

# Interrelationships of US Corn and Soybean Cash and Futures Markets with Storage Price Dependence: The Role of Futures Prices and Model Development

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**Abstract** This study develops a basic conceptual model of the link between supply, demand and international trade, storage and cash, future prices as well as basis (storage prices) of several popular crops, such as corn and soybean, grown in the United States. Corn and soybean are the most common crops in the United States and are the main source of feed grains for cattle, dairy, poultry and hog production. Basically, we provide a brief synopsis of the theoretical concepts that form the basis for the mathematical model of US corn and soybean sectors. The focus is on theories related to the conceptual framework of demand and supply response, as well as the theory of price-of-storage. The theoretical model of simultaneous equilibrium in cash, futures and storage markets is then developed. In particular, the role of futures price is of greater concern in model setting.

**JEL Classification:** F53, G13, Q02, Q11

**Keywords:** corn and soybean, storage, cash and futures markets, basis, rational expectations hypothesis

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## 1. Introduction

The interlinkage between futures trading and current production decisions of commodity producers facing price risk has been the subject of several ongoing investigations. The literature of Working [1,2], Stein [3], Peck [4], Sarris [5], Lapan and Moschini [6], Stoll and Whaley [7], Brunetti and Reiffen [8], Haase, Zimmermann and Zimmermann [9], Xu [10] and Xu [11] asserted that the issues of how production and futures trade are related to prevailing contract prices, the subjective distributions of the future cash price, futures price, basis and storage costs are the main focus. The difference between the current cash price and the futures price, i.e. basis, reflects the factors that influence the supply and demand for storage of these commodities. The reduced basis reflects the decline in storage costs as the delivery month approaches. Working's [1,2] studies on the theory of the price-of-storage highlighted the basic interdependence of the cash and futures markets. The theory of the price-of-storage is regarded as the basis ( $B_t = FP_t^{t+k} - P_t$ ) for the market to determine the storage price, whether it is positive or negative. It is postulated to be a direct function of current inventory levels. The theory of simultaneous determination of cash and futures prices and consumption and inventoried quantities can be derived from the

theory of hedging and speculation as well as theory of the price-of-storage.

This research deals with properties of a rational expectations equilibrium to provide the information about the expected price of the nearest futures contract price for future production based on when there is futures trading. A simple two-good model is analyzed in which the production of two commodities, corn and soybean, is affected by production risks. Hedging is an integral part of most futures market models. They are often considered to involve storage or production processes and attempt to avoid price risks associated with holding of underlying commodities through futures market transactions. Speculators challenge risk and make money by manipulating prices, rather than providing goods and services.

The model applied in this study is to allow simultaneous determination of quantity, cash price, futures price and basis. When we apply the futures price as the next spot price and price risk source, the effect of futures trading on the amount stored by the producer is the random disturbance affecting the demand. The price risk comes from the producer itself in the next period (rather than demand side), i.e., where the stochastic errors are disturbances in supply side. In fact, this research can be considered as an extension of the futures market and has been studied in relation to the spot market.

The rest of this study is organized as follows. The demand theory is discussed in Section 2. Theory of supply

response, especially futures price in supply analysis is described in Section 3. Theory and model of simultaneous equilibrium in the cash, futures and storage markets are contained in Section 4. Section 5 provides concluding remarks.

## 2. Demand Model

In the United States, most of the demand for soybeans comes from demand for soybean meal and soybean oil, which in turn comes from livestock and import and export needs of foreign sectors

### 2.1. Derived Demand

The term "derived demand" is used to denote a demand plan for the input of the final product. The demand for corn for raising livestock and the demand for cooking or salad oil for crushing into oil and soybeans for livestock feeding are examples of derived demand. The derivation of factor demand and its properties are well-documented throughout the literature [12,13].

Considering the demand for corn and soybean meal by the livestock industry, we assume that there is an implicit production relationship in which feed grains,  $Q_f$  are employed along with un-fattened feeder animals,  $D_{fr}$ , to produce fattened animals,  $A$ , and  $Z$  is other influencing factors, such that:

$$q(A, Q_f, D_{fr}, Z) = 0. \quad (1)$$

In this case, the livestock farmer's feed grain demands are derived from the underlying demand for the livestock which they produce. Assume that the producer maximizes his profits under the constraints of production relations and market conditions.

The optimization procedures for profit maximization by the farmer starts by assuming:

A profit function,

$$\pi = p_1 A - P_f Q_f - P_{fr} D_{fr} \quad (2)$$

Which is to be maximized subject to the production relationships,

$$q(A, Q_f, D_{fr}, Z) = 0. \quad (3)$$

And market demands for livestock:

$$A = h(P_1, P_{1s}, p_0, Z) \quad (4)$$

Where

$P_1$  = Price of slaughter animals,

$P_{1s}$  = Price of livestock product,

$P_0$  = Other prices affecting demand for livestock,

$P_f$  = Price of feed grains,

$P_{fr}$  = Price of feeder animals,

$Z$  = Other influencing factors.

