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Research on Inventory Decision of Loss-aversion Firms under Debt Financing

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Abstract By constructing the optimal inventory decision model of loss-aversion firm with and without debt, we analyze the influence of debt on firm's inventory decision, and discuss the influence of loss aversion, production cost, and shortage cost on the firm's optimal inventory. We show that when the firm is risk neutral, its optimal inventory is smaller under debt. The effects of debt on optimal inventory decreases with shortage cost and increases with production cost. However, the relationship of debt and optimal inventory is not clear when the firm is loss-averse.

Keywords: *inventory, loss aversion, risk neutral, shortage cost, production cost*

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1. Introduction

The study on the inventory decision of companies has been extensive. Most of them assume that the decision-makers are risk-neutral. However, some decision makers are loss-averse. In addition, companies need to raise funds through equity and debt financing before making inventory decisions. Debt will always affect the optimal inventory. In this paper, a theoretical model is established to describe the optimal capacity investment decision of the companies with different risk preferences, and to compare the optimal inventory of the companies under different capital structure.

Van Mieghem [1] regarded the study of inventory decision with risk aversion as a good direction. Ni J etc.[2] explored the coordination between operational decisions of risk-averse manager and financial hedging. Schweitzer and Cachon [3] found that the order quantity of the loss aversion newsvendor is less than the risk neutral one.

However, these studies do not consider debt financing, but the companies often need to use debt to provide fund support for their inventory investment. Ni J and others [5] studied the company's inventory investment decision under debt. Birge [6] pointed out that combine operational and financial decision can make researches closer to reality. Chod and others found that flexible capacity investment is positively correlated with its liabilities and negatively correlated with the cost of debt. These studies all assume the managers are risk neutral. However, many managers are loss-aversion. Considering loss-averse in the inventory decision-making process will lead to different conclusions.

Therefore, based on the loss aversion newsvendor model, this paper studies the optimal inventory investment decision of the manufacturing company under debt, solves the model with the optimization method, and verifies the results of the model theoretical analysis through numerical experiments.

2. Problem Description and Basic Assumption

We consider a one-product firm that invests in inventory. In the stage of capital rising, the company chooses the optimal equity and debt. Assume that the company's decision goal is to maximize the loss-aversion utility, we then discuss the impact of debt on the company's inventory decision.

Let w be the unit production cost, Q be the inventory, p be the retail price, x be the market demand, $F(x)$ be the probability distribution function, $f(x)$ be the probability density function, $G(x)$ be the primitive function of $F(x)$, $\overline{F}(x)=1-F(x)$, s be the shortage cost, r be the interest of debt, t be the corporate income tax rate, π be the business income, λ be the loss aversion coefficient, and $0<w<p$, $\lambda\geq 1$, $t<1$, thus the utility function of loss aversion decision makers is:

$$\mathrm{U}(\pi)=\begin{cases}\pi, & \pi\geq 0,\\ \lambda\pi, & \pi<0,\end{cases}\qquad (\lambda\geq 1) \tag{1}$$

3. Loss Aversion Company's Inventory Investment Decision Model with and without Debt

From the problem description and basic assumptions, we know that, when there is no debt, the utility function of loss aversion company is:

$$\pi_n(x,Q) = \begin{cases} \pi_{n1}(x,Q) = px - wQ, & x < wQ/p \\ \pi_{n2}(x,Q) = (1-t)(px - wQ), & wQ/p \le x < Q \\ \pi_{n3}(x,Q) = (1-t)(pQ - wQ) - s(x-Q), & x \ge Q \end{cases} \quad (2)$$

Let q_n be the demand when the income is 0, the following lemmas will thus be satisfied:

Lemma 1: When $s \ge (1-t)(p-w)Q/(I-Q)$, there are two break-even points $q_{n1}(Q) = wQ/p$, $q_{n2}(Q) = Q + (1-t)(p-w)Q/s$. When $x < q_{n1}(Q)$ or $x > q_{n2}(Q)$, the income is negative, otherwise it is positive. When $s < (1-t)(p-w)Q/(I-Q)$, there is only one break-even point $q_{n1}(Q)$, the income will be negative if and only if $x < q_{n1}(Q)$, vice versa.

