

# The Effect of Fiber Orientation and Laminate Stacking Sequences on the Torsional Natural Frequencies of Laminated Composite Beams

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**Abstract** The composite materials are well known by their excellent combination of high structural stiffness and low weight. The main feature of these anisotropic materials is their ability to be tailored for specific applications by optimizing design parameters such as stacking sequence, ply orientation and performance targets. Finding free torsional vibrations characteristics of laminated composite beams is one of the bases for designing and modeling of industrial products. With these requirements, this work considers the free torsional vibrations for laminated composite beams of doubly symmetrical cross sections. The torsional vibrations of the laminated beams are analyzed analytically based on the classical lamination theory, and accounts for the coupling of flexural and torsional modes due to fiber orientation of the laminated beams are neglected. Also, the torsional vibrations of the laminated beams analyzed by shear deformation theory in which the shear deformation effects are considered. Numerical analysis has been carried out using finite element method (FEM). The finite element software package ANSYS 10.0 is used to perform the numerical analyses using an eight-node layered shell element to describe the torsional vibration of the laminated beams. The rotary inertia and shear deformation effects of the element are taken. The influence of fiber directions and stacking sequences of laminates on torsional natural frequencies were investigated. Also, the effects of boundary conditions are demonstrated. Numerical results, obtained by the ANSYS 10.0, classical lamination theory, and shear deformation theory are presented to highlight the effects of fibers orientation and layers stacking sequence on torsional frequencies of the beams. The results obtained by ANSYS are compared against the classical lamination theory, as well as shear deformation theory.

**Keywords:** composite materials, laminated composite beams, torsional vibrations, shear deformation, finite element analysis

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## 1. Introduction

The composite beam members have been increasingly used over the past few decades in the fields of aerospace, civil and mechanical engineering due to their excellent engineering features. A variety of structural components made of composite materials such as turbine blades, vehicle axles, aircraft wing, and helicopter blade can be approximated as laminated composite beams, which requires a deeper understanding of the vibration characteristics of the composite beams as mentioned by [13]. The composite beams are generally used as structural components of light-weight heavy load bearing elements because of the high strength-to-weight and stiffness-to-weight ratios, the ability of being different strengths in different directions and the nature of being tailored to satisfy the design requirements of strength and stiffness in practical designs. The increased use of laminated

composite beams requires a better understanding of vibration characteristics of these beams; it is quite essential in the design of composite beams subjected to dynamic loads. Due to the composite beams widely used in a variety of structures as well as their substantial benefits and great promise for future application, the dynamic behaviors of the laminated composite beams have received widespread attention and have been investigated extensively by many researchers. A number of researchers have been developed numerous solution methods to analysis the dynamic behaviors of the laminated composite beams [14,16,19].

In designing structures, it is critical to know the natural frequencies of the structure. If a natural frequency of the structure is close to an excitation frequency, then severe vibration of the structure could occur. This condition is called resonance and to avoid resonance, the natural frequencies of the structure must be altered by making suitable adjustments in the design. The study of such free

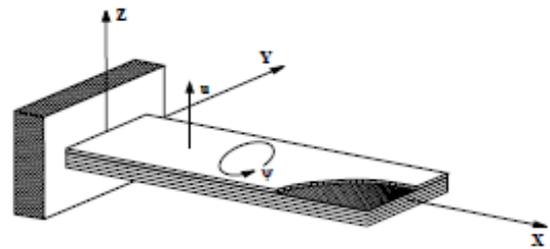
vibrations (free because the structure vibrates with no external forces) is very important in finding the dynamic response of the elastic structure. Thus, in the dynamic analyses, it is quite essential to consider an overview of the free vibration characteristics, including the natural frequencies of these composite structures.

