

Performance Evaluation of Multivariate Adaptive Regression Splines (MARS) and Multiple Linear Regression (MLR) for Forward Conversion of Geodetic Coordinates ($\varphi \lambda, h$) to Cartesian Coordinates (X, Y, Z)

M.S. Peprah*, I.O. Mensah, J. A. Akresi

Department of Geomatic Engineering, University of Mines and Technology, Ghana

*Corresponding author: mspeprah@st.umat.edu.gh

Abstract In Ghana's local Geodetic Reference Network, the standard forward transformation equation has played a major role in coordinate transformation between World Geodetic System 1984 (WGS84) and local geodetic datum. Thus, it is an initial step in forward conversion of geodetic coordinates ($\varphi \lambda, h$) to Cartesian coordinates (X, Y, Z) in transformation from global to local datum and vice versa. Several studies in the recent decades have been conducted on converting Cartesian coordinates to geodetic coordinates (reverse procedure) through the utilisation of iterative, approximate, closed form, vector-based and computational intelligence algorithms. However, based on the existing literature covered pertaining to this present study, it was found that the existing knowledge do not fully adhere to the issue of evaluating alternative techniques in the case of the forward conversion. Hence, the aim of this present study was to explore the coordinate conversion performance of the Multivariate Adaptive Regression Splines (MARS) and Multiple Linear Regression (MLR). The performance of each model was assessed based on statistical indicators of Mean Square Error (MSE), Root Mean Square Error (RMSE), Mean Bias Error (MBE), Mean Absolute Error (MAE), Standard Deviation (SD), Noise to Signal Ratio (NSR), Correlation Coefficient (R), and Correlation of Determination (R^2). The statistical findings revealed that the MARS and MLR offered satisfactory prediction of Cartesian coordinates. However, the MLR compared to MARS showed better stability and more accurate prediction results. From the results of this present study, the main conclusion drawn is that, MLR provides a promising alternative in the forward conversion of geodetic coordinates into Cartesian coordinates. Therefore, the capability of MLR as a powerful tool for solving majority of function approximation problems in mathematical geodesy has been demonstrated in this present study.

Keywords: *Multivariate Adaptive Regression Splines, Multiple Linear Regression, forward transformation equation, coordinate conversion*

Cite This Article: M.S. Peprah, I.O. Mensah, and J. A. Akresi, "Performance Evaluation of Multivariate Adaptive Regression Splines (MARS) and Multiple Linear Regression (MLR) for Forward Conversion of Geodetic Coordinates ($\varphi \lambda, h$) to Cartesian Coordinates (X, Y, Z)." *Journal of Geosciences and Geomatics*, vol. 5, no. 3 (2017): 109-118. doi: 10.12691/jgg-5-3-2.

1. Introduction

Global Navigation Satellite System (GNSS) such as the Global Positioning System (GPS) in capturing locations of stationary and non-stationary objects on, above, and beneath the Earth's surface has increased the possibility of obtaining coordinate positions with improved high accuracy and precision [1]. These satellite positioning technologies provide numerous amounts of spatio-temporal datums in either curvilinear geodetic coordinates ($\varphi \lambda, h$) or Cartesian coordinate (X, Y, Z) system. In order to solve most practical GPS navigation, geodetic, cartographical and astro-observation problems, it is important to convert geodetic coordinates ($\varphi \lambda, h$) into Cartesian coordinates (X, Y, Z) and vice versa [1,2,3,4,5].

The process of converting geodetic coordinates to Cartesian coordinates is known as the forward conversion [1].

The forward conversion is an initial step in converting the GPS position measurement to the local coordinate system [6,7]. This is because before the invention of the GPS, local geodetic datums were established based on traditional surveying techniques such as triangulation, trilateration, traversing, astronomical observation and among others [8,9]. Hence, the local geodetic datum involved data in only geodetic coordinates without the existence of Cartesian coordinates [1]. This prevalent situation has been problematic over the years to utilize GPS coordinates based on the global datum of world geodetic system 1984 (WGS84) into a local geodetic system. The first approach in utilizing GPS data locally requires the determination of transformation parameters.

The most widely adopted techniques in the past decades in literature for such application include the similarity models of Bursa-Wolf, Molodensky-Badekas, Veis model and three-dimensional Affine [10-16]. However, the major concern here is that, before these aforementioned similarity models could be applied, there is the need to convert all the geodetic dataset of common points to Cartesian coordinates. It must be well noted here that, without such conversion, the similarity models cannot be adopted in the transformation parameter determination for coordinate transformation between the global and local datums [1].

In order to accomplish this task, the standard forward transformation equation [17,18,19,20] given by Equation (1) to (3) are mainly widely used technique as the first step in the coordinate transformation procedure given as:

$$X = (N + h)\cos\varphi\cos\lambda \quad (1)$$

$$Y = (N + h)\cos\varphi\sin\lambda \quad (2)$$

$$Z = \left[N(1 - e^2) + h \right] \sin\varphi \quad (3)$$

Where φ , λ and h are the geodetic latitude, geodetic longitude and geodetic height while X , Y , Z are the Cartesian coordinates. N in Equation (1) to (3) is the radius of curvature in the prime vertical [1] defined by Equation (4) as:

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2\varphi}} \quad (4)$$

Where e is the first eccentricity expressed by Equation (5) as:

$$e = \frac{\sqrt{a^2 - b^2}}{a} \quad (5)$$

Where a and b are the semi-major axis and semi-minor axis of the geodetic ellipsoid [1].

Although Equation (1) to (3) are the mostly adopted technique in geodetic studies, there exist other numerical methods such as Artificial Neural Network (ANN), Multivariate Adaptive Regression Splines (MARS), Multiple Linear Regression (MLR) and many more that can serve as alternative approach. It is well acknowledged that the introduction of artificial intelligence techniques applications has revolutionized the field of mathematical geodesy in terms of its attainable accuracy and also in most instance, its dominance over the empirical methods. This assertion is well archived in existing technical papers of mathematical geodesy. For example, ANN has been applied to solve most coordinate transformation problems global and local datums [1,9,21-29], and transforming from geodetic coordinates to Cartesian coordinates [2] and many more. ANNs are been criticized for its long training process in achieving the optimal network's topology, and it is not easy to identify the relative importance of potential input variables, and certain interpretive difficulties [30,31].

There has been a surge of interests within the geodetic community in the recent decades in converting Cartesian coordinates into geodetic coordinates and this process is known as the reverse coordinate conversion technique.

Several studies have been conducted such as iterative, approximate, closed form, vector-based algorithms [32,33,34,35,36] and artificial intelligence algorithms [1,2]. Hence, Multivariate Adaptive Regression Splines (MARS) and Multiple linear regression (MLR) were applied in this study to ascertain its ability to convert geodetic coordinates to Cartesian coordinates (forward conversion).

