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Empirical Modelling of Refraction Error in Trigonometric Heighting Using Meteorological Parameters

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Abstract Refraction is a complex problem in terrestrial optical measurement and can be regarded as a major source of systematic error in the precise determination of height differences using trigonometric heighting. This paper deals with the development of an empirical model to estimate vertical refraction corrections from meteorological measurements gathered by freely available meteorological sensors. The proposed methodology can produce more realistic local estimates for the refraction coefficient than the typically used single generic value. Along with presentation of the proposed method, this study also presents experimental data to illustrate that the produced results are comparable to those obtained by surveying observations.

Keywords: *refraction coefficient, trigonometric heighting, meteorological parameters, empirical model*

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1. Introduction

In surveying, height differences are required for a wide range of measuring applications. Trigonometric heighting is a well established surveying technique for the determination of height differences. The method is however impaired by refraction effects. Refraction is detrimental to terrestrial optical measurements and can be regarded as major source of systematic error in the precise determination of distances and directions [1].

The refraction coefficient k represents a common method for quantifying refraction in surveying measurements. It is defined as the ratio of the radius of the Earth R (spheroid) to the radius of the line of sight (e.g., [2,3]). The physical model of the refraction coefficient depends on the wavelength of light, the meteorological parameters of the atmosphere's ground layer along the whole line of sight at the moment of light ray transition, and furthermore, the line of sight's distance and slope. For a detailed description about the theoretical aspects of the refraction coefficient, the interested reader is referred to [4,5,6].

Based on [2], the refraction coefficient varies between -5 to 15 depending on the height above sea level and micrometeorological conditions. However, for precise optical measurements through the atmosphere, the determination of k is crucial and remains a long-term topic of research in engineering geodesy. Notable studies are reported using reciprocal sights (e.g. [7-12]) or vertical temperature gradient measurements to focus on the short-term variations of the refraction coefficient (e.g. [13,14,15]). Because the determination of temperature gradient measurements along the line of sight is not practicable, other approaches to eliminate the impact of refraction have been developed. For example, [16] suggest statistical analysis of large amounts of data from one-sided zenith angle and distance measurements , combined with meteorological data. It is noted that, in this study, the meteorological data (temperature, relative humidity and air pressure) were measured with portable temperature measuring devices. Whilst these devices have no comparable accuracy with proper weather stations their use is particularly simple and can reportedly lead to reliable results when there is a high redundancy of observations.

Because of the complex process required to calculate an exact value for the refraction coefficient in every surveying task, it has been suggested, for simplicity, that specific values that approximate the average state of the atmosphere be applied. Such values include the global k = 0.13 for all latitudes and for all seasons. This specific value was introduced by C.F. Gauss for arc measurements in Hanover in 1823 and has been applied extensively ever since ([4,6]).

The above value for k is not representative of the Greek region as the method was developed from observations made in parts of Central Europe. For example, in Greece, the value of k = 0.16 is routinely implemented by surveyors (e.g. [17,18,19]). Specifically, [19] obtained values of the refraction coefficient between +0.12 to +0.20 from vertical angle measurements between two hills in Greece, indicating fairly small variability of k in regions well above sea level. The three above reported works agree in that the minimum value of k in Greece is usually recorded at the close to 13.00 hours. However, surveyors are not always aware of the risks involved in simplistically assuming an average value of k=0.16 for all regions of Greece. For example, whilst for short instrument-target distances up to 100m, application of k between 0.16 and 0.25 will not influence corrections to the vertical angle, this approximation will become more significant when the distance increases.

Inspired by the approach of [16], this paper proposes a method to derive an empirical model of the refraction coefficient as a function of meteorological parameters. The study is performed in a specific region of Greece (wider area of Athens). A significant aspect of the work is the fact that recent meteorological data are used to locally model the refraction coefficient. A further significant aspect is the simplicity of the proposed method because the meteorological data required can be freely acquired from national research organisations (in this work meteorological data from the Athens National Observatory were used). The main outcome of this work is the development of more suitable local values of the refraction coefficient for corrections to acquired angle measurements. It is important to note that whilst this work uses data from a specific region of Greece, the proposed method can be further improved and implemented in any other area globally so that local values of the refraction coefficient are estimated.

