

# Vibration and Parametric Instability of Functionally Graded Material Plates

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**Abstract** The present work aims the vibration and parametric instability of functionally graded material rectangular plates with simply supported boundary condition, subjected to a biaxial in-plane periodic loading. First order shear deformation theory is used for theoretical formulation of FGM plates. The properties of the functionally graded material plates are assumed to vary along the thickness direction according to a power law distribution in terms of the volume fractions of the constituents. Hamilton's principle is employed to convert the governing equations into a linear system of Mathieu–Hill equations from which the boundary of stable and unstable regions are determined by using Floquet's theory on the parameter space. Natural frequency and buckling analysis are also discussed. Numerical results are presented in both dimensionless parameters and graphical forms for FGM plates made of steel and alumina. The influences of various parameters such as index value, aspect ratio on the buckling load and natural frequencies are examined. Power law index value and aspect ratio effects on the dynamic stability regions also studied in detail.

**Keywords:** free vibrations, critical buckling load, functionally graded material plates, finite element method, instability regions

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## 1. Introduction

Functionally graded materials (FGM) are designed in such way to overcome the demerits of ordinary materials. It is so called as FGM because the properties changes as thickness changes which is not observed in ordinary materials. These materials have many advantages such as high resistance to temperature gradients, high wear resistance, reduction in residual and thermal stresses, and an increase in strength to weight ratio because of these inherent properties stability is also increases. Due to the outstanding properties of Functionally graded materials (FGMs) are used in many engineering applications such as the aerospace, aircraft, automobile, defence, biomedical and electronic industries. Many structural components can be modelled as plates like structural. Plate like structures are often subjected to various types of dynamic loads, among which the periodically in-plane time-varying force may cause dynamic instability, in which there is an unbounded exponential built up of the response. In practice the dynamic loads are dependent on time and may change their direction. It is enormous practical importance to clarify the dynamic stability of dynamic systems under periodic loads. Therefore, a broad understanding of the dynamic stability characteristics of structural materials in periodic loading environments is a matter of important weight for the design of the structural failure.

FGMs are made of a ceramic and a metal in such a way that the ceramic can resist the thermal loading from the high temperature environment. The material properties of FGMs vary continuously from one surface to the other surface, this results in eliminating surface problems of composite materials and achieving the smooth stress distribution. Theoretical modelling and analysis of FGM plates has become important topic to discussion at the present stage. The classical laminated plate theory has become the most widespread engineering application, a lot of articles about FGM plates and shells based on the classical laminated plate theory were reported. The recent advancement in the characterization, modelling and analysis of FGM has been reviewed Victor Birman and Larry [1] in this work focussed on research published since 2000. Due to the broad and rapidly developing field to various aspects of theory and applications of FGM are reflected in this effort. They reflect some of the observations of the authors based on the published research and their own analysis of the subject on FGM. The physical neutral surface and geometric middle surface are the same in homogenous symmetrical plates. The concept of neutral surface was derived based on the classical nonlinear plate theory for the FGM plates by Da-Guang and You-He [2]. Theoretical formulation and finite element modals for functionally graded plates based on the third-order shear deformation theory presented by Reddy [3]. In this the finite element model that accounts for the thermo mechanical coupling and geometric non-linearity.

Free vibration analysis of FGM rectangular plates has been numerically performed by number of researchers. Zhao X. et al. [4] have studied a method for analyzing the free vibration of FGPs with arbitrary boundary conditions using the element-free kp-Ritz method. In their analysis a mesh-free kernel particle functions were used to approximate the two-dimensional displacement fields. Refined two-dimensional shear deformation theory investigated by Fares et al. [5] for orthotropic FG Plates. For obtaining this theory a modified version of the mixed variational principle of Reissner was used. This theory does not require any shear correction factor. An exact analytical solution was developed by Hasani Baferani et al. [6] for free vibration analysis of thin FG rectangular plates by using the classical plate theory. In their analysis the effects of in-plane displacement on the vibration of FG rectangular plates are studied and also a closed-form solution for finding the natural frequency of FG simply supported rectangular plates. A 2-D higher-order theory is developed by Hiroyuki [7] for analyzing natural frequencies and buckling stresses of FG plates. Here the Hamilton's principle was used for dynamic analysis of a two-dimensional (2-D) higher-order theory for rectangular functionally graded (FG) plates. A finite element method (FEM) of B-spline wavelet on the interval (BSWI) was used to solve the free vibration and buckling analysis of plates by Zhibo Yang et al. [8]. In their analysis BSWI functions are considered for Structural analysis, the proposed method can obtained a fast convergence and a satisfying numerical accuracy with fewer degrees of freedoms (DOF). Serge [9] performing a direct analysis for natural frequencies of FG plates can be easily calculated from those known isotropic material results. Senthil and Batra [10] investigated an exact solution for the vibration of simply supported rectangular thick plates. They assumed that the plate is made of an isotropic material with material properties varying in the thickness direction only.

Hosseini Sh et al. [11] proposed a new exact closed-form approach for free vibration analysis of thick rectangular FG plates based on the third-order shear deformation theory of Reddy. In their analysis Hamilton's principle was used to extract the equations of dynamic equilibrium and natural boundary conditions of the plate. Mohammad and Singh [12] presented a higher order shear deformation theory with special modifications in the transverse displacement which contributes additional freedom to the displacements through the thickness and eliminates the over-correction. A meshless method was introduced by Ferreira et al. [13] for free vibration analysis of functionally graded plates with multiquadric radial basis functions to approximate the trial solution. The free vibration analysis of functionally graded material plates without enforcing zero transverse shear stress conditions on the top and bottom surfaces of the plate using higher order displacement model was presented by Suresh Kumar et al. [14]. Talha and Singh [15] studied the free vibration and static analysis of functionally graded material (FGM) plates using higher order shear deformation theory with special changes in the transverse displacement in conjunction with finite element models. In this the mechanical properties of the plate are assumed to vary continuously along the thickness direction by power-

law distribution in terms of the volume fractions of the constituents.