The above problem can be reformulated by using the Lagrangian function [12,14] as follows:

$$\begin{aligned} & \text{Max} L(A, Q_f, D_{fr}, \gamma, \delta) \\ & = P_1 A - P_f Q_f - P_{fr} D_{fr} + \gamma [0 - q(A, Q_f, D_{fr}, Z)] \\ & + \delta [A - h(P_1, P_{1s}, P_0, Z)] \end{aligned} \quad (5)$$

Where  $\gamma$  is a Lagrangian multiplier and is equal to the marginal profit to the representative livestock farmer due to an exogenous shift in the production relationship.  $\delta$  is another Lagrangian multiplier and is equal to the marginal profit to the representative livestock farmer due to an exogenous shift in demand for livestock. The first order conditions for the maximization of Equation (5) are:

$$\frac{\partial L}{\partial A} = P_1 - \gamma \frac{\partial q}{\partial A} + \delta = 0 \quad (5a)$$

$$\frac{\partial L}{\partial Q_f} = P_f - \gamma \frac{\partial q}{\partial Q_f} = 0 \quad (5b)$$

$$\frac{\partial L}{\partial D_{fr}} = -P_{fr} - \gamma \frac{\partial q}{\partial D_{fr}} = 0 \quad (5c)$$

$$\frac{\partial L}{\partial \gamma} = g(A, Q_f, D_{fr}, Z) = 0 \quad (5d)$$

$$\frac{\partial L}{\partial \delta} = A - h(P_1, P_{1s}, P_0, Z) = 0. \quad (5e)$$

Solving the Equations (5a) - (5e) simultaneously gives the optimal quantities of  $A^*$ ,  $Q_f^*$ ,  $D_{fr}^*$ ,  $\gamma^*$  and  $\delta^*$ :

$$A^* = A(P_1, P_{1s}, P_{fr}, P_f, P_0, Z) \quad (6a)$$

$$Q_f^* = Q_f(P_1, P_{1s}, P_{fr}, P_f, P_0, Z) \quad (6b)$$

$$D_{fr}^* = D_{fr}(P_1, P_{1s}, P_{fr}, P_f, P_0, Z) \quad (6c)$$

$$r^* = r(P_1, P_{1s}, P_{fr}, P_f, P_0, Z) \quad (6d)$$

$$\delta^* = (P_1, P_{1s}, P_{fr}, P_f, P_0, Z). \quad (6e)$$

Equation (6b) expresses the derived demand function for feed and provides the feed demand relationships for estimation purposes.

The market demand curve for feed grain is obtained by aggregating the individual livestock farmer's feed grain input demand curves. Hence, the derived demand curve for feed grains is  $Q_{fi}$  for any representative farmer  $i$ . The market derived demand curve  $Q_f^m$  is the summation of the demand curves derived by all farmers, i.e.,

$$Q_f^m = \sum_{i=1}^n Q_{fi}.$$

This curve shifts in response to changes in prices of other inputs as well as technological change. The derived demand function may include other possible shifters:

$$Q_f^m = f(P_1, P_{1p}, P_{fr}, P_f, \#LS, P_{oi}, T, Z) \quad (7)$$

where  $P_1$ ,  $P_{1p}$ ,  $P_{fr}$ ,  $P_f$ ,  $Z$  the same definition as described above,  $\#LS$  is the number of livestock,  $P_{oi}$  is the price of other feed input and  $T$  is technological change.

### 2.2. Import/Export Demand

The demand for U.S. exports is derived from the supply and demand conditions of the rest of the world (ROW). The traditional theory of import demand is based on the proposition that import and domestic goods are not perfect substitutes [15]. The quantity of imports purchased by any consumer in importing countries will depend on income,

the price of imports and the price of all other goods. The specification is analogous to conventional demand analysis given by the following equation:

$$M = f(P_m, P_0, Y) \quad (8)$$

Where

M = Average import demand

P<sub>m</sub> = Price of import good

P<sub>0</sub> = Price of all other goods

Y = Level of income in the importing countries

If imports and domestic goods are perfect substitutes for corn and soybeans, then under the assumption of perfect competition, the import demand for foreign goods is an excess demand, and prices are spatially competitive.

Worldwide equilibrium price and quantity determination for perfectly competitive goods can be illustrated with a generalization of a two-region market. Each region has a known supply and demand function and produces and consumes homogeneous products. The regions are separated but not isolated by known transfer costs. Given this knowledge, the problem is to determine the equilibrium levels of production, consumption and prices in each region and the equilibrium trade flows between regions.

In two-region model is reproduced, but cost of t<sub>12</sub>, is added to reflect the transportation costs. The effect of changes in transportation costs is to shift the excess demand schedule faced by importing countries. Equilibrium is accomplished by shifting ED<sub>2</sub> downward by t<sub>12</sub> since it represents an additional cost to ED<sub>2</sub>. Equilibrium is reached for region 2 at P'<sub>2</sub>, X'<sub>2</sub> and Y'<sub>2</sub> and for region 1 at P'<sub>1</sub>, X'<sub>1</sub>, Y'<sub>1</sub>. If the exchange rate effect is also considered, the only change in the equilibrium condition is:

$$P'_1 + t_{12} = eP'_2 \quad (9)$$

Where e is the exchange rate which is evaluated in terms of the unit of the exporter's currency per unit of the importer's currency. This price relationship will be discussed in more detail in the following section. Based on the above theoretical framework, the spatial equilibrium market model can be constructed.