Proof: When $x < wQ/p$, $\pi_{n1}(x,Q) < 0$,there is no break-even point. When $wQ/p \le x < Q$, let $x = wQ/p$, $\pi_{n2}(x,Q) = 0$, there is one break-even point $q_{n1}(Q) = wQ\ /p$, $\pi_{n2}(x,Q) \ge 0$. When $x \ge Q$, π_{n3} monotonically decreases, when $x \ge Q$, $\pi_{n3}(I,Q) \le 0$, $s \ge (1-t)(p-w)Q/(I-Q)$, $q_{n2}(Q) \le I$, there is break-even point. Let $\pi_{n3}(x,Q) = 0$,

$$q_{n2}(Q) = Q + (1-t)(p-w)Q/s,$$

when $Q \le x < q_{n2}(Q)$, $\pi_{n3} > 0$, otherwise $\pi_{n3} < 0$. If $\pi_{n3} > 0$, $s < (1-t)(p-w)Q/(I-Q)$, $q_{n2}(Q) > I$, there is no break-even point.

Substituting the company's revenue function (2)into the loss aversion function (1), we obtain:

$$E\left[U\left(\pi_n(x,Q)\right)\right] = E\left[\pi_n(x,Q)\right] + (\lambda-1) \left[\int_0^{q_{n1}(Q)} \pi_{n1}(x,Q) f(x)dx + \int_{q_{n2}(Q)}^{I} \pi_{n3}(x,Q) f(x)dx\right] \quad (3)$$

Theorem 1: A unique optimal inventory $Q_{n\lambda}^*$ exists with no debt, it satisfies the following first-order condition:

$$-wtF\left(wQ_{n\lambda}^*/p\right) + \left[p(1-t)+s\right]\overline{F}\left(Q_{n\lambda}^*\right) - w(1-t) + (\lambda-1) \begin{Bmatrix} -wF\left[q_{n1}\left(Q_{n\lambda}^*\right)\right] \\ +\left[(p-w)(1-t)+s\right]\overline{F}\left[q_{n2}\left(Q_{n\lambda}^*\right)\right] \end{Bmatrix} = 0 \quad (4)$$

If $\lambda = 1$, the optimal inventory Q_{n1}^* meets the following first-order condition:

$$-wtF\left(wQ_{n1}^*/p\right) + \left[p(1-t)+s\right]\overline{F}\left(Q_{n1}^*\right) - w(1-t) = 0 \quad (5)$$

Proof: When $q_{n2}(Q) \le I$, the expected utility is:

$$E\left[U\left(\pi_n(x,Q)\right)\right] = \int_0^{\frac{wQ}{p}} \pi_{n1}(x,Q) f(x)dx + \int_{\frac{wQ}{p}}^{Q} \pi_{n2}(x,Q) f(x)dx + \int_Q^I \pi_{n3}(x,Q) f(x)dx + (\lambda-1) \left[\int_0^{q_{n1}(Q)} \pi_{n1}(x,Q) f(x)dx + \int_{q_{n2}(Q)}^{I} \pi_{n3}(x,Q) f(x)dx\right] \quad (6)$$

combine chain rules with Lemma 1 we obtain: $d^2E\left[U\left(\pi_n(x,Q)\right)\right]/dQ^2 < 0$, so $E\left[U\left(\pi_n(x,Q)\right)\right]$ is a concave function of Q, there exists an optimal inventory $Q_{n\lambda}^*$ satisfies formula (4), when decision makers are risk neutral, the optimal inventory Q_{n1}^* satisfies formula (5). When $q_{n2}(Q) > I$, we can obtain the same result.

Under debt financing, the company's income is

$$\pi(x,Q) = \begin{cases} \pi_1(x,Q) = px - wQ - r, & x < (wQ+r)/p \\ \pi_2(x,Q) = (1-t)(px - wQ - r), & (wQ+r)/p \le x < Q \\ \pi_3(x,Q) = (1-t)(pQ - wQ - r) - s(x-Q), & x \ge Q \end{cases} \quad (7)$$

In order to analyze the influence of loss aversion, we need to obtain the demand q when the company's income is 0, it satisfies lemma 2.

Lemma 2. When $s \ge \left[(1-t)(p-w)Q - (1-t)r\right]/\left[I-Q\right]$, there are two break-even points $q_1(Q) = \left[wQ+r\right]/p$, $q_2(Q) = Q + \left[(1-t)(p-w) - (1-t)r\right]/s$. When $q_1(Q) < x < q_2(Q)$, the firm has positive income. Otherwise, it has negative income. When $q_2(Q) > I$, it has only one break-even point $q_1(Q)$. Its income will be negative if and only if $x < q_1(Q)$, vice versa.