Free vibration analysis of laminated beams has been conducted by significant amount of research. Yöldöröm and Kōral [27] studied the out-of-plane free vibration problem of symmetric cross-ply laminated beams using the transfer matrix method. Also, the effects of the rotary inertia and shear deformation are investigated under various boundary conditions. Banerjee, J. [2] investigated the free vibrations of axially loaded composite Timoshenko beams using the dynamic stiffness matrix method by developing an exact dynamic stiffness matrix of composite beams taking into account the effects of an axial force, shear deformation, and rotatory inertia. The theory includes the material coupling between the bending and torsional modes of deformations. Jun et al. [10] investigated the free vibration and buckling behaviors of axially loaded laminated composite beams having arbitrary lay-up using the dynamic stiffness method taking into account the influences of axial forces, Poisson effect, axial deformation, shear deformation, and rotatory inertia. They developed the exact dynamic stiffness matrix by directly solving the governing differential equations of an axially loaded laminated beam. Abramovich and Livshits [1] studied the free vibration of non symmetric Cross-ply laminated Composite Beams based on Timoshenko type equations. The effect of coupled longitudinal and transversal displacements, shear deformation and rotatory inertia are included in the analysis [1]. Eisenberger, M. et al. [6] used the dynamic stiffness analysis and the first-order shear deformation theory to study the free vibration of laminated beams. Calim, F. [3] make study intended to analyze free and forced vibrations of non-uniform composite beams in the Laplace domain. Khdeir and Reddy [14] have been studied free vibrations of cross-ply laminated beams with arbitrary boundary conditions. Krishnaswamy, S. et al. [16] gave analytical solutions for the free vibration problem of laminated composite beams. Song and Waas [23] have been studied both buckling and free vibration analyses of laminated composite beams. They Song and Waas [23] also investigated the shear deformation effects. Yildirim, V. [26] used the stiffness method for the solution of the purely in-plane free vibration problem of symmetric cross-ply laminated beams with the rotary inertia, axial and transverse shear deformation effects included by the first-order shear deformation theory. Chandrashekhara and Bangera [4] investigated the free vibration of angle-ply composite beams by a higher-order shear deformation theory using the shear flexible FEM. Teh and Huang [24] presented two finite element models based on a first-order theory for the free vibration analysis of fixed-free beams of general orthotropy. Qiao Pizhong and Zou Guiping [20] presented an analytical study for dynamic behavior of pultruded fiber-reinforced plastic (FRP) composite cantilever I-beams based on a Vlasov-type linear hypothesis.

Several researchers have carried out studies on experimental and theoretical evaluations of flexural-torsional vibration analysis for FRP structural members. Lee and Kim [18] studied free vibration of a thin-walled

laminated composite beam, where a general analytical model applicable to the dynamic behavior of a thin-walled channel section composite is developed. This model is based on the classical lamination theory, and accounts for the coupling of flexural and torsional modes for arbitrary laminate stacking sequence. Shadmehri, F. et al. [22] studied the flexural-torsional behavior of thin-walled composite beams with closed cross-section and a number of nonclassical effects, such as material anisotropy, transverse shear, are considered in the study. Kollar, LP. [15] investigated the analysis of Flexural-torsional vibration of open section composite beam with including shear deformation. Qiao et al. [21] presented a combined analytical and experimental approach to characterize the vibration behavior of pultruded Fiber-Reinforced Plastic (FRP) composite cantilever I-beams.

In engineering practice, we often come across the analysis of structures subjected to vibratory twisting loading, such as aerodynamic or asymmetric traffic forces. Also, composite structural elements consisting of a relatively weak matrix reinforced by stronger inclusions or of different materials in contact are of increasing technological importance in engineering. Steel beams or columns totally encased in concrete are most common examples, while construction using steel beams as stiffeners of concrete plates is a quick, familiar and economical method for long bridge decks or for long span slabs.



**Figure 1.** Geometry of the laminated composite beam of clamped-free

The extensive use of the aforementioned structural elements necessitates a rigorous dynamic analysis. Several researchers have dealt with torsional vibration of beams. Exact torsional vibration frequencies were presented by [9] for the case of circular cross-section shafts subjected to classical boundary conditions avoiding in this way warping effects. Eisenberger, M. [7] studied the torsional vibration of open and variable cross section bars by derive analytical method is to form the dynamic stiffness matrix of the bar, including the effect of warping. Kameswara and Mirza [11] studied the problem of free torsional vibration and buckling of doubly symmetric thin-walled beams of open section, subjected to an axial compressive static load and resting on continuous elastic foundation. Kameswara RC. [12] investigated the torsional frequencies and mode shapes of generally constrained shafts and piping, where the exact frequency and normal mode expressions are derived for the free torsional vibrations of circular shafts or piping systems constrained by unsymmetrical torsional springs and carrying unequal rotational masses at either end. Evangelos and Sapountzakis [8] studied the Torsional vibrations of composite bars by (BEM) boundary element method which is developed for the nonuniform torsional vibration

problem of doubly symmetric composite bars of arbitrary constant cross-section.

