \tag{15c}$$

$$w'(l) = 0 \tag{15d}$$

$$\upsilon(0) = 0, \tag{15e}$$

$$\upsilon(l) = 0 \tag{15f}$$

A closed-form solution of this eigenvalue problem is obtained by employing transfer matrix methods as established in [9]. The solutions of Equation (10) and (11) for the mast and jib are

$$w_i(x) = A_i \cos\lambda(x - x_{i-1}) + B_i \sin\lambda(x - x_{i-1}) + C_i \cosh\lambda(x - x_{i-1}) + D_i \sinh\lambda(x - x_{i-1}) \quad (16)$$

$$\upsilon_i(x) = E_i \cos\gamma(x - x_{i-1}) + F_i \sin\gamma(x - x_{i-1}) = E_i \cosh a\lambda^2(x - x_{i-1}) + F_i \sinh a\lambda^2(x - x_{i-1}) \quad (17)$$
$$x_{i-1} < x < x_i, i = 1, 2.$$

where A_i, B_i, C_i, D_i, E_i and F_i are constants associated with the jib segment. The constants in the mast segment are A_{i+1} B_{i+1} C_{i+1} D_{i+1} E_{i+1} and F_{i+1} are related to those in the jib segment through the compatibility conditions in Equations (14a) – (14f), which can be expressed as

$$\begin{Bmatrix} A_{i+1} \\ B_{i+1} \\ C_{i+1} \\ D_{i+1} \\ E_{i+1} \\ F_{i+1} \end{Bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} & t_{15} & t_{16} \\ \vdots & & & & & \\ \vdots & & & & & \\ t_{61} & t_{62} & t_{63} & t_{64} & t_{65} & t_{66} \end{bmatrix}^i \begin{Bmatrix} A_i \\ B_i \\ C_i \\ D_i \\ E_i \\ F_i \end{Bmatrix} = T^i{}_{6\times6} \begin{Bmatrix} A_i \\ B_i \\ C_i \\ D_i \\ E_i \\ F_i \end{Bmatrix} \quad (18)$$

Where $T^i{}_{6\times6}$ is the 6×6 transfer matrix which depends on the eigenvalue λ; for which the elements are given in reference [9]

Through repeated applications of Equation (18), the six constants in the mast A_1, B_1, C_1, D_1, E_1 and F_1 can be mapped into those of the Jib, hence the number of independent constants of the entire system are reduced to six as:

$$\{A_2 \quad B_2 \quad C_2 \quad D_2 \quad E_2 \quad F_2\}^T = T_{6\times6}\{A_1 \quad B_1 \quad C_1 \quad D_1 \quad E_1 \quad F_1\}^T \quad (19)$$

These six remaining constants A_1, B_1, C_1, D_1, E_1 and F_1 can be determined through the satisfaction of the boundary conditions in Equation (15a) - (15f). For the system, Equations (16), (17), (15a), (15c) and (15e) lead to

$$B_1 + D_1 = 0, \quad (20a)$$

$$A_1 + C_1 = 0, \quad (20b)$$

$$F_1 = 0 \quad (20c)$$

The Satisfaction of those boundary conditions of Equations (16) and (17) at the mast support, Equations (15b), (15d) and (15f), requires

$$-A_2 \sin\lambda l_2 + B_2 \cos\lambda l_2 + C_2 \sinh\lambda l_2 + D_2 \cosh\lambda l_2 = 0 \quad (20d)$$

$$A_2 \cos\lambda l_2 + B_2 \sin\lambda l_2 + C_2 \cosh\lambda l_2 + D_2 \sinh\lambda l_2 = 0 \quad (20e)$$

$$E_2 \sinh a\lambda^2 l_2 + F_2 \cosh a\lambda^2 l_2 = 0 \quad (20f)$$

this is expressed in matrix form as

$$B_{3\times6}\{A_2 \quad B_2 \quad C_2 \quad D_2 \quad E_2 \quad F_2\}^T = 0 \quad (21)$$