2. Fundamentals

The basic equation that estimates of the refraction coefficient k when the height difference ΔH between two points is known is [4]:

$$k=1+\frac{2R}{D}\times\left(\frac{1}{\tan z}\times\frac{\Delta H}{D}\right)+\left(i-j\right)\times\frac{2R}{D^{2}} \qquad (1)$$

where, D is the spheroidal distance , z is the measured zenith angle, R is the radius of earth, and i, j are the instrument height and target height respectively. In Eq. 1, parameters D and ΔH must be known to a high accuracy to ensure confident estimation of k.

The refraction coefficient of a particular point is also a function of the measured vertical temperature-gradient dT/dH (in degrees Kelvin/m), the pressure P (in mmHg), and the temperature T (in degrees Kelvin), as expressed by the well-known simplified equation (e.g., [20,21,22]):

$$k=670.87\times\frac{P}{T^{2}}\times\left(0.034+\frac{dT}{dH}\right)\times\sin z \qquad (2).$$

Here, z is the zenith angle between two measuring points ([23,24,25,26]). In Eq. (2) the temperature gradient dT/dH is the most difficult parameter to estimate. In comparison with Eq. 1, Eq. (2) provides less accurate results for estimation of k but its application is clearly preferable when there is a lack of other information or when is not possible to make angular measurements in the field for testing.

This is important in many surveying projects when neither geometric levelling nor reciprocal trigonometric heighting methods are appropriate. Applications may include the observation of targets on near vertical high dam walls from a number of control stations in a one-way trigonometrical mode, with no possibility of reciprocal observations, or observations to tall buildings, chimneys and other inaccessible structures.

3. Experimental Procedure

3.1. Field Data

For the practical field experiments, a TS30 total station by Leica Geosystems was used (http://hds.leica-geosystems.com). The specific instrument has motorization and automatic target recognition at an accuracy of 1". The accuracy of the vertical angle measurements is quoted as 0.5". The telescope has a magnification of 30X and a focusing range from 1.7m to infinity. In addition, a weather monitoring system Kestrel meter 4250 tracker (http://kestrelmeters.com) was used for the field tests and was located next to the surveying instrument. The specific weather system was used to collect data for temperature (accuracy ±0.5C), relative humidity (accuracy 3%), and barometric pressure (accuracy ±1.0hPa/mbar).

A number of field tests were performed involving trigonometric measurements of one-sided and two-sided zenith angles using two different baselines in the wider area of Athens. Two baseline lengths were chosen since the refraction error is a function of the square of the sight length. The baseline locations were selected in order to enable good visibility between the two stations of the baselines, easy access and to ensure that the instrument beam path radius travelled over land only.

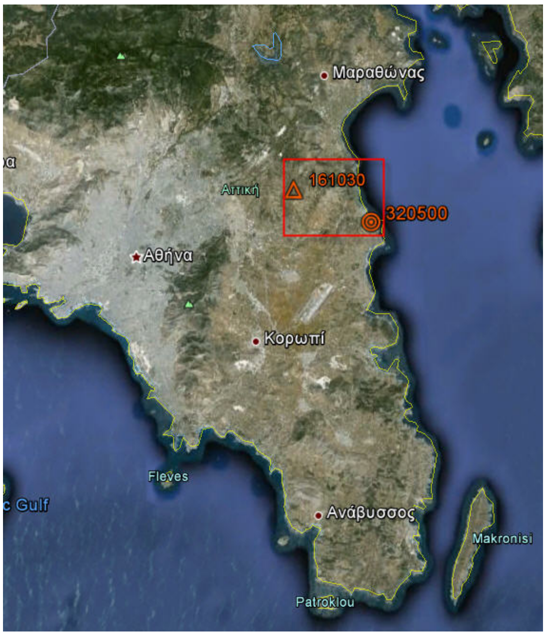

Figure 1. Location of baseline 1 in the wider area of Athens (Google)

The first baseline comprises two stations named as 161030 and 322500 (Figure 1) and is located in the eastern part of Attica. Station 161030 is a trigonometric pillar of 2nd order belonging to the national Greek network. Station 322500 is a point established with static GNSS surveying for the purposes of this work. Using GNSS measurements collected by Topcon HyperPro GNSS receivers, the spheroidal distance between the two stations was computed as D = 8582.705m ± 4mm and the orthometric height difference between the two points was ΔH = 645.166m ± 6mm. In this baseline, single-sided vertical angle observations only were collected.

The second baseline is of shorter length and comprises two stations established at the campus of the Technological Educational Institute, in the western part of Attica. The distance in the order of 500m and reciprocal vertical angle observations were collected. The area was moderately flat with a general inclination of less than 5 degrees.