Exporting Country

Supply

$$S_i = S_i(P_i, X) \quad (10a)$$

Demand

$$D_i = D_i(P_i, X) \quad (10b)$$

Excess Supply

$$ES_i = S_i(p_i, X) - D_i(p_i, X) \quad (10c)$$

$$= ES_i(p_i, X)$$

Importing Country

Supply

$$S_j = S_j(P_j, X) \quad (10d)$$

Demand

$$D_j = D_j(P_j, X) \quad (10e)$$

Excess Demand

$$ED_j = D_j(p_j, X) - S_j(p, X) \quad (10f)$$

$$= ED_j(p_j, X)$$

Price Linkage

$$P_i + t_{ij} = eP_j \text{ or } \frac{P_i + t_{ij}}{e} = P_j \quad (10g)$$

Spatial Equilibrium

$$\sum_i ES_i = \sum_j ES_j \quad (10h)$$

Where

S<sub>i</sub> = Supply of corn (soybeans) in exporting country i,

D<sub>i</sub> = Demand of corn (soybeans) in exporting country i,

P<sub>i</sub> = Price of corn (soybeans) which related to demand and supply in country i,

ES<sub>i</sub> = Excess supply of corn (soybeans) in country i,

S<sub>j</sub> = Supply of corn (soybeans) in importing country j,

D<sub>j</sub> = Demand of corn (soybeans) in importing country j,

P<sub>j</sub> = Price of corn (soybeans) which related to demand and supply in country j,

ED<sub>i</sub> = Exchange demand of corn (soybeans) in country j,

e = Exchange rate evaluated in terms of the units of the exporter's currency per unit of the importer's currency,

t<sub>ij</sub> = Transportation costs between country i and j,

X = Other exogenous variables.

In the above spatial model, the equilibrium occurs where  $\sum_j ED_j = \sum_i ES_i$ .

The Price-linkage between countries i and through transportation costs (t<sub>ij</sub>) and the exchange rate (e) as shown in equation (10g). Trade theory suggests that importers view international prices in terms of their own currency. This captures the exchange rate effect. The market model consists of eight equations, (10a) through (10h), and eight endogenous variables, S<sub>i</sub>, D<sub>i</sub>, S<sub>j</sub>, D<sub>j</sub>, ES<sub>i</sub>, ED<sub>j</sub>, P<sub>i</sub> and P<sub>j</sub>. Substitutions can be made to obtain an expression for the import (excess) demand equation:

$$ED_j = f\left(\frac{P_i + t_{ij}}{e}, X\right) \quad (11)$$

Excess demand for corn (soybeans) is a demand derived from the importing country's derived demand for corn (soy-beans). Thus, import demand for corn (soybeans) is affected by the price of livestock P<sub>l</sub>, the price of substitute feed P<sub>lf</sub>, the number of livestock #LS<sub>j</sub>, and income levels Y<sub>j</sub>, in the importing country j. Corn (soybean) supply in exporting countries Q<sub>1</sub> and other competing supply of corn (soybeans) S<sub>0</sub> would also affect U.S. corn (soybean) exports. The above function (11) is further modified to incorporate those factors above, so that world demand for U.S. exports of corn (soybeans) can be expressed as:

$$ED_j = f\left(\frac{P_i + t_{ij}}{e_{ij}}, P_{jf}, P_j, \#LS_j, Y_j, Q_1, S_0, X\right) \quad (12)$$

### 3. Model of Supply Response-Futures Price in Supply Analysis

One of the objectives of this study is to examine the role futures prices in production decisions and supply response for corn and soybean producers. In this section,

special consideration is given to the role that futures prices may play as a market-determined expected price observation in shaping output decisions.

Since the appropriate price for supply analysis is the price that the producer expects when making a production decision, the current futures price at the time of the production decision provides a reasonable candidate for the direct observable of the expected product price in the supply analysis. A theoretically well-grounded hypothesis that extends to futures prices is an unbiased estimate of subsequent cash prices that has been explored in many theoretical and empirical studies, such as: Working [16], Brennan [17], Telser [18], Tomek and Gray [19], Peck [4] and Gardner [20]. In each of these cases, the hypothesis was maintained that futures prices provide an exact measure of the price expectations. The conclusions they draw from this empirical analysis, especially the storable commodity, conclude that futures prices can be considered as unbiased forecasts of subsequent spot prices. It indicates that the price of a futures contract for the new crop reflects the market estimate of the new crop cash price.

$$\begin{aligned} P_t^e &= E_{t-k}(P_t) = FP_{t-k}^t \text{ or} \\ P_{t+k}^e &= E_t(P_{t+k}) = FP_t^{t+k} \end{aligned} \quad (13)$$

Where  $FP_{t-k}^t$  (or  $FP_t^{t+k}$ ) is the price at time  $t-k$  (or  $t$ ) quote for futures deliverable in period  $t$  (or  $t+k$ ).

The use of futures price as an observation of expected crop price raises two issues of concern. First, the market's estimate as given by a futures prices reflects the expectations of nonfarm speculators as well as crop producers. Thus, do farmers use information efficiently if they have access to the same information available to the informed speculators acting in futures markets? Second, which futures price is most appropriate and at what date this price become producers' expected price?

Both of the above issues can be answered by Muth's theory of rational expectations [21]. Muth provided a link between survey studies of expectations and econometric approaches. The rational expectations theory is based on the following hypothesis about individual behavior ([21], p.316), "...information is scarce, and the economic system generally does not waste it; ... the way expectations are formed depends specifically on the structure of the relevant system describing the economy..."

In order to derive the price expected to prevail at period  $t$  ( $P_t^e$ ), based on information through  $t-k$  periods, Muth [21] further assumed the actual and expected price relationships to be:

$$P_t = P_t^e + V_t \quad (14)$$

Where  $V$  is error in period  $t$  and the structure of  $V_t$  is assumed to be known,

$$E(V_t) = 0 \text{ and } E(P_t^e V_t) = 0.$$

These expectations are rational if the expectations formed to predict future events are the same as those predicted by relevant economic theories. Mathematically, the expected value of the market price at time  $t$  equals the market price expected to prevail at time  $t$  based on information available at time  $t-k$  or,

$$E_{t-k}(P_t/I_{t-k}) = P_t^e \quad (15)$$

Where  $I_{t-k}$  is information available in period  $t-k$ .