Multivariate Adaptive Regression Splines (MARS) is an adaptive modelling process invented by [37] used for non-linear relationships. In addition, MARS divides the predictor variables into piece-wise linear segments to describe non-linear relationships between the predictor and the dependent variable [31,38]. There is limited availability of literature of MARS in transformation of coordinates studies but many studies have successfully applied MARS for solving different problems in engineering. Some of the areas of applications include estimating energy demand [39], slope stability analysis [31,40,41,42], landslide susceptibility mapping [43], water pollution prediction [44], earthquake modelling [45], studying ecological variables [46], region spatio-temporal mapping [47], modelling of the ionosphere [48], geothermal prospect [49] and so on.

Multiple Linear Regression technique (MLR) was also applied in this study as an alternative mathematical procedure to carry out the forward conversion from geodetic coordinates to Cartesian coordinates. Several studies have been carried out using MLR and Simple Linear Regression (SLR) procedures in coordinate transformation from global to local datum and vice versa [1,50,51,52,53]. It is well acknowledged that, the regression techniques achievable are applicable to surveying and mapping related works [1]. In addition, this present study adopted the MLR over the SLR based on the multiple input parameters utilized in Equation (1) to (3) to carry out the forward coordinate conversion. This will help maintain system consistency between MLR and Equation (1) to (3). The objective of this present study is to evaluate and compare the performance of MARS and MLR models in converting geodetic coordinates to Cartesian coordinates.

MLR have been applied in coordinate transformation but MARS have not been applied in both coordinate transformation and in coordinate conversion. The difference is that, in coordinate conversion as in the case of this present study, there is no change of datum and the geodetic ellipsoid parameters chosen for the transformation process are all based on the same geodetic datum [1]. That is, the geodetic datums used in coordinate transformation, the source and target coordinate reference systems are all based on different datums and the transformation parameters are empirically determined and thus maybe subject to measurement errors [1].

The existing knowledge and publications have not fully addressed the issue of applying alternative techniques in the forward conversion of geodetic coordinates to Cartesian coordinates. In addition, upon careful review of existing studies, the authors realized that the utilization of the MARS and MLR techniques have not applied as a practical alternative technology to the existing approaches. This present study for the first time explored the utilization of the MARS and MLR in the forward conversion of geodetic coordinates to Cartesian

coordinates. To achieve the aim of this present study, the MARS and MLR methods were applied. This study also highlights the comparison between MARS and MLR through the use of a set of training and test data based on the results of statistic performance indicators such as mean (M), mean square error (MSE), root mean square error (RMSE), mean bias error (MBE), mean absolute error (MAE), standard deviation (SD), noise to signal ratio (NSR), correlation of coefficient (R), and correlation of determination (R^2). The finding of these two models will reveal the working efficiency of the two models for converting geodetic coordinate to Cartesian coordinates. Hence this study will serve as an added contribution to existing knowledge of MARS and MLR in mathematical geodesy.

2. Resources and Methods Used

The study area (Figure 1) is situated in the mining town of Tarkwa which is the administrative capital of the Tarkwa Nsuaem Municipal Assembly in the Western Region of Ghana. It is found in the Southwest of Ghana

with geographical coordinates between longitudes $1^{\circ} 59' 00''$ W and latitude $5^{\circ} 18' 00''$ N and is 78 m above mean sea level. It is about 85 km from Takoradi, which is the regional capital, 233 km from Kumasi and about 317 km from Accra [52]. The topography is generally described as remarkable series of ridges and valleys. The ridges are formed by the Banket and Tarkwa Phyllites whereas upper quartzite and Huni Sandstone are present in the valleys. Surface gradients of the ridges are generally very close to the Banket and Tarkwa Phyllites. Its environs generally lie within the mountain ranges covered by thick forest interjected by undulating terrain with few scarps. The study area has a South-western Equatorial climate with seasons influenced by the moist South-West Monsoon winds from the Atlantic Ocean and the North-East Trade Winds. The mean rainfall is approximately 1500 mm with peaks of more than 1700 mm in June and October. Between November and February, the rainfall pattern decrease to between 20 mm to 90 mm [54]. The mean annual temperature is approximately 25°C with small daily temperature variations. Relative humidity varies from 61 % in January to a maximum of 80 % in August and September [52,55].