Where

$$B_{3\times6} = \begin{bmatrix} -\sin\lambda l_{k+1} & \cos\lambda l_{k+1} & \sinh\lambda l_{k+1} & \cosh\lambda l_{k+1} & 0 & 0 \\ \cos\lambda l_{k+1} & \sin\lambda l_{k+1} & \cosh\lambda l_{k+1} & \sinh\lambda l_{k+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sinh a\lambda^2 l_{k+1} & \cosh a\lambda^2 l_{k+1} \end{bmatrix} \quad (22)$$

Substituting Equation (19) into Equation (21) and use of Equations (20a) - (20c) gives

$$B_{3\times6}\, T_{6\times6}\{A_1 \quad B_1 \quad C_1 \quad D_1 \quad E_1 \quad F_1\}^T = 0$$

Further written as

$$B_{3\times6}\, T^i{}_{6\times6}\{A_1 \quad B_1 \quad -A_1 \quad -B_1 \quad E_1 \quad 0\}^T = 0 \quad (23)$$

Where

$$R_{3\times6} = B_{3\times6} T_{6\times6}{}^k T_{6\times6}{}^{k-1} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ \vdots & & & & & \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} & r_{36} \end{bmatrix}$$

Hence, this gives a non-trivial solution which requires

$$\det\begin{bmatrix} r_{11}(\lambda) - r_{13}(\lambda) & r_{12}(\lambda) - r_{14}(\lambda) & r_{15}(\lambda) \\ r_{21}(\lambda) - r_{23}(\lambda) & r_{22}(\lambda) - r_{24}(\lambda) & r_{25}(\lambda) \\ r_{31}(\lambda) - r_{33}(\lambda) & r_{32}(\lambda) - r_{34}(\lambda) & r_{35}(\lambda) \end{bmatrix} = 0 \quad (24)$$

The determinant gives the characteristic equation for the solution of the eigenvalues λ_n. The solution of this equation with Newton–Raphson iterations, using the method shown in Reference [9] gives the eigenvalues.

This coefficients of the Eigen functions of the equation, $w_n(x)$ and $\upsilon_n(x)$; can then be obtained using back-substitution into Equation (23), (18) and then Equations (16) and (17).

Applying Equations (15a – f) into Equation (20d - f) for the Jib crane leads to,

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ -\sin\lambda l_{k+1} & -\cos\lambda l_{k+1} & \sinh\lambda l_{k+1} & \cosh\lambda l_{k+1} & 0 & 0 \\ -\cos\lambda l_{k+1} & \sin\lambda l_{k+1} & \cosh\lambda l_{k+1} & \sinh\lambda l_{k+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\cos a\lambda^2 l_{k+1} & \sin a\lambda^2 l_{k+1} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \\ E_1 \\ F_1 \end{bmatrix} = \begin{bmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \\ E_2 \\ F_2 \end{bmatrix} \quad (25)$$

For a non-trivial solution of Equation (25), determinant of coefficients of constants A_1, B_1, C_1, D_1, E_1 and E_1 must be zero, that is

$$\begin{vmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ -\sin\lambda l_{k+1} & -\cos\lambda l_{k+1} & \sinh\lambda l_{k+1} & \cosh\lambda l_{k+1} & 0 & 0 \\ -\cos\lambda l_{k+1} & \sin\lambda l_{k+1} & \cosh\lambda l_{k+1} & \sinh\lambda l_{k+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\cos a\lambda^2 l_{k+1} & \sin a\lambda^2 l_{k+1} \end{vmatrix} = 0 \quad (26)$$

Expanding the determinant gives the frequency equation

$$-2\cos a\lambda^2 l_{k+1}(\cos\lambda l_{k+1}\cosh\lambda l_{k+1}+1)=0 \quad (27)$$

This is similar to equation of a cantilever beam in [8], thus $\lambda l_{k+1}=\lambda l$ for this case and same solution applies as:

$$\cos\lambda l\cosh\lambda l+1=0 \quad (28)$$

The possible solution of Equation (28) is $\cos\lambda l=0$, which gives

$$\lambda_n=\frac{(2n-1)\pi}{2}. \quad (29)$$

From Equation (29) and Equation (13), The natural frequencies are:

$$\omega_n=\left(\frac{(2n-1)\pi}{2}\right)^2\frac{1}{l^2}\sqrt{\frac{EI}{\rho A}}. \quad (30)$$