The vertical angle measurements were performed in two telescope positions (dual face observations) and for two periods in order to reduce the impact of instrumental zero offset variations with time (e.g. [3,27]). Each full vertical angle measurement was repeated every 5 minutes. Table 1 provides the details of the vertical angle measurement experiments for both baselines.

The meteorological data (i.e., temperature, pressure, relative humidity) were collected from two sources; using a Kestrel meter weather system and from the data provided by the nearest permanent weather stations of the extensive meteorological network of the National Athens Observatory (www.noa.gr). The majority of the stations are equipped with Davis sensors (www.davisnet.com).

Table 1. Details of the experimental tests

Date	Temp (°C)	Start local time (h)	End local time (h)	Net time (h)	Collected epochs
Baseline 1 (stations 161030 and 322500) single-sided observations					
25/01/2013 Mostly cloudy, humid	8-12	8.45	10.00	1.15	14
26/01/20013 Mostly cloudy, average visibility and rain	8-12	8.00	11.00	3.0	15
27/01/2013 Sunny, excellent visibility	8-11	7.50	14.40	6.9	97
Baseline 2 (campus points) reciprocal observations					
21/05/2015 Sunny, good visibility	22-25	11.15	13.40	2.25	27

3.2. Data Processing

Based on Eq. (1) and (2), k was computed using surveying data (vertical angle measurements) and meteorological data (temperature gradient) acquired each day from the portable meteorological sensor. Figure 2 shows an example of the variations experienced in the k values between the two computations, for baseline 1 on day 3 (cf. Table 1).

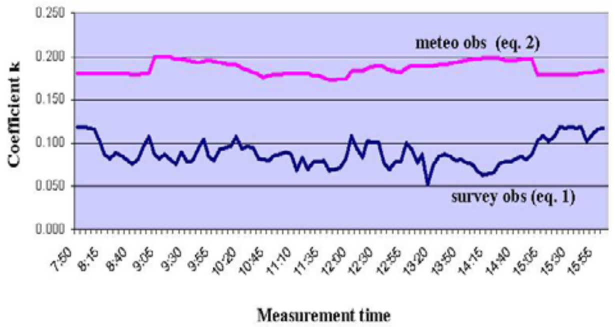

Figure 2. Estimation of coefficient k from survey oobservations (Eq. 1) and meteorological data (Eq. 2) (baseline 1, day 3)

In Figure 2 it is clear that the k values calculated from angle sightings differ greatly from those calculated from the meteorological data. The estimation of k using the equation of the temperature gradient does not give satisfactory results. The average values obtained by the two approaches vary significantly (i.e. $k_{average}$ (single sightings) = 0.089 vs. $k_{average}$ (temp. gradient) = 0.186). It should be noted that during angle observations on long baselines, atmospheric fluctuations can alter the image of the target as seen through the telescope, thus affecting the estimation of k. Also, it was noticed that for all three days of the experiments, the temperature gradient method results in estimated values of k with no considerable "sensitivity". Thus, the values of k remain largely stable despite fluctuations in the values of the meteorological parameters.

Further to the above, a correlation analysis was performed between the estimated k and the temperature (T), pressure (P) and relative humidity (RH). The correlation analysis referred only to the estimated coefficient using the survey observations method (cf. Eq. 1) because the method of meteorological data contains the above parameters as independent variables (cf. Eq. 2). The results of the correlation analysis for the three observation days gave very low values for the correlation coefficient r equal to 0.12, 0.12 and 0.01 for (k, T), (k, P) and (k, RH) respectively, indicating practically any absence of correlation.

3.3. Proposed Model

The above results indicate that is there is indeed a difficulty in directly estimating k from Eq. (2). Therefore, an effort was made to establish a mathematical relationship between k and the meteorological parameters. For this, a data fit approach was implemented; the parameters included being pressure (P), temperature (T) and relative humidity (RH %). The RH is used instead of the vertical temperature gradient (dT/dH). Measuring the temperature gradient along a line is neither practical nor economical in routine surveying applications [25]. The dependent variable, i.e. the associated values of k, was known from the surveying observations described in section 3.2.

The construction of the new equations described below was based on multiple regression estimation using the SPSS Statistics v.17.0 software package. A large number of multiple regression trials were run in order to optimally adapt the variables to the collected data. The validity of the proposed equations was checked using cross validation.