In the past the stability analysis problems of functionally graded material plates have been dealt by some of the researchers. Buckling behaviour of simply supported functionally graded material (FGM) plates under constant and linearly varying in-plane compressive loads was investigated by Rohit and Maiti [16]. In their analysis the effect of shear deformation was studied using third order shear deformation theory and first order shear deformation theory. They concluded the influence of transverse shear on buckling loads is almost similar for all types of FGMs. Xinwei et al. [17] obtained the buckling analysis of thin rectangular plates with cosine-distributed load along two opposite plate edges, it was considerably complicated. They followed the first the plane elasticity problem to solve the distribution of in-plane stresses and then the buckling problem. Mokhtar et al. [18] investigated the Buckling analysis of rectangular thin functionally graded plates under uniaxial and biaxial compression by using classic plate theory and Navier's solution. Andrzej [19] studied the parametric vibrations or dynamic stability of functionally graded rectangular plate described by geometrically nonlinear partial differential equations using the direct Liapunov method. In their analysis an oscillating temperature causes generation of in-plane time-dependent forces destabilizing plane state of the plate equilibrium. Review on free, forced vibration analysis and dynamic stability of ordinary and functionally grade material plates presented by Ramu and Mohanty [20,21]. Finite element method for functionally graded material thick plates discussed by Lucia and Paolo [22]. Ng et al. [23] found the parametric resonance or dynamic stability of functionally graded cylindrical shells under periodic axial loading, by using Bolotin's [24] first approximation. In their work motivated by the increased general use of functionally graded materials and also need to understand their dynamic responses. Rath and Dash [25] studied the parametric instability of woven fiber laminated composite plates under in-plane periodic loadings in hygrothermal environment.

The paper conducts the vibration and parametric instability of functionally grade material plates under in-plane time-varying pulsating force. Four node rectangular elements are used for modelled as the FGM plate by using finite element method. Hamilton's principle is employed to convert the governing equations into a linear system of Mathieu-Hill equations from which the boundary of stable and unstable regions are determined by using Floquet's theory. For this analysis steel and alumina materials are used to make the FGM plate. Free vibration and static stability analysis are also discussed as parting problems. Numerical analyses are presented in both dimensionless parameters and graphical forms. The influences of various parameters on parametric instability of FGM plate studied in detail.

## 2. Methodology

### 2.1. Formulation of the problem

The FGM plate is of uniform rectangular cross-section having a length  $L$ , width  $W$  and thickness  $h$ . The plate is

subjected to a pulsating axial force, acting along its axial axis.  $\Omega$  is the excitation frequency of the dynamic load component,  $P_s$  is the static and  $P_t$  is the amplitude of the time dependent component of the load. A typical FGM plate subjected to in plane biaxial in plane dynamic loads as show in Figure 1.

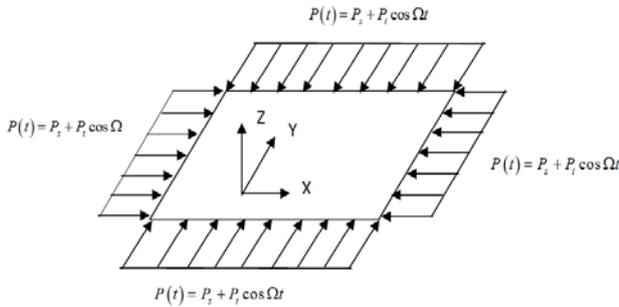


Figure 1. FGM plate subjected to in plane biaxial in plane dynamic loads

### 2.2. Functionally Graded Material Plates

The extensive use of plates the various types of functionally graded material plates was considerable interests to many researchers in the field of modelling, analysis and design of these structures. Accurate prediction of structural response characteristics is a demanding problem for the analysis of functionally graded materials due to the anisotropic structural behaviour and the presence of various types of complicated constituents. This is possible because the material composition of an FGM changes gradually through-the-thickness. The graphical representation  $V_c$  and  $V_m$  (volume fraction of ceramic and metal) of FGM through along the thickness direction of the specimen.

### 2.3. The Simple Power Law

More common in the analysis for functionally graded materials with two constituent materials the variations through the thickness of material properties  $P$  can be expressed as

$$P(z) = P_m + (P_c - P_m) \times V_c(z) \quad (1)$$

Here  $P$  can represent Young's modulus  $E$ , Poisson ratio  $\mu$ , and the mass density  $\rho$ , and  $V_c(z)$  is the volume fraction variation of the ceramic material, and it is assumed to follow a simple power-law distribution as

$$V_c = (1/2 + z/h)^n \quad (2)$$

Where  $-h/2 \leq z \leq h/2$  is coordinate through the thickness from the middle surface to ceramic and metal sides, and  $n$  is a gradient index. Working range of design requirements in this case is based on a grading indexed.