Incorporating equation (15) into his model gives

$$\text{Demand: } D_t = -bP_t$$

$$\text{Supply: } S_t = cP_t^e + U_t \quad (16)$$

$$\text{Market equilibrium: } D_t = S_t$$

$$\text{Expectation function: } P_t^e = E_{t-k}(P_t)$$

Where,

$D_t$  = Quantity demanded in period  $t$ ,

$S_t$  = Quantity supplied in period  $t$ ,

$P_t$  = Market price in period  $t$ ,

$P_t^e$  = Price expected to prevail in period  $t$  on the basis of information through period  $t-k$ ,

$b, c$  = Coefficients of the equation related to variables  $P_t$  and  $P_t^e$

$U_t$  = Stochastic error.

According to the rational expectations hypothesis, the price expectations in this model are unbiased and the expected price is treated as endogenous to the system.  $D_t, S_t$ , and  $P_t$  are endogenous to this system. The single expression of the expected price ( $P_t^e$ ) is solved by system (16) as

$$P_t^e = \frac{b}{c} = \sum_{i=0}^{\infty} \frac{1}{\left(1 + \left(\frac{b}{c}\right)^i\right)} P_{t-i}. \quad (17)$$

Given the rational expectations, we can answer the first issue of using futures prices as described above. This theory implies that economic behavior underlies the formulation of expectations. Expectations are based on information and generated according to perceptible forces affecting the economic activity to be investigated. According to this hypothesis, if a producer operating under free competition understands market conditions, he will use the information he has about the demand and supply conditions to generate his expectations of the relevant variables for decision making. Therefore, it may be hypothesized that farmers have no different prices expectations from futures speculators, nor do those farmers who make no futures transactions have expectations different from those who do. If the price expectations of those not participating in the futures markets differ from the futures price, there is great incentive for them to enter. Thus, those out of the market are likely to have expected prices equal to the futures market price.

The rational expectations hypothesis also allows the resolution of the second issue related to the use of futures prices. The essence of the farmer's decision problem is to forecast the price for time  $t$  (the harvest price), given the information available through  $t-k$  (the time which the planting decision must be made). From a theoretical point of view, we can say that futures quotes during the pre-planned harvest period will provide usable information and allow producers to observe future (harvest) prices at the time of planting. Peck ([4], p. 407) noted that, "...futures markets, with simultaneous trading in successive maturities, provide forward price(s) that could be used by a producer in formulating his production decisions, one contract, the first in the new crop year, is of particular concern. Generally, this new crop futures can be traded well in advance of the time that the production

decision must be made. Unlike a price forecast, this is a forward price that is a market price...the producer could sell his expected output at that price."

Thus, we may incorporate equation (16) into system (19) since at planting time, a new crop futures price represents an observable realization of farmer's expectations. This is consistent with rational expectations as rational expectations hypothesis (Muth's extended version) asserts that "...expectations of individual market participants (the subjective probability distribution of price outcomes) tend to be distributed for the same information set about the predictions of the theory, (the objective probability distribution of price outcome)."

Expectations are unbiased, and producers are assumed to use futures prices in their production decisions. Therefore, the decision is endogenous to the system and depends on the futures price. By using futures prices as decision variables, we can establish a framework for empirically testing the hypothesis that producers react to futures prices when making production decisions. The model also allows for partial adjustments to capture technical constraints, institutional rigidities, and the persistence of habits involved in farmers switching from one crop to another. That is,

$$A_t = a + bP_t^e + \sum_{i=1}^n c_i Z_{it} + U_t \quad (18)$$

Where,  $A_t$ = Output or acreage planted at time t,  
 $P_t^e$ = The expected price at time t formed at t-k,  
 $Z_{it}$ = Other inputs i or shifter variables at time t  
 $c_i$  = Coefficient of the equation related to variable  $Z_{it}$   
 $U_t$ = The random disturbance terms.

$$P_t^e = FP_{t-k}^t \quad (19)$$

$$A_t^e = A_{t-1}^t + r(A_t - A_{t-1}), 0 < r \leq 1 \quad (20)$$

all variables are as defined previously. Solving this system (Equations 18-20) gives

$$A_t = ar + brFP_{t-k}^t + (1 - r)A_{t-1} + r \sum_{i=1}^n C_i Z_{it} + rU_t \quad (21)$$

Equation (21) is to be statistically estimated and tested.

The appealing feature of using futures prices in supply response analysis is that they are "forward looking." However, prices generated by futures markets provide not only a forward pricing function but also the function of inventory (storage) allocation. In the next section, we discuss the theory of storage behavior that leads to a simultaneous equilibrium of cash, future, and storage markets.

## 4. Model of Simultaneous Equilibrium in the Cash, Futures and Storage Markets

### 4.1. Comprehensive Discussions

A theory of simultaneous determination of cash and futures prices and consumption and inventoried quantities can be derived from the theory of hedging and speculation as well as theory of the price of storage. The development of hedging and speculative theory began with the view that the main function of the futures market is to shift the price risk of producers, traders, processors and other stockholders to speculators [22,23]. Speculators will buy

(sell) contracts for future delivery only when the futures price (FP) is greater (less) than the expected cash price  $E(P)$  by an amount equal to or greater than the risk premium (R). Algebraically,

$$FP - E(P) \geq R \quad (22)$$

Working [1,2] argues that Keynes-Hicks' hedging theory does not adequately explain the behavior of hedgers. He claims that hedging is not always insurance, risk is transferred to premium, but when the expected cash-future price relationship changes, real hedges are used. Hedging is often used to take advantage of changes in cash-future price differentials rather than simply ensuring that overall price changes are prevented. Since hedgers take positions in both cash and futures markets, it is important to understand the basis ( $B_t = FP_t^{t+k} - P_t$ ) to convert a specific futures price into a possible price for cash delivery. The researches of Turnovsky [24], Peterson and Tomek [25], Fantasia, Marcuzzo and Sanfilippo [26] and Fische, Janzen and Smith [27] had verified these statements by their investigations and/or empirical evidences.