Prior to running the multiple regression analysis, the input variables were checked to follow certain criteria such as being continuous, independent, showing homoscedasticity, with no significant outliers or highly influential points and

the residuals of the random errors being normally distributed. The final selection of the derived mathematical equations shown in equations 3 - 6 was based on their relevant statistics that measure the quality of the prediction of k. Besides the linear relationship (Eq. 3), there was also an effort to define alternative models in which the variables are related non-linearly. Parameters for equations of polynomial, logarithmic, and exponential type were derived as follows:
linear:

$$k = -0.0077P - 0.0066T - 0.0024RH + 6.2414 \qquad (3)$$

polynomial 2nd order:

$$k = 0.0004\left(P + 0.86T + 0.32T\right)^2 \\ -0.7182\ \left(P + 0.86T + 0.32T\right) + 287.46 \qquad (4)$$

logarithmic:

$$k = -6.1343\ln\left(P + 0.86T + 0.32\ RH\right) + 41.049 \qquad (5)$$

exponential:

$$k = 2^{10}21e^{-0.0646(P+0.86T+0.32RH)}. \qquad (6)$$

The associated multiple *correlation coefficients* R were 0.87 (linear), 0.92 (logarithmic) and 0.80 (exponential). The residual standard deviations were 0.028 (linear), 0.023 (logarithmic) and 0.073 (exponential). The polynomial model (Eq. 4) produced unacceptable results with the coefficient k having erroneous values of about -30 and thus was excluded from any further analysis.

The next stage involved a value calculation for the k using Equations (3), (5), (6). For this process, an independent set of meteorological data was used, obtained from the nearest (to survey station 322500) weather station of the meteorological network of the National Athens Observatory.

Figure 3 shows an example of a comparison made between the calculated values for k using five different methods (day 1, cf. Table 1). Similar results were obtained from all the other days of measurements. It is seen that all three approaches k (log), k (exp) and k (linear) better follow k derived from the survey data than those calculated from the method of temperature gradient k (meteo). The RMS of the differences between the estimated values of k from the proposed equations and the k values from the survey data are 0.0267 (linear), 0.0259 (logarithmic) and 0.0278 (exponential).

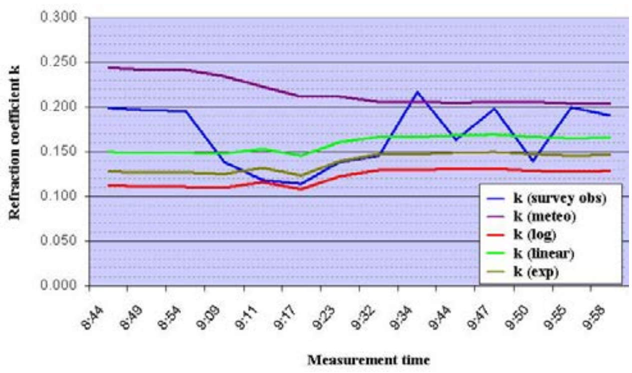

Figure 3. Computed refraction coefficient k with proposed models versus measured from survey data

From the above, it is seen that the logarithmic approach as described in Eq. (5) produced marginally better results than the others. It is interesting to note that this is in agreement with definitive literature. For example, [28] discuss the approach of Lallemand who, in 1896, suggested a logarithmic function to solve the refraction error problem in geodetic levelling. Reference [24] also suggested an exponential model for refraction modelling. The same authors pointed out that the polynomials of greater than second degree failed to work properly, which again corroborates with the results of this study.

3.4. Verification of Models

In order to assess the results obtained with the derived logarithmic model (cf. Eq. 5), a number of comparisons were made using (a) reciprocal zenith angle observations from an independent baseline and (b) meteorological data from various stations distributed in the region of Attica.

The first part of the assessment used reciprocal zenith angle observations. These observations provide the most reliable method for estimating k. The derived values of k were compared to the values of k produced by the logarithmic model (Eq. 5) using meteorological data from the Kestrel meter located next to the total station instrument at baseline 2. The measurements took place on 21st May 2015 on a relatively clear and dry day and the measured baseline distance was about 0.5km (cf. Table 1). The reciprocal observations give a mean of k = 0.061 and a standard deviation of 0.011 whilst the use of the logarithmic model gives a mean of 0.063 and a standard deviation of 0.003 (Figure 4).