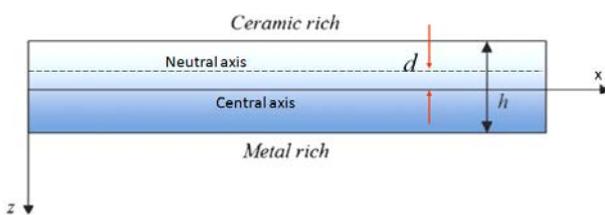


Figure 2. Geometry of the FGM plate

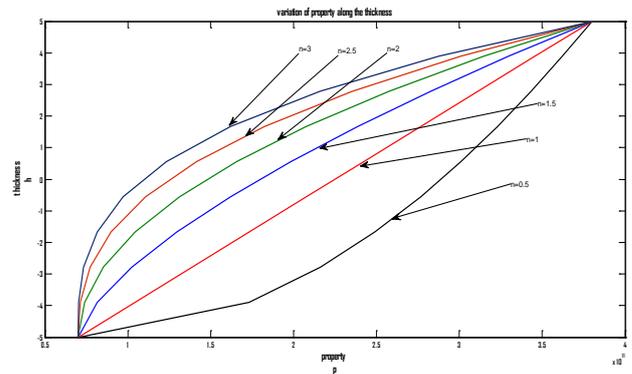


Figure 3. Variation of young's modulus along the thickness of the FGM plate

### 2.4. Physical Neutral Surface of the FGM Plate

In the present work neutral axis concept has been employed for analysis. For a FGM plate due to the variation of the material properties along thickness, the neutral plane does not coincide with the geometrical mid-plane of the plane shown in Figure 2. The distance of the neutral surface ( $d$ ) from the geometric mid-surface may be expressed as

$$d = \frac{\int_{-h/2}^{h/2} zE(z)dz}{\int_{-h/2}^{h/2} E(z)dz} \quad (3)$$

### 2.5. Kinematics

Plate structures made of functionally graded materials are characterized by first order shear deformation theory so that the extension of the classical model suggested by (Reissner and Mindlin 2004) to the case of graded material plates provides a good compromise between numerical accuracy and computational load. In-plane displacements  $u$  and  $v$  and the normal displacement  $w$  are therefore given the form:

$$\begin{aligned} u(x, y, z) &= z\theta_x(x, y), \\ v(x, y, z) &= z\theta_y(x, y), \\ w(x, y, z) &= w(x, y) \end{aligned} \quad (4)$$

where  $\theta_x$  and  $\theta_y$  are the rotations of the normal to the undeformed middle surface in the  $x-z$  and  $y-z$  planes, respectively.

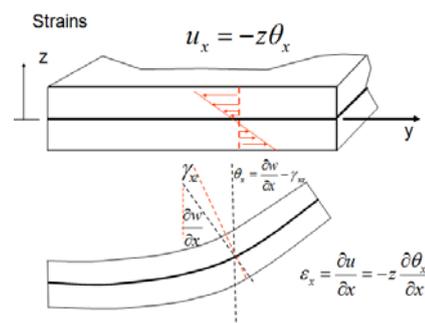


Figure 4. Plate structure before and after deformation

In-plane and out-of-plane strain-displacement constitutive law relations may be written as

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{bmatrix} = z \begin{bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \end{bmatrix}, \begin{bmatrix} \gamma_x \\ \gamma_y \end{bmatrix} = \begin{bmatrix} w_{,x} + \theta_x \\ w_{,y} + \theta_y \end{bmatrix} \quad (5)$$

The normal stress-strain and shear stress-strain relationships of the functionally graded plate in the global x-y coordinates system can be written as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = zD \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{bmatrix}, \begin{bmatrix} \tau_x \\ \tau_y \end{bmatrix} = S \begin{bmatrix} \gamma_x \\ \gamma_y \end{bmatrix} \quad (6)$$

whereas grading matrices  $D$  and  $S$  are defined as

$$D = \frac{E(z)}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}, \quad S = \frac{E(z)}{1-\nu^2} \begin{bmatrix} \frac{1-\nu}{2} & 0 \\ 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Moments and shear forces are obtained via standard integration over the thickness, i.e.

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} z dz = \begin{bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \end{bmatrix} \int_{-h/2}^{h/2} D z^2 dz \quad (7a)$$

$$\int_{-h/2}^{h/2} \begin{bmatrix} \tau_x \\ \tau_y \end{bmatrix} dz = \begin{bmatrix} w_{,x} + \theta_x \\ w_{,y} + \theta_y \end{bmatrix} \int_{-h/2}^{h/2} S dz \quad (7b)$$

## 2.6. Energy equations

The potential strain energy  $U^{(e)}$  of the plate can be written in terms of the stress resultants and strain components through integrating by parts with respect to coordinates as follows

$$U^{(e)} = \frac{1}{2} \iiint_v \varepsilon^T \sigma dv \quad (8)$$

$$U^e = \iiint (\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \tau_{xy} \delta \gamma_{xy}) dx dy dz$$

The kinetic energy of the plate follows as

$$V^{(e)} = \frac{1}{2} \iint_A \rho h \dot{w}^2 dA \quad (9)$$

Work done by in plane loading of plate is

$$W_e = \frac{1}{2} \int_A N_x \left( \frac{\partial w}{\partial x} \right)^2 dA + \frac{1}{2} \int_A N_y \left( \frac{\partial w}{\partial y} \right)^2 dA \quad (10)$$

where  $N_x$  and  $N_y$  are the in-plane compression loads.