To illustrate the transactions in the cash and futures markets, it is assumed that a hedge is carried from  $t_1$  to  $t_2$ . The cash price and futures price at  $t_1$  and  $t_2$  are respectively  $CP_1, CP_2$  and  $FP_1, FP_2$ . The total profit (loss) on the hedged position is

$$\begin{aligned} & [\text{Profit (loss) in futures}] - [\text{loss (profit) in cash}] \\ &= (FP_2 - FP_1) - (CP_2 - CP_1) \\ &= (FP_2 - CP_2) - (FP_1 - CP_1) = B_2 - B_1 \quad (23) \\ &= \Delta B \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \end{aligned}$$

Where  $B_1$  and  $B_2$  are the basis at time  $t_1$  and  $t_2$ .  $\Delta B$  is the change of the basis. The basis is the focal point for identifying profitable opportunities.

Profit motives lead the futures market to reflect the spot market. Cash and futures prices tend to move up and down together, but as noted above, they do not show the same volatility. Different fluctuations are caused by various market conditions, including:

- a) The total supply and demand for commodities;
- b) The total supply and demand of the substitute commodities;
- c) Available storage spaces;
- d) The behavior of hedgers and speculators;
- e) Quality and transportation issues;
- f) Expected risks and uncertainties.

Hence, a change in the basis ( $\Delta B$ ) can be positive, negative or zero. Although traders still face basis risk, it is lower than price risk which is why traders watch basis behaviors and prefer basis to price risk.

The difference between current cash prices and futures prices reflects the factors that influence the supply and demand of these commodities. The narrowing basis reflects the decreasing cost of storage as the delivery month approaches. Working [2] argued that less (more) inventory is carried if the market shows a forward premium (discount) in relation to a futures delivery price. This premium is not a prediction of price increases (falls)

but a market-determined storage price. In his theory of storage prices, this basis is used as a directly observable current storage price. As Working [2] stated, "... the origin and prevalence of the term "carrying charge" in trade usage reflects the designated price differences as in fact equivalent to the price for "carrying" the commodity, or what may be called for economic analysis a price of storage."

Working's studies [1,2] on the theory of the price-of-storage highlighted the basic interdependence of the cash and futures markets. The theory of the price-of-storage views the basis ( $B_t = FP_t^{t+k} - P_t$ ), whether positive or negative, as the market determined storage price. In addition, it is assumed to be a direct function of the current inventory ( $I_t$ ) level. Working hypothesized that the supply-of-storage curve

$$FP_t^{t+k} - P_t = f(I_t) \tag{24}$$

It would have the shape depicted by  $ST_s$  in Figure 1.

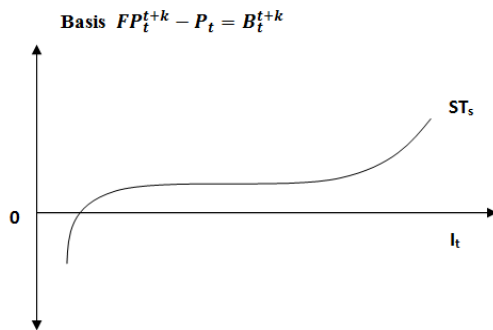


Figure 1. The supply-of-storage curve

Working's [1,2] model allows for:

- (i) Storage which may have a convenience yield (treated as a negative price of storage); and
- (ii) As the level of storage increases or the available storage facilities limit, the marginal cost of storage increases.

The essence of the price-of-storage (supply-of-storage) theory gives the direct connection between the cash and futures markets.

As Working ([28], p. 455) stated, "... the main reason for listing the price of storage concept as making a significant step in the advance of economic science is that it seems capable of displacing the belief that spot prices are commonly less affected by changing distant expectations. That opinion is not found on any factual observation, but has its "basis" in an assumption embodied in that conventional exposition of price formulation, an exposition that is ordinarily taken to mean that "supply" as a determination of spot price, means currently available physical supply".

Clearly, the usefulness of the theory of the price-of-storage is that it provides a direct explanation of intertemporal price relationships, and it can serve as a basis for the hypothesis of the interdependence of the three markets: cash market, futures market and storage market. In order to fully understand the role of the futures market in determining cash and storage prices, futures prices must be treated as endogenous variables whose value is determined simultaneously with cash and storage prices. For seasonally produced commodities, the average

cash price for the year depends upon the demand and supply for the commodities [13,21,26,27,29,30].

However, futures prices are determined directly by the demand and supply of futures contracts by hedgers and speculators. The market equilibrium represented by futures prices is the balance between net hedging and net speculation. The greater concern is that the futures price for the commodity and contract period is dependent upon the perceived demand and supply situation in the commodity market. The spread between the cash and futures prices is defined above as the storage price. Theoretically, the equilibrium level of the price of storage ( $FP_t^{t+k} - P_t$ ) and the size of the inventory ( $I_t$ ) at a point in time are jointly determined by the supply of and demand for storage (Figure 2). The demand or storage is related to consumption demand and the demand for inventory at the end of the period [25]. The quantity of storage demanded is inversely related to the price of storage. The price of storage influences the amount of storage throughout the season. If the difference between futures and cash price exceeds the cost of storage, this will motivate the sale of commodities by selling futures contracts and holding commodities for future delivery. A negative price of storage, referred to as an inverted market situation, is usually considered to reflect a current shortage of the commodity.