In the present study, the torsional vibration behaviors of symmetrical laminated composite beams are studied. The laminated beam is modeled and analyzed by the FEM. The commercial finite element program ANSYS 10.0 is used to perform a dynamic modelling to the laminated beams by performing an eigenvalue analysis. Mindlin eight-node isoparametric layered shell elements (SHELL 99) are employed in the modeling for describing the torsional vibrations of these beams. Also, analytical models are developed by classical lamination theory and shear deformation theory to study the torsional vibrations of the beams. In the analytical models, the flexural-torsional coupling effects are ignored and pure torsional vibrations are taken. The effects of fiber direction and laminate stacking sequence on the frequencies of torsional vibrations were investigated. Also, the effects of boundary conditions on the torsional frequencies of the laminated beams are demonstrated.

**Table 1. Material elastic properties**

Material	Properties	Value
Glass fiber	Fiber longitudinal modulus in $\ell$ direction $E_{f\ell}$ (GPa)	74
	Fiber transverse modulus in $t$ direction $E_{ft}$ (GPa)	74
	Fiber shear modulus $G_{f\ell t}$ (GPa)	30
	Density $\rho_f$ (kg/m <sup>3</sup> )	2600
	Fiber Poisson ratio $\nu_{f\ell t}$	0.25
Epoxy resin	Elastic modulus $E$ (GPa)	4.5
	Shear modulus $G$ (GPa)	1.6
	Density $\rho_m$ (kg/m <sup>3</sup> )	1200
	Poisson ratio $\nu$	0.4
orthotropic Laminae	Lamina longitudinal modulus $E_1$ (GPa)	46.2
	Lamina transverse modulus $E_2$ (GPa)	14.70
	Lamina transverse modulus $E_3$ (GPa)	14.70
	Density of composite $\rho_c$ (kg/m <sup>3</sup> )	2040
	Lamina shear modulus in plane 1-2 $G_{12}$ (GPa)	5.35
	Lamina shear modulus in plane 1-3 $G_{13}$ (GPa)	5.35
	Lamina shear modulus in plane 2-3 $G_{23}$ (GPa)	5.22
	Major Poisson ratio in plane 1-2 $\nu_{12}$	0.31
	Major Poisson ratio in plane 1-3 $\nu_{13}$	0.31
	Major Poisson ratio in plane 2-3 $\nu_{23}$	0.41
	Fiber volume fraction $\nu_f$	60%

## 2. Material and Geometry

A generally laminated composite beam with a solid rectangular cross-section of doubly symmetrical cross sections, as shown in Figure 1, is considered to be studied. The laminated beam is made of many plies of orthotropic materials, and the principal material axes of a ply may be oriented at an arbitrary angle with respect to the  $x$ -axis. In the right-handed Cartesian coordinate system, the  $x$ -axis is coincident with the beam axis and its origin is on the mid-plane of the beam. The length, breadth and thickness of the beam are represented by  $L$ ,  $b$  and  $h$ , respectively.

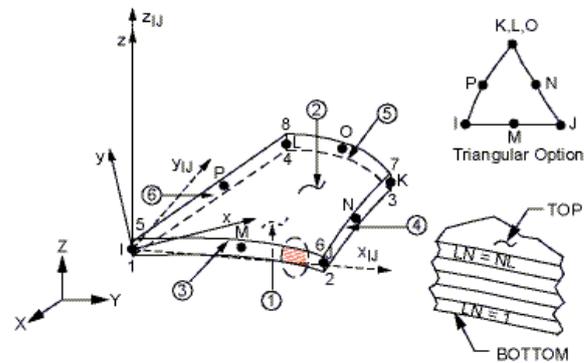
Glass fiber (E-Glass) is used as reinforcement in the form of unidirectional fibers with epoxy resin as matrix for the laminated composite beams. The mechanical properties for fiber and matrix are presented in Table 1 [5].

For all finite element and analytical models, their associate material elastic properties were calculated analytically using the simple rule-of-mixtures as given in [25]. More accurate values can be further obtained with some mechanical testing.

The constituent laminae were considered to be linear elastic and generally orthotropic therefore the concept of engineering constants was used to describe the laminae elastically. A certain set of elastic properties is required as input parameters for the finite element code and for the analytical models. The set of properties required as an input parameter at a material level were  $E_1$ ,  $E_2$ ,  $E_3$ ,  $G_{12}$ ,  $G_{13}$ ,  $G_{23}$ ,  $\nu_{12}$ ,  $\nu_{13}$  and  $\nu_{23}$  as shown in Table 1; Where 1, 2, and 3 are principal material directions.