## 2.7. FE formulation of a 4-noded Rectangular Element

Rectangular four node element is having one node at each corner as shown in Figure 5. There are three degrees of freedom at each node, the displacement component along the thickness ( $w$ ), and two rotations along X and Y directions in terms of the ( $x, y$ ) coordinates (Ramu I and Mohanty S C 2012). The element consists of four nodes 1, 2, 3 and 4 with  $w, \theta_x$  and  $\theta_y$  as the  $w$  is the transverse displacement and  $\theta_x$  and  $\theta_y$  represents the rotations about x and y axis.

$$w, \theta_x = \frac{\partial w}{\partial x} - \gamma_{xz}, \theta_y = \frac{\partial w}{\partial y} - \gamma_{yz} \quad (11)$$

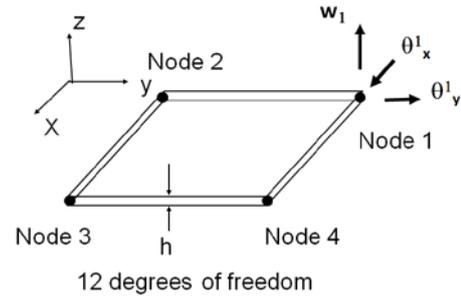


Figure 5. Geometry of the rectangular element

$$\begin{aligned} w &= \sum_{i=1}^n N_i w_i \\ \theta_x &= \sum_{i=1}^n N_i \theta_x^i \\ \theta_y &= \sum_{i=1}^n N_i \theta_y^i \end{aligned} \quad (12)$$

Here  $w^e = [w_i \ \theta_x^i \ \theta_y^i]$ ,  $N_i$  displacement vector and the appropriate shape functions,  $n$  number of nodes in the element.

The element stiffness matrix, geometric stiffness and mass matrices are derived on the basis of principle of minimum potential, kinetic energy and work done.

The element stiffness matrix is derived as

$$[k]_e = \iiint [B]^T [D] [B] dv \quad (13)$$

where

$$[B] = z \begin{bmatrix} \frac{\partial^2}{\partial x^2} \\ \frac{\partial^2}{\partial y^2} \\ 2 \frac{\partial^2}{\partial x \partial y} \end{bmatrix} [N], D = \frac{Eh^3}{12(1-\nu^2)}$$

Element mass matrix is derived as

$$[M]_e = \iiint [N]^T \rho [N] dv \quad (14)$$

The element geometric stiffness matrix is derived as

$$[k_g] = N_x \iint \left( \frac{\partial^2 w}{\partial x^2} \right) da + N_x \iint \left( \frac{\partial^2 w}{\partial y^2} \right) da \quad (15)$$

### 3. Governing Equations of Motion

The element equation of motion for a plate subjected to axial force is obtained by using Hamilton's principle.

$$\delta \int_{t_1}^{t_2} (U - V + W) dt = 0 \quad (16)$$

By dividing the plate in to number of elements and assembling the element matrices, the potential energy and the kinetic energy for element can be written in terms of global displacement vector as

$$U^{(e)} = \frac{1}{2} \{q^{(e)}\}^T [K_s^{(e)}] \{q^{(e)}\} - \frac{1}{2} \{q^{(e)}\}^T P(t) [K_g^{(e)}] \{q^{(e)}\} \quad (17)$$

Here  $\{q^e\} = \{w^e\}$

$$V^{(e)} = \frac{1}{2} \{q^{(e)}\}^T [M^{(e)}] \{q^{(e)}\} \quad (18)$$

The equation of motion of plate element in matrix form for the axially loaded discretised system is obtained as follows

$$[M^{(e)}] \{\ddot{q}\} + [K_s^{(e)}] \{q\} - P(t) [K_g^{(e)}] \{q\} = 0 \quad (19)$$

The governing equation of motion of plate in terms of global displacement matrix obtained as follows

$$[M] \{\ddot{q}\} + [K_s] \{q\} - P(t) [K_g] \{q\} = 0 \quad (20)$$

Here  $[K_s]$ ,  $[M]$  and  $[K_g]$  are global stiffness, global mass and geometric stiffness matrix respectively.

Where  $P_s$  is the static and  $P_t$  the amplitude of time dependent component of the load, can be represented as function of fundamental static buckling load  $P^{cr}$  of the plate and having all sides simply supported boundary conditions. Hence substituting,  $P(t) = \alpha P^{cr} + \beta P^{cr} \cos \Omega t$  with  $\alpha$  and  $\beta$  are called static and dynamic load factors respectively

$$[M] \{\ddot{q}\} + \left( [K_s] - \alpha P^{cr} [K_g]_s \right) \{q\} - \beta P^{cr} \cos \Omega t [K_g]_t \{q\} = 0 \quad (21)$$

Where  $[K_g]_s$  and  $[K_g]_t$  reflect the influence of  $p_s$  and  $p_t$  respectively. If the static and time dependent components of the load are applied in the same manner then,

$$[K_g]_t = [K_g]_s = [K_g]$$

#### 3.1. Parametric Instability Regions

From the above equation (21) represents a system of second order differential equations with periodic coefficients the Mathieu-Hill type. From the theory of Mathieu function is evident that the nature of solution is dependent on the choice of load frequency and load amplitude. The frequency amplitude domain is divided in two regions, which give raise to stable solutions and to regions, which cause unstable solutions. According to the Floquet's theory the periodic solutions characterize the boundary conditions between the dynamic stability and instability zones. So the periodic solution can be expressed as Fourier series.