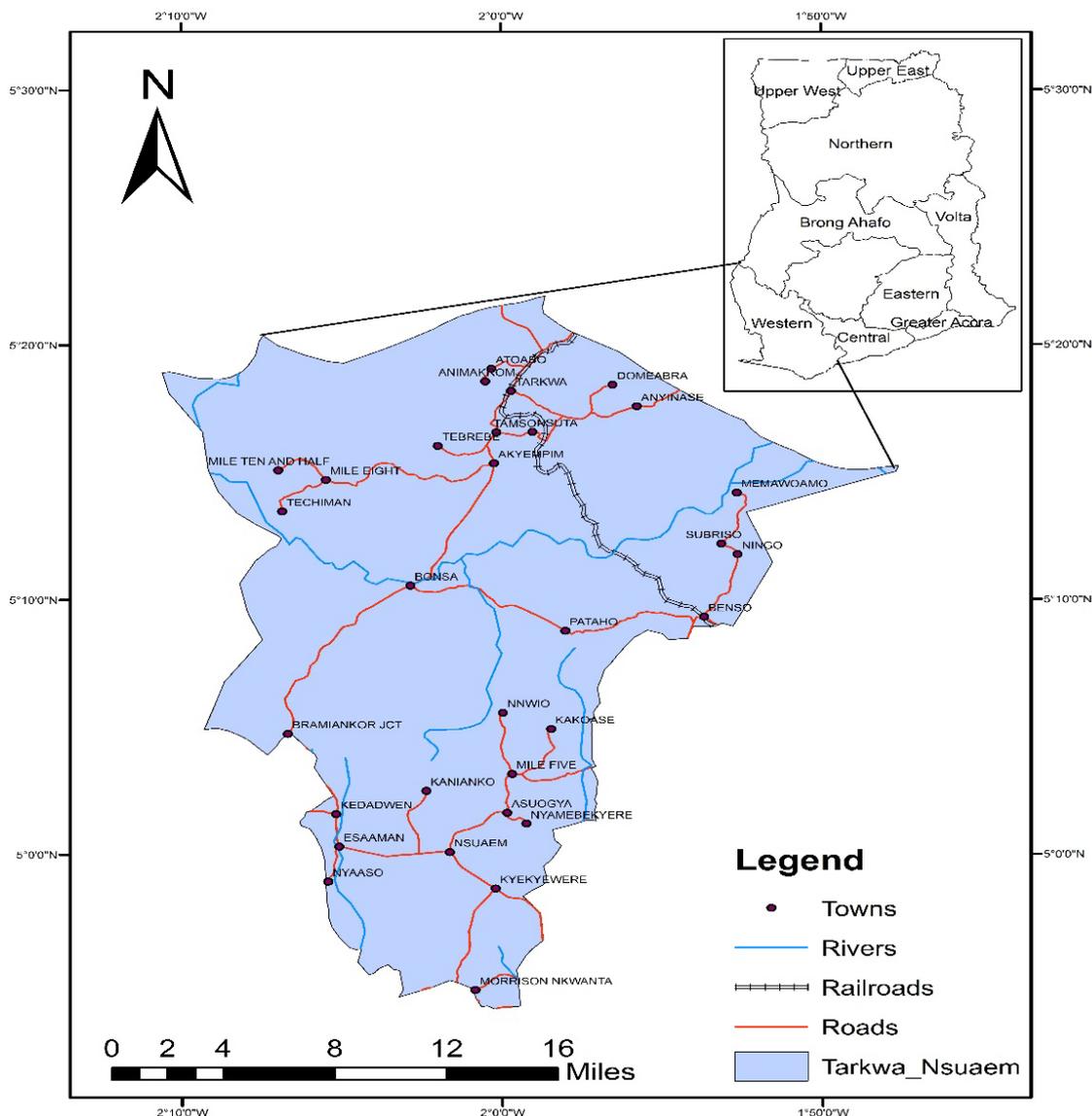


Figure 1. Study Area

In this present study, a total of 328 GPS geodetic coordinates (φ, λ, h) collected by field measurement in Tarkwa, Ghana, situated in West Africa, were used in the MARS and MLR model formulation. It is well acknowledged that one of the contributing factors affecting the estimation accuracy of models is related to the quality of datasets used in model-building and selection of appropriate inputs parameters [56,57]. Therefore, to ensure that the obtained geodetic coordinate (φ, λ, h) data from the GPS receivers are reliable and accurate, several factors such as checking of overhead obstruction, obstruction, observation period, observation period, observation principles and techniques as suggested by many researchers [58]. In addition, all potential issues relating to GPS survey work were also considered. The next factor considered in this present study was the identification of the input parameters for the MARS and MLR training. It must be noted here that the input parameters act as control variables with an influence on the desired output of the models. Hence, the input variable which are the (independent variables) should represent the condition for which training of the two models are done [59]. Consequently, the 328 GPS points measurement was first transformed into Cartesian coordinates (X, Y, Z) using Equation (1) to (3). The GPS world geodetic system 1984 (WGS84) semi-major axis, a and semi-minor axis, b parameter values of 6378137.0 m and 6378299.99899 m were implemented in Equation (1) to (3). It is important to note that, in order for the MARS and MLR models to have consistency with Equation (1) to (3), the radius of curvature in the prime vertical (N) and the square of the square of the first eccentricity (e^2) values were estimated separately. With this in mind, ($\varphi, \lambda, h, N, e^2$) was used as the input layer data while (X,Y,Z) was used as the output layer data. Table 1 and Table 2 are sample of the dataset that was used for the study.

Table 1. GPS Coordinates

POINT ID	LONGITUDE	LATITUDE	HEIGHT
P1	-2.001587	5.299648	105.0657
P2	-2.001814	5.299471	106.1565
P3	-2.001831	5.299346	93.3399
P4	-2.001931	5.299505	106.3422
P5	-2.002049	5.299301	109.1203
P6	-2.002202	5.299204	107.6099
P7	-2.002393	5.299211	105.6151
P8	-2.001292	5.299563	106.3142
P9	-2.001984	5.298738	103.9401
P10	-2.002044	5.298839	106.0406

Table 2. Cartesian Coordinates

POINT ID	X	Y	Z
P1	6347283.0440	221828.0487	585195.8577
P2	6347285.0490	221853.3320	585176.5450
P3	6347273.5130	221854.7280	585161.5417
P4	6347284.4380	221866.2369	585180.2757
P5	6347288.8300	221879.4586	585158.0634
P6	6347287.7270	221896.3946	585147.2065
P7	6347284.9220	221917.5324	585147.8686
P8	6347286.2870	221795.4442	585186.7128
P9	6347289.6670	221872.3381	585095.6314
P10	6347290.5010	221879.0197	585106.8719

2.1. Methods

2.1.1. Multivariate Adaptive Regression Splines (MARS)

The MARS model uses a nonparametric modelling approach that does not require assumptions about the form of the relationship between the independent and dependent variables [37,60]. The MARS model works by dividing the ranges of the explanatory variables into regions and by producing for each of these regions a linear regression equations [60]. The breaks values between each regions are called knots, while the term basis functions (BFs) are used to demonstrate each distinct interval of the predictors [45,60]. BFs are functions of the following form as denoted by Equation (1):

$$\max(0, x - k) \text{ or } \max(0, k - x) \tag{6}$$

Where x is an independent variable and k is a constant corresponding to a knot [60]. The general formula for the MARS model is given by Equation (7) as denoted by [31]:

$$y = f(x) = a_0 + \sum_{n=1}^N \alpha_n \beta_n(x) \tag{7}$$

Where, y is the dependent variable predicted by the function $f(x)$, a_0 is a constant, and N is the number of terms, each of them formed by a coefficient α_n and $\beta_n(x)$ is an individual basis functions or a product of two or more BFs. The MARS model was developed in two steps. In the first step, the forward algorithm, basis functions are presented to define Equation (7). Many basis functions are added in Equation (7) in order to get a better result [45,60]. The developed MARS may experience overfitting due to the large number of basis functions. In order to mitigate this problem, the second step that is the backward algorithm prevents overfitting by removing redundant basis functions from Equation (7). The MARS model adopts Generalized Cross-Validation (GCV) to delete the redundant basis functions [61]. The expression of GCV is written as given by Equation (8) [62]:

$$GCV = \frac{\frac{1}{N} \sum_{i=1}^N [y_i - \hat{f}(x_i)]^2}{\left[1 - \frac{C(H)}{N}\right]^2} \tag{8}$$

Where N is the number of data and $C(H)$ is a complexity penalty that increases with the number of basis function (BFs) in the model and which is defined as denoted by Equation (9):

$$C(H) = (h + 1) + dH \tag{9}$$

Where d is a penalty for each BFs included into the model and H is the number of basis functions in Equation (7) [37,61].

2.1.2. Multiple Linear Regression (MLR)

The multiple linear regression (MLR) is an extensively used techniques in geoscientific studies for articulating the dependence of a response variable on several explanatory

variables [1]. It fits a linear combination of the components of multiple input parameters to a single output parameter defined by Equation (10) as:

$$y = \alpha_0 + \sum_{i=1}^M \beta_i x_i \quad (10)$$

Where α_0 is the intercept (values when all the independent variables are zero) with β_i values denoting the regression coefficients which were obtained in this present study using the least square technique. In Equation (10), i is an integer varying from 1 to M , where M is the total number of observations. Since there are several variables that can be used as candidates for predictor variables in the MLR model formulation, it would be demanding having to try every possible combination of variables [1].