## 3. Modal Analysis by Finite-Element Method, ANSYS

The beams were discretized using (type shell 99) finite element as shown in Figure 2, available in the commercial package ANSYS 10.0. This element has 8 nodes and is constituted by layers that are designated by numbers (LN - Layer Number), increasing from the bottom to the top of the laminate; the last number quantifies the existent total number of layers in the laminate (NL - Total Number of Layers). The element has six degrees of freedom at each node: translations in the nodal  $x$ ,  $y$ , and  $z$  directions and rotations about the nodal  $x$ ,  $y$ , and  $z$ -axes. The choice of shell 99 element type is based on layered applications of a structural shell model, and the type of results that need to be calculated.



**Figure 2. Shell 99 geometry**

$x_{11}$  = Element  $x$ -axis if ESYS is not supplied.

$x$  = Element  $x$ -axis if ESYS is supplied.

A modal analysis will be carried out using ANSYS 10.0 finite element software to study the frequencies of free torsional vibrations of the mentioned composite laminated beam.

## 4. Dynamic Modeling by Analytical Methods

In the present paper, the free torsional vibrations of symmetric laminated beams are studied by the classical lamination theory and shear deformation theory.

When the cross-section has one plane of symmetry, one of the circular frequencies belongs to a flexural mode and the other two circular frequencies to flexural-torsional

modes; while when the cross-section has two planes of symmetry, the three circular frequencies belong respectively to the flexural modes in the two planes of symmetry and to the pure torsion mode (when the axis of the beam does not bend). The bending–torsion coupling due to stiffness coupling presented in composite beams due to fiber orientation and stacking sequence is neglected.

#### 4.1. Classical Beam Theory

The oldest and the well-known beam theory is the Euler–Bernoulli beam theory (or classical beam theory—CBT), in which the shear deformation not included. Although this theory is useful for slender beams, it does not give accurate solutions for thick beams. The beams to be studied are orthotropic and its cross section has two axes of symmetry  $y$  and  $z$ . The mass is also symmetrical with respect to these axes, and, accordingly, the center of mass coincides with the origin of the  $y$ - $z$  coordinate system, so that the flexural-torsional coupling not occurs.

A beam with two cross-sectional planes of symmetry may undergo flexural vibration in either of the two planes of symmetry and torsional vibration [17]. Pure torsional vibrations are focused in this study.

Expressions for the torsional vibration  $\omega_{\psi_i}^B$  of long ( $\overline{GI}_t \gg \overline{EI}_\omega / L^2$ ) and short ( $\overline{GI}_t \ll \overline{EI}_\omega / L^2$ ) orthotropic beams are:

Torsional vibration of long beam is given by:

$$(\omega_{\psi_i}^B)^2 = \frac{\overline{GI}_t}{2\pi * \theta} \frac{\mu_{Gi}^2}{L^2} \quad (1)$$

Torsional vibration of short beam is given by:

$$(\omega_{\psi_i}^B)^2 = \frac{\overline{EI}_\omega}{2\pi * \theta} \frac{\mu_{Bi}^4}{L^4} \quad (2)$$

$$\theta = \int_{(A)} \rho_{comp} (y^2 + z^2) dA \quad (3)$$

Where  $\overline{GI}_t$  is the torsional stiffness of the beam; in  $N.m^2$ ,  $\overline{EI}_\omega$  is the warping stiffness of the beam; in  $N.m^4$ ,  $\theta$  is the polar moment of mass per unit length about the shear center,  $\rho_{comp}$  is mass per unit volume,  $A$  is the area of the cross section, and  $\mu_{Bi}^4$  and  $\mu_{Gi}^4$  are parameters in the calculation of natural frequencies, which are given in Table 2.