There are infinite number of natural frequencies associated with a continuous structure as it possesses infinite degrees of freedom. The associate mode shapes are obtained by

$$W_n(x)=\sinh\lambda_n x-\sin\lambda_n x-\alpha(\cosh\lambda_n x-\cosh\lambda_n x)$$

$$\text{where } \alpha=\left[\frac{\sinh\lambda_n l+\sin\lambda_n l}{\cosh\lambda_n l+\cosh\lambda_n l}\right] \qquad n=1,\ 2,\ 3,\ldots \quad (31)$$

The response of the beam is given as linear combination of mode shapes.

2.3. Forced Vibration of the Jib

The principle of superposition is used to solve the forced vibration of the Jib as a cantilever beam as shown in Figure 4b. Thus, the deflection of the beam is assumed as

$$w(x,t)=\sum_{n=1}^{\infty}W_n(x)q_n(t) \quad (32)$$

where $q_n(t)$ is the generalized coordinate in the nth mode and $W_n(x)$ is the nth normal mode or characteristic function satisfying the differential equation [9]

$$EI\frac{d^4W_n}{dx^4}(x)-\omega^2\rho AW_n(x)=0; n=1,2. \quad (33)$$

Substituting Equation (32) in Equation (33), it gives

$$\sum_{n=1}^{\infty}EI\frac{d^4W_n(x)}{dx^4}+\sum_{n=1}^{\infty}\rho A(x)\,W_n(x)\frac{d^2q_n(t)}{dt^2}=f(x,t) \quad (34)$$

From Equation (32), Equation (33) can be written as

$$\sum_{n=1}^{\infty}\omega^2W_n(x)q_n(t)+\sum_{n=1}^{\infty}W_n(x)\frac{d^2q_n(t)}{dt^2}=\frac{1}{\rho A}f(x,t) \quad (35)$$

Multiplying Equation (35) throughout by W_m, integrating from 0 to l and using orthogonality condition, it gives

$$\frac{d^2q_n(t)}{dt^2}+\omega^2q_n(\mathrm{t})=\frac{1}{\rho Ab}Q_n(\mathrm{t}) \quad (36)$$

where $Q_n(\mathrm{t})$ is the generalized force corresponding to $q_n(t)$

$$Q_n(\mathrm{t})=\int_0^l f(x,\mathrm{t})\,W_n(x)dx \quad (37)$$

And the constant is given by

$$b=\int_0^l f(x,\mathrm{t})\,W_n(x)dx \quad (38)$$

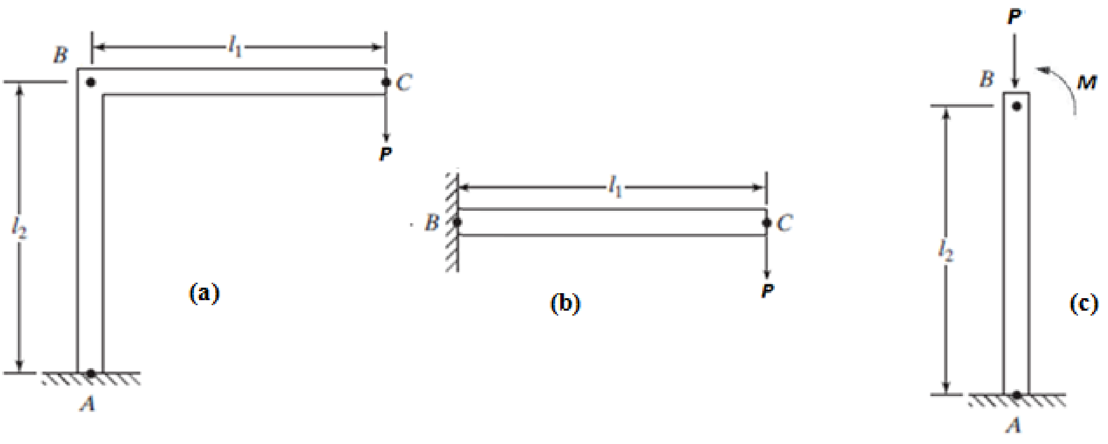

Figure 4. (a) The Jib Crane System, (b) The Jib Under Loading, (c) The Mast Under Loading