A solution with period  $2T$  is represented by:

The boundaries of the principal instability regions with period  $2T$  are of practical importance.

$$q(t) = \sum_{k=1,3,\dots}^{\infty} \left[ \{c_k\} \sin \frac{k\Omega t}{2} + \{d_k\} \cos \frac{k\Omega t}{2} \right] \quad (22)$$

If the series expansions of eq. (22), term wise comparisons of the sine and cosine coefficients will give infinite system of homogeneous algebraic equations for the vectors  $\{c_k\}$  and  $\{d_k\}$  for the solutions on the stability borders. Non-trivial solutions exist if the determinant of the coefficient matrices of these equation systems of infinite order vanishes.

Substituting the first order ( $k=1$ ) Fourier series expansion of eq. (22) in eq. (21) and comparing the coefficients of  $\cos \frac{\Omega t}{2}$  and  $\sin \frac{\Omega t}{2}$  terms, the condition for existence of these boundary solutions with period  $2T$  is given by

$$\left( [K_s] - \left( \alpha \pm \frac{\beta}{2} \right) P^{cr} \times [K_g] - \frac{\Omega^2}{4} [M] \right) \{q\} = 0 \quad (23)$$

The above equation represents an eigenvalue problem for known values of  $\beta$  and  $p^{cr}$ . This equation gives two sets of eigenvalues  $\Omega$  bounding the regions of stability due to the presence of plus and minus sign. The instability boundaries can be determined from the solution of the equation.

$$\left| [K_s] - \left( \alpha \pm \frac{\beta}{2} \right) P^{cr} \times [K_g] - \frac{\Omega^2}{4} [M] \right| \{q\} = 0 \quad (24)$$

Also the equation represents the solution to a number of related problems

(1) For natural frequencies:

$$\alpha = 0, \beta = 0 \text{ and } \omega = \frac{\Omega}{2}.$$

The equation becomes

$$\left( [K_s] - \omega^2 [M] \right) \{q\} = 0 \quad (25)$$

(2) For static stability or buckling analysis:

$$\alpha = 1, \beta = 0 \text{ and } \omega = 0$$

The equation becomes

$$\left( [K_s] - P^{cr} \times [K_g] \right) \{q\} = 0 \quad (26)$$

(3) For dynamic instability, when all terms are present  
Let

$$\Omega = \left(\frac{\Omega}{\omega_1}\right) \times \omega_1$$

Where the  $\omega_1$  is the fundamental frequency of the plate having the different boundary conditions, equation then becomes

$$\left( [K_s] - \left( \alpha \pm \frac{\beta}{2} \right) P^{cr} \times [K_g] - \psi \frac{\omega_1^2}{4} [M] \right) \{q\} = 0 \tag{27}$$

where,  $\psi = \left(\frac{\Omega}{\omega_1}\right)^2$

The region of parametric instability can be determined by using equation (27).

### 4. Results and Discussions

#### 4.1. Validation of Results

The results for FGM plate free vibration and buckling analysis obtained by applying first order shear deformation theory in this study are compared by the ref.

results Talha & Singh [15]. The natural frequencies are obtained by considering a combination of Al/ZrO<sub>2</sub> and SUS304/Si<sub>3</sub>N<sub>4</sub> where the top surface is ceramic rich and the bottom surface is metal rich. In according to the  $\varpi$  the dimensionless frequency parameter from Talha & Singh [15] is:

$$\varpi = \omega \times \sqrt{12 \times (1 - \nu^2) \times \rho_c \times L^2 \times W^2 / (\pi^4 E_c h^2)}$$

where  $E_c$  and  $\rho_c$  are young's modulus and density of ceramic material. Validation has been done by considering the values of thickness, length, width, Poisson's ratio, density and young's modulus in the ceramic and metal as:

- Al,  $\rho=2702\text{kg/m}^3$ ,  $E=70 \times 10^9\text{Pa}$ ,  $\nu=0.3$
- ZrO<sub>2</sub>,  $\rho=3,000\text{kg/m}^3$ ,  $E=151 \times 10^9\text{Pa}$ ,  $\nu=0.3$
- SUS304,  $\rho=8,166\text{kg/m}^3$ ,  $E=207.78 \times 10^9\text{Pa}$ ,  $\nu=0.3177$
- Si<sub>3</sub>N<sub>4</sub>,  $\rho=2,370\text{kg/m}^3$ ,  $E=322.27 \times 10^9\text{Pa}$ ,  $\nu=0.24$

Table 1 and 2 shows the natural frequency parameter obtained from the present study using first order shear deformation theory and Talha & Singh [15]. There is a good matching between the presented results and those from ref. [15], particularly for simply supported case. There is less difference between the results predicted by first order shear deformation theory and higher order theories.

Table 1. Variation of the frequency parameter ( $\varpi$ ) with the volume fraction index n for (CCCC) square (Al/Al<sub>2</sub>O<sub>3</sub> FGM plates (a/h = 10)

S.N.	Ceramic		n=0.5		n=1		n=5	
	Ref.[15]	Present	Ref.[15]	Present	Ref.[15]	Present	Ref.[15]	Present
1	3.41	3.46	3.089	3.11	2.94	2.88	2.71	2.25
2	6.70	6.68	6.078	5.99	5.78	5.55	5.29	4.33
3	6.70	6.68	6.078	5.99	5.78	5.55	5.29	4.33
4	9.46	9.28	8.593	8.33	8.17	7.72	7.44	6.02
5	11.6	11.2	10.57	10.0	10.0	9.34	9.11	7.29

Table 2. Variation of the frequency parameter ( $\varpi$ ) with the volume fraction index n for (SSSS) square (Al/Al<sub>2</sub>O<sub>3</sub> FGM plates (a/h = 10)