The torsional frequencies of a beam of arbitrary length can be approximated by:

$$(\omega_{\psi_i}^B)^2 = (\omega_{\psi_i}^B)^2_{short} + (\omega_{\psi_i}^B)^2_{long} \quad (4)$$

By using the previous two equations of torsional vibration, the torsional frequencies will be:

$$(\omega_{\psi_i}^B)^2 = \frac{\overline{EI}_\omega}{2\pi * \theta} \frac{\mu_{Bi}^4}{L^4} + \frac{\overline{GI}_t}{2\pi * \theta} \frac{\mu_{Gi}^2}{L^2} \quad (5)$$

For symmetric orthotropic laminated beam previously mentioned; the torsional stiffness of the beam  $\overline{GI}_t$  can be obtained by this relation,

$$\overline{GI}_t = \frac{4b}{d_{66}} \text{in} (N.m^2) \quad (6)$$

and Warping stiffness of the beam  $\overline{EI}_\omega$  can be obtained by this relation,

$$\overline{EI}_\omega = \frac{b^3}{a_{11}} \frac{h^2}{144} \text{in} (N.m^4) \quad (7)$$

Where:

$a_{11}$ : element 1–1 of the laminate extensional compliance matrix (m/N).

$d_{66}$ : element 6–6 of the laminate bending compliance matrix (1/N. m).

**Table 2. The constants  $\mu_{Bi}$  and  $\mu_{Gi}$  for for different types of end supports**

Geometry	$\mu_B$	$\mu_G$
Clamped-Free	$\mu_{B1} = 1.875$ $\mu_{B2} = 4.694$ $\mu_{Bi} \approx (i-0.5)$	$\mu_{Gi} \approx (i-0.5)\pi$
Clamped-Clamped	$\mu_{B1} = 4.730$ $\mu_{B2} = 7.853$ $\mu_{Bi} \approx (i+0.5)\pi$	$\mu_{Gi} = i\pi$
Clamped-Simply supported	$\mu_{B1} = 3.927$ $\mu_{B2} = 7.069$ $\mu_{Bi} \approx (i+0.25)\pi$	$\mu_{Gi} = i\pi$
Simply supported-Simply supported	$\mu_{Bi} = i\pi$	$\mu_{Gi} = i\pi$

#### 4.2. Shear Deformation Theory

The theory, based on the assumption that cross sections remain plane but not perpendicular to the axis is frequently called first-order shear theory. A beam, in which shear deformation is taken into account, is called a Timoshenko beam. In shear deformation theory the effect of the shear deformation is considered in torsional frequencies calculation as given by (La's zlo' and George, 2003).

Torsional vibration with shear deformation  $\omega_{\psi_i}$  of short ( $\overline{GI}_t L^2 \ll \overline{EI}_\omega$ ) and long

( $\overline{GI}_t L^2 \gg \overline{EI}_\omega$  and  $\overline{S}_{\omega\omega} L^2 \gg \overline{EI}_\omega$ ) orthotropic beams are:

Torsional vibration of short beam is given by,

$$(\omega_{\psi_i})^2 = \left[ \frac{\theta}{\overline{EI}_\omega} \left( \frac{L}{\mu_{Bi}} \right)^4 + \frac{\theta}{\overline{S}_{\omega\omega}} \left( \frac{L}{\mu_{Si}} \right)^2 \right]^{-1} \quad (8)$$

Torsional vibration of long beam is given by,

$$(\omega_{\psi_i})^2 = \frac{\overline{GI}_t}{\theta} \frac{\mu_{Gi}^2}{L^2} \quad (9)$$

The torsional circular frequency of a beam of arbitrary length can be approximated by:

$$(\omega_{\psi_i})^2 = (\omega_{\psi_i})^2_{short} + (\omega_{\psi_i})^2_{long} \quad (10)$$

By using Esq. (8) and (9) then,

$$(\omega_{\psi_i})^2 = \left[ \frac{\theta}{\overline{EI}_\omega} \left( \frac{L}{\mu_{Bi}} \right)^4 + \frac{\theta}{\overline{S}_{\omega\omega}} \left( \frac{L}{\mu_{Si}} \right)^2 \right]^{-1} + \frac{\overline{GI}_t}{\theta} \frac{\mu_{Gi}^2}{L^2} \quad (11)$$

Where the torsional shear stiffness is given by,

$$\bar{S}_{\omega\omega} = \frac{bh^2}{1.2a_{66}} \quad (12)$$

Where  $a_{66}$  is element 6–6 of the laminate extensional compliance matrix (m/N)

## 5. Numerical Results and Discussion

### 5.1. Influence of Fiber Angle on Torsional Natural Frequencies

The influences of fiber orientation are investigated by modeling laminated beams of different lay-up construction of clamped – free boundary condition as shown in Figure 1. The analysis was performed to 8-layered symmetrically

laminated beam with length 400 mm, width 40 mm and thickness 3.2 mm and the lamination scheme of beams is ranging from  $\theta=0^\circ$  to  $90^\circ$ , in increments of  $5^\circ$ .