Figure 4. Refraction ccoefficient k estimated by the logarithmic model and reciprocal zenith observations (data of 21/05/2015, local time 11.00 - 14.00hrs)

The visual agreement between the two curves is also verified by the fact that their residuals are generally small indicating a reasonably good agreement between the two computations of k. Specifically, their average difference is 0.0015 with a standard deviation of 0.01.

Considering that the logarithmic model using the Kestrel meteorological data produced k values reasonably close to the values obtained by reciprocal observations, it is suggested in this work to create a daily model of k whereby the interested user can interpolate k values for a specific location in the map. These values can be created using Eq. (5) and meteorological values acquired from a network of permanent weather stations in the surrounding area. Specifically, there are 53 meteorological permanent stations of Attica established by the Athens National Observatory network and other parties (Figure 5).

Figure 5. Map of permanent meteorological stations in the region of Attica (www.metar.gr)

From data obtained by the above network, a contour model for k has been created using Kriging gridding methods. Figure 6 gives an example of the refraction k contour map of the north-west area of Attica that was produced using the meteorological data from six stations and the logarithmic model (Eq. 5) between 7.30 to 14.00 hrs local time. The map depicted in Figure 6 includes the area of the experiment and a small part of the sea (shown by the isolated "bull's eye" feature). It is noted that the selected weather stations are on elevations up to 100m above mean sea level. The data interval is every 5sec and thus the map has been created with averages from a large sample of data. The contour map has been produced for 21 May 2015 in order to have a comparison with the "true" values (cf. Figure 4). The two axes (X, Y) of the map refer to the planar coordinate system of Greece (i.e. Hellenic Geodetic Reference System of 1987 using a transverse mercator projection). Taking into account from the analysis of Figure 4 that the estimates of k using Eq. (5) are fairly acceptable, the user can achieve a level of precision for the refraction coefficient k between 0.01 and 0.03 which is sufficient for most practical surveying applications.

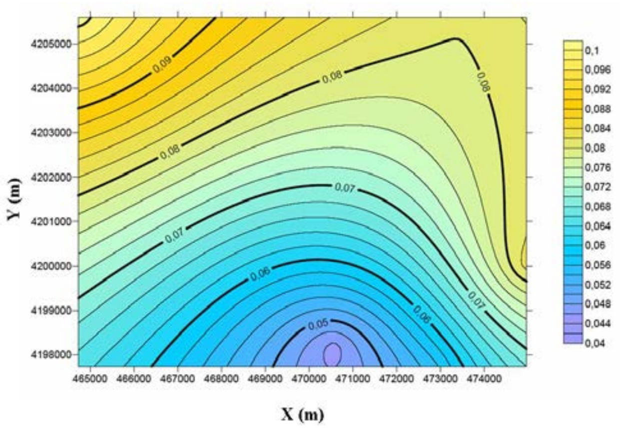

Figure 6. Contour map of refraction coefficient k derived from meteorological data using the logarithmic model

Figure 7 shows graphically the comparison between the values of k derived from the Kestrel weather station and the contour map from about 7.00- 9.30 hrs local time. It is seen that there is a good agreement between the Kestrel derived values of k with the interpolated from the map derived values of k. The interpolated values are slightly more fluctuating because they are derived from a large number of meteorological stations. It is interesting to note that the linear model as described with Eq. (3) gave better results than the logarithmic function using the Kestrel data. Using correlation analysis, a correlation coefficient between the two close estimates of k was computed equal to 0.882 which indicates a strong correlation.

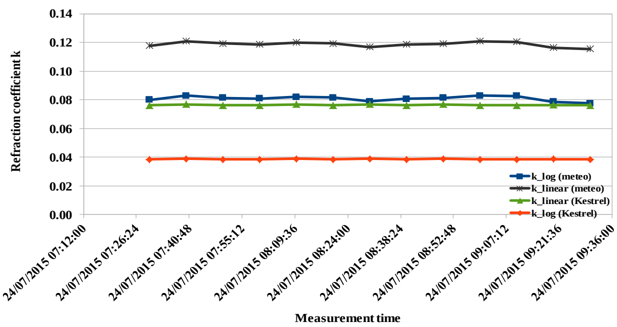

Figure 7. Refraction coefficient k derived from Kestrel data vs. interpolated values from contour map

In order to illustrate the effect of k on vertical angle measurements, the following table (Table 2) is included. The table gives the correction values of the measured vertical angle (in cc), for given distances and different coefficient values. The coefficient value of 0.16 is the one used uniquely across Greece in practical surveying, the value of 0.08 is the mean value as derived from the refraction map for Attica and 0.07 is the mean value of k as derived from the Kestrel data. The calculations below assume a mean radius of Earth equal to 6371km. It is seen that even for short distances the value of 0.16 as used by surveyors imposes about 50% more vertical angle correction than is actually required in heighting calculations. This is as expected since the refraction error is a function of the square of the sight length.