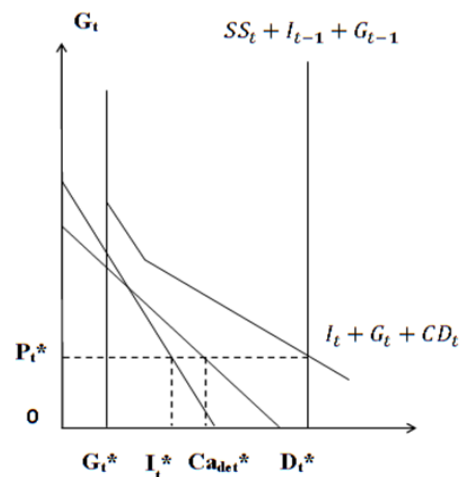


Figure 2.a Cash market equilibrium

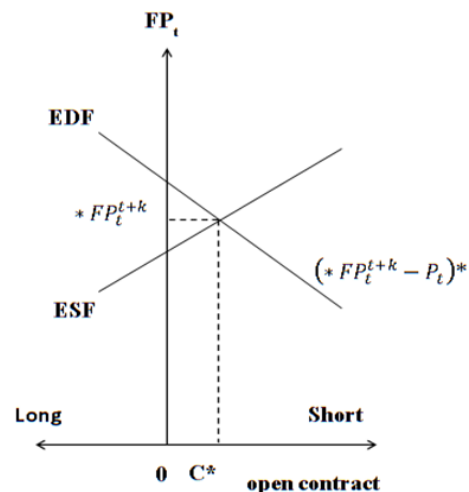


Figure 2.b Future market equilibrium

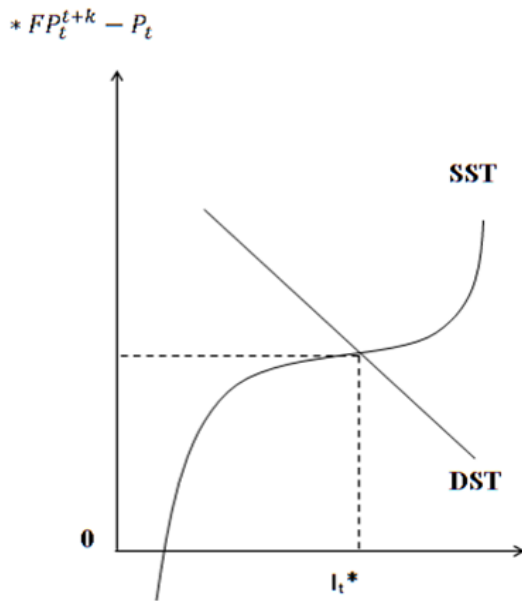


Figure 2.c Storage market equilibrium

Within this framework, the equilibrium situation is presented diagrammatically in Figure 2.a, Figure 2.b and Figure 2.c. The cash market equilibrium is shown in Figure 2.a. The average cash price for the year is dependent on the total supply ( $SS_t + I_{t-1} + G_{t-1}$ ) and total demand ( $CD_t + I_t + G_t$ ).  $P_t^*$  is the equilibrium price and  $I_t^*$ ,  $G_t^*$ ,  $CD_t^*$  are equilibrium quantities of private inventory, government stock and current disappearance, respectively.

In the futures market, traders can be classified into two groups: hedgers and speculators. In this discussion, hedgers are described as traders who have taken the opposite position in the cash and futures markets [3,21,27,29,31]. Speculators are traders who have both bought and sold futures at different times. Both hedgers and speculators can be in "long" or "short" or "both" positions. Buying futures contracts is "long" while selling futures is "short".

The futures market equilibrium can be achieved when the demand and supply of futures contracts are equal, i.e.,

$$H_L + S_L = H_S + S_S \tag{25}$$

Where

$H_L$  = net long hedging

$H_S$  = net short hedging

$S_L$  = net long speculation

$S_S$  = net short speculation

Buying futures contracts is "long" while selling futures is "short".

Review equation (25) as

$$S_L - S_S = H_S - R_L \text{ or} \tag{26}$$

$$EDF = ESF \tag{27}$$

Where

EDF = excess demand of futures contracts for speculators

ESF = excess supply of futures contracts for hedgers.

The futures market equilibrium is shown in Figure 2.b. The intersection of the EDF curve with the ESF curve gives the equilibrium level of the futures price  $*FP_R^{t+k}$ ,

and the net commitment  $C^*$  in terms of open contracts measured in units of the cash commodity in that period. The storage market equilibrium is shown in Figure 2.c. Efficiency in storage occurs when supplies are distributed through time so that the marginal return to storage in each period is equal to the marginal cost of bringing a commodity to that period.

Simultaneous equilibrium occurs when there exists an appropriate price-quantity linkage between the three markets. In general, if all three markets are in competitive equilibrium, then in Figure 2.a, Figure 2.b, and Figure 2.c, the difference between the equilibrium cash price,  $(P_t^*)$  and the futures price,  $(FP_t^{t+k})$  is equal to  $(FP_t^{t+k} - P_t)^*$ . The equilibrium level of inventory,  $I_t^*$ , in Figure 2.a must be equal to the equilibrium level of inventory,  $I_t^*$  in Figure 2.c. Since the basis will affect the level of inventory and the economic return to storage, the simultaneous determination of cash and futures prices reflects the interdependence of the three markets.