The results obtained after modeling the beams are presented in Figure 3. It presents the variation of the lowest three torsional natural frequencies of the beam with respect to fiber angle. From the results, It is seen that the torsional frequencies increase with increasing the fiber angle of the laminated beams until reach to significant values in the range from about  $\theta=35^\circ$  to  $\theta=45^\circ$ , then the torsional frequencies decrease gradually with increasing the fiber angle until reach to minimum value at  $\theta=90^\circ$ .

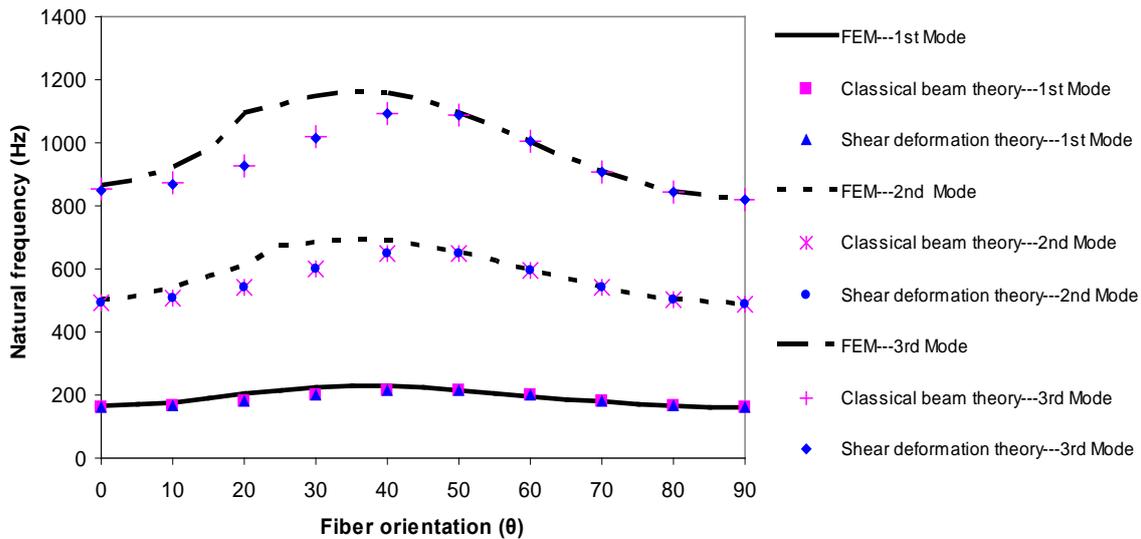


Figure 3. Variation of 1st, 2nd, and 3rd torsional frequencies of Clamped–Free composite beam with respect to fiber angle change

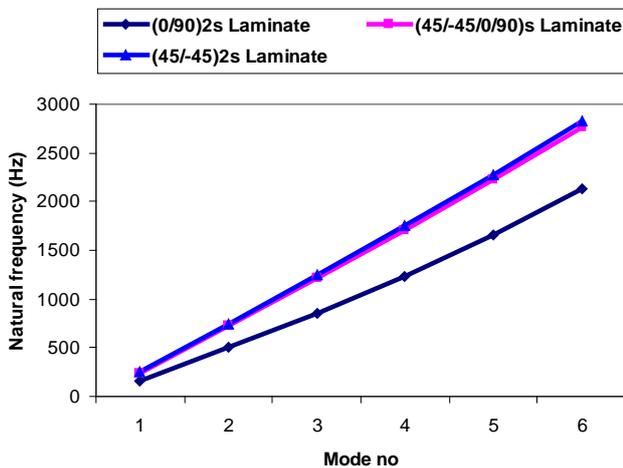


Figure 4. Influence of laminate stacking sequence on torsional natural frequencies for clamped free boundary condition [More Explanation of Figure 4 is required regarding Observations and Comparison]

### 5.2. Influence of Laminate Arrangement on Torsional Frequencies

To investigate the influence of laminate stacking sequence, dynamic modeling is performed to 3 set of symmetrical laminates with a total of 8 layers and dimension of 400 mm length, 40 mm width and total thickness 3.2 mm. Each layer in the laminate has the same thickness. The lamination schemes of the beams are as

follow: (0/90) 2S, (45/-45) 2S, and (45/ 45/0/90) S. The torsional frequencies are obtained by the analytical methods and by FE package, Table 3. [More Explanation of Table 3 is required regarding Observations and Comparison].