Equation (36) is an equation of motion of undamped single degree of freedom system. Using Duhamel integral, the solution of the Equation (36) can be expressed as

$$q_n(t) = A_n \cos\omega_n t + B_n \sin\omega_n t + \frac{1}{\rho Ab\omega_n}\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)d\tau \quad (39)$$

2.4. Response of the Jib Subjected to Harmonic Force

From Equation (36), the generalized coordinate $q_n(t)$ is given by

$$\frac{d^2 q_n(t)}{dt^2} + \omega^2 q_n(t) = \frac{1}{\rho Ab} Q_n(t) \quad (40)$$

Where

$$Q_n(t) = \int_0^l f(x,t)W_n(x)dx = F(t)W_n(l). \quad (41)$$

The steady state solution of Equation (36) is given by

$$q_n(t) = A_n \cos\omega_n t + B_n \sin\omega_n t + \frac{1}{\rho Ab\omega_n}\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)d\tau \quad (42)$$

Where

$$b = \int_0^l W^2{}_n(x)dx = l$$

$$\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)d\tau = PW_n(l)\int_0^t e^{\omega\tau}\sin\omega_n(t-\tau)d\tau = PW_n(l)e^{\omega\tau}\int_{t-\tau=0}^{t-\tau=t} e^{\omega\tau}\sin\omega_n(t-\tau)(-d\tau) \quad (43)$$

Using the formula

$$\int e^{ax}\sin bx dx = \frac{1}{a^2+b^2}e^{ax}\{a\sin bx - b\sin bx\} \quad (44)$$

Equation (43) can be evaluated to obtain

$$q_n(t) = \frac{PW_n(l)}{\rho Al\omega_n(\omega_n^2+\omega^2)}\begin{Bmatrix}\omega_n e^{\omega t} + \omega\sin\omega_n t \\ -\omega_n\cos\omega_n t\end{Bmatrix}. \quad (45)$$

2.5. Response of Mast Subjected to a Moment at the Free End

From Equation (37), the generalized force $Q_n(t)$ becomes

$$Q_n(t) = M_0 \left.\frac{dW_n}{dx}\right|_{x=l} \quad (46)$$

Where

$$\left.\frac{dW_n}{dx}\right|_{x=l} = \beta_n(\cos\beta_n l - \cosh\beta_n l) + \alpha_n\beta_n(\sin\beta_n l + \sinh\beta_n l). \quad (47)$$

The steady state response of the beam under the action of the moment is given by Equation (32) with

$$q_n(t) = \frac{1}{\rho Ab\omega_n}\int_0^t Q_n(\tau)\sin\omega_n(t-\tau)d\tau$$

$$q_n(t) = \frac{1}{\rho Ab\omega_n}M_0\left.\frac{dW_n}{dx}\right|_{x=l}\int_0^t \sin\omega_n(t-\tau)d\tau \quad (48)$$

Where

$$b = \int_0^l W^2{}_n(x)dx = l.$$

Noting that

$$\int_0^t \sin\omega_n(t-\tau)d\tau = \frac{1}{\omega_n}(1-\cos\omega_n t).$$

Thus, the steady state response can be written as

$$q_n(t) = \frac{1}{\rho Ab\omega_n}M_0\left.\frac{dW_n}{dx}\right|_{x=l}(1-\cos\omega_n t) \quad (49)$$

Thus, the combined response of the Jib Crane due to the harmonic force and moment is

$$q_n(t) = \frac{pW_n(l)}{\rho Al\omega_n(\omega_n^2+\omega^2)}\begin{Bmatrix}\omega_n e^{\omega t}+\omega\sin\omega_n t\\ -\omega_n\cos\omega_n t\end{Bmatrix} + \frac{1}{\rho Ab\omega_n}M_0\left.\frac{dW_n}{dx}\right|_{x=l}(1-\cos\omega_n t) \quad (50)$$

2.6. Numerical Analysis of the Jib Crane Using ANSYS

Finite element method is used to analyse the crane frame-structure mode shape, frequency, and amplitude using ANSYS [10] software in other to compare and verify it with the theoretical solution. Its main advantage is that it employs a different technique in its solution to the theoretical solution. However, results must be subjectively analysed.