S.N.	Ceramic		n=0.5		n=1		n=5	
	Ref.[15]	Present	Ref.[15]	Present	Ref.[15]	Present	Ref.[15]	Present
1	1.93	1.96	1.75	1.76	1.685	1.63	1.569	1.27
2	4.72	4.75	4.26	4.26	4.060	3.95	3.765	3.08
3	4.72	4.75	4.26	4.26	4.060	3.95	3.765	3.08
4	7.27	7.24	6.58	6.50	6.275	6.02	5.786	4.70
5	9.11	9.00	8.27	8.07	7.876	7.48	7.049	5.84

Table 3. Comparison of the critical buckling load (MN/m) for a FGM plate (L = 1, h = 0.01)

Index value n	L/W	Uniaxial compression		Biaxial compression	
		Ref [18]	Present	Ref [18]	Present
n=0	0.5	2.14655	2.1375	1.71724	1.71621
	1	1.37379	1.3673	0.686896	0.68366
	1.5	1.61230	1.4824	0.49609	0.49288
n=1	0.5	1.06993	1.0654	0.85594	0.85233
	1	0.68475	0.68293	0.34238	0.33879
	1.5	0.80363	0.74048	0.24727	0.24567
n=2	0.5	0.83488	0.83136	0.66791	0.66509
	1	0.53433	0.5329	0.26716	0.26437
	1.5	0.62709	0.57781	0.19295	0.19176

FGM plate buckling analysis has been compared and studied. The buckling analysis is performed for FGM rectangular plate's different values of power law index, for aluminium-alumina FGM. The Young's modulus and Poisson's ratio for aluminium are: 70 GPa and 0.3 and for alumina: 380 GPa and 0.3, respectively. To validate the derived equation, the obtained critical buckling loads of simply supported FGM plates shown in Table 3. They are good matching between the calculated and results of Mokhtar et al. [18] for low values of L/h.

#### 4.2. Numerical Results

##### 4.2.1. Natural Frequency and Buckling Analysis

The following numerical results are obtained by considering the steel as the bottom surface and alumina as

the top surface in FGM plate according to index value. The geometry of plate and material properties is as follows:

- L=1 m (length), W=1 m (width), h=0.1 m (thickness).
- SUS304,  $\rho=7,800\text{kg/m}^3$ ,  $E=201\times 10^9\text{Pa}$ ,  $\nu=0.3$
- $\text{Al}_2\text{O}_3$ ,  $\rho=2,707\text{kg/m}^3$ ,  $E=380\times 10^9\text{Pa}$ ,  $\nu=0.3$ .

The variations of natural frequency parameter in FGM (SUS304/ $\text{Al}_2\text{O}_3$ ) plate with different boundary conditions are shown in Figure 6 and Figure 7. The effect of power

law index  $n$  on the frequencies can be seen by different boundary conditions. As expected, the increasing index value leads to reduce the natural frequency. The increase power law index reduces the ceramic volume fraction; it affects the effective material properties.

In Figure 8 shows the results of critical buckling load of a FGM rectangular plate based CPT was presented. This figure shows that the critical buckling load decreases when the power law index value increases.

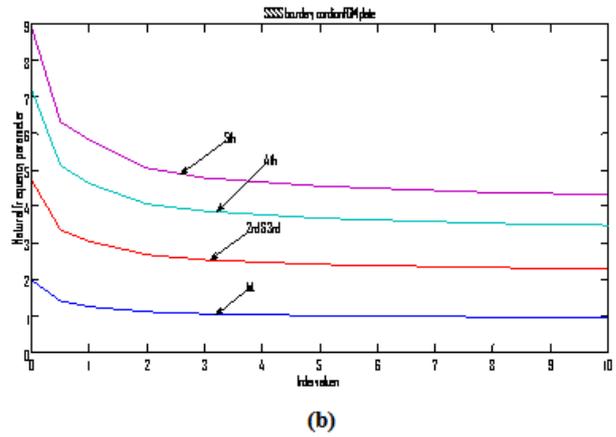
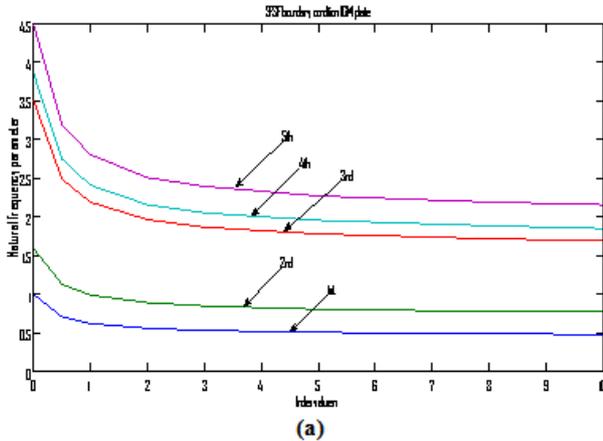


Figure 6. Variation of frequency parameter with index value (a) SFSF and (b) SSSS boundary conditions