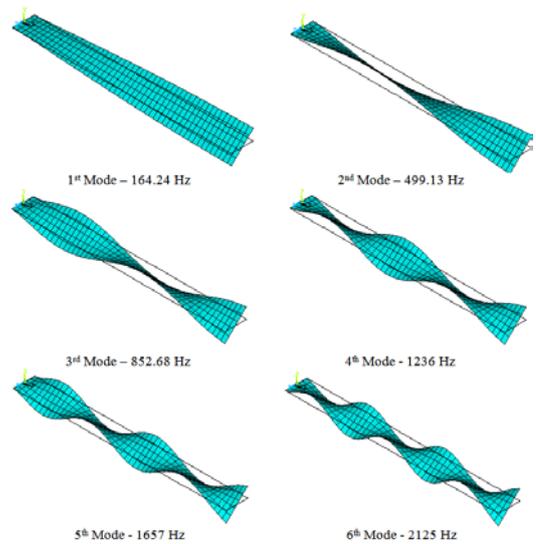
Figure 4, give the variation of the torsional natural frequencies of the laminates with respect to mode number for clamped free end condition, from the results it is already possible to verify the influence of the stacking sequence of the laminate on torsional vibration: the laminate with fibers at  $\pm 45^\circ$  has a larger torsional natural frequencies than the laminate of  $\pm 45/0/90^\circ$  and of  $0/90^\circ$  fibers. This was expected, since the natural frequencies are related to the stiffness of the structure and the  $(\pm 45^\circ)$  is much stiffer on torsion than the  $\pm 45/0/90^\circ$  and than  $0/90^\circ$  laminate.

The laminate of  $0/90^\circ$  fibers have the lowest torsional natural frequencies than the other lamination schemes and thus, because 50% of the fibers are oriented at  $0^\circ$  direction for  $0/90^\circ$  laminate, and thus appropriate for bending (Flexural Modes). This can be explained by the fact that the fibers oriented at  $0^\circ$  are more appropriate to flexural loads and the fibers oriented at  $45^\circ$  are more appropriate to torsional loads.

The mode shapes associated with the torsional natural frequencies of (0/90) 2S laminated beam are illustrated in Figure 5. They are deduced by FEM ANSYS for the first six torsional frequencies.

**Table 3. Torsional natural frequencies (Hz) for different stacking sequences laminate**

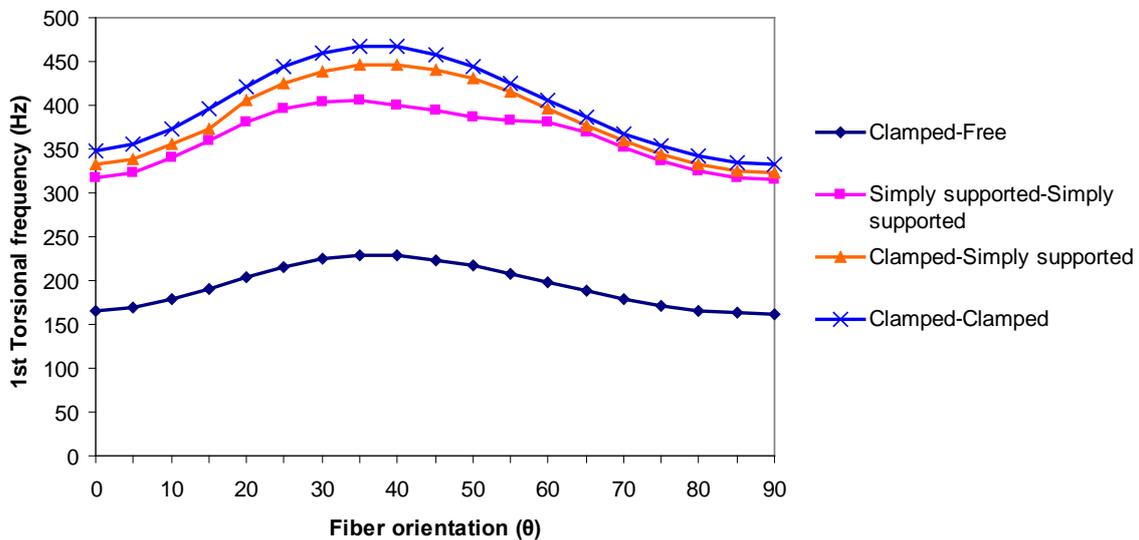
Lamination schemes	Theory	Modes					
		1	2	3	4	5	6
(0/90) <sub>2S</sub>	FEM ANSYS	164.24	499.13	852.68	1236.00	1657.00	2125.00
	Classical Beam Theory	161.90	490.40	836.00	1208.40	1616.30	2067.40
	Shear deformation Theory	161.80	490.30	835.20	1204.00	1602.70	2033.00
(45/-45) <sub>2S</sub>	FEM ANSYS	246.40	741.30	1242.00	1754.00	2280.00	2824.00
	Classical Beam Theory	250.30	752.70	1261.20	1779.40	2311.00	2860.00
	Timoshenko	250.30	752.70	1261.10	1779.00	2310.00	2856.80
(45/-45/0/90) <sub>S</sub>	FEM ANSYS	239.00	720.00	1208.00	1709.00	2227.00	2767.00
	Classical Beam Theory	242.50	730.10	1227.00	1738.70	2270.90	2828.70
	Shear deformation Theory	242.50	730.00	1226.80	1737.60	2267.00	2819.00