2.6.1 Element Type

The selection of the type of element is of paramount importance and time, precision and application are some of the variables that must be considered when choosing an element. The elements selected is SOLID187.

2.6.2. SOLID187

SOLID187 as shown in Figure 5 is a higher order 3-D, 10-noded element that considers quadratic displacement behavior throughout the cross-section suited for irregular mesh. This was used because it has three degree of freedom at each nodal x, y, and z direction and gives significant accuracy to the Euler-Bernoulli beam under consideration, section control and accommodate material properties needed for this analysis [10].

2.6.3. Material Property

The material of the crane structure is a structural steel with Modulus of Elasticity of 200 GPa, Poisson ratio of 0.3 and Density of 7800 kg/m^3.

2.6.4. Geometry

2.6.4.1. Jib and Mast

The jib has an I-cross section and keypoints are used to define the edges. Lines are then created through the keypoints, then area and finally extruding the cross-section along the normal to the length of the jib and the mast is then attached to the jib by defining a new working plan due to the difference in plan between the jib and mast as shown in Figure 6.

2.6.5. Meshing

The created volume is then meshed. First, the element size is specified as 0.04 because of the complexity of the model which will not allow for AutoMesh and then specifying mesh attributes with Element type as SOLID187 for the Jib as shown in Figure 7.

2.6.6. Solution

2.6.6.1. Modal Solution

Modal analysis of the jib crane structure was carried out with frequency range of 0 to 100 Hz and natural frequencies and mode shapes were extracted.