The theory developed is particularly relevant to those commodities that are produced once but stored and utilized throughout the year. The model is restricted to the inter-harvest period of a periodically produced storable commodity. The model could be made more general by adding a supply equation to incorporate harvest changes. This will be considered in a later section. The behavior of hedgers, merchants and speculators affects cash and storage markets. The prices generated by cash and futures markets further give market-determined storage prices, provide incentives or determine stored stocks. In essence,

a. Hedgers carry stock and reduce their risk by selling futures in an amount equal to the quantity of stock they intend to carry over to the next period. Hedger's behavior depends on cash and futures prices.

b. Merchants hold unhedged stocks and react to current cash prices and expected cash prices for futures. They are really cash speculators.

c. The speculator buys a futures contract based on the futures price relative to the expected cash price. They do not carry merchandise inventory.

It is assumed that markets are perfectly competitive. Expectations about the spot price for future periods are given. The model has three markets distinguished as follows:

1) The Storage Market

The demand for storage arises from those who plan to hold stocks in the next period. This includes merchants holding unhedged stocks and speculators who plan to deliver on futures contracts. The storage supply is provided by the person holding the stock in the current period, including the hedger holding the hedged stock and the trader holding the unhedged stock.

2) The Futures Market

The supply of futures contracts is provided by short hedgers and speculators. The demand comes from long hedgers and speculators in futures contracts.

3) The Cash Market

Demand comes from current consumption, and supply in the spot market must be allocated to current consumption and storage.

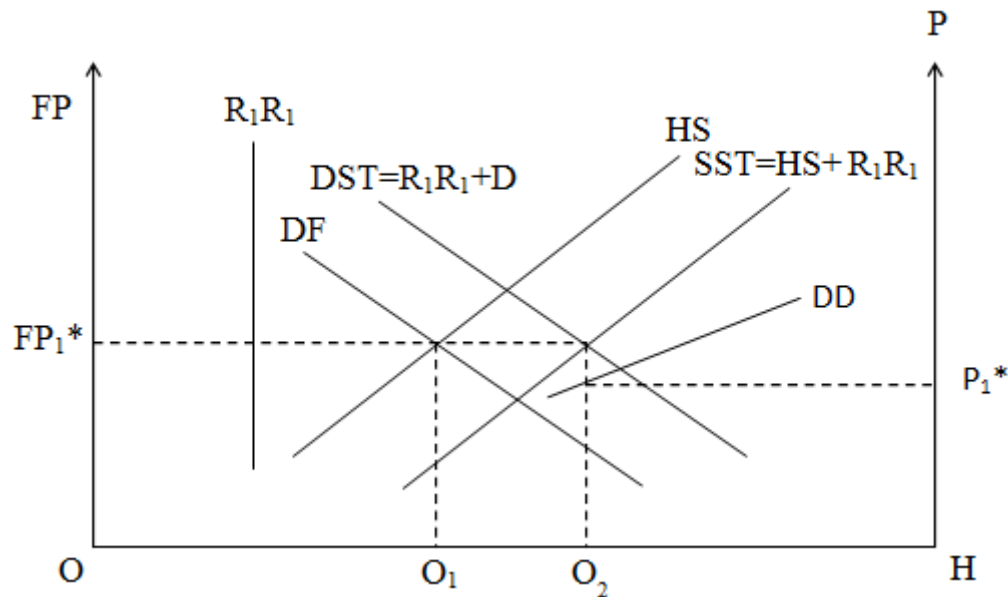


Figure 3. Determination of cash and futures prices with storage market interdependence

Market interdependencies and three market equilibriums can be graphically presented in Figure 3. In the diagram, consumption demand  $DD$  is a decreasing function of the spot price. It is not affected by a change in the futures price.  $R_1R_1$  is the supply of (and demand for) unhedged storage by merchants. It is drawn at a given cash price  $P_1$  and with a higher cash price it would shift to the left.  $HS$  is the supply of hedged stock by hedgers and is drawn for a given cash price  $FP_1^*$ . For higher cash price, it would also shift to the left.  $DF$  is the demand of speculators for futures contracts. The total demand for storage by speculators and merchants ( $DST$ ) is the horizontal summation of  $R_1R_1$  and  $DF$  at each future price level. The total supply of storage by hedgers and merchants ( $SST$ ) is the horizontal summation of  $R_1R_1$  and  $HS$ .

The equilibrium futures price  $FP_1^*$  is determined by the intersection of  $DF$  and  $HS$ , i.e., the equality of demand for and supply of futures contracts, or the equality of  $DST$  and  $SST$ . The total quantity of storage forthcoming at  $FP_1^*$  is  $O_1O_2$ , of which  $O_1O_2$  is supplied by hedgers and  $O_1O_2$  is supplied by merchants. Since total stock of the commodity to be allocated is  $OH$ , this leaf  $O_2H$  for current consumption. Given the consumption demand curve  $DD$ , the cash price is  $P_1^*$  and the equilibrium price of storage is  $B_1^*$  which is equal to  $FP_1^* - P_1^*$ . This price difference also affects storage behavior, affecting other components in the system, as these markets interact dynamically at the same time.