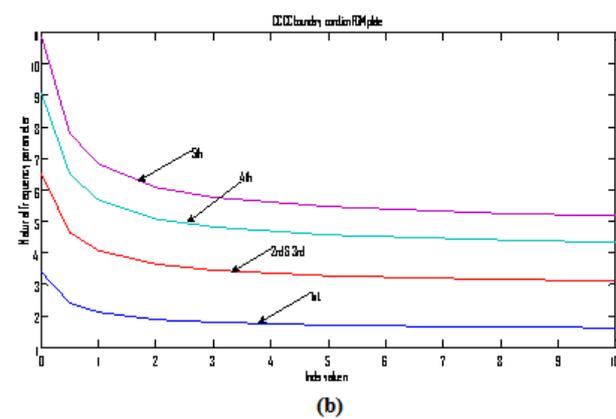
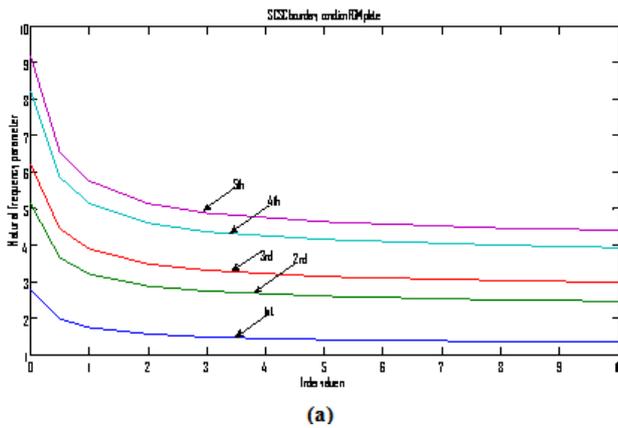


Figure 7. Variation of frequency parameter with index value (a) SCSC and (b) CCCC boundary conditions

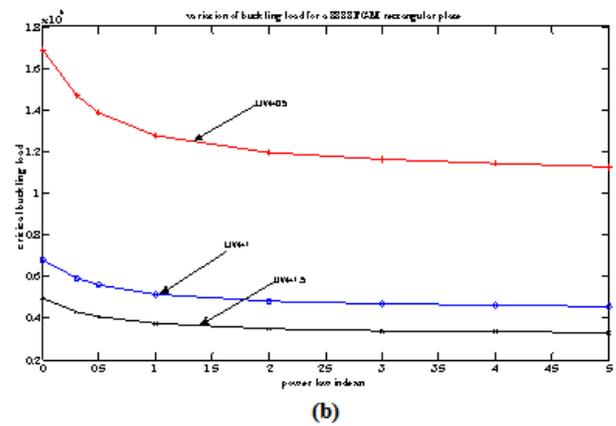
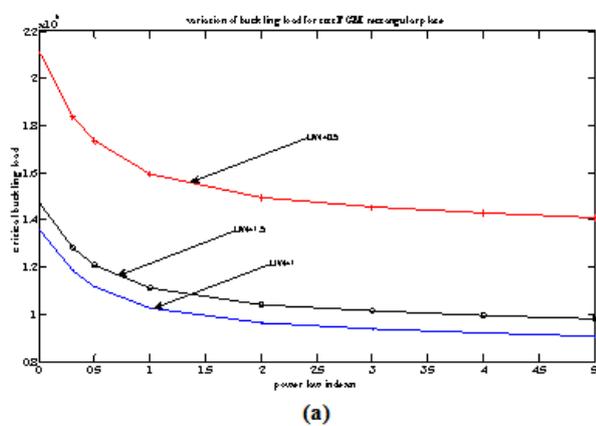


Figure 8. Variation of critical buckling load versus index value with (a) uniaxial and (b) biaxial compression

#### 4.2.2. Dynamic Stability Analysis

The dynamic stability of FGM plates under parametric excitation was investigated. The power law index value, the length, the width and the thickness of the FGM plates

were varied to assess their effects on the parametric instability behaviour of the FGM plates. Figure 9 shows the dynamic stability of simply supported FGM plate aspect ratio  $L/W=0.5, 1, 1.5$  and plate thickness  $h=0.1m$ . The effects of aspect ratios on the first three instability

regions for simply supported plate are presented in Figure 9a, Figure 9b, Figure 9c. Increase the aspect ratio  $L/W$  the unstable region is located farther from dynamic load axis, it shows that reduces the dynamic instability of simply supported rectangular FGM plate. Figure 10a, Figure 10b and Figure 10c shows the first three parametric instability regions of FGM plate of rectangular cross-section with

various index values and aspect ratio is examined. It can be seen that the instability regions are shifted towards the dynamic load axis with increases power law index value at lower excitation frequency. The effect is much more significant on another three instability regions than on the first region. Increase in power law index increases the dynamic stability.