Table 2. Algebraic correction of vertical angle due to refraction (in cc)

	S (km)				
K	**0.1**	**0.5**	**1**	**2**	**5**
0.07	0. 34	1.7	3.49	7.01	18.00
0.08	0.40	2.01	4.01	8.02	20.00
0.16	0.80	4.01	8.02	16.00	40.00

4. Discussion

The first part of the analysis referred to the comparison between the refraction coefficient k being calculated from single-sighted vertical angle observations and the coefficient estimated by the equation of the temperature gradient. This process led to the conclusion that there is quite a considerable variation between the two methods. The estimated coefficient was systematically overestimated and the value range was much smaller than in reality. When k was calculated by the equation of the temperature gradient, there was very little "sensitivity", despite marked fluctuations in atmospheric conditions.

This led to an effort to investigate the change in the rate of refraction compared to the changes that occur in atmospheric conditions. Based on the calculated correlation coefficients, the refraction coefficient showed poor correlation between the atmospheric temperature, humidity and pressure.

The main goal of the study was to create a mathematical function that relates the coefficient of refraction to the variables of temperature, atmospheric pressure and relative humidity. Using multiple regression analysis, three linear, exponential and logarithmic equations were computed. Amongst the three, the logarithmic equation was selected because, when compared with the values of k derived by the survey data, it produced a marginally smaller difference and associated RMS compared to the other two.

The removal of the requirement of a vertical temperature gradient between the measurement points is the main difference in the derived equations. This is beneficial in many surveying applications. Unlike the equation of the temperature gradient that requires a priori knowledge of the height difference between points, thus making its use suitable only in limited cases, the derived equations are independent of this parameter. It was shown that the judicious removal of this parameter maintains the robustness of the estimated k values (cf. Figure 4).

Whilst the measurements that were used to develop the derived functions were taken in the eastern part of Attica, cross validation of the estimated k values was performed with actual sighting measurements in a different part of Attica, namely in western Attica. The comparison was satisfactory, demonstrating the generality of the derived logarithmic function and its implementation in areas with similar climatic characteristics, as in Attica. However, more data and further investigation is needed to optimise the number of meteorological stations being used for the calculation of the model map. Specific issues still exist regarding station elevations. In the results shown in this work, six meteorological stations of Attica were used to create the map of Figure 6 and were on a zone of elevations up to 100m above mean sea level.

Clearly, the strength of the proposed empirical refraction model lies in its directness and simplicity. For a surveying job where knowledge of the geodetic refraction coefficient is necessary, a good solution is provided by the proposed model where only the meteorological data (T,P,RH) are required along with the vertical angle observations.

Although this technique provides only an approximate estimation of the refraction coefficient in terms of easily accessible parameters, it still allows correction of the bulk of the refraction effect in single observations. It is important to bear in mind that determination of k by means of a model may turn out to be somewhat inaccurate, but still remains better than the blind use of a universal k of 0.16 (for Greece) or 0.13 (for other parts of Europe).

5. Conclusions

Despite the progress and continuous modernization of total stations, the crucial factor in the trigonometric measurement of elevations is the impact of vertical refraction. In this paper the aim was to model vertical refraction by means of a mathematical model expressed as a function of meteorological parameters and derived from data provided by a number of freely available sensors over a wider area.

The empirical modelling approach could prove particularly useful for height traverses in an engineering environment under changing weather conditions.. It is important to bear in mind that determination of the refraction coefficient by means of an empirical model may turn out to be somewhat inaccurate, but still remains better than the blind use of a single value of k for all the area of Greece. Clearly there is a need to use more data to verify and improve the proposed models for the specific area. Nevertheless, this paper lays the foundation for future development of a practical method. As a further extension of the refraction experiments, meteorological data such as wind speed, wind direction, and cloud cover could be automatically acquired from meteorological sensors. Then precise cloud cover observations could be related to the variability of the refraction coefficient and the proposed models could be improved. The response time of the refraction coefficient could also be investigated. Finally, instead of using simple data fitting as in this work, approaches such as neural network computing may be implemented.

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