2.1.3. Model Performance Assessment

In order to compare the MARS and MLR models results, the residuals calculated between the desired outputs and the outputs produced by the various techniques were utilized. Hence, to make an objective assessment of the models, performance criteria indices (PCI) of mean error (ME), mean square error (MSE), root mean square error (RMSE), mean bias error (MBE), standard deviation (SD), noise to signal ratio (NSR), correlation coefficient (R), correlation of determination (R^2) were adopted. Their mathematical representation is given by Equation (11) as:

$$ME = \frac{1}{n} \sum_{i=1}^n (\alpha_i - \beta_i) \quad (11)$$

$$MSE = \frac{1}{n} \sum_{i=1}^n (\alpha_i - \beta_i)^2 \quad (12)$$

$$RMSE = \sqrt{\sum \frac{E^2}{n}} \quad (13)$$

$$MBE = \sqrt{\sum \frac{E}{n}} \quad (14)$$

$$MAE = \sqrt{\sum \frac{|E|}{n}} \quad (15)$$

$$SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (e - \bar{e})^2} \quad (16)$$

$$SNR = \frac{\text{Average of the residuals}}{SD \text{ of the residuals}} \quad (17)$$

With reference to Equations (11) to (15), n is the total number of points, α_i and β_i are the measured variable and estimated variable produced by the models. α represent the residuals between the measured and estimated variables and $\bar{\alpha}$ is the mean value of the residuals. $||$ in Equation (15) represents the absolute value of the residuals estimated between α_i and β_i . The correlation coefficient (R) is given by Equation (18) as:

$$R = \left(\frac{\sum_{i=1}^N (\alpha_i - \bar{\alpha})(\beta_i - \bar{\beta})}{\sqrt{\sum_{i=1}^N (\alpha_i - \bar{\alpha})^2} \times \sqrt{\sum_{i=1}^N (\beta_i - \bar{\beta})^2}} \right) \quad (18)$$

Where N is the total number of test examples presented to the learning algorithms, α and β are the measured and predicted plane coordinates from the various procedures, while $\bar{\alpha}$ and $\bar{\beta}$ are the mean of the predicted plane coordinates. e in Equation (16) denotes the residual between the measured and predicted plane coordinates, and \bar{e} is the average of the residual. The R^2 was computed by Equation (19) as:

$$R^2 = \left(\frac{\sum_{i=1}^N (\alpha_i - \bar{\alpha})(\beta_i - \bar{\beta}_i)}{\sqrt{\sum_{i=1}^N (\alpha_i - \bar{\alpha})^2} \times \sqrt{\sum_{i=1}^N (\beta_i - \bar{\beta}_i)^2}} \right)^2 \quad (19)$$

3. Results and Discussions

The training and testing results for the two models are represented by Figure 2 to Figure 7, sample of the results is shown in Table 3 to Table 5 and their statistical analysis are tabulated in Table 6 and Table below. From the figures below, it was observed that, the performance of MLR was encouraging as compared to MARS. MLR achieved satisfactory results with high accuracy as compare to MARS which achieved good results but with less accuracy predictive performance.

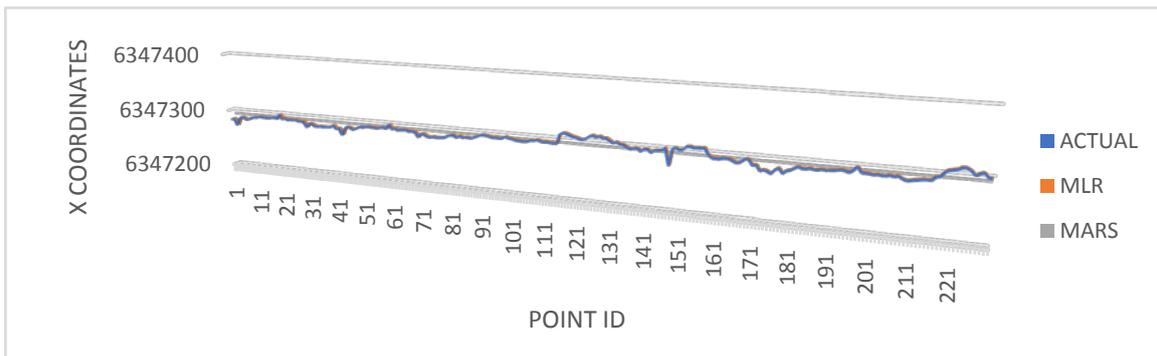


Figure 2. Training results by the two models

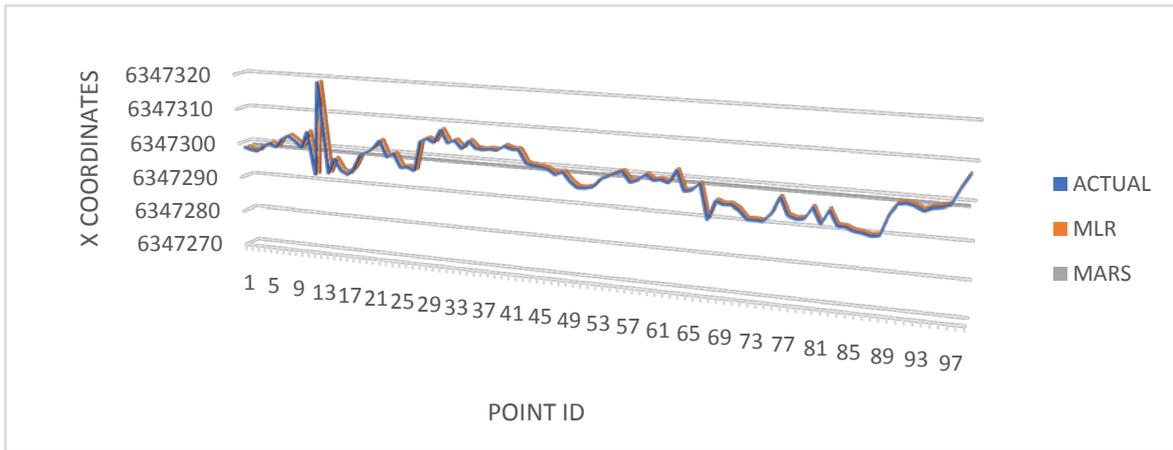


Figure 3. Testing results by the two models

Table 3. Sample of the results by the two models

ACTUAL	MLR	RESIDUAL	ACTUAL	MARS	RESIDUAL
6347283.044	6347283.050	-0.00607	6347283.044	6347293	-9.7013
6347285.049	6347285.050	-0.00536	6347285.049	6347293	-7.6963
6347273.513	6347273.520	-0.00667	6347273.513	6347293	-19.2323
6347284.438	6347284.440	-0.00556	6347284.438	6347293	-8.3073
6347288.830	6347288.830	-0.00400	6347288.830	6347293	-3.9153
6347287.727	6347287.730	-0.00461	6347287.727	6347293	-5.0183
6347284.922	6347284.930	-0.00494	6347284.922	6347293	-7.8233
6347286.287	6347286.290	-0.00600	6347286.287	6347293	-6.4583
6347289.667	6347289.670	-0.00414	6347289.667	6347293	-3.0783
6347290.501	6347290.510	-0.00408	6347290.501	6347293	-2.2443