There are three equilibrium conditions in this model. Total stocks must be allocated between storage and current consumption demand and the cash and futures prices must equate demand and supply in the cash and futures markets respectively. For empirical purposes, the role of futures prices can be considered in the storage market because futures-cash price differences reflect storage prices. Therefore, the simultaneous equilibrium in the cash and storage markets also captures the interdependence of the cash, futures and storage markets.

For the general equilibrium that exists, these three markets must have a set of prices that allow all of these markets to be clear. In this case, consumers and firms can

maximize their utility. In the context of risk management, the simultaneous equilibrium of the three markets is indicative of the process of finding an equilibrium point between risk-bearers and risk-aversers. As shown in Figure 3, it should be noted that the quantities of storage and of futures and of futures contracts are measured from left to right on the horizontal axis and consumption is measured in the opposite direction. The futures price is shown on the left-hand ordinate and the cash price is now on the right-hand side.

## 4.2. Mathematical Model Specification

Along with the theoretical background and the review of previous modeling efforts, a model which considers the role of futures price and captures the simultaneous equilibrium in the cash and storage markets for corn and soybeans can be constructed and represented in mathematical form. In the cash market, the supply equations need to contain both the prices of corn and soybeans. Demand side equations, need to contain not only the commodity prices but also the price of livestock since both corn and soybeans are used as feeds.

The interdependence of cash, futures and storage markets can be captured by treating the futures price as endogenous in the demand and supply of storage equations (since the difference between cash and futures price refers to price of storage). The mathematical model which is consistent with the conceptual framework of Figure 3 for the interrelated corn and soybean sectors can be expressed as:

Corn supply

$$Q_c^s = f_1[P_c, FP_c, G, X] + \varepsilon_1 \quad (28)$$

Corn demand

$$Q_c^d = f_2[P_c, R_s, X] + \varepsilon_2 \quad (29)$$

Soybean supply

$$Q_s^s = f_3[P_c, FP_s, G, X] + \varepsilon_3 \quad (30)$$



Soybean demand

$$Q_s^d = f_4 [P_s, P_{1S}, X] + \varepsilon_4 \quad (31)$$

Storage supply of corn

$$ST_c^s = f_5 [FP_c - P_c, FP_s - P_s, G, X] + \varepsilon_5 \quad (32)$$

Storage demand for corn

$$ST_c^d = f_6 [FP_c - P_c, G, X] + \varepsilon_6 \quad (33)$$

Storage supply of soybean

$$ST_s^s = f_7 [FP_s - P_s, FP_c - P_c, G, X] + \varepsilon_7 \quad (34)$$

Storage demand of soybean

$$ST_s^d = f_8 [FP_s - P_s, G, X] + \varepsilon_8 \quad (35)$$

Storage basis identities (Price of storage)

$$B_c = FP_c - P_c \quad (36)$$

$$B_s = FP_s - P_s \quad (37)$$

Storage market equilibrium

$$ST_c^s = ST_c^d \quad (38)$$

$$ST_s^s = ST_s^d \quad (39)$$

Market clearing identities (equilibrium)

$$Q_c^d + ST_c^d = Q_c^s \quad (40)$$

$$Q_s^d + ST_s^d = Q_s^s \quad (41)$$

Where

$Q_c^s$  = quantity supplied of corn

$Q_c^d$  = quantity demanded for corn

$Q_s^s$  = quantity supplied of soybeans

$Q_s^d$  = quantity demanded for soybeans

$ST_c^s$  = quantity supplied of storage for corn

$ST_c^d$  = quantity demanded for storage for corn

$ST_s^s$  = quantity supplied of storage for soybeans

$ST_s^d$  = quantity demanded for storage for soybeans

$P_c$  = cash price of corn

$P_s$  = cash price of soybeans

$FP_c$  = futures price of corn

$FP_s$  = futures price of soybeans

$B_c$  = corn basis (price of storage)

$B_s$  = soybean basis (price of storage)

$P_{1S}$  = price of livestock and its products

$G$  = government policy variables

$X$  = other exogenous variables

The variables endogenous to this model are

$Q_c^s, Q_c^d, ST_c^s, ST_c^d, ST_s^s, ST_s^d, P_c, P_s, FP_c, FP_s, B_c$  and  $B_s$ .

The model specified above disregards the temporal distribution of adjustments. However, due to the inherent time lag in agricultural production, the model should be formulated under the assumption that the current price will not have a significant influence on current production. Instead, producers base their production decisions on expected price. Several hypotheses can be made that consider formation of price expectations. If the distributed lag hypothesis is used, the supply equations (28) and (30) may apply lagged prices as expected prices. However, within the spirit of rational expectations hypothesis, in this research, futures prices ( $FP_c, FP_s$ ) will be utilized as

expected prices in making production decisions for corn and soybean producers.

## 5. Concluding Remarks

In this study, the corn and soybean sectors are modeled. The econometric model consists of 8 behavioral equations with 2 identities and 4 equilibrium conditions. These equations explain demand for and supply of corn and soybeans, demand for and supply of storage and price linkage relationships. In the system, the basis, or the cash and futures price difference, and storage are endogenous. Storage, a component of the total supply and derived demand for future utilization, is an adjustment channel in temporal equilibrium which simultaneously determines storage and cash prices and thus, futures prices.

Futures prices also enter the production decisions. Since producers behave as if they are using the futures prices in production decisions, the futures prices and production decisions must be viewed as simultaneously determined either through interacting with demand and storage subsectors or via identities and market equilibrium conditions. In short, corn and soybean production, consumption, storage, cash and future prices are endogenous, so the dynamics of the interactions between corn and soybean markets can be analyzed and the simultaneous equilibrium of cash, futures and storage markets can be accomplished.

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