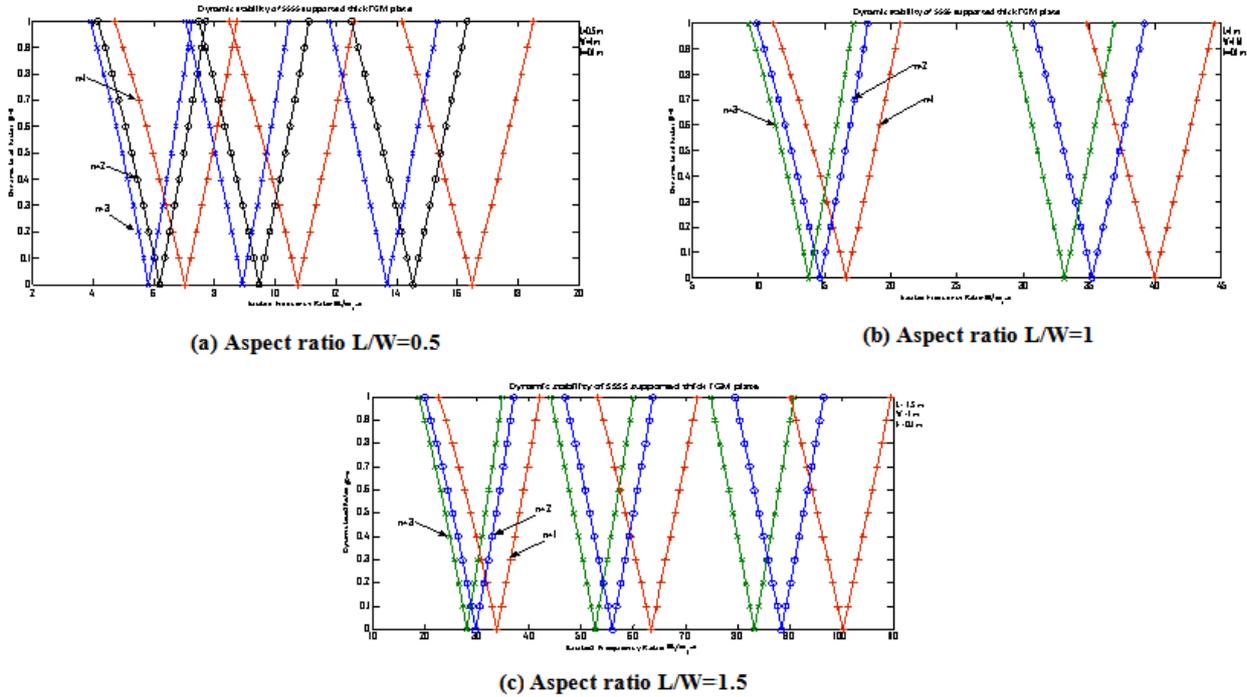


Figure 9. Dynamic stability regions for simply supported FGM plate with various aspect ratios  $L/W=0.5, 1, 1.5$

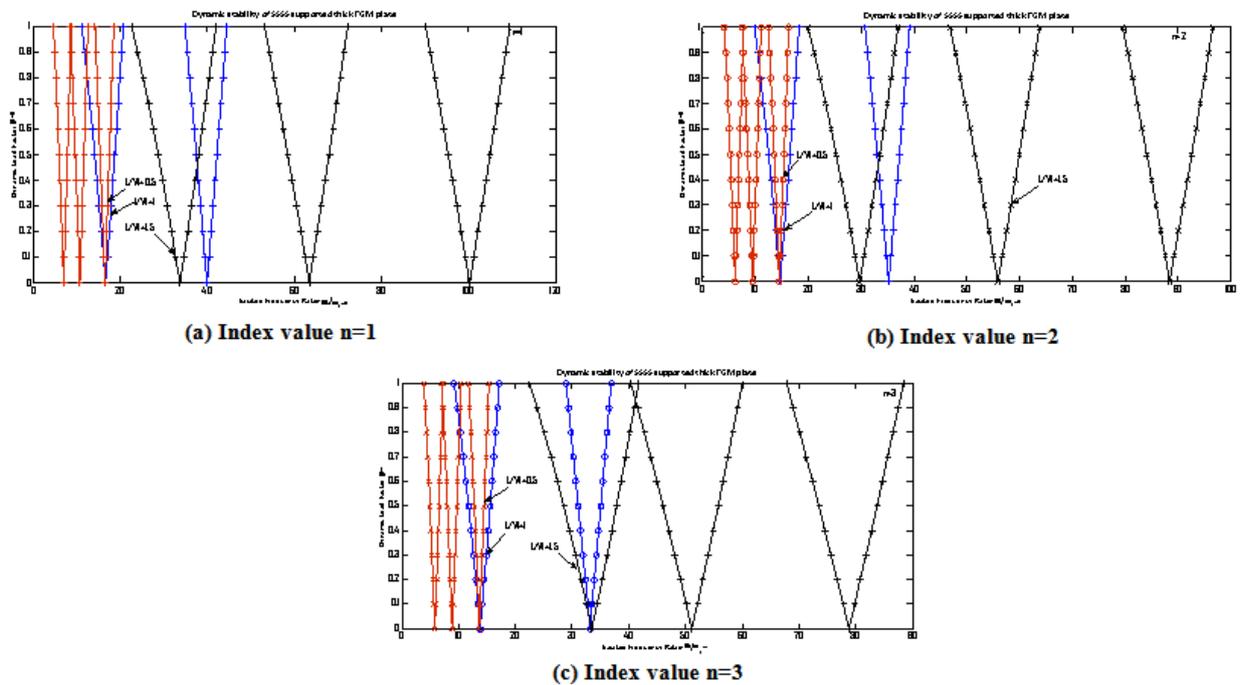


Figure 10. Dynamic stability regions for simply supported FGM plate with different index values  $n=1, 2, 3$

### 5. Conclusions

Finite element modelling of rectangular FGM plate has been developed using first order shear deformation theory.

Based on the above formulation various types of analyses i.e. free vibration, buckling and dynamic stability have been carried out. In case of FGM plate with increase of power law index value, the first five natural frequencies decrease. If  $L/W$  ratio is increased the critical buckling load decreases for each case of loading and it is also

observed that as the power law index value increases critical buckling load decreases. FGM plate vibration and static characteristics can be improved by developing proper material grading. Increase in aspect ratio  $L/W=0.5, 1$  and  $1.5$  of rectangular plate results in overall enhancement of instability of the plate. With increase of the power index value  $n=1, 2$  and  $3$  instability regions moves closer to dynamic load axis with the different aspect ratios, it shows that there is deterioration of the dynamic stability.

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