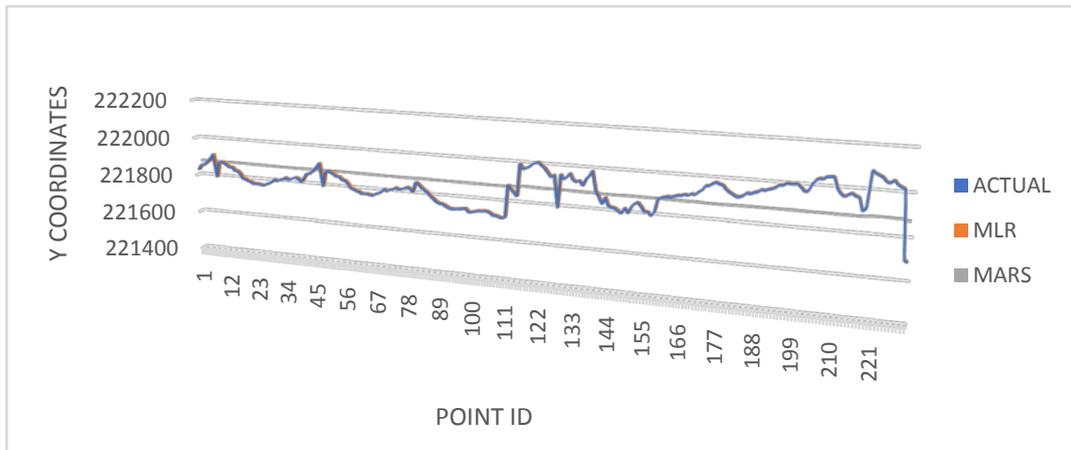


Figure 4. Training results by the two models

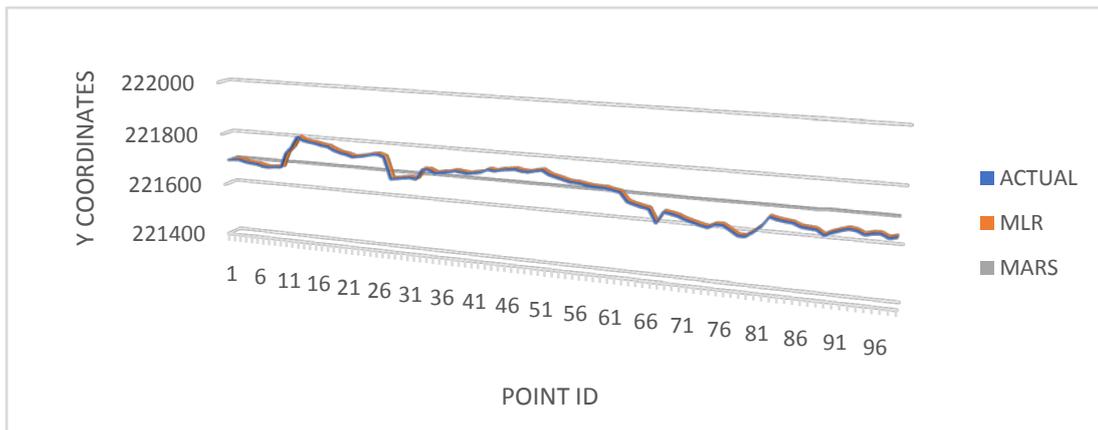


Figure 5. Testing results by the two models

From Figure 2 and Figure 3, it can be observed that MLR model has achieved better results in estimating the X coordinates as compared to the MARS model. 25 basis function was used in the formulation of the MARS model. The final equation that was obtained for the training and testing by the MARS model is given by Equation 20:

$$y(i) = a_0 + \sum_{n=1}^1 \alpha_i \beta_i. \quad (20)$$

The optimal equation developed for the training and testing by the MLR model is given by Equation 21:

$$y(i) = a_0 - (a_1 \times \phi_i) - (a_2 \times \lambda_i) + (a_3 \times h_i) - (a_4 \times N_i). \quad (21)$$

The results achieved by the two models in estimating Y coordinates is represented by Figure 4 and Figure 5. Sample of the results is tabulated in Table 4 below. From the results, it was observed that, the MLR achieved better results in estimating the Y coordinates as compared to the MARS model.

The results achieved by the two models in estimating the Z coordinates is represented by Figure 6 and Figure 7. Sample of their results is tabulated in Table 5 below. From the Figure 6 and Figure 7, it can be observed that MLR model achieved higher results in estimating the Z coordinates as compare to the MARS model which achieved satisfactory results but with less accuracy.

Table 4. Sample results of the two models

ACTUAL	MLR	RESIDUAL	ACTUAL	MARS	RESIDUAL
221828.049	221828.050	-0.00082	221828.049	221873.000	-44.985
221853.332	221853.333	-0.00083	221853.332	221873.400	-20.110
221854.728	221854.729	-0.00068	221854.728	221873.700	-19.005
221866.237	221866.238	-0.0090	221866.237	221873.400	-7.127
221879.459	221879.460	-0.00093	221879.459	221873.800	5.621
221896.395	221896.396	-0.00093	221896.395	221874.100	22.332
221917.532	221917.533	-0.00091	221917.532	221874.000	43.488
221795.444	221795.445	-0.00077	221795.444	221873.200	-77.785
221872.338	221872.339	-0.00085	221872.338	221875.100	-2.803
221879.020	221879.021	-0.00088	221879.020	221874.900	4.111