**Figure 5.** The torsional vibration modes of (0/90°) clamped-free laminated beam

### 5.3. Effect of Boundary Conditions on Torsional Frequencies

The FE analysis by using ANSYS is performed to investigate the influences of boundary conditions on torsional frequencies of the laminated beams. The analysis can be applied to the laminated beam previously mentioned with the same dimensions and geometry. The boundary conditions to be investigated in this study are as follow: C-F Clamped-Free, C-C Clamped-Clamped, C-S Clamped-Simply-Supported, and S-S Simply Supported - Simply Supported.



**Figure 6.** Influence of boundary conditions on 1st torsional natural frequency

The influence of boundary conditions on torsional natural frequencies is investigated for the 1st torsional frequency as shown in Figure 6. From the results, It is seen that the clamped-clamped condition has a larger torsional frequency than other boundary fixations, and thus for all fiber angles. The torsional frequency for clamped-simply supported condition comes to be lower than clamped-clamped condition, then simply supported-simply supported comes to be lower than clamped-simply

supported, and finally clamped-free condition comes to be lower than other supports.

## 6. Conclusion

This study is the first attempt to deal with the torsional natural frequencies of laminated composite beam with different lamination. For this, the influences of ply angle as well as laminate stacking sequences on the torsional natural frequencies of the laminated beams with doubly

symmetrical cross-sections are investigated. Also, effects of boundary conditions on torsional natural frequencies are demonstrated.

It is observed that the results obtained by FEM using ANSYS are in good agreement with the analytical solutions, classical lamination theory and shear deformation theory. The analytical results by both classical lamination theory and shear deformation theory; the coupling effects due to bending–twisting coupling stiffness result from fiber orientation and laminate stacking sequences are excluded.

From the results, it is clear that changes in fiber angle as well as laminate stacking sequences yield to different dynamic behavior of the component, that is, different torsional natural frequencies for the same geometry, mass and boundary conditions. This gives the designer one additional degree of freedom to design the laminate - the possibility to change fiber orientations in order to get more (or less) structure stiffness in torsion. Different layers (plies) have different contributions to the overall stiffness of the beam depending on their locations from the mid-plane.

This possibility makes once more these materials very attractive since it makes possible to obtain the desired torsional natural frequencies without increasing mass or changing geometry. In practical applications, it means that if a torsional natural frequency excites the structure, the designer can change the material properties by changing the laminate stacking sequence, instead of re-design the complete structure. It is seen that the torsional frequencies increase with increasing the fiber angle of the laminated beams until reach to significant value at about  $\theta=45^\circ$ , then the torsional frequencies decrease gradually with increasing the fiber angle until reach to minimum value at  $\theta=90^\circ$ .

The finite element software package ANSYS is an efficient vibration prediction tool, because of its ability to model the laminated composite beam and reveal fundamental modal frequencies and modal shapes. Finally this study is useful for the designer in order to select the fiber orientation angle to shift the torsional natural frequencies as desired or to control the vibration level.

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