Proof: If $x < (wQ+r)/p$, $\pi_1(x,Q) < 0$. If $(wQ+r)/p \le x < Q$, let $x = (wQ+r)/p$, then $\pi_2(x,Q) = 0$, because $\pi_2(x,Q)$ monotonically increases with x, so there exists one break-even point $q_1(Q) = (wQ+r)/p$, and when $(wQ+r)/p < x < Q$, $\pi_2(x,Q) > 0$. When $x \ge Q$, $\pi_3(x,Q)$ decreases with x, so $\pi_3(I,Q) \le 0$, $s \ge \left[(1-t)(p-w)Q - (1-t)r\right]/(I-Q)$, $q_2(Q) \le I$, there is break-even points. Let $\pi_3(x,Q) = 0$, then $q_2(Q) = \left\{\left[(1-t)(p-w)+s\right]Q - (1-t)r\right\}/s$. When $Q \le x \le I$, $\pi_3(x,Q) > 0$, $\pi_3(x,Q)$ monotonically decreases with

x, $s < \left[(1-t)(p-w)Q-(1-t)r\right]/(I-Q)$, $q_2(Q) > I$, there is no break-even point. Combine (7) and (1), we get

$$E\left[U\left(\pi(x,Q)\right)\right] = E\left[\pi(x,Q)\right] + (\lambda-1)\left[\int_0^{q_1(Q)} \pi_1(x,Q) f(x)dx + \int_{q_2(Q)}^{I} \pi_3(x,Q) f(x)dx\right] \quad (8)$$

Proposition 2. There is a unique optimal inventory Q_λ^* when the company has debt, it satisfies:

$$-wtF\left(wQ_\lambda^* + r\right)/p + \left[p(1-t)+s\right]\overline{F}\left(Q_\lambda^*\right) - w(1-t) + (\lambda-1)\left\{-wF\left[q_1\left(Q_\lambda^*\right)\right] + \left[(p-w)(1-t)+s\right]\overline{F}\left[q_2\left(Q_\lambda^*\right)\right]\right\} = 0$$

When $\lambda = 1$, Q_1^* satisfies:

$$-wtF\left(wQ_1^* + r\right)/p + \left[p(1-t)+s\right]\overline{F}\left(Q_1^*\right) - w(1-t) = 0.$$

Proof; When $q_2(Q) \le I$, the expected utility is:

$$E\left[U\left(\pi(x,Q)\right)\right] = \int_0^{\frac{wQ+r}{p}} \pi_1(x,Q) f(x)dx + \int_{\frac{wQ+r}{p}}^{Q} \pi_2(x,Q) f(x)dx + \int_Q^{I} \pi_3(x,Q) f(x)dx + (\lambda-1) \quad (9)$$
$$\left[\int_0^{q_1(Q)} \pi_1(x,Q) f(x)dx + \int_{q_2(Q)}^{I} \pi_3(x,Q) f(x)dx\right]$$

Use the chain rule to find the derivative of Q in (9) and combine it with Lemma 1 we get:

$$d^2E\left[U\left(\pi(x,Q)\right)\right]/dQ^2 < 0,$$

so $E\left[U\left(\pi(x,Q)\right)\right]$ is a concave function, Q_λ^* satisfies proposition 2. When $q_{2n}(Q) > I$, the result is also the same.

We further assume the probability distribution function $F(x)$ follows the uniform distribution in the interval of $x \in (0, I)$ to study the relationship between these two, then $F(x) = x/I$, $f(x) = 1/I$. So the relationship between these two satisfies the under theorem.

Proposition 3. When the company has two break-even points, the relationship between the optimal inventory $Q_{n\lambda}^*$ without debt and Q_λ^* with debt is as follow: $Q_{n\lambda}^* - Q_\lambda^* = z/m$, where

$$z = (\lambda-1+t)wr/p - (\lambda-1)(1-t)r\left[(p-w)(1-t)+s\right]/s,$$

$$m = (\lambda-1+t)w^2/p + s + (\lambda-1)\left[(p-w)(1-t)+s\right]^2/s + p(1-t) > 0.$$

When there is only one break-even points, the relationship is:

$$Q_{n\lambda}^* - Q_\lambda^* = (\lambda-1+t)wr/p\Big/(\lambda-1+t)w^2/p + p(1-t) > 0.$$

Proof: Bring $F(x) = x/I$, $f(x) = 1/I$ into formula (4) and (11) respectively, we get: $Q_{n\lambda}^* - Q_\lambda^* = z/m$.