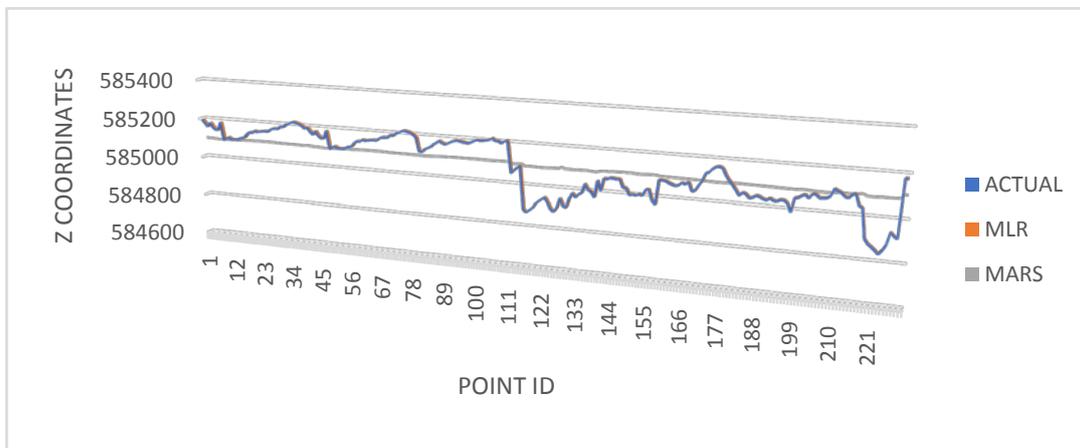


Figure 6. Training results obtained by the two models

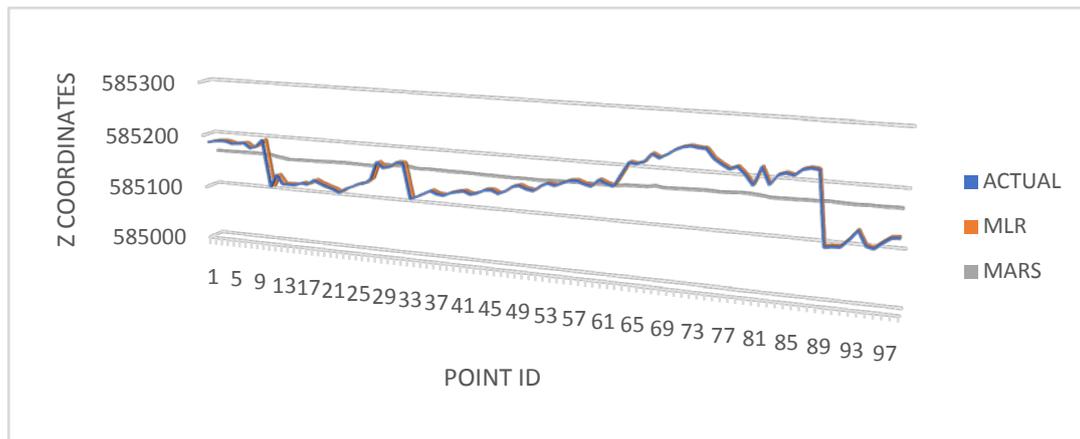


Figure 7. Testing results obtained by the two models

Table 5. Sample results by the two models

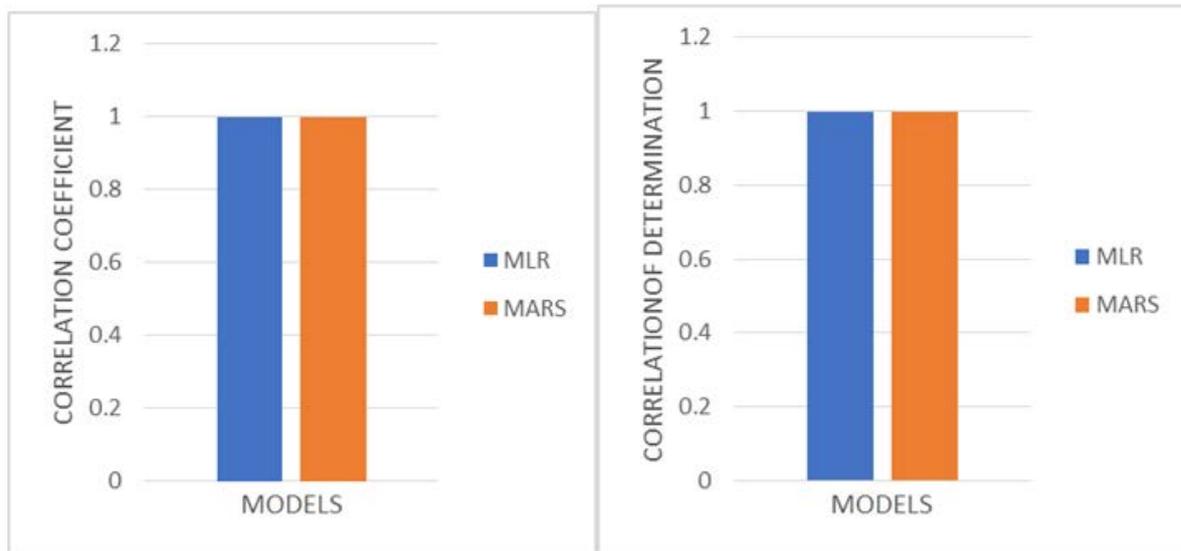
ACTUAL	MLR	RESIDUALS	ACTUAL	MARS	RESIUDAL
585195.8577	585195.8520	0.005745	585195.8577	585096.1000	99.7340
585176.5450	585176.5391	0.005860	585176.5450	585094.8000	81.7640
585161.5417	585161.5366	0.005063	585161.5417	585094.7000	66.8570
585180.2757	585180.2698	0.005856	585180.2757	585094.1000	86.1840
585158.0634	585158.0573	0.006052	585158.0634	585093.4000	64.6680
585147.2065	585147.2005	0.005986	585147.2065	585092.5000	54.7150
585147.8686	585147.8627	0.005876	585147.8686	585091.4000	56.5090
585186.7128	585186.7070	0.005791	585186.7128	585097.9000	88.8460
585095.6314	585095.6256	0.005799	585095.6314	585093.8000	1.8550
585106.8719	585106.8659	0.005983	585106.8719	585093.4000	13.4500

Table 6. Statistical results of the MLR model (Units in metres)

PCI	ME	MSE	RMSE	MAE	MBE	SD	NSR
X TRAIN	-0.00513	2.75E-05	0.005248	0.071624	0.071624	0.000959	5.35163
X TEST	-0.07045	0.004964	0.070458	0.265424	0.265424	0.000767	91.8605
Y TRAIN	-0.00091	8.40E-07	0.000917	0.030166	0.030166	0.0001	9.09749
Y TEST	-0.00567	3.22E-05	0.005678	0.075299	0.075299	0.000191	29.7381
Z TRAIN	0.005944	3.53E-05	0.005947	0.077098	0.077098	0.000162	36.62004
Z TEST	0.993016	0.986081	0.993016	0.996502	0.996502	0.00019	5220.215

Table 7. Statistical results by the MARS model (Units in metres)

PCI	ME	MSE	RMSE	MAE	MBE	SD	NSR
X TRAIN	-9.38E-08	55.63725	7.459038	0.000306	0.000306	7.479038	1.25E-08
X TEST	3.99E-09	23.69043	4.867281	6.31E-05	6.31E-05	4.892306	8.15E-10
Y TRAIN	-0.00057	8261.355	90.89199	0.023875	0.023875	79.71755	7.10E-06
Y TEST	-0.00013	2921.090	54.04711	0.011402	0.011402	54.32499	2.41E-06
Z TRAIN	-0.00052	7939.477	89.10374	0.0228035	0.022804	89.10374	5.81E-06
Z TEST	-0.00011	1651.373	40.63709	0.010488	0.010488	40.84602	2.78E-06

**Figure 8.** Correlation results

The statistical performance of both models is shown in Table 6 and Table 7. From the statistical findings, it was observed that MLR achieved higher performance in terms of accuracy.

The correlation coefficient and correlation of determination which shows the relationship between the dependent variables (output) and independent variables (input) is represented by Figure 8. From the Figure 8, it was observed that, there was a higher relationship between the dependent and independent variables in estimating the Cartesian coordinates.

4. Conclusions

Transformation of geodetic coordinates whose reference is the World Geodetic System 1984 (WGS84) into a local system have been a common practice in mathematical geodesy for solving major problems of astronomic, geodetic, cartographic, navigation and datum related issues. This procedure is generally carried out using the standard forward transformation equation given Equation (1) to Equation (3). However, existing knowledge has shown that, little or no alternative

techniques has been carried out to serve as a substitute for the standard forward transformation equation. The main contribution in this present study is to evaluate the working efficiency of MARS and MLR models as a realistic alternative technology for converting geodetic coordinates to Cartesian coordinates. The utilization of MARS and MLR for conversion of geodetic coordinates to cartesian coordinates have been presented in this study. The statistical findings revealed that the MLR and MARS offered satisfactory results in estimating Cartesian coordinates. However, MLR compared to MARS showed superior stability and more accurate results. It can be therefore be proposed that, the MLR should be used instead of the MARS within the study area in the forward conversion of geodetic coordinates to Cartesian coordinates. On the basis of the analysis achieved in this present study, it has demonstrated that geodetic longitude (λ), geodetic latitude (ϕ), geodetic height (h), radius of curvature (N) in the prime vertical and first eccentricity (e) squared, combined into MLR could produce accurate estimate of Cartesian coordinates. Therefore, this study does not only have a localized significance but will also open up more scientific discourse into the applications of MLR and MARS in mathematical geodesy within the geoscientific community.