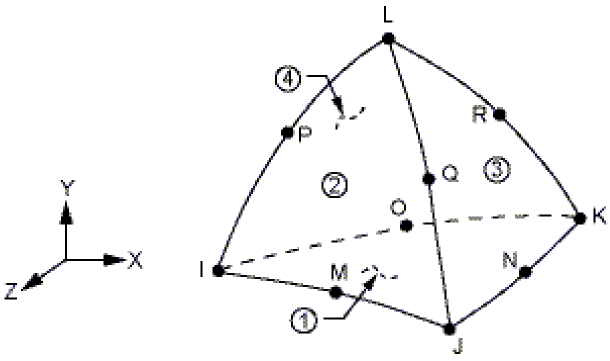

Figure 5. SOLID187

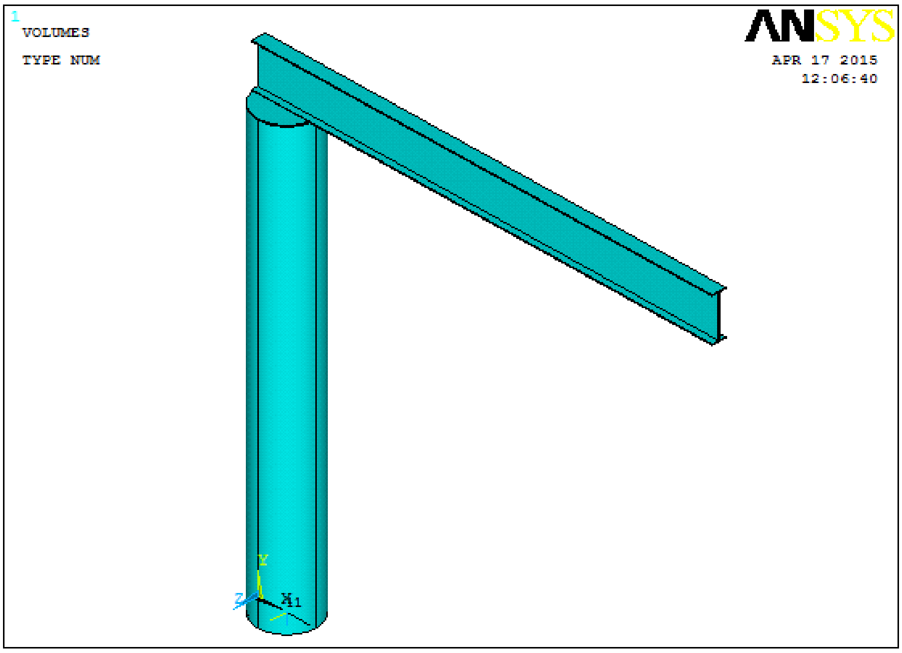

Figure 6. Jib Crane Model in ANSYS

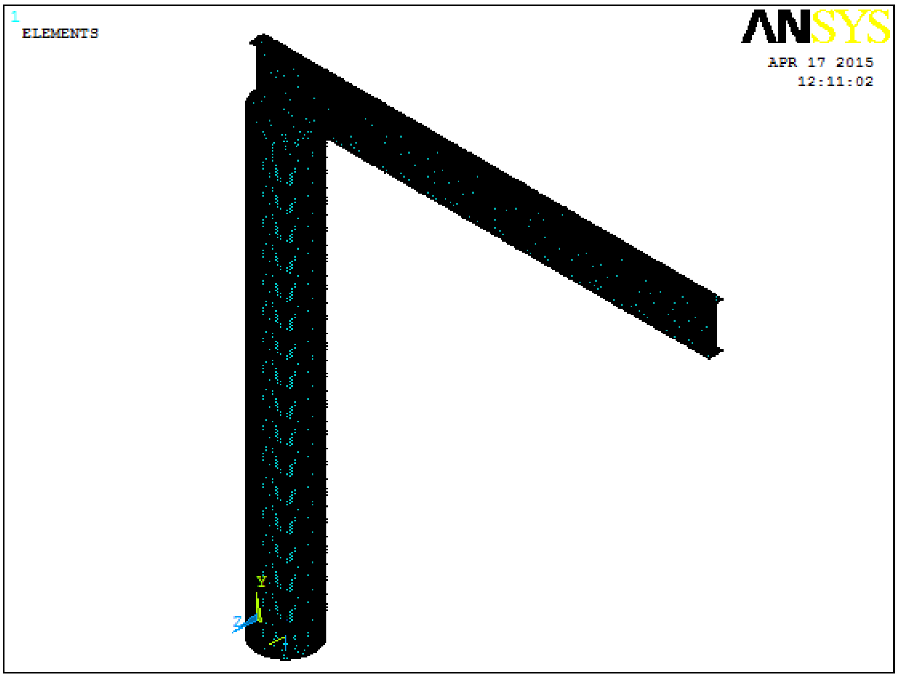

Figure 7. Meshed Jib Crane

3. Results and Discussion

3.1. Numerical Parameters

The following are standard numerical data obtained from [8] for a 5ton model. I-cross section beam, $L = 6.096\text{m}$, $A = 0.603\text{m}$, $B = 0.179\text{m}$, $C = 0.015\text{m}$, and $D = 0.011\text{m}$ with structural steel (A36) having $\rho = 7800Kg/\text{m}^3$ $E = 2\times10^{11}GPa$ and ϑ=0.3. The force acting is $P = 49050N$ and a motor speed of $7.4\ m/\min$ on 0.326m drum.

3.2. The Natural Frequencies of the Jib Crane

The following parameters were calculated as $\omega = 0.3783 rad/s$, area $A = 0.011673m^2$, $M = 555.4kg$, $I = 6.3672\times10^{-4}m^4$, $k = 1683.1\ kN/m$, $\omega_n = 55.049 rad/s$, $\omega/\omega_n = 0.0069$, and maximum displacement as $X_p = 29.144mm$. The first three natural frequencies are obtained from Equation (30) and tabulated as in Table 1.