Proposition 1-3 studies the optimal inventory of loss aversion firm with and without debt ,and their relationship. We can see that the optimal inventory is affected by many factors. Because the results are too indebted, we will conduct the numerical experiments.

4. Numerical Experiments

Let $I = 1000$, $t = 0.25$, $p = 1$, $w = 0.5$, $r = 20$, $\lambda = 1, 1.2, 1.5, 2, 2.5$, $s = 0.1:1:10$, $g(s,w) = Q_{n\lambda}^* - Q_\lambda^*$, the relationship between $g(s, 0.5)$ and s is as Figure 1.

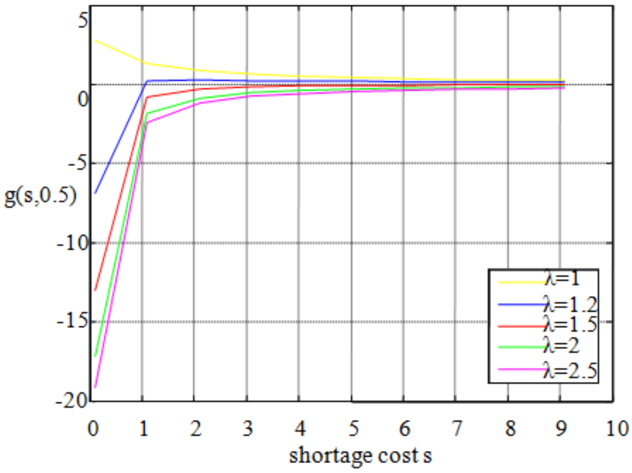

Figure 1. Difference between optimal inventory of levered and unlevered firm under different shortage cost

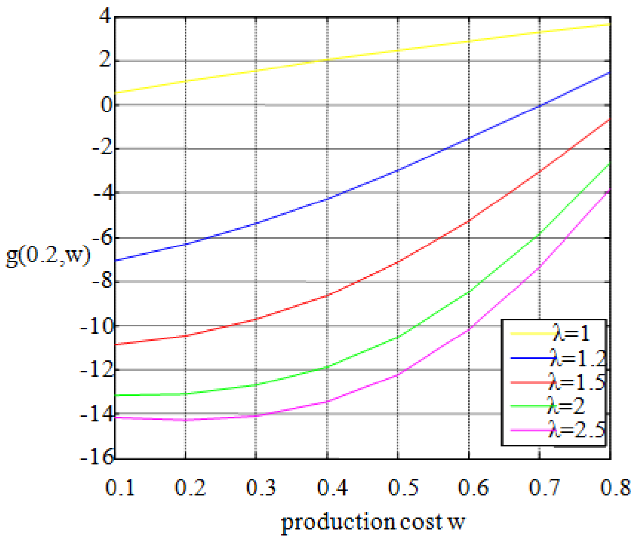

Figure 2. Difference between optimal inventory of levered and unlevered firm under different production cost

From Figure 1 we can see that when $\lambda = 1$, the levered firm's optimal inventory is less than the unlevered firm, and the impact of debt on the optimal inventory reduces with the shortage cost. When $\lambda > 1$, there exists a λ', when $\lambda > \lambda'$, the levered firm's optimal inventory is always more than the unlevered firm, when $\lambda < \lambda'$, there exists a s', if $s < s'$, the optimal inventory of levered firm is more than the unlevered firm, if $s > s'$, the optimal inventory of levered firm is less than the unlevered firm.

Let $I = 1000,\ t = 0.25,\ p = 1,\ r = 20,\ s = 0.2,\ \lambda = 1, 1.2,\ 1.5,\ 2,\ 2.5$, $w = 0.1:0.1:0.8$. Then the relationship of $g(0.2, w)$ and w is as Figure 2.

From Figure 2 we can see that when $\lambda = 1$, the optimal inventory of levered firm is less than the unlevered firm, and the impact of debt on optimal inventory increases with w.

5. Conclusion

This paper studies the optimal inventory of the risk- neutral and loss aversion firm. We find that when the decision- maker is risk-neutral, the optimal inventory is smaller with debt, and the effect of debt decreases with the shortage cost, increases with the production cost. When the decision-maker is loss averse, the relationship between levered and unlevered firm's optimal inventory is not clear.

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