Acknowledgements

The authors would like to thank the anonymous reviewers for their helpful comments and also to thank all Professors across the globe for their impartation of knowledge in all fields of study. Special thanks go to the staff members of the Department of Geomatic Engineering for providing us the required instruments used for the collection of data that was used to undertake this study.

References

- [1] Ziggah, Y. Y., Youjian, H., Yu, X., & Basommi, L. P., "Capability of Artificial Neural Network for forward Conversion of Geodetic Coordinates (Φ , λ , h) to Cartesian Coordinates (X , Y , Z)", *Math Geosci*, 48, 687-721, 2016.
- [2] Civicioglu, P., "Transforming geocentric Cartesian coordinates to geodetic coordinates by using differential search algorithm", *Comput Geosci*, 46, 229-247, 2012.
- [3] Ligas, M., and Banasik, P., "Conversion between Cartesian and geodetic coordinates on a rotational ellipsoid by solving a system of nonlinear equations", *Geod Cartogr*, 60(2), 145-159, 2011.
- [4] Shu, C., and Li, F., "An iterative algorithm to compute geodetic coordinates", *Comput Geosci*, 36, 1145-1149, 2010.
- [5] Zhu, J., "Conversion of earth-centered earth-fixed coordinates to geodetic coordinates", *IEEE Trans Aerosp Electron Syst*, 30(3), 957-961, 1994.
- [6] Vanicek, P., and Steeves, R. R., "Transformation of coordinates between two horizontal geodetic datums", *J Geod*, 70, 740-745, 1996.
- [7] Cai, G., Chen, B. M. and Lee, T. H., *Unmanned rotorcraft systems, Advances in industrial control*, Springer-Verlag London Limited, 2011, 23-34.
- [8] Andrei, O. C., "3D affine coordinate transformations", *Masters of Science Thesis in Geodesy*, No. 3091 TRITA-GIT EX 06-004, School of Architecture and the Built Environment, Royal Institute of Technology (KTH), 100 44 Stockholm, 2006, 1-4.
- [9] Tierra, A., Dalazoana, R., and De Freitas, S., "Using an artificial neural network to improve the transformation of coordinates between classical geodetic reference frames", *Comput Geosci*, 34, 181-189, 2008.
- [10] Ge, Y., Yuan, Y., and Jia, N., "More efficient methods among commonly used robust estimation methods for GPS coordinate transformation", *Surv Rev*, 45(330), 229-234, 2013.
- [11] Pan, G., Zhou, Y., Sun, H., and Guo, W., "Linear observation based total least squares", *Surv Rev*, 47(340), 18-27, 2015.
- [12] Solomon, M., "Determination of transformation parameters for Montserrado County, Republic of Liberia", *Masters Thesis*, Faculty of Civil and Geomatic Engineering, College of Engineering, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana, 2013.
- [13] Zeng, H.E., "Geodetic datum transformation and inverse transformation", *Appl Mech Mater*, 501(504), 2154-2157, 2014.
- [14] Zeng, H. E., "Analytical algorithm of weighted 3D datum transformation using the constraint of orthonormal matrix", *Earth Planets Space*, 67, 1-10, 2015.
- [15] Ziggah, Y. Y., Youjian, H., Odutola, A. C., and Fan, D.L., "Determination of GPS coordinate transformation parameters of geodetic data between reference datums: a case study of Ghana geodetic reference network", *Int J Eng Sci Res Technol*, 2(4), 2277-9655, 2013.
- [16] Annan, R. F., Ziggah, Y. Y., Ayer, J., Odutola, C. A., "A Hybridized Centroid Technique for 3D Molodensky-Badekas Coordinate Transformation in the Ghana Reference Network using Total Least Squares Approach", *South African Journal of Geomatics*, 5(3), 269-284, 2016.
- [17] Hoar, G. J., "Satellite surveying", *Magnavox Advanced Products and Systems Company*, 2829 Maricopa Street. Torrance, California, 1982, 1-3.
- [18] Leick, A., "GPS satellite surveying", Wiley, Hoboken, NJ, 2004, 1-5.
- [19] Schofield, W., "Engineering surveying: theory and examination problems for students", 5th edn. Butterworth-Heinemann, Linacre House, Jordan Hill, Oxford OX2 8DP, UK, 2001.
- [20] Sickle, J. V., "Basic GIS coordinates", 2nd edn. CRC Press, Taylor and Francis Group, New York, 2010.
- [21] Gullu, M., "Coordinate transformation by radial basis function neural network", *Sci Res Essays*, 5(20), 3141-3146, 2010.
- [22] Gullu, M., Yilmaz, M., Yilmaz, I., and Turgut, B., "Datum transformation by artificial neural networks for geographic information systems applications", *In: International symposium on environmental protection and planning: geographic information systems (GIS) and remote sensing (RS) applications (ISEPP)*, Izmir-Turkey, 2011, 13-19.
- [23] Lin, L. S., and Wang, Y. J., "A study on cadastral coordinate transformation using artificial neural network", *In: Proceedings of the 27th Asian conference on remote sensing*, Ulaanbaatar, Mongolia, 2006, 1-5.
- [24] Mihalache, R. M., "Coordinate transformation for integrating map information in the new geocentric European system using artificial neural networks", *GeoCAD*, 2012, 1-9.
- [25] Tierra, A. R., De Freitas, S. R. C., and Guevara, P. M., "Using an artificial neural network to transformation of coordinates from PSAD56 to SIRGAS95", *Geodetic reference frames, international association of geodesy symposia*, Springer, 134, 173-178, 2009.
- [26] Tierra, A., and Romero, R., "Planes coordinates transformation between PSAD56 to SIRGAS using a multilayer artificial neural network", *Geod Cartogr*, 63(2)199-209, 2014.
- [27] Turgut, B., "A back-propagation artificial neural network approach for three-dimensional coordinate transformation", *Sci Res Essays*, 5(21), 3330-3335, 2010.
- [28] Yilmaz, I., Gullu, M., "Georeferencing of historical maps using back propagation artificial neural network", *Exp Tech*, 36(5), 15-19, 2012.
- [29] Zaletnyik, P., "Coordinate transformation with neural networks and with polynomials in Hungary", *International Symposium on Modern Technologies, Education and Professional Practice in Geodesy and Related Fields*, Sofia, Bulgaria, 2004, 471-479.
- [30] Lee, T. S., and Chen, I. F., "A two-stage hybrid credit scoring model using artificial neural networks and multivariate adaptive regression splines", *Expert Syst Appl*, 28, 743-752, 2005.
- [31] Samui, P., "Multivariate Adaptive Regression Spline (MARS) for prediction of Elastic Modulus of jointed Rock Mass", *Geotech Geol Eng*, 31, 249-253, 2013.