Table I. Modal Analysis frequencies

Mode	Theoretical Frequency (Hz)	ANSYS Frequency (Hz)
1	17.81	18.86
2	111.43	110.93
3	312.49	289.65

The result shown in Table 1 shows the first three natural frequencies of the jib crane. This result is almost the same as the solution of a cantilever beam with same boundary conditions as derived in Equation 30. The result tallies with Reference [8] as a good indication of the effective of this approaching of treating the Jib Crane as a Frame Structure.

3.3. Mode Shapes of the Jib Crane

The mode shapes of the jib crane are shown in Figure 4a-c from Equation 31. It shows that each of the mast and the jib behaves as an independent cantilever beam. The mast is fixed at the base, but the other end is free in its axis while forming a fixed support for the jib.

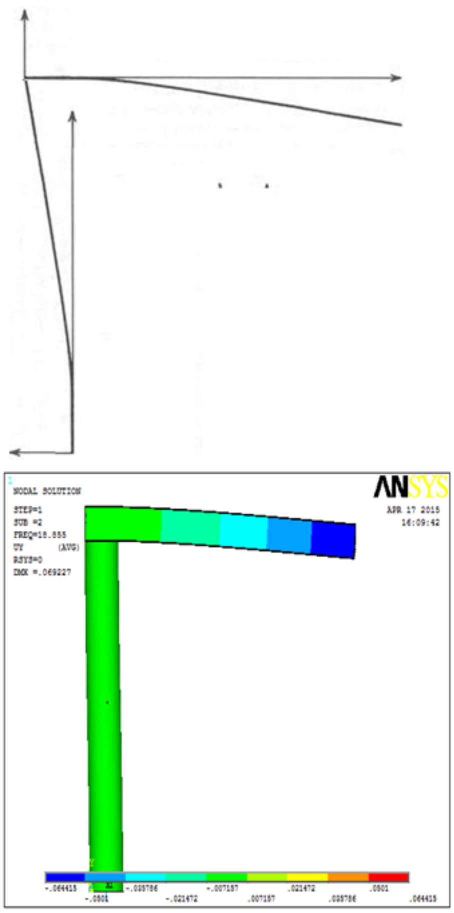

Figure 8. (a): First Mode Shape

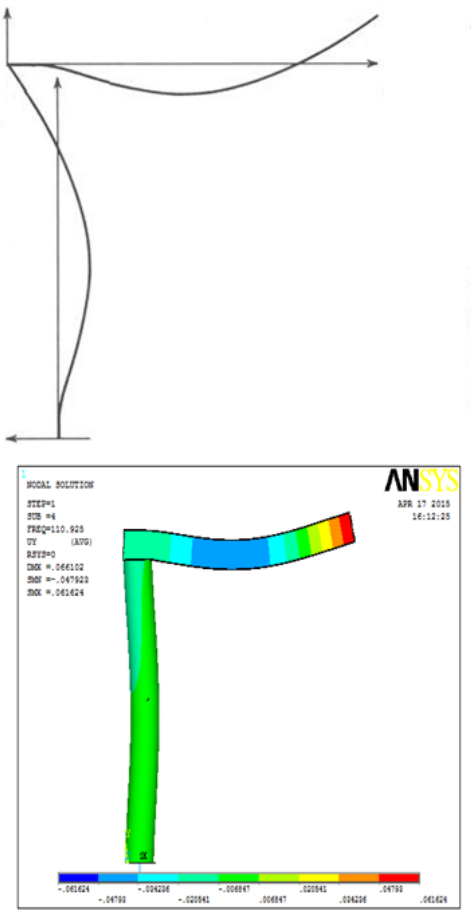

Figure 8. (b): Second Mode Shape

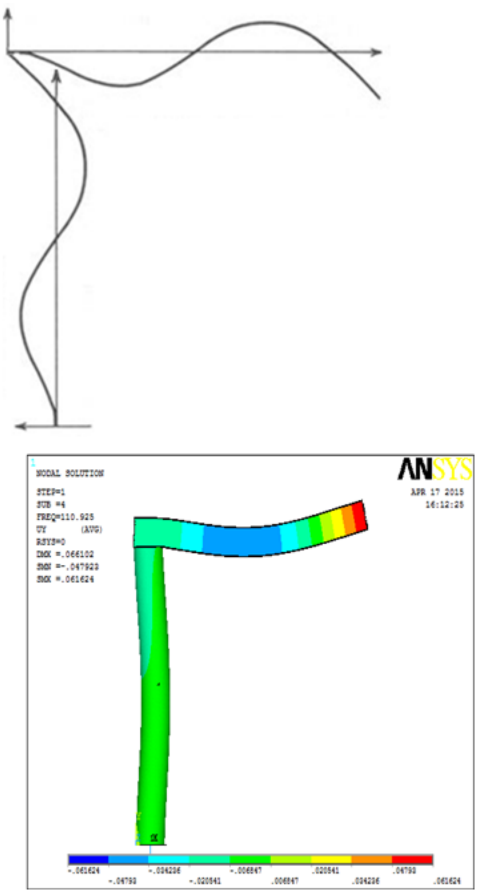

Figure 8. (c): Third Mode Shape

3.4. The Response of the Jib Crane to the Force

The response of the Jib to the force P at the end of the Jib which is the load carried by the Jib Crane from Equation 45 is shown in Figure 9(a) with a maximum displacement of about 26mm and a mean steady displacement of less that 20mm while the response of the Mast due to the combined effect of the harmonic force and the moment acting on it from Equation 50 is shown in Figure 9(b) with a mean displacement of about 32mm. The response of the mast is higher due to the combined displacement of the jib and mast from the superposition effect.

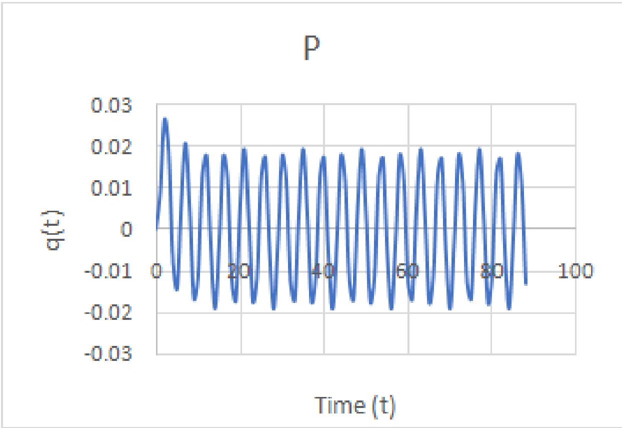

Figure 9. (a): Response of the Jib

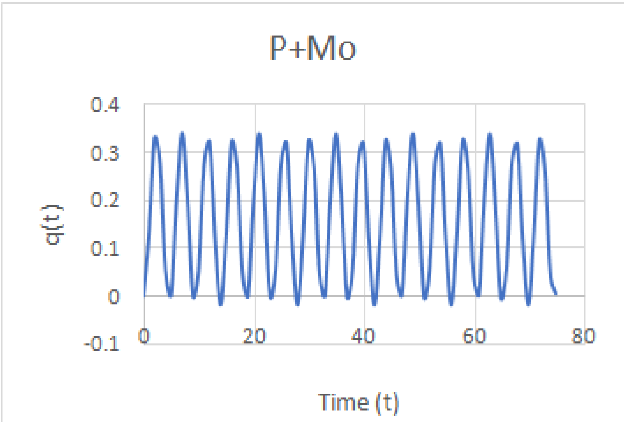

Figure 9. (b): Response of the Mast

4. Conclusion

A comparative approach of analytical and numerical solution of a jib crane system was explored. The analytical method gave an efficient analysis of the jib crane using planar serial-frame structure approach to evaluate the eigensoulutions of the system and other vibration parameters. This approach shows that the jib crane can be modelled and treated as a frame structure in analysing the vibration of the system. The numerical method utilizes ANSYS in modelling the derived system using the established boundary conditions and solved. The ANSYS results conform with the analytical solution which is a good indicator for this study.

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