- [32] Feltens, J., "Vector methods to compute azimuth, elevation, ellipsoidal normal, and the Cartesian (X, Y, Z) to geodetic (φ, λ, h) transformation", *J Geod*, 82(8), 493-504, 2007.
- [33] Feltens, J., "Vector method to compute the Cartesian (X, Y, Z) to geodetic (φ, λ, h) transformation on a triaxial ellipsoid", *J Geod*, 83(2), 129-137, 2009.
- [34] Fok, S. H., and Iz, H. B., "A comparative analysis of the performance of iterative and non-iterative solutions to the Cartesian to geodetic coordinate transformation", *J Geospatial Eng*, 5(2), 61-74, 2003.
- [35] Gerdan, G. P., and Deakin, R. E., "Transforming Cartesian Coordinates (X, Y, Z) to Geographic Coordinates (φ, λ, h)", *Aust Surv*, 44(1), 53-63, 1999.
- [36] Shu, C., and Li, F., "An iterative algorithm to compute geodetic coordinates", *Comput Geosci*, 36, 1145-1149, 2010.
- [37] Friedman, J. H., "Multivariate adaptive regression splines", *Annals Statistics*, 19, 1-67, 1991.
- [38] Leathwick, J. R., Rowe, D., Richardson, J., Elith, J., and Hastie, T., "Using multivariate adaptive regression splines to predict the distributions of New Zealand's freshwater diadromous fish", *Freshw Biol*, 50, 2034-2051, 2005.
- [39] Alreja, J., Parab, S., Mathur, S., and Samui, P., "Estimating hysteretic energy demand in steel moment resisting frames using Multivariate Adaptive Regression Spline and Least Square Support Vector Machine", *Ains Shams Engineering Journal*, 2015, 1-7.
- [40] Lall, U., Sangoyomi, T., Abarbanel, H. D. I., "Nonlinear dynamics of the Great Salt Lake: nonparametric short term forecasting", *Water Resour Res*, 32, 975-985, 1996.
- [41] Attoh-Okine, N. O., Mensah, S., Nawaiseh, M., "A new technique for using multivariate adaptive regression splines (MARS) in pavement roughness prediction", *Proc ICE Trans*, 156(1), 51-55, 2003.
- [42] Attoh-Okine, N. O., Cooger, K., Mensah, S., "Multivariate Adaptive Regression (MARS) and hinged hyperplanes (HHP) for doweled pavement performance modelling", *Constr Build Mater*, 23(9), 3020-3023, 2009.
- [43] Wang, L. J., Guo, M., Sawada, K., Lin, J., and Zhang, L., "Landslide Susceptibility mapping in Mizunami city, Japan: A comparison between logistic regression, bivariate statistical analysis and multivariate adaptive regression spline models", *Journal of Catena*, 135, 271-282, 2015.
- [44] Kisi, O., and Parmar, K. S., "Application of Least Square Support Vector Machine and Multivariate Adaptive Regression Spline Models in Long Term Prediction of River Water Pollution", *Journal of Hydrology*, 2015, 1-28, Accessed: February 10, 2017.
- [45] Samui, P. and Kim, D., "Modelling of reservoir-induced earthquakes: a multivariate adaptive regression spline", *Journal of Geophysics and Engineering*, 9, 494-497, 2012.
- [46] Laurin, G. V., Puletti, N., Chen, Q., Piermaria, C., Papale, D., and Valentini, R., "Above ground biomass and tree species richness estimation with airborne Lidar in tropical Ghana forest", *International Journal of applied Earth Observation and Geoinformation*, 52, 371-379, 2016.
- [47] Durmaz, M. and Karslioglu, M. O., "Non-parametric regional VTEC modelling with Multivariate Adaptive Regression B-Splines", *Advances in Space Research*, 48, 1523-1530, 2011.
- [48] Durmaz, M., Karslioglu, M. O., and Nohutcu, M., "Regional VTEC modelling with multivariate adaptive regression splines", *Advances in Space Research*, 46, 180-189, 2010.
- [49] Chen, M., Tompson, A. F. B., Mellors, R. S., Ramirez, A. L., Dyer, K. M., Yang, X., and Wagoner, J. L., "An efficient Bayesian inversion of a geothermal prospect using a multivariate adaptive regression spline method", *Journal of Applied Energy*, 136, 619-627, 2014.
- [50] Dawod, G. M., Mirza, N. M., Al-Ghamdi, A. K., "Simple precise coordinate transformations for geomatics applications in Makkah metropolitan area, Saudi-Arabia", *Bridging the gap between cultures FIG working week*, Marrakech Morocco, 2010, 18-22.
- [51] Featherstone, W. E., "A comparison of existing co-ordinate transformation models and parameters in Australia", *Cartogr*, 26(1), 13-26, 1997.
- [52] Ziggah, Y. Y., Hu, Y., and Odutola, A. C., "Regression models for 2-dimensional Cartesian coordinates prediction: a case study at University of Mines and Technology (UMaT), Tarkwa-Ghana", *Int J Comput Sci Eng Surv*, 3(6), 61-79, 2012.
- [53] Odutola, A. C., Beiping, W., and Ziggah, Y. Y., "Testing simple regression model for coordinate transformation by comparing its predictive result for two regions", *Acad Res Int*, 4(6), 540-550, 2013.
- [54] Forson, K. I., "Design of distribution network for University of Mines and Technology", *Unpublished BSc Project Report*, University of Mines and Technology, Tarkwa, Ghana, 2006, 1-10.
- [55] Seidu, M., "GIS as a Tool in Water Monitoring for Public Health and Safety Management", *Unpublished BSc Report*, University of Mines and Technology (UMaT), Tarkwa, Ghana, 2004, pp. 1-10.
- [56] Dreiseitl, S., and Ohno-Machado, L., "Logistic regression and artificial neural network classification models: a methodology review", *J Biomed Inf*, 35(5-6), 352-359, 2002.
- [57] Ismail, S., Shabri, A., and Samsudin, R., "A hybrid model of self-organizing maps and least square support vector machine for river flow forecasting", *Hydrol Earth Syst Sci*, 16, 4417-4433, 2012.
- [58] Yakubu, I., and Kumi-Boateng, B., "Control position fix using single frequency global positioning system receiver technique-a case study", *Res J Environ Earth Sci*, 3(1), 32-37, 2011.
- [59] Konaté, A. A., Pan, H., Khan, N., and Ziggah, Y. Y., "Prediction of porosity in crystalline rocks using artificial neural networks: an example from the Chinese continental scientific drilling main hole", *Stud Geophys Geod*, 59(1), 113-136, 2015.
- [60] Zabihi, M., Pourghasemi, H. R., Pourtjhi, Z. S., and Behzadfar, M., "GIS-based multivariate adaptive regression spline and random forest models for groundwater potential mapping in Iran", *Environ Earth Sci*, 75(665), 646-665, 2016.
- [61] Samui, P. and Kothari, D. P., "A Multivariate Adaptive Regression Spline Approach for Prediction of Maximum Shear Modulus (G_{max}) and Minimum Damping Ratio (ξ_{min})", *Engineering Journal*, 16(5), 1-10, 2012.
- [62] Craven, P. and Wahba, G., "Smoothing noisy data with spline function: estimating the correct degree of smoothing by the method of generalized cross-validation", *Numer Math*, 31, 317-403, 1979.